A MASS BALANCING THEOREM FOR THE ECONOMICAL NETWORK FLOW MAXIMISATION

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ABSTRACT

A mass balancing theorem (MBT) was recently introduced, concerning the role of 'unbalanced nodes' in the optimization of network flow. The MBT discovers and proves a flow-balancing property, which can be exploited in the design of network flow algorithms. Subsequently a number of such applications of the MBT have been explored for various types of flow-networks. These have included, in particular, single and multiple commodity networks with additional equipment of separators, which are present in various real world scenarios including the oil and gas industry. In this paper, the mass balancing theorem is revisited, and further developed to consider new network examples with embedded cycles. In doing so, algorithms based on the mass balancing method are extended to remove any undesirably saturated edges in the network, consequently reducing economic costs for flow-maximization in such networks.

KEY WORDS

Mass Balancing Theorem, Graph Theory, Flow Networks, Optimisation

1.INTRODUCTION

Flow maximization through networks has been a major problem under study for last several decades [1]. This is because many real world problems can be formulated as a network problem such as optical networks [2], wireless networks [3], reliability networks [4,5], biological networks [6], production assembly networks [7] and social networks [8] etc.

A typical flow network is a directed graph with a number of nodes connected through a number of edges with a limited capacity. A flow network also has a source node and a sink node. It is assumed that source node can produce a flow of unlimited quantity. The problem is to push maximum flow through the network from the source node to the sink node such that capacity of any edge is not violated and all nodes must be balanced nodes *i.e.* incoming and outgoing flow at the node is equal.

In 1956 a remarkable theorem on this network was developed which is popularly known as max-flow-min-cut theorem [9, 10]. According to this theorem maximum flow through the network is equal to minimum cut of the network. The minimum cut of the network is defined as a cut of minimum size through the network that disconnects completely the source from the sink such that no flow from the source could pass to the sink. Based on this theorem, a number of approaches

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have been discovered to solve this problem. These approaches can be divided into two main branches, *i.e.* augmentation paths algorithms, [9-16] and pre-flow push algorithms [17-25]. Some novel ideas have also been discovered such as pseudo flows [26], draining algorithm [27], postflow-pull method [28] and the mass balancing theorem (MBT) [29].

The MBT discovered an interesting network property that the fully saturated network is actually balance of certain easily computable flow load on the either side of the minimum cut. Utilizing this property, a flow dissipation algorithm was developed to maximize the flow through the network. An interesting aspect of this algorithm is that it visits only unbalanced nodes rather than the whole network to maximize the flow. Therefore this algorithm has very important role to play in dynamic networks where the network continues changing its state. A change is marked by removal of E^- edges and/or addition of E^+ edges. Due to these changes each time maximum of $2(\mathrm{E}^+ + \mathrm{E}^-)$ number of nodes becomes unbalanced. This number is only a small fraction of total number of nodes in the network and by visiting only those unbalanced nodes flow can be re-maximized for the modified network.

The usefulness of this theorem has already been proved in various types of networks in different application areas [30, 31]. In this paper mass balancing theorem is revisited again with intent to study its applicability in new example networks with embedded cycles. These cycles sometimes have undesirably saturated edges which may incur economic costs without contributing to any flow maximisation. Therefore mass balancing method is extended to remove these undesirably saturated edges in those cycles to produce most economical solution of the flow maximisation problem. This paper is structured as follows. Section 2 revisits the theorem with explanation through network examples with cycles. Section 3 extends mass balancing method to produce most economical solution for the flow maximisation on networks with cycles and finally section 4 concludes the paper and discusses the future work.

2.MASS BALANCING THEOREM-A REVISIT

MBT states that minimum cut in the flow network is a balance of certain flow load on either side of the minimum cut. This flow balance can be represented through the following fundamental equation.

$$\sum_{k\in\overrightarrow{V_s}} c_{sk} - \underset{a \in N_s}{\forall} \sum d_a = C_0 = \sum_{k\in\overleftarrow{V_t}} c_{kt} + \underset{b \in N_t}{\forall} \sum d_b \qquad (1)$$

Where

$\overrightarrow{V_s}$ = Set of nodes adjacent to source node *s*
$\overleftarrow{V_t}$ = Set of nodes adjacent to sink node *t*
C_0= Capacity of the minimum cut
N_s= Network on the source side of the minimum cut
N_t = Network on the sink side of the minimum cut
d_a = Dissipative flow at node a. Dissipative flow means amount of unbalanced flow at a node.

Figure 1 provides an example network to understand equation 1.

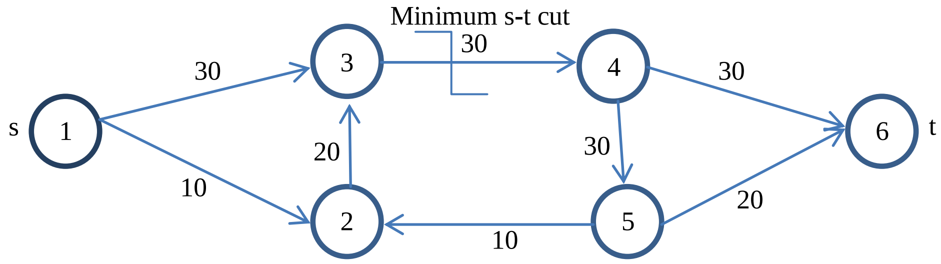

Figure 1: An example network with a minimum s-t cut

In Figure 1, edge $e_{2,3}$ represents the minimum cut. If this edge is blocked or removed then no flow can pass from the source to the sink. To describe equation 1, let us divide this network into two disjoint subnetworks *i.e.*, source subnetwork (network *A*) and sink subnetwork (network *B*) according to the following procedure.

1. Disconnect the edges representing minimum s-t cut so that no flow can pass from the source to the sink.
2. Connect broken edges of minimum cut from the source side with the newly created sink.
3. Connect broken edges of the minimum cut on the sink side with the newly created source.
4. Remove any additional edges that are not on the any path of the flow from source to the sink of the same subnetwork.

Following above procedure, the set of two disjoint subnetworks shown in Figure 2 can be deduced from the network in Figure 1.

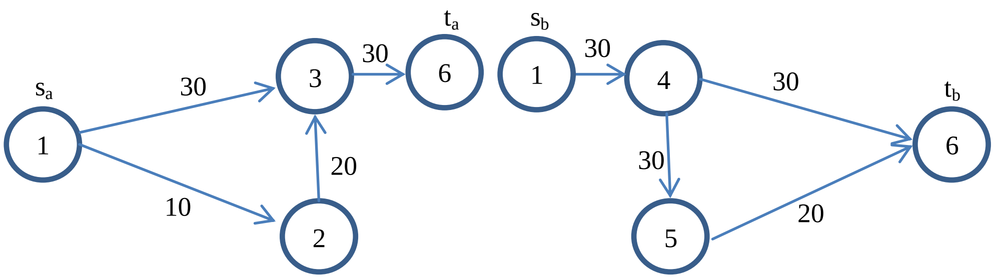

Figure 2: Two networks *A* and *B* deduced from network in Figure 1

By substituting parameters from the networks in Figure 2 in equation 1 we get

$$\left(c_{1,2}+c_{1,3}\right)-\left(d_2+d_3\right)=c_0=\left(c_{4,6}+c_{5,6}\right)+\left(d_4+d_5\right) \tag{2}$$

Where

$c_{i,j}$ = capacity of edge $e_{i,j}$
d_i = dissipative flow at node i

By substituting the values of above parameters from Figure 2 in equation 2 we get

$$(10+30)-(-10+20)=30=(30+20)+(-30+10)$$

From the above substitution of values it can be seen that fundamental equation 1 of the theorem holds on this example. The theorem holds true on any complex network.

2.1. MASS BALANCING METHOD FOR SINGLE COMMODITY NETWORK

Based on MBT a flow dissipation algorithm was devised to maximize the flow through the network which is re-described here briefly. Readers are requested to refer mass balancing theorem [29] for details. It is necessary to reintroduce some of the terminology related to MBT [29] to explain the algorithm. These terms are dissipative flow of node, dissipative flow of edge, dissipative flow of path, and the act of flow dissipation on path.

The dissipative flow of node is the amount of unbalanced flow on the node. The dissipative flow of the source and the sink is considered $+\infty$ and $-\infty$ respectively. The dissipative flow of edge in forward direction is equal to the flow present in that edge, while in backward direction it is equal to residual capacity of that edge. The dissipative flow of path is equal to the minimum of dissipative flows of initial and final nodes of the path and dissipative flows of all edges present in that path. Furthermore the dissipative flow of the path is taken as negative if initial node is negative node else it is taken as positive. The act of flow dissipation on the path from initial to final node means adding dissipative flow of the path to each forward edge and deducting dissipative flow of the path from each backward edge of that path. The single commodity algorithm is shown in procedure 1.

1. The Algorithm starts with the fully saturated network not observing the flow conservation law at nodes.
2. The algorithm computes dissipative flow at all nodes.
3. The algorithm scans the node list to locate the node N^- with negative dissipative flow.
4. The algorithm finds path from node N^- to node N^+ with positive dissipative flow and dissipates the flow along this path.
5. The algorithm repeats the steps 3-4 iteratively until it fails to find a feasible path with non-zero dissipative flow.
6. The algorithm repeats step 3 and finds the path from current N^- node to sink node through the network and dissipates flow along this path.
7. The algorithm repeats the step 6 iteratively until there remains no node with negative dissipative flow.
8. The algorithm scans the node list again to find the node N^+ with positive dissipative flow.
9. The algorithm finds the path from N^+ node to source node through the network and dissipates flow along this path.
10. The algorithm repeats the steps 8-9 iteratively until there remains no node with positive dissipative flow.
11. End

Procedure 1: MBT Method for Single Commodity Network [29]

If method in procedure 1 is applied on network in Figure 1 then algorithm will follow steps mentioned below.

-The algorithm starts with the fully saturated network (Figure 1) not observing the flow conservation law at nodes (Step 1)

-The algorithm computes the dissipative flow at each node except source and sink nodes as follows (Step 2)

$d_2 = 10 + 10 - 20 = 0$
$d_3 = 20 + 30 - 30 = 20$
$d_4 = 30 + 30 - 30 = -30$
$d_5 = 30 - 20 - 10 = 0$

-The algorithm scans the node list to locate the node with negative dissipative flow. *i.e.*, node N_4 (Step 3)

-The algorithm finds path $P_{4,5,2,3}$ from node N_4 to node with positive dissipative flow *i.e.*, node N_3. (Steps 4-5)

-The algorithm dissipates the flow equal to -10 units along this path and updates the network as follows (Steps 4-5). Symbol q in the following equations represents the resulting flows in the respective edges.

$q_{4,5} = 30 - 10 = 20$
$q_{5,2} = 10 - 10 = 0$
$q_{2,3} = 20 - 10 = 10$
$d_4 = -30 + 10 = -20$
$d_3 = 20 - 10 = 10$

-The algorithm finds another path from node N_4 to sink node N_6, *i.e.*, the path comprising only one edge $e_{4,6}$ and dissipates the flow equal to -20 units along this path and updates the network flows as follows (Steps 6-7)

$q_{4,6} = 30 - 20 = 10$
$d_4 = -20 + 20 = 0$

-The algorithm locates positive node N_3 and finds path from node N_3 to source node N_1 *i.e.*, path comprising only one edge $e_{3,1}$, and dissipates the flow equal to +10 units along this path and updates the network flows as follows (Steps 8-10)

$q_{1,3} = 30 - 10 = 20$
$d_3 = 10 - 10 = 0$

At this point the algorithm terminates as all the nodes are balanced. Figure 3 shows resultant network, representing optimized flow through the network.

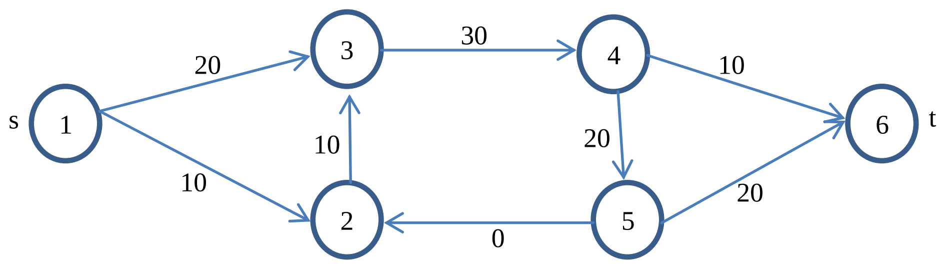

Figure 3: An optimized flow for the network in Figure 1

2.2. MASS BALANCING METHOD FOR MULTI-COMMODITY NETWORK

In mass balancing theorem [29], the method in section 2.1 was also extended to multi-commodity problem where network consists of more than one source with each source delivers mixture of number of commodities and each source has mixture of commodities in different proportions. The objective is to maximize output of one of those commodities, *i.e.*, commodity of interest (COI). The proportion of this commodity in overall mixture in each source is called COI ratio. The algorithm can be summarized in the following steps.

1. Calculate COI ratio for each source and sort the sources from the minimum to the maximum COI ratio and numbering them from 1 to *n*.
2. Apply first 7 steps of algorithm for single commodity presented in procedure 1.
3. The algorithm scans the node list to find the node $N^+$with positive dissipative flow.
4. The algorithm tries to find the path from node $N^+$to source S_1 through the network. If it fails to find then it looks for source S_2 and so on until it find such a path and dissipates the flow along that path.
5. The algorithm repeats the steps 3-4 iteratively until there remains no node with positive dissipative flow.
6. End

Procedure 2: MBT Method for Multi-Commodity Network [29]

To see the applicability of above procedure let us expand the network in Figure 1 from one source to two sources as shown in Figure 4.

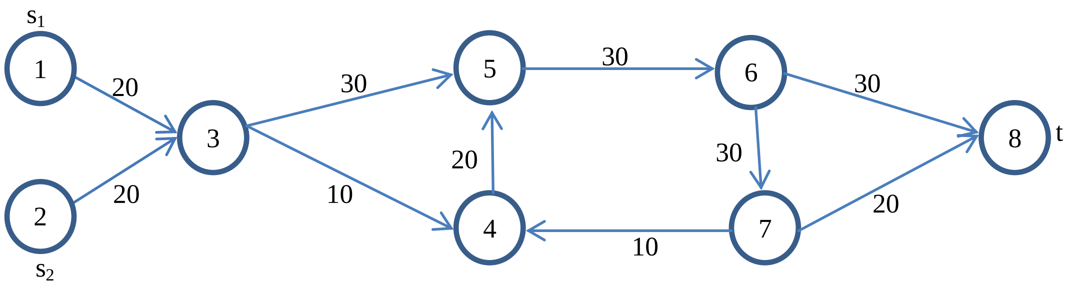

Figure 4: A multi-commodity network

According to first step of algorithm, the ratio of commodity of interest in the overall mixture of each source is computed and the sources are sorted accordingly in the ascending order *i.e.* from lowest to highest COI ratio.

First 7 steps of procedure 1, as explained earlier, are applied to maximize the overall flow (Step 2). This ends up with only one node *i.e.*, node N_3 with positive dissipative flow equal to +10 units (Step 3). At this point decision needs to be made that to which source this excess flow should be dissipated. The natural choice would be the source s_1 as this source contain the minimum COI ratio (Steps 4-5). Therefore the optimised network is shown in Figure 5.

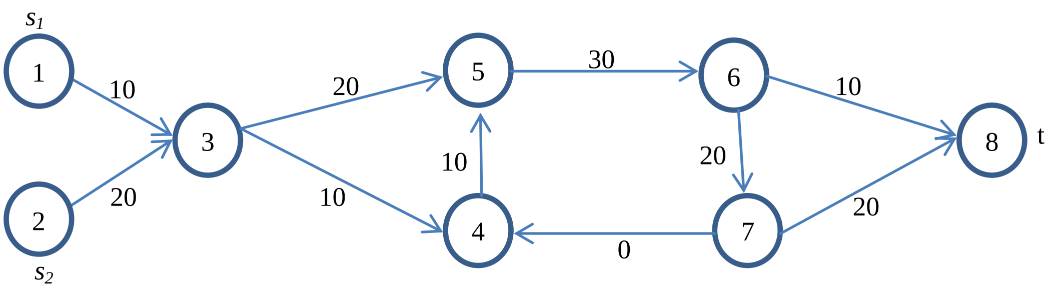

Figure 5: An optimised multi-commodity network

2.3. MBT METHOD FOR MULTI-COMMODITY NETWORK WITH A SEPARATOR

An application of mass balancing theorem was further extended to multi-commodity network with a separator [31].

Let the multi-commodity network consists of n sources and m commodities, and each source j produces a unique mix of commodities in quantity Q^j such that

$$\underset{j=1,n}{\forall} Q^j = \sum_{i=1}^{i=m} q_i^j = \sum_{i=1}^{i=m} \gamma_i^j Q^j \tag{3}$$

q_i^j= flow of commodity i in source j
γ_i^j = proportion of flow of commodity i in source j such that

$$0 \leq \gamma_i^j \leq 1 \tag{4}$$

Equation 3 shows that total quantity of mixture is sum of all the quantities of individual commodities in the mixture, where quantity of each individual commodity can be determined from its proportion in the mixture. The value of proportion varies between 0 and 1 (expression 4). Flow from all the sources ultimately terminates onto a separator that separates the commodity mixes. At the output of the separator, there are m commodity networks each corresponding to a single commodity, carrying i^{th} commodity to the i^{th} sink. The goal is to maximize the output of commodity of interest (COI) while obeying the capacity constraints of multi-commodity network and each of the m single commodity networks.

Figure 6 shows a multi-commodity network, namely, N_0 connected to n sources S_1, S_2, … S_n, and a separator U. In addition, there are m commodity networks, namely N_1, N_2, … N_m, which originate from the separator U and each of these networks has its own sink, *i.e.* T_1 through T_m, respectively. For the problem formulation, following subsections define some notions.

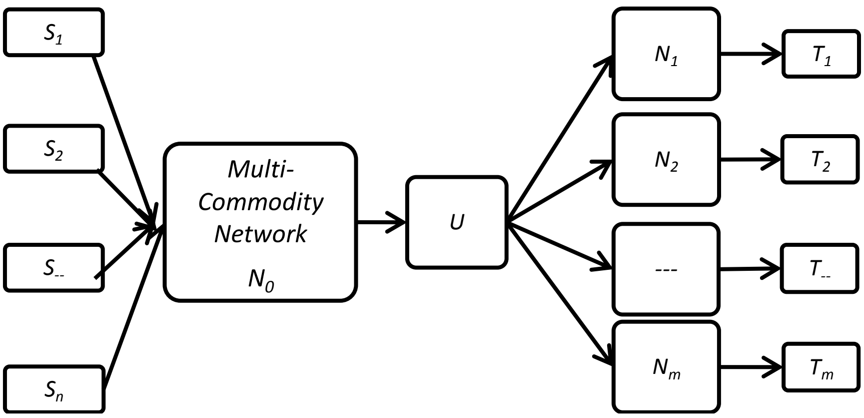

Figure 6. Multi-Commodity Network with Separator

2.3.1 UNIFIED AND INDIVIDUAL SOURCE NETWORKS (USN AND ISNS)

Let us modify the network in Figure 6 by connecting its source nodes S_1, S_2, … S_n with the universal source node S_0 of unlimited capacity through the edges e_1, e_2, … e_n of unlimited capacity respectively. Furthermore considering the separator as the ultimate sink, the network of Figure 6 can be reduced to the network shown in Figure 7. The network hereby referred to as Unified Source Network (USN). The USN in Figure 7 can be calibrated into the individual source networks. The individual source network corresponding to the source i, ISN_i is the network with $c_i = \infty$ and $c_j = 0$ where $j \in \{1, \dots\dots, n/j \neq i\}$. This means that in the individual network of source i all the other sources will be disconnected from the network except source i itself. Furthermore the capacity of source i is also considered unlimited.

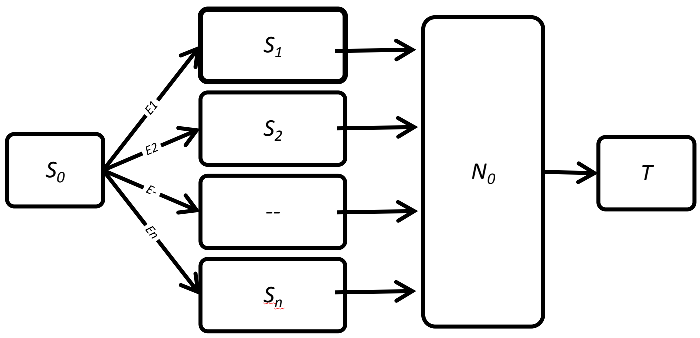

Figure 7. Unified Source Network and n Individual Source Networks

2.3.2 INDIVIDUAL COMMODITY NETWORK (ICN)

Considering the separator U as the primary source for each commodity network, the network of Figure 6 can be reduced to the network shown in Figure 8. In Figure 8 since there are m commodities hence there are m ICNs, such that for ICN_i, $c_i = \infty$ and $c_j = 0$ where $j \in \{1, \dots\dots, n/j \neq i\}$. This means that in the individual commodity network of commodity i all the other commodities are disconnected from the network except commodity i itself. Furthermore the capacity of primary source of commodity i is also considered unlimited.

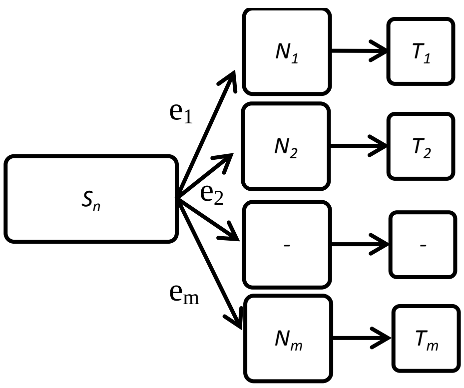

Figure 8. m Individual Commodity Networks

Let us consider

C_0 = Minimum cut of the USN in Figure 7
$C_{si\in\{1,\dots,n\}}$= Minimum cut of the $ISN_{i\in\{1,\dots,n\}}$ respectively in Figure 7
$C_{ci\in\{1,\dots,m\}}$ = Minimum cut of the $ICN_{i\in\{1,\dots,m\}}$ respectively in Figure 8

Therefore the maximum flow Q_0 in the multi-commodity network of Figure 1 is given by

$$Q_0 \leq min\left(C_0, \sum_{i=1}^{i=n} C_{si}, \sum_{i=1}^{i=m} C_{ci}\right) \tag{5}$$

This means that maximum flow in the network can be only be minimum of the following three quantities.

1. Minimum cut of the unified source network
2. Sum of minimum cuts of individual source networks
3. Sum of minimum cuts of individual commodity networks

Since in any case

$$C_0 \leq \sum_{i=1}^{i=n} C_{si} \tag{6}$$

Therefore expression 5 reduces to

$$Q_0 \leq min\left(C_0, \sum_{i=1}^{i=n} C_{ci}\right) \tag{7}$$

Expression 7 shows that maximum flow through the multi-commodity network of Figure 6 cannot be greater than lesser of the two quantities *i.e.*, minimum cut of USN of Figure 7, and sum of minimum cuts of all the individual commodity networks ICNs of Figure 8. The $\leq$ sign in this expression indicates that there are other constraints too that may restrict the flow. Those constraints are shown in expressions 8 and 9. Suppose q_i^j is flow of commodity i in source j then

$$\forall_{j=1,n}\ Q^j \leq C_{sj} \tag{8}$$

and

$$\forall_{i=1,m}\ Q_i = \sum_{j=1}^{j=n} q_i^j \leq C_{ci} \tag{9}$$

Expression 8 shows that flow from any source j must not be greater than minimum cut of its individual source network and expression 9 shows that total flow of any commodity i must not be greater than the minimum cut C_{ci} of its respective individual commodity network. The $\leq$ sign signifies the fact that if one of the commodities k exhausts the capacity of its individual network C_{ck}, then flow cannot be further increased for other commodities $i \in \{1, \ldots, m/i \neq k\}$ as increase in the overall mixture would also increase the flow of the commodity k. Therefore objective is to maximize the flow of COI, Q_{coi} *i.e.*,

$$max(Q_{coi}) = max\left(\sum_{j=1}^{j=n} q_{coi}^j\right) \tag{10}$$

Substituting the values of q_{coi}^j from expression 2 into expression 10 gives the following linear function

$$Z = \sum_{j=1}^{j=n} \gamma_{coi}^j Q^j \tag{11}$$

The linear function in equation 11 is to be maximized under the constraints of expressions 7 through 9.

A method for maximization of flow of a commodity of interest through the network with a separator has been devised by keeping problem formulation presented above in mind. The method hybridizes mass balancing theorem with simplex method. Simplex method is used to maximize linear function shown in equation 11 under the constraints in expressions 7-9. However to determine the value of constraints mass balancing method of flow maximization (procedure 1) is used. This hybridized method is termed as simplex mass balancing (SMB) method.

The algorithm is explained in the following steps.

1. Create all the networks including USNs, ISNs and ICNs
2. Maximize the flow through each USNs, ICNs and ISNs to determine the values of x_0, $x_{i\in\{1,....,n\}}$, $y_{i\in\{1,....,m\}}$ respectively, corresponding to n sources and m commodities by using procedure 1.
3. Design linear programming formulation (equation 11) under the constraints 7-9 from output of step 2.
4. Maximize the linear function of equation 11 using Simplex method to determine $Q^{j\in\{1,...,n\}}$.
5. Maximize the flow through the USN by equating capacity of $E_{j\in\{1,2,....,n\}}$ with $Q^{j\in\{1,...,n\}}$ respectively by using procedure 1.
6. Compute quantity of each commodity q_i using equation 11 from output of step 4.
7. For all i, maximize the flow in ICN_i by equating capacity of $e_{j\in\{1,2,....,m\}}$ with $q_{j\in\{1,...,m\}}$ respectively by using procedure 1.
8. Join USN obtained from step 5 and ICNs obtained from step 7 and remove additional edges and universal source node to represent actual network with maximized flow of COI.
9. End

Procedure 3: MBT Method for Multi-Commodity network with a separator [31]

From the above procedure it can be seen that in the second step procedure 1 is used to determine minimum cuts (maximum flows) of various conceptual networks introduced in Figures 7-8. The values of those minimum cuts are later used in design of linear programming formulation in step 3. The designed linear function is then optimized in step 4 using simplex method to determine optimal flows from all sources. In step 5, a flow through multi-commodity network is again maximized by restricting flow from sources to optimal flows obtained in previous step. In step 6, quantity of each commodity is computed in the resultant output mixture from all the sources. In step 7, flow in each commodity network is maximized by restricting quantity of each commodity equal to the commodity quantities obtained in previous step. In the final step, all the conceptual networks are joined together to form original network. The resultant network represents the optimal flow solution with respect to commodity of interest.

Readers are referred to simplex mass balancing method [31] to see proof of optimality, complexity analysis and solved example of multi-commodity network with a separator. In the next section, the MBT method for single commodity is extended to produce most economical maximised flow solution in the networks with embedded cycles.

3.MBT METHOD FOR THE ECONOMICAL FLOW MAXIMISATION

In the real world sometimes we may encounter with networks having embedded cycles, such as cyclic path $P_{2,3,4,5,2}$ in the network of Figure 1. The MBT method shown in procedure 1 produced not only optimal but also most-economic solution of maximum flow on that network as shown in Figure 3. The solution is most economical because there is no extra-saturated edge not contributing to maximum flow through the network. However if edge capacities are reassigned in the same network as shown in Figure 9, then MBT method described in procedure 1 will definitely produce optimal solution on this network but the solution may not be most economical as it may contain extra saturated edge not contributing to the maximum flow.

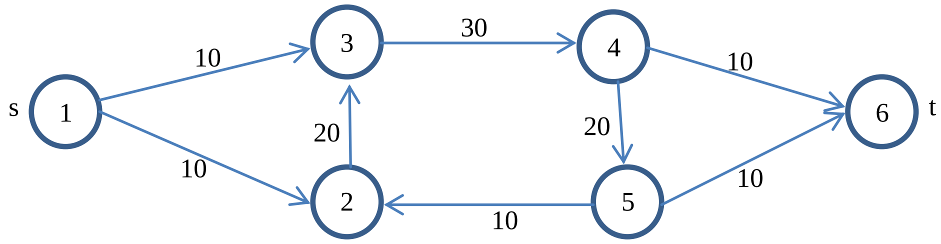

Figure 9: An example network with embedded cycle

In the above example it can be seen that all the nodes are balanced nodes. Since MBT method mentioned in procedure 1 will not find any unbalanced nodes, it will return fully saturated network as a solution to the maximum flow problem. It is true that fully saturated network in the Figure 9 is a valid maximum flow solution of the network. However, it is not the most economical solution. This is because there are some saturated edges which do not contribute to the maximum flow such as edge $e_{5,2}$. To address this, MBT method in procedure 1 is modified to produce most economical maximum flow solution for the networks with embedded cycles. The modified algorithm is shown in sequential steps in procedure 4 bellow.

1. Apply procedure-1 on the network.
2. Mark all the nodes as unvisited
3. Count $i = 0$
4. $i = i + 1$
5. if $i > n$ go to step 13
6. if node N_i is not a candidate node[a], go to step 4.
7. Find cyclic path[bc] (cycle) P_c from node N_i back to node N_i and mark all the unvisited nodes in this path as visited
8. If cyclic path P_c is not found go to step 4.
9. Locate the edge e_j having minimum flow q_{min} among all the edges on the cyclic path P_c.
10. Remove entire flow q_{min} from the edge e_j. The resultant network has one negative node N^- and one positive node N^+ of dissipative flow equal to $\mp q_{min}$ units at two ends of the edge e_j
11. Find the path P_k from negative node N^- to positive node N^+, such that path $P_k \subset P_c$ *i.e.*, P_k includes all the edges as that of P_c excluding edge e_j.
12. Dissipate flow of $-q_{min}$ on the path P_k.
13. if $i < n$ go to step 4.
14. end

Note a: Candidate node is any unvisited node except source or sink node
Note b: Except node N_i, other nodes on the cyclic path P_c can be already visited nodes
Note c: The cyclic path P_c should not contain an empty edge without any flow

Procedure 4: MBT method for the economical maximum network flow

Now let us apply procedure 4 on the network in Figure 9. The step 1 of procedure 4 *i.e.*, application of procedure 1 returns the fully saturated network of Figure 9. In step 2, all the nodes are marked as unvisited. In step 3, counter of variable i starts ($i = 0$). In step 4, counter i is incremented to ($i = 1$). In step 5, it is checked that whether the whole network has already been scanned. In step 6, node N_1 is found as not a candidate node because it is a source node therefore control of algorithm goes back to step 4. In step 4, counter i is incremented to ($i = 2$). In step 5, again it is checked that whether the whole network has already been scanned. In step 6, node N_2 is found as a candidate node *i.e.*, it is unvisited and not a source or sink node. In step 7 a cyclic path from node N_2 to node N_2 *i.e.*, $P_c = P_{2,3,4,5,2}$ is found and all the nodes on this path *i.e.*, N_2, N_3, N_4, N_5

are marked as visited. In step 8, '*if*' condition fails as cyclic path is found in step 7. In step 9, edge $e_j = e_{5,2}$ of minimum flow $q_{min} = 10$ units, is found on cyclic path P_c. In step 10, the entire flow of 10 units from edge $e_{5,2}$ is removed. The resultant network is shown in Figure 10.

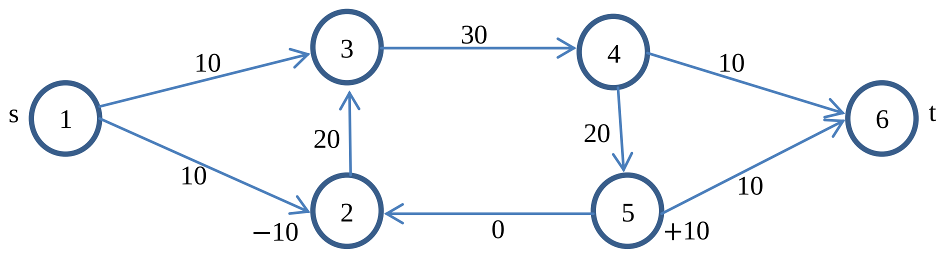

Figure 10: A network of Figure 9 after application of step 10 of procedure 4

It can be seen that after application of step 10, two nodes of the network become unbalanced *i.e.* node ($N_2 = -10$) and node ($N_5 = +10$). In step 11, a path $P_k = P_{2,3,4,5}$ from node $N^- = N_2$ to node $N^+ = N_5$ is established, which is subset of cyclic path $P_c = P_{2,3,4,5,2}$. In step 12, a flow of -10 units is dissipated along path P_k. The resultant network is shown in Figure 11.

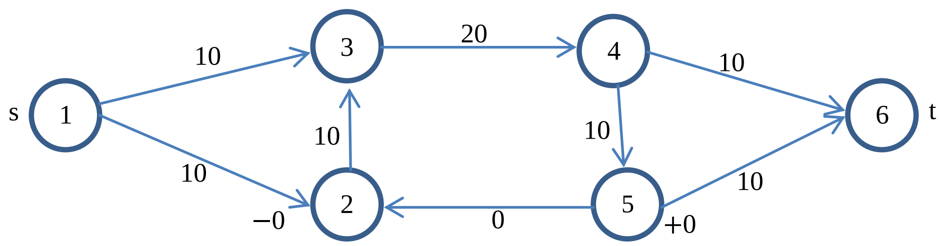

Figure 11: A network of Figure 9 after application of step 12 of procedure 4

In Figure 11, it can be seen that after application of step 12, the nodes N_2 and N_5 become balanced. In step 13, '*if*' condition becomes true because $i = 2 < n = 6$. Therefore the control of algorithm goes back to step 4. The algorithm iterates between step 4 and step 6 as no other candidate node could be found as all the nodes except source and sink node have been marked visited. Finally step 5 sends control to step 13 when $i = 7$. In step 13, '*if*' condition fails to materialise because $i > n$. Finally the algorithm terminates at step 14. The network in Figure 11 represents the maximised flow with minimum costs. The solution in Figure 11 does not have any extra saturated edges.

4.CONCLUSIONS

In this paper mass balancing theorem on the flow networks has been revisited. The theorem is explained on new example networks with embedded cycles. The flow maximisation method based on this theorem is described again on single and multiple commodity network examples with embedded cycles. Simplex MBT method for the flow networks with a separator is also revisited with a view of how it is coupled with the earlier methods. Finally MBT single commodity flow maximisation method is extended to produce most economical solution of flow maximisation on the networks with embedded cycles. In future this method can be extended to multi-commodity networks with embedded cycles and multi-commodity networks with embedded cycles and a separator.

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