THE LEFT AND RIGHT BLOCK POLE PLACEMENT COMPARISON STUDY: APPLICATION TO FLIGHT DYNAMICS

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ABSTRACT

It is known that if a linear-time-invariant MIMO system described by a state space equation has a number of states divisible by the number of inputs and it can be transformed to block controller form, we can design a state feedback controller using block pole placement technique by assigning a set of desired Block poles. These may be left or right block poles. The idea is to compare both in terms of system's response.

KEYWORDS

MIMO, Block Controller Form, State Feedback Controller, Block Pole Placement Technique, Left and/or Right Block Poles.

1. INTRODUCTION

In most applied mathematical research, the aim is to investigate and control a given system. A system is defined to mean a collection of objects which are related by interactions and produce various outputs in response to different inputs.Examples of such systems are chemical plants, aircrafts, spacecraft, biological systems, or even the economic structure of a country or regions see [1, 4 and 17]. The control problems associated with these systems might be the production of some chemical product as efficiently as possible, automatic landing of aircraft, rendezvous with an artificial satellite, regulation of body functions such as heartbeat or blood pressure, and the ever-present problem of economic inflation [18].To be able to control a system, we need a valid mathematical model. However practical systems are inherently complicated and highly non-linear. Thus, simplifications are made, such as the linearization of the system. Error analysis can then be employed to give information on how valid the linear mathematical model is, as an approximation to the real system [17].

A large-scale MIMO system, described by state equations, is often decomposed into small subsystems, from which the analysis and design of the MIMO system can be easily performed. Similarity block transformations are developed to transform a class of linear time-invariant MIMO state equations, for which the systems described by these equations have the number of inputs dividing exactly the order of the state, into block companion forms so that the classical lines of thought for SISO systems can be extended to MIMO systems [19].

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Such systems can be studied via the eigen structure, eigen values and eigenvectors, of the state matrix A. The eigenvalues and eigenvectors can determine system performance and robustness far more directly and explicitly than other indicators. Hence their assignment should improve feedback system performance and robustness distinctly and effectively [1].Eigen structure assignment (EA) is the process of applying negative feedback to a linear, time-invariant system with the objective of forcing the latent-values and latent-vectors to become as close as possible to a desired eigen structure. EA, in common with other multivariable design methodologies, is inclined to use all of the available design freedom to generate a control solution. It is a natural choice for the design of any control system whose desired performance is readily represented in terms of an ideal eigen structure. Many research works has been done on EA [20, 21, 22, 23 and 24] and more specifically on flight control systems [25, 26, and 27].

The design of state feedback control in MIMO systems leads to the so-called matrix polynomials assignment [2]. The use of block poles constructed from a desired set of closed-loop poles offers the advantage of assigning a characteristic matrix polynomial rather than a scalar one [3]. The desired characteristic matrix polynomial is first constructed from a set of block poles selected among a class of similar matrices, and then the state feedback is synthesized by solving matrix equations. The forms of the block poles used in our work are the diagonal, the controller and the observer forms. Robustness is assessed, in each case, using the infinity norm, the singular value of the closed loop transfer matrix and the condition number of the closed-loop transfer matrix. Time response is assessed by plotting the step response and comparing the time response characteristics [1]. A comparison study is conducted to determine, in light of the above criteria, the best choice of the form of the block poles.

In thepresent paper, firstly we have started the work by introducing some theoretical preliminaries on matrix polynomials, after that a theoretical background on robustness and sensitivity analysisin term of responses is illustrated and briefly discussed, it is then followed by an application to flight dynamic system by doing a comparison study in term of block roots form. As a fifth section a discussion of the obtained results is performed, and finally the paper is finished by a comparison study and a conclusion.

2. PRELIMINARIES

2.1. Definition of a polynomial matrix

Definition 1: given a set of $m \times m$complex matrices $\{A_0, A_1, \dots, A_l\}$the following matrix valued function of the complex variableλ is called matrix polynomial of degree (index)l and order$m$:
($A(\lambda)$: is called also λ-matrix.)

$$A(\lambda) = A_0\lambda^l + A_1\lambda^{l-1} + \cdots + A_{l-1}\lambda + A_l \qquad (1)$$

Consider the system described by the following dynamic equation:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ \quad y(t) = Cx(t) \end{cases} \qquad (2)$$

Assuming that the system can be transformed to a block controller form, this means:

i. The number $\frac{n}{m} = l$ is an integer.
ii. The matrix $W_c = \{B, AB, \dots, A^{l-1}B\}$is of full rank n.

Then we use the following transformation matrix:

$$T_c = \begin{bmatrix} T_{cl} \\ T_{cl}A \\ \vdots \\ T_{cl}A^{l-2} \\ T_{cl}A^{l-1} \end{bmatrix} \text{Where } T_{cl} = [O_m O_m, \dots, I_m][B, AB, \dots, A^{l-1}B]^{-1} \qquad (3)$$

The new system becomes:

$$\begin{cases} \dot{x}(t) = A_c x(t) + B_c u(t) \\ y(t) = C_c x(t) \end{cases} (4)$$

With

$$A_c = T_c A T_c^{-1}, \quad B_c = T_c B, \quad C_c = C T_c^{-1}$$

$$A_c = \begin{bmatrix} O_m & I_m & \cdots & O_m \\ O_m & O_m & \cdots & O_m \\ \vdots & \vdots & \cdots & \vdots \\ O_m & O_m & \cdots & I_m \\ -A_l & -A_{l-1} & \cdots & -A_1 \end{bmatrix}, \qquad B_c = \begin{bmatrix} O_m \\ O_m \\ \vdots \\ O_m \\ I_m \end{bmatrix}, \qquad C_c = [C_l \quad C_{l-1} \quad \cdots \quad C_1]$$

2.2.Matrix transfer function

The matrix transfer function of this open-loop system is given by:

$$TF_R(s) = N_R(s) D_R^{-1}(s) (5)$$

Where:

- $N_R(s) = [C_1 s^{l-1} + \cdots + C_{l-1} s + C_l]$
- $D_R(s) = [I_m s^l + A_1 s^{l-1} + \cdots + A_l]$

This transfer function is called the Right Matrix Fraction Description (RMFD); we need to use it in the block controller form.

It should be noted that the behavior of the system depends on the characteristic matrix polynomial$D_R(s)$.

2.3. Concept of solvents (block roots)

A root for a polynomial matrix is not well defined. If it is defined as a complex number it may not exist at all. Then we may consider a root as a matrix called block root.

2.3.1. Right solvent

Given the matrix polynomial of order m and index l defined by:

$$D_R(s) = I_m s^l + A_1 s^{l-1} + \cdots + A_l = \sum_{i=0}^{l} A_i s^{l-i} \text{ where } A_0 = I_m \qquad (6)$$

A right solvent, denoted by R, is a $m \times m$ matrix satisfying:

$$D_R(R) = A_0 R^l + A_1 R^{l-1} + \cdots + A_{l-1} R + A_l = 0_m \qquad (7)$$

2.3.2. Left solvent

A left solvent of the matrix polynomial $D(s)$ defined above, denoted by L, is $m \times m$ matrix satisfying:

$$D_R(L) = L^l A_0 + L^{l-1} A_1 + \cdots + L A_{l-1} + A_l = 0_m \qquad (8)$$

A right solvent, if exist, is considered as a right block root. A left solvent, if exist, is considered as a left block root.

2.3.3. Latent root and latent vector

- A complex number λ satisfying $\det\left(D_R(\lambda)\right) = 0$ is called a latent root of $D_R(\lambda)$.
- Any vector $\boldsymbol{x_i}$ associated with the latent root satisfying $D_R(\lambda_i)\boldsymbol{x_i} = 0_m$ is a right latent vector of $D_R(\lambda)$.

The relationship between latent roots, latent vectors, and the solvents can be stated as follows:

Theorem: If $D(\lambda)$ has n linearly independent right latent vectors $p_1, p_2, \dots, p_n$ (left latent vectors $q_1, q_2, \dots, q_n$) corresponding to latent roots $p_1, p_2, \dots, p_n$, then $P \Lambda P^{-1}$, $(Q \Lambda Q^{-1})$ is a right (left) solvent. Where: $P = [p_1, p_2, \dots, p_n]$, $(Q = [q_1, q_2, \dots, q_n]^T)$ and $\Lambda = diag(\lambda_1, \lambda_2, \dots, \lambda_n)$.

Proof: see [16]

Theorem: If $D(\lambda)$ has n latent roots $\lambda_1, \lambda_2, \dots, \lambda_n$ and the corresponding right latent vectors $p_1, p_2, \dots, p_n$ has as well as the left latent vectors $q_1, q_2, \dots, q_n$ are both linearly independent, then the associated right solvent R and left solvent L are related by: $R = WLW^{-1}$, Where $W = PQ$ and $P = [p_1, p_2, \dots, p_n]$, $(Q = [q_1, q_2, \dots, q_n]^T)$. and "$T$ "stands for transpose.(**Proof:** see [16])

2.3.4. Complete set of solvents

Definition 1: Consider the set of solvents $\{R_1, R_2, \dots, R_l\}$ constructed from the eigenvalues $(\lambda_1, \lambda_2, \dots, \lambda_n)$ of a matrix A_c, $\{R_1, R_2, \dots, R_l\}$ is a complete set of solvents if and only if:

$$\begin{cases} \cup\, \sigma(R_i) = \sigma(A_c) \\ \sigma(R_i) \cap \sigma(R_j) = \emptyset \\ \det\big(V_R(R_1, R_2, \dots, R_l)\big) \neq 0 \end{cases} \tag{10}$$

Where:

σ denotes the spectrum of the matrix.

V_R is the block Vandermonde matrix corresponding to$\{R_1, R_2, \dots, R_l\}$ given as:

$$V_R(R_1, R_2, \dots, R_l) = \begin{bmatrix} I_m & I_m & \dots & I_m \\ R_1 & R_2 & \dots & R_l \\ \vdots & \vdots & \ddots & \vdots \\ {R_1}^{l-1} & {R_2}^{l-1} & \dots & {R_l}^{l-1} \end{bmatrix} \tag{11}$$

The conditions for the existence and uniqueness of the complete set of solvents have been investigated by P. Lancaster [15] and Malika Yaici [3].

Remark: We can define a set of left solvents in the same way as in the previous theorem.

2.4. Constructing a matrix polynomial from a complete set of solvents

We want to construct the matrix polynomial defined by $D(\lambda)$from a set of solvents or a set of desired poles which will determine the behavior of the system that we want.Suppose we have a desired complete set of solvents. The problem is to find the desired polynomial matrix or the characteristic equation of the block controller form defined by:

$$D(\lambda) = D_0\lambda^l + D_1\lambda^{l-1} + \dots + D_{l-1}\lambda + D_l$$

We want to find the coefficients$D_i$for $i = 1, \dots, l$

a. Constructing from a complete set of right solvents:

Consider a complete set of right solvents $\{R_1, R_2, \dots, R_l\}$for the matrix polynomial$D(\lambda)$, If $R_i$is a right solvent of $D(\lambda)$so:

$${R_i}^l + D_1{R_i}^{l-1} + \dots + D_{l-1}R_i + D_l = O_m \;\Rightarrow\; D_1{R_i}^{l-1} + \dots + D_{l-1}R_i + D_l = -{R_i}^l$$

Replacing i from 1 $to\ l$ we get the following:

$$\left[D_{dl}, D_{d(l-1)}, \dots, D_{d1}\right] = -\left[R_1^l, R_2^l, \dots, R_l^l\right]{V_R}^{-1} \tag{12}$$

Where $V_R$is the right block Vander monde matrix

b. Constructing from a complete set of left solvents:

Consider a complete set of left solvents$\{L_1, L_2, \dots, L_l\}$for the matrix polynomial $D(\lambda)$ If $L_i$is a left solvent of $D(\lambda)$so:

$$L_i{}^l + L_i{}^{l-1}D_1 + \cdots + L_iD_{l-1} + D_l = O_m \;\Rightarrow\; L_i{}^{l-1}D_1 + \cdots + L_iD_{l-1} + D_l = -L_i{}^l$$

Replacing i from 1 $to\ l$ we get the following:

$$\begin{bmatrix} D_{dl} \\ D_{d(l-1)} \\ \vdots \\ D_{d1} \end{bmatrix} = -V_L{}^{-1}\begin{bmatrix} L_1{}^l \\ L_2{}^l \\ \vdots \\ L_l{}^l \end{bmatrix} \qquad (13)$$

Where $V_L$is the left block Vandermonde matrix defined by:

$$V_L(L_1, L_2, \dots, L_l) = \begin{bmatrix} I_m L_1 \dots L_1{}^{l-1} \\ I_m L_2 \cdots L_2{}^{l-1} \\ \vdots \;\; \vdots \;\; \ddots \;\; \vdots \\ I_m L_l \cdots L_l{}^{l-1} \end{bmatrix} \qquad (14)$$

2.5. State feedback design

Consider the general linear time-invariant dynamic system described by the previous state space equation (2). Now applying the state feedback $u = -Kx(t)$to this system, where: K is a $m \times n$gain matrix.

After using the block controller form transformation for the system, we get:$u = -K_c x_c(t)$
Where: $K = K_c T_c = [K_{cl}, K_{c(l-1)}, \dots, K_{c1}]T_candK_{ci} \in R^{m\times m}$for $i = 1, \dots, l$

Then the resulting closed loop system is shown below:

$$\begin{cases} \dot{x}_c = (A_c - B_cK_c)x_c \\ \quad y_c = C_c x_c \end{cases}$$

Where:

$$(A_c - B_cK_c) = \begin{bmatrix} O_m & \cdots & O_m \\ O_m & \cdots & O_m \\ \vdots & \cdots & \vdots \\ O_m & \cdots & I_m \\ -(A_l + K_{cl}) & \cdots & -(A_1 + K_{c1}) \end{bmatrix}$$

The characteristic matrix polynomial of this closed loop system is:

$$D(\lambda) = I_m\lambda^l + (A_1 + K_{c1})\lambda^{l-1} + \cdots + (A_l + K_{cl})$$

From a set of desired eigenvalues, we construct the solvents then we construct the desired characteristic matrix polynomial in the form:

$$D_d(\lambda) = I_m\lambda^l + D_{d1}\lambda^{l-1} + \cdots + D_{dl} \qquad (15)$$

By putting $D_d(\lambda) = D(\lambda)$we get the coefficients K_{ci}as follows:

$$K_{ci} = D_{di} - A_i \text{for } i = 1, \dots, l \qquad (16)$$

After that we find the gain matrix by the following formula $K = K_c T_c$.

3. Robustness And Sensitivity Analysis

3.1. The sensitivity of eigenvalues (robust performance)

Robust performance is defined as the low sensitivity of system performance with respect to system model uncertainty and terminal disturbance. It is well known that the eigenvalues of the dynamic matrix determine the performance of the system then from that the sensitivities of these eigenvalues determine the robustness of the system (2).

Theorem: Let $\lambda\ and\ \lambda'$be the eigenvalues of the matrices $A\ and\ A + \Delta A$ respectively, and let V be the right eigenvectors matrix of A , then Wilkinson has derived the variation in eigenvalues as follows:

$$\min_i(\lambda_i - \lambda_i{}') \triangleq \min_i\big(\Delta(\lambda_i)\big) \le \kappa(V).\|\Delta A\| \qquad (17)$$

$\|.\|$ Stands for the matrix norm and $\kappa(.)$Is the condition number. (**Proof:**see[1])

Theorem: Let $\lambda_i, \boldsymbol{v_i}and\boldsymbol{t_i}$be the i^{th} eigenvalue, right and left eigenvectors of a matrix A respectively$(i = 1, \dots, n)$, let$\lambda_i + \Delta\lambda_i$be the i^{th} eigenvalue of the matrix $A + \Delta A$, then for small enough$\|\Delta A\|$

$$\Delta\lambda_i \le \|\boldsymbol{v_i}\|\|\boldsymbol{t_i}\|\|\Delta A\| \triangleq s(\lambda_i)\|\Delta A\| \qquad (18)$$

Such that: $s(\lambda_i) = \|\boldsymbol{v_i}\|\|\boldsymbol{t_i}\|$.(**Proof:** see [1].)

This theorem shows that the sensitivity of an eigenvalue is determined by its corresponding left and right eigenvectors and it is valid for small perturbations in the matrix$A$.

3.2. Relative change

Let $\lambda_i\ and\ \lambda_i{}'$be the eigenvalues of the matrices $A\ and\ A + \Delta A$ respectively.The relative change$r_i$of the eigenvalue $\lambda_i$is defined as follows:

$$r_i = \frac{|\lambda_i - \lambda_i'|}{|\lambda_i|} = \frac{|\Delta\lambda_i|}{|\lambda_i|} \qquad i = 1, .., n \tag{19}$$

3.3. Robust Stability

The stability of a system is the most wanted property. So its sensitivity to uncertainties is very important when analyzing and designing the system. Stability is affected by the system eigenvalues of the dynamic matrix so the sensitivity of these eigenvalues directly affects the robust stability of the system (2). There are three robust stability measures using the sensitivity of this system eigenvalues defined as follows:

Definition: Let$\{\lambda_1, \lambda_2, \dots, \lambda_n\}$be the set of eigenvalues of an$n \times n$matrix denoted by $A$and assuming that all the eigenvalues are stable ($i.e$: $Re\{\lambda_i\} < 0\ \forall i$) and all the eigenvalues are already arbitrary assigned for guaranteed performance, the three robust stability measures are defined by:

1. $M_1 = \min_{0<\omega<\infty}\{\underline{\sigma}(A - j\omega I)\}$, $\underline{\sigma}$denotes the smallest singular value.
2. $M_2 = (\kappa(V))^{-1}.|Re\{\lambda_n\}|$such that: $|Re\{\lambda_n\}| \le \cdots \le |Re\{\lambda_1\}|$ and $V$is the right eigenvector matrix of A .
3. $M_3 = \min_{0\le i\le n}\left\{(s(\lambda_i))^{-1}|Re\{\lambda_i\}|\right\}$

4. Left/Right Block Pole Placement Comparison Study Application To The B747Aircraft

When the controlled system is ***multi-input multi-output***then an infinite number of gain matrices **K** may be found which will provide the required stability characteristics. Consequently, an alternative and very powerful method for designing feedback gains for auto-stabilization systems is the right and/or left block pole placement method. The method is based on the manipulation of the equations of motion in block state space form and makes full use of the appropriate computational tools in the analytical process.

The motion of the aircraft is described in terms of force, moment, linear and angular velocities and attitude resolved into components with respect to the chosen aircraft fixed axis system. For convenience it is preferable to assume a generalized body axis system in the first instance [5]. Whenever the aircraft is disturbed from equilibrium the force and moment balance is upset and the resulting transient motion is quantified in terms of the perturbation variables. The perturbation variables are shown in Fig. 1.

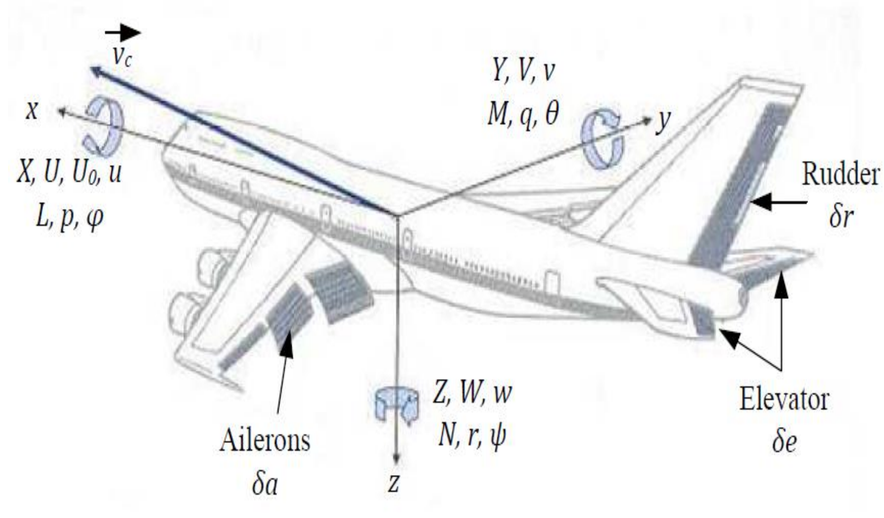

Figure: 1 Motion variables notation

The basic dynamics using Newton's laws are, assuming that the mass isn't variable:

$$\vec{F} = m\,\dot{\overrightarrow{\mathcal{V}_c^I}} \quad and \quad \vec{M} = \dot{\overrightarrow{H^I}} \qquad (20)$$

Where: $I$is indicating that the vector is written in the base of the inertial axes.

$\vec{F}$is the resultant force.m is the mass of the body.$\dot{\overrightarrow{\mathcal{V}_c}}$is the acceleration of the body.

$\overrightarrow{H^I}$is the angular momentum.$\vec{M}$is the resultant moment.

Referring to the body fixed frame the equations can be rewritten as

$$\vec{F} = m\,\dot{\overrightarrow{\mathcal{V}_c^B}} + \overrightarrow{\mathcal{W}^{BI}} \times \overrightarrow{\mathcal{V}_c^B} \quad and \quad \vec{M} = \dot{\overrightarrow{H^B}} + \overrightarrow{\mathcal{W}^{BI}} \times \overrightarrow{H^B} \qquad (21)$$

With $$\mathcal{W}^{BI} = \begin{bmatrix} P \\ Q \\ R \end{bmatrix} \quad and \quad \mathcal{V}_c^B = \begin{bmatrix} U \\ V \\ W \end{bmatrix}$$

Where:B denotes the body axes and $\overrightarrow{\mathcal{W}^{BI}}$indicates the angular vector from base $I$to base B.

By symmetry, we have$I_{xy} = I_{yz} = 0 \ and \ H^B = \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} = \begin{bmatrix} H_{xx} & 0 & H_{xz} \\ 0 & H_{yy} & 0 \\ H_{xz} & 0 & H_{zz} \end{bmatrix} \begin{bmatrix} P \\ Q \\ R \end{bmatrix}$

The state description of the lateral motion of B747 was developed in the ref [5]. In this section we will apply the results to the case of the Boeing 747 for an altitude of $12192\mathrm{m}$ and a speed of $235.9\mathrm{m/s}$ and assuming that $\Theta_0 = 0$.The state space model of its lateral motion (Lateral motion of the B747) is given by the next state space equations:

$$\begin{bmatrix} \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} -0.0558 & 0 & -235.9 & 9.81 \\ -0.0127 & -0.4351 & 0.4143 & 0 \\ 0.0036 & -0.0061 & -0.1458 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v \\ p \\ r \\ \phi \end{bmatrix} + \begin{bmatrix} 0 & 1.7188 \\ -0.1433 & 0.1146 \\ 0.0038 & -0.4859 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix}$$

Where:$y_1 = v$:Lateral speed rate perturbation$y_2 = p$:Roll rate perturbation
$y_3 = r$:Yaw rate perturbation$y_4 = \phi$:Roll angle.
$u_1 = \delta_a$:Ailerons deflection$u_2 = \delta_r$:Rudder deflection

The dimension of the matrix A is 4×4 and the number of inputs is2 . The rank of the matrix $[B\ AB]$is 4, and then the system is block controllable of index 2. Therefore we can convert the system into block controller form by the following transformation matrixT_c:

$$T_c = \begin{bmatrix} [O_2 I_2][B\ AB]^{-1} \\ [O_2 I_2][B\ AB]^{-1}A \end{bmatrix} = \begin{bmatrix} 0.0070 & 0.0007 & 0.0250 & -7.0199 \\ 0.0088 & 0.0008 & 0.0312 & -0.0538 \\ -0.0003 & -7.0203 & -1.6573 & 0.0688 \\ -0.0004 & -0.0543 & -2.0723 & 0.0860 \end{bmatrix}$$

We obtain the following:

$$A_c = \begin{bmatrix} O_2 & I_2 \\ -A_{c2} & -A_{c1} \end{bmatrix} = \begin{bmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ \begin{pmatrix} -0.0561 & 9.6066 \\ 0.0041 & -0.7736 \end{pmatrix} & \begin{pmatrix} -0.4589 & 1.7627 \\ -0.0161 & -0.1778 \end{pmatrix} \end{bmatrix}$$

$$B_c = \begin{bmatrix} O_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{bmatrix}$$

$$C_c = C \begin{bmatrix} [O_2 I_2][B\ AB]^{-1} \\ [O_2 I_2][B\ AB]^{-1}A \end{bmatrix}^{-1} = \begin{bmatrix} \begin{pmatrix} -0.8588 & 114.8295 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 1.7188 \\ -0.1433 & 0.1146 \end{pmatrix} \\ \begin{pmatrix} -0.0058 & -0.0167 \\ -0.1433 & 0.1146 \end{pmatrix} & \begin{pmatrix} 0.0038 & -0.4859 \\ 0 & 0 \end{pmatrix} \end{bmatrix}$$

We want to design a state feedback using block pole placement for the following set of desired eigenvalues:$-1 \pm 1.5i,\ -1.5$ and -2 or $\lambda_{1,2,3,4} = \{-1 + 1.5i\ , -1 - 1.5i\ , -1.5\ , -2\}$

4.1. State Feedback Design Using Right Block Poles

4.1.1. Right solvents in diagonal form

- **Construction of the feedback gain matrix:**

The desired right block poles in diagonal form are constructed as follows:

$$R_1 = \begin{pmatrix} -1 & 1.5 \\ -1.5 & -1 \end{pmatrix}, \qquad R_2 = \begin{pmatrix} -1.5 & 0 \\ 0 & -2 \end{pmatrix}$$

Then the corresponding controllable feedback gain matrix, which is obtained using the equation (16), is:

$$K_c = \left[\begin{pmatrix} 2.0576 & 6.8794 \\ 2.6632 & 0.4082 \end{pmatrix} \begin{pmatrix} 2.4502 & 0.3990 \\ 1.7566 & 2.4131 \end{pmatrix}\right]$$

$$K = K_c T_c \Leftrightarrow K = \left[\begin{pmatrix} 0.0738 & -17.2161 \\ 0.0208 & -12.4610 \end{pmatrix} \ - \begin{pmatrix} 4.6217 & 14.6110 \\ 7.8327 & 18.3891 \end{pmatrix}\right]$$

The norm of the feedback gain matrix is$\|K\|_2 = 32.5970$.

- **Time specifications:**

The following points summarize the time specifications (Settling time$T_s$and Maximum peakM_p) for the closed-loop system described by equation (15 and 16) and the response of the system to an initial state$x_0 = [1 \ \ 1 \ \ 1 \ \ 1]^T$.

a. Lateral speed rate perturbation (v) : $T_s = 10.1587$ s and $M_p = 396$
b. Roll rate perturbation(p) : $T_s = 4.2328$ s and $M_p = 1$
c. Yaw rate perturbation(r) : $T_s = 4.9735$ s and $M_p = 2.43$
d. Roll angle(ϕ): $T_s = 3.5979$ s and $M_p = 1.27$

The settling time$T_s$is defined in our case as the time required for the outputs to remain within ± 0.05 unity.

- **Robustness:**

Robust stability: Robust stability is determined using the measures defined in the previous section. First we find the norms of the left and right eigenvectors associated to each eigenvalue. The right eigenvector matrix associated to the closed-loop system is:

$$V = \begin{bmatrix} 1.0000 & 1.0000 & -0.9575 & 1.0000 \\ (-0.0029 + 0.0003i) & (-0.0029 - 0.0003i) & 0.2397 & -0.0021 \\ (0.0040 - 0.0065i) & (0.0040 + 0.0065i) & -0.0127 & 0.0086 \\ (0.0011 + 0.0012i) & (0.0011 - 0.0012i) & -0.1598 & 0.0010 \end{bmatrix} \text{and} \|V\| = 1.9844$$

The left eigenvector matrix has norm equal to:

$$\|T\| = \|V^{-1}\| = 1018.8 \implies s(V) = \|V\|\|V^{-1}\| = 2021.6$$

The norms of all the right eigenvectors are 1. The associated left eigenvectors have norms as follows: $\|\boldsymbol{t_1}\|_2 = 473.0835$, $\|\boldsymbol{t_2}\|_2 = 473.0835$, $\|\boldsymbol{t_3}\|_2 = 3.6939$, $\|\boldsymbol{t_4}\|_2 = 774.7516$.

The sensitivity of each eigenvalue is: $s(\lambda_i) = \{473.0835\ , 473.6939\ , 3.6939\ , 774.7516\}$

Now the stability measures are:

- ✓ $M_1 = \min_{0\leq\omega<\infty}\{\underline{\sigma}(A - BK - j\omega I)\} = 0.0025$
- ✓ $M_2 = \left(s(V)\right)^{-1}\left|Re\{\lambda_{1,2}\}\right| = 4.9464 \times 10^{-4}$
- ✓ $M_3 = \min_{0\leq i\leq 4}\left\{\left(s(\lambda_i)\right)^{-1}|Re\{\lambda_i\}|\right\} = 0.0023$

Robust performance: We generate a random small perturbation using MATLAB software, we get the following:

$$\Delta A = \begin{bmatrix} 0.42180.65570.65550.6787 \\ 0.91570.03570.75770.1712 \\ 0.79220.84910.74310.7060 \\ 0.95950.93400.39220.0318 \end{bmatrix} \times 10^{-4}$$

The eigenvalues of the matrix $(A - BK + \Delta A)$ are:

$$-1.0595 + 1.4677i;\ -1.0595 - 1.4677i;\ -1.4999;\ -1.8809.$$

The relative change of the each eigenvalue is given below by the following:

$$r_i(\lambda_i) = \{0.0751\ , 0.0751\ , 6.6667 \times 10^{-4}\ , 0.0595\}$$

4.1.2. Right solvents in controllable form

- **Construction of the feedback gain matrix:**

The desired right block poles in controllable form are constructed as follows:

$$R_1 = \begin{pmatrix} -2 & -3.25 \\ 1 & 0 \end{pmatrix}, \qquad R_2 = \begin{pmatrix} 0 & 1 \\ -3 & -3.5 \end{pmatrix}$$

Then the corresponding controllable feedback gain matrix, which is obtained using the equation (16), is:

$$K_c = \left[\begin{pmatrix} 4.1627 & 11.8410 \\ -2.0584 & 0.44510 \end{pmatrix} \begin{pmatrix} 2.2286 & 2.1689 \\ -0.6411 & 2.6347 \end{pmatrix}\right]$$

$$K = K_c T_c \Leftrightarrow K = \left[\begin{pmatrix} 0.1315 & -15.7512 \\ -0.0114 & 4.3567 \end{pmatrix} - \begin{pmatrix} 7.7148 & 29.5186 \\ 4.4348 & -14.6081 \end{pmatrix}\right]$$

The norm of the feedback gain matrix is $\|K\|_2 = 37.0470$.

- **Time specifications:**

The following points summarize the time specifications (Settling time T_s and Maximum peak M_p) for the closed-loop system described by equation (15 and 16) and the response of the system to an initial state $x_0 = [1\ \ 1\ \ 1\ \ 1]^T$.

a. Lateral speed rate perturbation (v) : $T_s = 8.8889$ s and $M_p = 346$
b. Roll rate perturbation (p) : $T_s = 3.7037$ s and $M_p = 1.58$
c. Yaw rate perturbation (r) : $T_s = 4.7619$ s and $M_p = 2.06$
d. Roll angle (ϕ): $T_s = 3.0688$ s and $M_p = 1.06$

- **Robustness:**

Robust stability: Robust stability is determined using the measures defined in the previous section. First we find the norms of the left and right eigenvectors associated to each eigenvalue. The right eigenvector matrix associated to the closed-loop system is:

$$V = \begin{bmatrix} 1.0000 & 1.0000 & 1.0000 - 1.0000 \\ (0.0006 + 0.0053i) & (0.0006 - 0.0053i) & -0.0028\ \ 0.0033 \\ (0.0041 - 0.0066i) & (0.0041 + 0.0066i) & 0.0064 - 0.0086 \\ (0.0022 - 0.0019i) & (0.0022 + 0.0019i) & 0.0019 - 0.0017 \end{bmatrix} \text{and} \|V\| = 2$$

The left eigenvector matrix has norm equal to:

$$\|T\| = \|V^{-1}\| = 2221 \implies s(V) = \|V\|\|V^{-1}\| = 1238.7$$

The norms of all the right eigenvectors are 1. The associated left eigenvectors have norms as follows: $\|\boldsymbol{t_1}\|_2 = 345.5263$, $\|\boldsymbol{t_2}\|_2 = 345.5263$, $\|\boldsymbol{t_3}\|_2 = 958.4084$, $\|\boldsymbol{t_4}\|_2 = 875.6205$. The sensitivity of each eigenvalue is: $s(\lambda_i) = \{345.5263\,, 345.5263\,, 958.4084\,, 875.6205\}$

Now the stability measures are:

- ✓ $M_1 = \min_{0\le\omega<\infty}\{\underline{\sigma}(A - BK - j\omega I)\} = 0.0031$
- ✓ $M_2 = \left(s(V)\right)^{-1}\left|Re\{\lambda_{1,2}\}\right| = 8.0732 \times 10^{-4}$
- ✓ $M_3 = \min_{0\le i\le 4}\left\{\left(s(\lambda_i)\right)^{-1}|Re\{\lambda_i\}|\right\} = 0.0016$

Robust performance: The eigenvalues of the matrix $(A - BK + \Delta A)$ are:

$$-0.9594 + 1.5188i;\ -0.9594 - 1.5188i;\ -1.6042;\ \ -1.9770.$$

The relative change of the each eigenvalue is given below by the following:

$$r_i(\lambda_i) = \{0.0248\,, 0.0248\,, 0.0695\,, 0.0115\}$$

4.1.3. Right solvents in observable form

- **Construction of the feedback gain matrix:**

The desired right block poles in observable form are constructed as follows:

$$R_1 = \begin{pmatrix} -2 & 1 \\ -3.25 & 0 \end{pmatrix}, \qquad R_2 = \begin{pmatrix} 0 & -3 \\ 1 & -3.5 \end{pmatrix}$$

Then the corresponding controllable feedback gain matrix, which is obtained using the equation (16), is:

$$K_c = \left[\begin{pmatrix} 3.9439 & 7.6066 \\ 2.0041 & 0.6639 \end{pmatrix} \begin{pmatrix} 3.5411 & 0.7627 \\ 1.7964 & 1.3222 \end{pmatrix} \right]$$

$$K = K_c T_c \Leftrightarrow K = \left[\begin{pmatrix} 0.0929 & -24.8926 \\ 0.0188 & -12.6812 \end{pmatrix} - \begin{pmatrix} 7.1135 & 27.7859 \\ 5.6464 & 13.8672 \end{pmatrix} \right]$$

The norm of the feedback gain matrix is $\|K\|_2 = 42.7086$.

- **Time specifications:**

The following point summarizes the time specifications (Settling time T_s and Maximum peak M_p) for the closed-loop system described by equation (15 and 16) and the response of the system to an initial state $x_0 = [1 \ \ 1 \ \ 1 \ \ 1]^T$.

a. Lateral speed rate perturbation (v) : $T_s = 9.8413$ s and $M_p = 205$
b. Roll rate perturbation(p) : $T_s = 2.6455$ s and $M_p = 1$
c. Yaw rate perturbation(r) : $T_s = 4.6561$ s and $M_p = 1.49$
d. Roll angle(ϕ): $T_s = 2.1164$ s and $M_p = 1.12$

- **Robustness:**

Robust stability: Robust stability is determined using the measures defined in the previous section. First we find the norms of the left and right eigenvectors associated to each eigenvalue. The right eigenvector matrix associated to the closed-loop system is

$$V = \begin{bmatrix} 1.0000 & 1.0000 & -1.0000 & 1.0000 \\ (-0.0015 + 0.0004i) & (-0.0015 - 0.0004i) & -0.0023 & 0.0018 \\ (0.0040 - 0.0065i) & (0.0040 + 0.0065i) & -0.0062 & 0.0085 \\ (0.0006 + 0.0006i) & (0.0006 - 0.0006i) & 0.0016 & -0.0009 \end{bmatrix} \text{and} \|V\| = 2$$

The left eigenvector matrix has norm equal to:

$$\|T\| = \|V^{-1}\| = 2314.8 \implies s(V) = \|V\|\|V^{-1}\| = 4628.8$$

The norms of all the right eigenvectors are 1. The associated left eigenvectors have norms as follows: $\|\boldsymbol{t_1}\|_2 = 393.0204$, $\|\boldsymbol{t_2}\|_2 = 393.0204$, $\|\boldsymbol{t_3}\|_2 = 1464.9$, $\|\boldsymbol{t_4}\|_2 = 1725.8$.

The sensitivity of each eigenvalue is: $s(\lambda_i) = \{393.0204\,, 393.0204\,, 1464.9\,, 1725.8\}$

Now the stability measures are:

- ✓ $M_1 = \min_{0\leq\omega<\infty}\{\underline{\sigma}(A - BK - j\omega I)\} = 0.0042$
- ✓ $M_2 = \left(s(V)\right)^{-1}\left|Re\{\lambda_{1,2}\}\right| = 2.1604 \times 10^{-4}$
- ✓ $M_3 = \min_{0\leq i\leq 4}\left\{\left(s(\lambda_i)\right)^{-1}\left|Re\{\lambda_i\}\right|\right\} = 0.0010$

Robust performance: The eigenvalues of the matrix $(A - BK + \Delta A)$ are:

$$-1.0240 + 1.4553i;\ -1.0240 - 1.4553i;\ -1.7260 - 0.2201i;\ \ 1.7260 + 0.2201i.$$

The relative change of the each eigenvalue is given below by the following:

$$r_i(\lambda_i) = \{0.0281\,, 0.0281\,, 0.2103\,, 0.1757\}$$

4.2. State feedback design using left block poles

4.2.1. Left solvents in diagonal form

- **Construction of the feedback gain matrix:**

The desired left block poles in diagonal form are constructed as follows:

$$L_1 = \begin{pmatrix} -1 & 1.5 \\ -1.5 & -1 \end{pmatrix}, \qquad L_2 = \begin{pmatrix} -1.5 & 0 \\ 0 & -2 \end{pmatrix}$$

Then the corresponding controllable feedback gain matrix, which is obtained using the equation (16), is:

$$K_c = \left[\begin{pmatrix} 2.0576 & 6.9475 \\ 2.7314 & 0.4082 \end{pmatrix}\begin{pmatrix} 2.4502 & -0.4502 \\ 1.3475 & 2.4131 \end{pmatrix}\right]$$

$$K = K_c T_c \Leftrightarrow K = \left[\begin{pmatrix} 0.0746 & -17.1938 \\ 0.0214 & -9.5890 \end{pmatrix} \ -\begin{pmatrix} 3.7718 & 14.6499 \\ 7.1530 & 18.8959 \end{pmatrix}\right]$$

The norm of the feedback gain matrix is $\|K\|_2 = 31.3234$.

- **Time specifications:**

The following points summarize the time specifications (Settling time T_s and Maximum peak M_p) for the closed-loop system described by equation (15 and 16) and the response of the system to an initial state $x_0 = [1\ \ 1\ \ 1\ \ 1]^T$.

a. Lateral speed rate perturbation (v) : $T_s = 10.1587$ s and $M_p = 424$
b. Roll rate perturbation (p) : $T_s = 4.1270$ s and $M_p = 1$

c. Yaw rate perturbation(r) : $T_s = 5.0794$ s and $M_p = 2.44$
d. Roll angle(ϕ): $T_s = 3.4921$ s and $M_p = 1.25$

- **Robustness:**

Robust stability: Robust stability is determined using the measures defined in the previous section. First we find the norms of the left and right eigenvectors associated to each eigenvalue. The right eigenvector matrix associated to the closed-loop system is:

$$V = \begin{bmatrix} 1.0000 & 1.0000 & 1.0000 & -0.9999 \\ (-0.0027 + 0.0004i) & (-0.0027 - 0.0004i) & -0.0003 & 0.0096 \\ (0.0040 - 0.0065i) & (0.0040 + 0.0065i) & 0.0063 & -0.0087 \\ (0.0007 + 0.0014i) & (0.0007 - 0.0014i) & 0.0002 & -0.0048 \end{bmatrix} \text{and} \|V\| = 2$$

The left eigenvector matrix has norm equal to:

$$\|T\| = \|V^{-1}\| = 1026.1 \implies s(V) = \|V\|\|V^{-1}\| = 2052.1$$

The norms of all the right eigenvectors are 1. The associated left eigenvectors have norms as follows: $\|\boldsymbol{t_1}\|_2 = 496.8456$, $\|\boldsymbol{t_2}\|_2 = 496.8456$, $\|\boldsymbol{t_3}\|_2 = 737.9024$, $\|\boldsymbol{t_4}\|_2 = 201.6491$.

The sensitivity of each eigenvalue is: $s(\lambda_i) = \{496.8456, 496.8456, 737.9024, 201.6491\}$

Now the stability measures are:

- ✓ $M_1 = \min_{0\le\omega<\infty}\{\underline{\sigma}(A - BK - j\omega I)\} = 0.0021$
- ✓ $M_2 = (s(V))^{-1}|Re\{\lambda_{1,2}\}| = 4.8731 \times 10^{-4}$
- ✓ $M_3 = \min_{0\le i\le 4}\left\{(s(\lambda_i))^{-1}|Re\{\lambda_i\}|\right\} = 0.0020$

Robust performance: The eigenvalues of the matrix$(A - BK + \Delta A)$are:

$$-1.0665 + 1.4825i;\ -1.0665 - 1.4825i;\ -1.3861;\ -1.9808.$$

The relative change of the each eigenvalue is given below by the following:

$$r_i(\lambda_i) = \{0.0381, 0.0381, 0.0759, 0.0096\}$$

4.2.2. Left solvents in controllable form

- **Construction of the feedback gain matrix:**

The desired left block poles in controller form are constructed as follows:

$$L_1 = \begin{pmatrix} -2 & -3.25 \\ 1 & 0 \end{pmatrix}, \qquad L_2 = \begin{pmatrix} 0 & 1 \\ -3 & -3.5 \end{pmatrix}$$

Then the corresponding controllable feedback gain matrix, which is obtained using the equation (16), is:

$$K_c = \left[\begin{pmatrix} 3.9439 & 11.6066 \\ -1.9959 & 0.6639 \end{pmatrix} \begin{pmatrix} 3.5411 & 3.5752 \\ -1.0161 & 1.3222 \end{pmatrix}\right]$$

$$K = K_c T_c \Leftrightarrow K = \left[\begin{pmatrix} 0.1269 & -25.0421 \\ -0.0084 & 7.0609 \end{pmatrix} - \begin{pmatrix} 12.8170 & 27.7592 \\ 1.0850 & -14.0189 \end{pmatrix}\right]$$

The norm of the feedback gain matrix is $\|K\|_2 = 41.9890$.

- **Time specifications:**

The following points summarize the time specifications (Settling time T_s and Maximum peak M_p) for the closed-loop system described by equation (15 and 16) and the response of the system to an initial state $x_0 = [1\ \ 1\ \ 1\ \ 1]^T$.

a. Lateral speed rate perturbation (v) : $T_s = 10.0529$ s and $M_p = 407$
b. Roll rate perturbation (p) : $T_s = 4.4444$ s and $M_p = 1.57$
c. Yaw rate perturbation (r) : $T_s = 4.8677$ s and $M_p = 2.46$
d. Roll angle (ϕ): $T_s = 3.9153$ s and $M_p = 1.05$

- **Robustness:**

Robust stability: Robust stability is determined using the measures defined in the previous section. First we find the norms of the left and right eigenvectors associated to each eigenvalue. The right eigenvector matrix associated to the closed-loop system is:

$$V = \begin{bmatrix} 1.0000 & 1.0000 & 1.0000 - 0.9368 \\ (-0.0012 + 0.0034i) & (-0.0012 - 0.0034i) & 0.0041\ \ 0.3127 \\ (0.0040 - 0.0066i) & (0.0040 + 0.0066i) & 0.0062 - 0.0145 \\ (0.0019 - 0.0005i) & (0.0019 + 0.0005i) & -0.0027 - 0.1563 \end{bmatrix} \text{and} \|V\| = 1.9763$$

The left eigenvector matrix has norm equal to:

$$\|T\| = \|V^{-1}\| = 884.8156 \implies s(V) = \|V\|\|V^{-1}\| = 1784.7$$

The norms of all the right eigenvectors are 1. The associated left eigenvectors have norms as follows: $\|\boldsymbol{t_1}\|_2 = 390.9862$, $\|\boldsymbol{t_2}\|_2 = 390.9862$, $\|\boldsymbol{t_3}\|_2 = 698.5479$, $\|\boldsymbol{t_4}\|_2 = 16.5739$.

The sensitivity of each eigenvalue is: $s(\lambda_i) = \{390.9862\,, 390.9862\,, 698.5479\,, 698.5479\}$

Now the stability measures are:

- ✓ $M_1 = \min_{0 \le \omega < \infty}\{\underline{\sigma}(A - BK - j\omega I)\} = 0.0026$
- ✓ $M_2 = (s(V))^{-1}|Re\{\lambda_{1,2}\}| = 5.7187 \times 10^{-4}$
- ✓ $M_3 = \min_{0 \le i \le 4}\left\{(s(\lambda_i))^{-1}|Re\{\lambda_i\}|\right\} = 0.0021$

Robust performance: The eigenvalues of the matrix $(A - BK + \Delta A)$ are:

$$-0.9529 + 1.5250i;\ -0.9529 - 1.2550i;\ -1.5919;\ \ -2.0022.$$

The relative change of the each eigenvalue is given below by the following:

$$r_i(\lambda_i) = \{0.0296\,, 0.0296\,, 0.0613\,, 0.0011\}$$

4.2.3. Left solvents in observable form

- **Construction of the feedback gain matrix:**

The desired left block poles in observable form are constructed as follows:

$$L_1 = \begin{pmatrix} -2 & 1 \\ -3.25 & 0 \end{pmatrix}, \qquad L_2 = \begin{pmatrix} 0 & -3 \\ 1 & -3.5 \end{pmatrix}$$

Then the corresponding controllable feedback gain matrix, which is obtained using the equation (16), is:

$$K_c = \left[\begin{pmatrix} 4.1627 & 7.5441 \\ 2.2385 & 0.4451 \end{pmatrix} \begin{pmatrix} 2.2286 & 1.1377 \\ 0.3901 & 2.6347 \end{pmatrix}\right]$$

$$K = K_c T_c \Leftrightarrow K = \left[\begin{pmatrix} 0.0942 & -15.6987 \\ 0.0185 & -2.8802 \end{pmatrix} - \begin{pmatrix} 5.7119 & 29.3762 \\ 6.0367 & 15.4846 \end{pmatrix}\right]$$

The norm of the feedback gain matrix is$\|K\|_2 = 37.3973$.

- **Time specifications:**

The following points summarize the time specifications (Settling time$T_s$and Maximum peakM_p) for the closed-loop system described by equation (15 and 16) and the response of the system to an initial state$x_0 = [1\ \ 1\ \ 1\ \ 1]^T$.

a. Lateral speed rate perturbation (v) : $T_s = 8.9947$ s and $M_p = 302$
b. Roll rate perturbation(p) : $T_s = 3.8095$ s and $M_p = 1$
c. Yaw rate perturbation(r) : $T_s = 4.8677$ s and $M_p = 1.69$
d. Roll angle(ϕ): $T_s = 3.1746$ s and $M_p = 1.12$

- **Robustness:**

Robust stability: Robust stability is determined using the measures defined in the previous section. First we find the norms of the left and right eigenvectors associated to each eigenvalue. The right eigenvector matrix associated to the closed-loop system is:

$$V = \begin{bmatrix} 1.0000 & 1.0000 & -1.0000 & 1.0000 \\ (-0.0016 - 0.0021i) & (-0.0016 + 0.0021i) & 0.0007 & -0.0013 \\ (0.0039 - 0.0065i) & (0.0039 + 0.0065i) & -0.0063 & 0.0086 \\ (-0.0005 + 0.0014i) & (-0.0005 - 0.0004i) & -0.0004 & 0.0007 \end{bmatrix} \text{and} \|V\| = 2$$

The left eigenvector matrix has norm equal to:

$$\|T\| = \|V^{-1}\| = 1383.9 \implies s(V) = \|V\|\|V^{-1}\| = 2767.8$$

The norms of all the right eigenvectors are 1. The associated left eigenvectors have norms as follows: $\|\boldsymbol{t_1}\|_2 = 402.7999$, $\|\boldsymbol{t_2}\|_2 = 402.7999$, $\|\boldsymbol{t_3}\|_2 = 1180.7$, $\|\boldsymbol{t_4}\|_2 = 717.5067$.

The sensitivity of each eigenvalue is:

$$s(\lambda_i) = \{402.7999\,, 402.7999\,, 1180.7\,, 717.5067\}$$

Now the stability measures are:

- ✓ $M_1 = \min_{0\le\omega<\infty}\{\underline{\sigma}(A - BK - j\omega I)\} = 0.0025$
- ✓ $M_2 = \left(s(V)\right)^{-1}\left|Re\{\lambda_{1,2}\}\right| = 3.6129 \times 10^{-4}$
- ✓ $M_3 = \min_{0\le i\le 4}\left\{\left(s(\lambda_i)\right)^{-1}|Re\{\lambda_i\}|\right\} = 0.0013$

Robust performance: The eigenvalues of the matrix$(A - BK + \Delta A)$are:

$$-1.0497 + 1.4968i;\ -1.0497 - 1.4968i;\ -1.3717;\ -2.0288.$$

The relative change of the each eigenvalue is given below by the following:

$$r_i(\lambda_i) = \{0.0276\,, 0.0276\,, 0.0855\,, 0.0144\}$$

4. DISCUSSION

Small gains are desirable because they minimize the control energy and prevent saturation of the controller elements and noise amplification. For time specifications, the smaller the settling time and maximum peak the better the time response. The right solvents in canonical forms give smaller settling times. The maximum peaks are practically the same except the case of the lateral velocity $v$where it is better in the case of right solvents.For the sensitivities of the eigenvalues, we choose the one that has the lowest sensitivity taking into account the distance of the eigenvalue from the $j\omega$axis. For the robust stability the greater the value of its measure the more robustly stable the system, where $M_3$is more accurate than $M_1$and M_2[1]. For robust performance, the smaller the value of relative change the better the performance. In our case, the crucial criterion is the robustness, because of the linearization of the model.

6. COMPARISON OF THE RESULTS

In order to do a comparison study between left and right block roots placement we gather the preceding results in table: 1as shown below.To make the analysis easier and be clear table: 1 can be summarized into table: 2.

From table: 2the choice of the block roots in our case is as follows:

- In diagonal form: the right solvents are more suitable.
- In controllable form: The left solvents are suitable.
- In observable form: The left solvents are more suitable.

Table: 1 shows the collected data to be compared.

			Diagonal form		Controllable form		Observable form	
			Right	Left	Right	Left	Right	Left
$\|K\|_2$			32.5970	31.3234	37.0470	41.9890	42.7086	37.3973
Time specifications	v	t_s	10.1587 s	10.1587 s	8.8889 s	10.0529 s	9.8413 s	8.9947 s
		Max peak	396	424	346	407	205	302
	p	t_s	4.2328 s	4.1270 s	3.7037 s	4.4444 s	2.6455 s	3.8095 s
		Max peak	1	1	1.58	1.57	1	1
	r	t_s	4.9735 s	5.0794 s	4.7619 s	4.8677 s	4.6561 s	4.8677 s
		Max peak	2.43	2.44	2.06	2.46	1.49	1.69
	ϕ	t_s	3.5979 s	3.4921 s	3.0688 s	3.9153 s	2.1164 s	3.1746 s
		Max peak	1.27	1.25	1.06	1.05	1.12	1.12
Robust stability	$s(\lambda_i)$	$-1+1.5i$	437.0835	496.8456	345.5263	390.9862	393.0204	402.7999
		$-1-1.5i$	437.0835	496.8456	345.5263	390.9862	393.0204	402.7999
		-1.5	3.6939	737.9024	958.4084	698.5479	1464.9	1180.7
		-2	774.7516	201.6491	875.6205	698.	3.5979	3.5979
	M_1		0.0025	0.0021	0.0031	0.0026	0.0042	0.0025
	$M_2(\times 10^{-4})$		4.9464	4.8731	8.0732	5.7187	2.1604	3.6129
	M_3		0.0023	0.0020	0.0016	0.0021	0.0010	0.0013
Robust performance	$r_i(\lambda_i)$	$-1+1.5i$	0.0751	0.0381	0.0248	0.0296	0.0281	0.0276
		$-1+1.5i$	0.0751	0.0381	0.0248	0.0296	0.0281	0.0276
		-1.5	6.6667×10^{-5}	0.0759	0.0695	0.0613	0.2103	0.0855
		-2	0.0595	0.0095	0.0115	0.0011	01757	0.0144

Table: 2 show the comparative study.

<table>
<tr><td colspan="2" rowspan="2"></td><td colspan="2">Diagonal form</td><td colspan="2">Controllable form</td><td colspan="2">Observable form</td></tr>
<tr><td>Right</td><td>Left</td><td>Right</td><td>Left</td><td>Right</td><td>Left</td></tr>
<tr><td colspan="2">$\|K\|_2$</td><td colspan="2">Left</td><td colspan="2">Right</td><td colspan="2">Left</td></tr>
<tr><td colspan="2">Time specifications</td><td colspan="2">Right</td><td colspan="2">Right</td><td colspan="2">Right</td></tr>
<tr><td rowspan="4">Robust stability</td><td>$s(\lambda_i)$</td><td colspan="2">Right</td><td colspan="2">Right</td><td colspan="2">Left</td></tr>
<tr><td>M_1</td><td colspan="2" rowspan="3">Right</td><td colspan="2" rowspan="3">Left</td><td colspan="2" rowspan="3">Left</td></tr>
<tr><td>M_2</td></tr>
<tr><td>M_3</td></tr>
<tr><td>Robust performance</td><td>$r_i(\lambda_i)$</td><td colspan="2">Left</td><td colspan="2">Left</td><td colspan="2">Left</td></tr>
</table>

7. CONCLUSION

The aim of the present work in this paper is to apply block pole placement technique to a multi-input multi-output system with an application to a lateral motion of a Boeing 747, in order to determine the best feedback gain matrix. It has been shown that this technique is due to the fact that the characteristic matrix polynomial of the system has block poles as roots. These block poles can be chosen to be right or left as specified before. Their forms are not unique, but we restricted our study to the case of the canonical forms (diagonal, controllable and observable).

The choice of feedback gain matrix is done by the comparison between the two forms of the block poles in terms of best response characteristics and system robustness. In our case we can say that the choice between left and right solvents is made according to the form of these solvents. We can conclude that the choice of the feedback gain matrix is made according to the application itself from a desirable system's response.

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