FILTER-BASED ACTIVE SUSPENSION SYSTEM WITH ADAPTED REFERENCE INPUT

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ABSTRACT

In this paper, the Active Suspension system is controlled using a PID controller with an adapted reference point. After the derivation of the quarter car suspension model. Three approaches were applied: passive suspension system, Active Suspension system with constant reference and with adapted reference. The proposed approach was focusing on system life span; how to reduce brutal controller actions, that can cause car body damage, and assure a certain ride comfort? Simulation of three approaches has been done using the quarter car system and Matlab simulation model to implement the proposed technique and compare performance variation in different cases: road bump and other road disturbances.

KEYWORDS

Active Suspension system, PID controller, Quarter car model, Passive Suspension system

1. INTRODUCTION

Conventionally, automotive suspension designs have to compromise between three criteria: road holding, load carrying, and passenger comfort. Usually, the suspension must support the vehicle, improve handling while steering, and assure certain isolation of passengers and loads from road disturbances. Therefore, a Good ride needs a soft suspension whereas applied load requires stiff suspension. According to ISO 2631-1 standard, exposing the human body to vibrations with frequencies between 0.5 - 80 Hz could probably cause a great risk of injury to the vertebrae in the lumbar region and the nerves connected to these segments [1].

Due to these conflicting demands, suspension design had to be determined by the type of use for which the vehicle was designed. However, Active suspensions are considered a way to auto adapt suspension stiffness according to the road needs. Therefore, Active suspension is a very fertile topic with innovations each year.

In this paper, the mathematical model of vehicle, disturbance, etc. will be presented. Then, the PID controller used to control suspension has been discussed. Afterward, model simulation and comparison between Passive Suspension and Active Suspension with constant and adapted reference. Later, simulation results are analyzed. Finally, the work is concluded with a general study of performances of different techniques, and future works are shown.

2. STATE OF THE ART


Two common assumptions are taken into account in all these works; constant reference for all control techniques, and vehicle representation using the so-called ‘quarter-car’ model [13]. In this work, adaptive reference is proposed to reduce brutal controller actions that can cause damage to the vehicle system.

3. MATHEMATICAL MODELLING

In this part, a full model of the suspension dynamics and road disturbances is described. This shall describe the full model for a rigorous computer simulation study to be carried out. The mathematical model for the suspension system is given using the Newtonian mechanics, the suspension system is model based on a quarter car configuration.

The Active Suspension system is designed and model using the feedback control element embedded into the system. For simplification purposes, the active system is modeled as an action applied directly to the chassis and the tire.

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3.1. Quarter Car

Quarter car model [14] is frequently used for suspension system analysis and design due to its simplicity and yet ability to present many important parameters. Extension to the one-half car model has been described in [5] and a full car model in [7].

Figure 1(a) presents the Quarter car Passive Suspension system. Car body is denoted as sprung mass and the tire is denoted un-sprung mass. The single wheel axle is connected to the quarter portion of the car body through a passive spring and damper. The tire is assumed to have only the spring constant and is in permanent contact with the road at the other end. The road serves as an external disturbance to the system.

Based on Newtonian mechanics, the equations of the motion for the Passive Suspension system are given as [15]:

$H_\infty$ state-space optimization techniques
\[ m_s \frac{d^2 z_s}{dt^2} = -k_s(z_s - z_u) - C_s \left( \frac{dz_s}{dt} - \frac{dz_u}{dt} \right) \]  

(1)

\[ m_u \frac{d^2 z_u}{dt^2} = -k_s(z_s - z_u) + k_t(z_u - z_r) + C_s \left( \frac{dz_s}{dt} - \frac{dz_u}{dt} \right) \]  

(2)

Where,
- \( z_r \) Road displacement
- \( z_s \) Car body displacement
- \( z_u \) Un-sprung mass displacement
- \( C_s \) Damping coefficient
- \( m_s \) Sprung mass
- \( m_u \) Un-sprung mass
- \( k_s \) Spring stiffness constant
- \( k_t \) Tire stiffness constant

The equations of motion for active suspension system are given as [15]:

\[ m_s \frac{d^2 z_s}{dt^2} = -k_s(z_s - z_u) - C_s \left( \frac{dz_s}{dt} - \frac{dz_u}{dt} \right) + f_a \]  

(3)

\[ m_u \frac{d^2 z_u}{dt^2} = -k_s(z_s - z_u) + k_t(z_u - z_r) + C_s \left( \frac{dz_s}{dt} - \frac{dz_u}{dt} \right) - f_a \]  

(4)

Where, \( f_a \) is the actuator force.

The quarter car model for the Active Suspension system can be obtained by adding an actuator parallel to spring and damper. Figure 1(b) shows a schematic of a quarter car model for the Active Suspension system.

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(3)

\[ m_u \frac{d^2 z_u}{dt^2} = -k_s(z_s - z_u) + k_t(z_u - z_r) + C_s \left( \frac{dz_s}{dt} - \frac{dz_u}{dt} \right) - f_a \]  

(4)

Where, \( f_a \) is the actuator force.

3.2. Disturbance Model

A road bump will be used as a disturbance to the vehicle system in this work. The bump was assumed as a sinusoidal form. Some works [6, 9] used a road disturbance in the function of time, this type of bump does not take into account the vehicle speed. However, in our work, we will study the case of different car speeds.
Figure 2 illustrates the dimension of the disturbance model according to [10].

![Figure 2 Disturbance Model](image)

3.3. Filter Model

For standard Passive Suspension systems, if the comfort is improved, then handling stability is deteriorating. This problem is the main reason why Active Suspension systems are used. However, most active suspension systems [10, 9, 2] aim to bring car body into equilibrium value, which always needs high energy, or in some cases, can cause damage to the tire and/or suspension system. Therefore, in this work, the active suspension tries to bring the car body to a comfortable situation according to tire movements in such a way that annoying frequencies must be eliminated (according to [4], they can be ranged between 4 and 8 Hz).

Figure 3 illustrates a control configuration based in a filtered wheel displacement [16].

![Figure 3 Proposed Active Suspension system](image)

The filter was chosen in such a way to ensure a comfortable ride and taking into account sudden displacements. The figure below shows the step response of the proposed filter. For a step disturbance, the desired displacement must stabilize after about 1 second so the passengers will feel a minor annoyance.

The filter transfer function is as follows:

\[
F(s) = \frac{50}{s^2 + 15s + 50}
\]
3.4. Model parameters

Modeling parameters are taken for a sedan vehicle. The parameters used in the study are taken from [9]:

<table>
<thead>
<tr>
<th>Parameter name</th>
<th>Parameter symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sprung mass</td>
<td>$m_s$</td>
<td>282 kg</td>
</tr>
<tr>
<td>Unsprung mass</td>
<td>$m_u$</td>
<td>45 kg</td>
</tr>
<tr>
<td>Tire stiffness constant</td>
<td>$k_t$</td>
<td>165790 N/m</td>
</tr>
<tr>
<td>Spring stiffness</td>
<td>$k_x$</td>
<td>17900 N/m</td>
</tr>
<tr>
<td>Damping constant</td>
<td>$C_s$</td>
<td>1000 N.s/m</td>
</tr>
</tbody>
</table>

With these parameters and the given kinematic equations, and with assuming that the initial values are zero. The system can be written as follows:

$$z_x(s) = \frac{1000s + 17900}{282s^2 + 1000s + 17900} z_u(s) + \frac{1}{282s^2 + 1000s + 17900} f_a(s)$$  \hspace{1cm} (6)

$$z_u(s) = \frac{165790}{45s^2 + 1000s + 183690} z_r(s) + \frac{1000s + 17900}{45s^2 + 1000s + 183690} z_x(s) - \frac{1}{45s^2 + 1000s + 183690} f_a(s)$$  \hspace{1cm} (7)
4. **INSTRUMENTATION OF ACTIVE SUSPENSION SYSTEM**

4.1. **Measurements required**

As in [9], the only required variable for implementation of the proposed controller is the vertical displacement of the car body and the vertical displacement of the wheel. These variables are needed to control actuators according to measured values.

4.2. **Using sensors**

In [1], the use of sensors in experimental and commercial vehicles is detailed. The most common sensors used for measuring the vertical displacement of the body and the wheels are laser sensors [9]. Accelerometers and other sensors that measure and are not needed since it is possible to integral reconstruct them from the knowledge of body and wheel displacements. The schematic diagram of the instrumentation of the Active Suspension system is illustrated as follows:

![Diagram of Active Suspension System](image)

Figure 6. Schematic diagram of the instrumentation of the active suspension system

4.3. **Linear Electric Motor**

In [4], Linear electric motor has been used as an actuator to generate needed forces. Figure 7 shows the basic principle of the linear motor and an example of a commercialized linear motor (GSM Series by Exlar). The main interest of linear motors is that they directly translate electrical energy into usable linear mechanical force and motion.

![Linear Motor Image](image)

Figure 7 GSM Series Electric Linear Actuator by EXLAR.
Linear motor movements take place with high velocities (more than 1000 mm/s), large accelerations (up to \( G \) multiples), and forces (up to 10 kN). However, there is an important question of whether the linear motor model could be omitted or not? The answer is 'yes' [8]. Both electrical and mechanical constants are small (just above 1ms).

5. **PID Controller Design**

PID controller has been widely used in industry because of its simplicity and effectiveness. Despite many uses of the PID controller, its standard structure has constant gain parameters and is not good to decrease velocity control error [7]. Therefore, the exponential function is added to the derivative component of a conventional PID controller.

The used PID controller was adapted from [7] and can be described as follows:

\[
fa(t) = G \left( K_p e(t) + K_i \int e(t) dt + K_d \frac{de(t)}{dt} (K_N e^{-K_N t}) \right) \tag{8}
\]

where \( K_p, K_i, K_d \) and \( K_N \) are the PID controller gains.

Random tuning technique has been used to optimize controller parameters using step response. The Fitness function, that has to be minimized, is defined as follows:

\[
J = \frac{1}{T} \int_0^T (z_f - z_f) dt \times \frac{1}{T} \int_0^T (z_c - z_u) dt \tag{9}
\]

With \( z_f \) is filtered output for Active Suspension with adapted reference. For the constant reference, \( z_f \) is assumed to be zero. And \( T \) is the simulation period, which in our case equals to 5 seconds. Method of trial and error is used to determine the parameters of the PID controller.

After 5000 iterations, the parameters for adapted reference were set to:

\[
\begin{align*}
G &= 15000 \\
K_p &= 4.9336 \\
K_i &= 4.7032 \\
K_d &= 0.4736 \\
K_N &= 283.5086
\end{align*} \tag{10}
\]

And for constant reference were set to:

\[
\begin{align*}
G &= 15000 \\
K_p &= 4.9751 \\
K_i &= 4.9489 \\
K_d &= 0.3614 \\
K_N &= 414.1968
\end{align*} \tag{11}
\]
6. **Model Simulation**

To validate the proposed control technique, the mathematical model is transformed into a computer simulation model. As in [10], model variable-step continuous solver (ODE45 or Dormand-Prince) is used which is based on an explicit Runge-Kutta formula. It is a one-step solver. Therefore, to compute $y(t_n)$, it needs only the solution at the immediately preceding time point $y(t_{n-1})$.

6.1. **Passive Suspension System**

The Passive Suspension system was modeled in Simulink as shown in Figure 8. this model was built using the equations 1 and 2 by supposing that there is no external actuator ($f_a = 0$). The model has one input ($z_r$) and two outputs ($z_s$ and $z_u$).

![Figure 8 Passive Suspension system simulation model](image)

6.2. **Active Suspension System with constant Reference**

Active Suspension system requires actuator force to provide external control, the goal of this actuator to provide a better ride and handling. The force $f_a$ is given by a PID controller that aims to bring the body position ($z_s$) to constant value (zero). The Simulink model was built based on equations 3 and 4. The system has one input ($z_r$) and three outputs ($z_s$, $z_u$ and $f_a$).

![Figure 9. Active Suspension system with constant reference](image)

6.3. **Active Suspension System with Adapted Reference**

As it is seen, the previous Active suspension system tries to bring the car body to the position that ensures a comfortable ride without taking into account sudden road disturbances that can
cause damage to the wheel and/or suspension system. In the following configuration, the actuator tries to bring the car body to a comfortable ride following wheel displacements.

**Figure 10 Active Suspension system with Adapted reference**

### 7. Simulation Results

In the simulation, two cases were taken into account: different car speeds and different road disturbances. Comparisons of the car body displacement and needed force for different cases are discussed.

#### 7.1. Different car speed

In this case, road disturbance presented in Figure 2 is taken as reference and car speed will be varied for the three suspension systems. Figure 11 shows the displacement of sprung mass for different speeds to show the speed influence into car displacement and Figure 12 shows actuator force for different speeds.

**Figure 11 body displacement for different car speeds**

(a) Car speed 5 km/h

(b) Car speed 30 km/h
From results, it can be seen that the Passive Suspension system suffers from high displacement rates (up to 20 cm) for low speeds. Though the classical active suspension system reduced disturbance to 3 cm, and the proposed system gave a displacement up to 5 cm. However, if the car speed is increased, the body displacement for the three systems are similar. Moreover, the proposed system needed less force for small speeds. This is due to the control system that tries to reduce disturbance by taking into account sudden vibrations.

To have a more elaborate idea on the tested techniques, the following table contains a comparison between different results.

Table 2. displacement, force and wheel deformation for different speeds

<table>
<thead>
<tr>
<th>Speed (km/h)</th>
<th>Maximum displacement speed (m/s)</th>
<th>Maximum Force (N)</th>
<th>Wheel deformation (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Passive</td>
<td>Active 1</td>
<td>Active 2</td>
</tr>
<tr>
<td>5</td>
<td>1.2035</td>
<td>0.2534</td>
<td>0.3857</td>
</tr>
<tr>
<td>15</td>
<td>0.9409</td>
<td>0.5924</td>
<td>0.5797</td>
</tr>
<tr>
<td>25</td>
<td>0.9531</td>
<td>0.7153</td>
<td>0.6473</td>
</tr>
<tr>
<td>35</td>
<td>0.8652</td>
<td>0.6396</td>
<td>0.6198</td>
</tr>
<tr>
<td>45</td>
<td>0.7547</td>
<td>0.5640</td>
<td>0.5609</td>
</tr>
<tr>
<td>55</td>
<td>0.6546</td>
<td>0.5014</td>
<td>0.4970</td>
</tr>
</tbody>
</table>

Table 2 is a comparison between studied systems for different car speeds from 5 km/h to 55 km/h. the studied systems are passive suspension (Passive), Active Suspension with constant reference (Active 1) and Active Suspension with adapted reference (Active 2). the presented parameters are:

- Maximum displacement speed in m/s, which is the maximum vertical body speed;
- Maximum force in N, which is the maximum value of exerted force during bump passing;
- Wheel deformation in cm, which is the difference between wheel $z_u$ and road $z_r$ position or

$$W_d = z_u - z_r$$
Two aspects can be retrieved from Table 2:

1) Car speed influence on system behavior is very important; wheel deformation is very important from about 20 km/h, body displacement speed is stable between 15 and 30 km/h and the maximal exerted force is at its highest value between 20 and 30 km/h. Therefore, 15 km/h is the best car speed to pass through road bump.

2) For car speeds between 5 and 20 km/h, the proposed approach needed less force (up to 40% less), but body displacement speed was greater than classic Active Suspension. However, between 20 and 40 km/h, body displacement speed and maximal exerted force for the proposed approach were the lowest. For car speeds more than 40 km/h, systems gave more or less very near performances.

To have a more approximate view of body speed influence on systems, Figure 13 shows maximum displacement speed and maximum exerted force for different car speeds.

![Figure 13 Maximum displacement speed and force for different car speeds](image)

7.2. Different road disturbances

In this case, the systems were tested for different road types: random disturbance, road bumps, step, ramp, etc. this comparison is used to study control robustness for different inputs.
Random disturbance

The first case is a random disturbance with given maximum acceleration ($\ddot{z}_{\text{max}} = 100 \text{ m/s}^2 \approx 10G$). It can be seen (from Figure 14) that the passive suspension system moves with high velocity (up to 0.591 m/s). While active suspension with constant reference moves with low speed (0.139 m/s vs. 0.254 m/s for our approach). However, if we observe the force efficiency (Figure 15), active suspension with adapted reference needed less force (30% less).

![Figure 14 Random disturbance ($\ddot{z}_{\text{max}} \approx 10G$)]

Step input

The second test was made using step input (final value = 10 cm). In this case, the disturbance is a sudden displacement of 10 cm, it can be seen that passive suspension has large displacement speed (up to 0.712 m/s) and overshoot (up to 16 cm) which causes ride inconvenience. However, active suspension with constant reference tries to stay at zero reference, which needs a lot of energy (see exerted force in Figure 17). Moreover, the proposed approach has less displacement speed (up to 0.481 m/s vs 0.478 m/s for classic Active Suspension) and needed less maximal force (up to 15% less).

![Figure 16. Step response (final value = 10 cm)]
7.3. Comparison Between suspension systems

From previous tests, it can be seen that Passive Suspension gave a very disturbing ride from maximum displacement to displacement speed. Nevertheless, Active suspension with constant reference gave very good results for the best ride parameters. However, Active suspension with adapted reference, which is proposed in this paper, gave more or less acceptable results, but if the system lifespan and energy efficiency are taken into account, the proposed configuration gave better results because it takes into account wheel displacement to determine best body displacement.

8. CONCLUSIONS

This study intended to present a new approach to control the Active Suspension system of a quarter car system. Simulation results show that the technique performs better than the Passive Suspension system for different disturbances. However active suspension system with constant reference gave better results compared to the proposed technique, this is due to the controller tending to bring the body displacement to a constant value which is the best comfort option. Though, this last technique does not take into account brutal road displacements that can cause system damage. Therefore, the proposed technique tries to bring the car body to a certain comfort which takes into account sudden road displacement in such a way to assure a certain ride comfort.

Although, the study has some limitations that can be considered as future perspectives:

- Improving PID controller design and use other controllers (e.g. Fractional PID) to improve system results.
- Try adaptive control techniques for the case where system parameter changes like body weight.
- Improve filter design so it can be adapted for different road profiles.
- Quarter car suspension gives a simple representation of car suspension. However, a more elaborate model would be closer to the real car suspension.
- Experimental work is also needed to evaluate the proposed with real system parameters.

REFERENCES


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