AN OPTIMAL FUZZY LOGIC SYSTEM FOR A NONLINEAR DYNAMIC SYSTEM USING A FUZZY BASIS FUNCTION

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ABSTRACT

The impetus for this paper is the development of Fuzzy Basis Function “FBF” that assigns in an optimal fashion, a function approximation for a nonlinear dynamic system. A fuzzy basis function is applied to find the best location of the characteristic points by specifying the set of fuzzy rules. The advantage of this technique is that, it may produce a simple and well-performing system because it selects the most significant fuzzy basis functions to minimize an objective function in the output error for the fuzzy rules. The fuzzy basis function is a linguistic fuzzy IF_THEN rule. It provides a combination of the numerical information and the linguistic information in the form input-output pairs and in the form of fuzzy rules. The proposed control scheme is applied to a magnetic ball suspension system.

KEYWORDS

Fuzzy Logic Control “FLC”, Fuzzy Basis Function “FBF”, Orthogonal Least Squares ”OLS”.

1. INTRODUCTION

The fuzzy logic controller comprises three stages namely fuzzifier, rule-based assignment tables and the defuzzifier. In this system, the input signals are converted to fuzzy representations, the rule base will produce a consequent fuzzy region for each solution variable, and the consequent fuzzy region are defuzzified to find the expected solution variable [1-7].

In this paper, an optimal approach to function approximation is presented using fuzzy systems. The proposed approach does not optimize the function approximation, but also gives some insight to the role of different parts of fuzzy systems employing the new concepts of characteristic point.

A fuzzy basis function is then applied to find the best location of the characteristic points specifying the optimal rule-set [8-12]. This paper is organized as follows, the second section describes the fuzzy logic system. The third section is devoted to discuss the optimal fuzzy system. The fourth section is concerned with the fuzzy control procedures. In fifth section, a propose design for magnetic ball system is introduced. The magnetic ball system is designed. Finally the last section is dedicated to the computer simulation, and the conclusions.
2. FUZZY LOGIC SYSTEMS

The fuzzy logic system with fuzzifier and defuzzifier has many attractive features. First, it is suitable for engineering systems because its input and output are real-valued variables. Second, it provides a natural framework to incorporate fuzzy IF-THEN rules from human experts. Finally, there is much freedom in the choices of fuzzifier, fuzzy rules, assembled in which as the fuzzy inference engine and defuzzifier. Fig. 1 shows the Fuzzy Logic System "FLs" [13,14].

![Figure 1. Fuzzy Logic System](image)

The Fuzzy-Logic Control "FLC" comprises three stages namely Fuzzifier, Rule-base and Defuzzifier. Fig. 2 shows the configuration of a fuzzy logic controller. The Fuzzy Control Theory uses the fuzzy knowledge to describe the characteristics of the system, because the nonlinear system is difficult to establish for traditional methods [12].

![Figure 2. The basic Configuration of a Fuzzy Logic Controller](image)

The signal $e(k)$ is the error signal and it is the difference between the reference signal and the actual signal. The signal $\Delta e(k)$ is defined as the rate of change of the error signal.

$$e(k) = \Delta r(k) - \Delta c(k)$$  \hspace{1cm} (1)$$

$$\Delta e(k) = e(k) - e(k-1)$$  \hspace{1cm} (2)$$

Where:

- $\Delta r(k)$ is the reference speed at k-th sampling interval
- $\Delta c(k)$ is the speed signal at k-th sampling interval
- $e(k)$ is the error signal
The workable range is divided into intervals, these intervals are named for example, Positive "P", Zero "Z", and Negative "N". These names for the intervals are called labels [14,20]. The rules relate the input variables to the output of the fuzzy controller. These rules can be expressed in a table with the inputs in the horizontal Cartesian and the output in the vertical Cartesian. However, the output would be inside each cell as shown in Table 1.

Table 1: Rules Table

<table>
<thead>
<tr>
<th>Δe / e</th>
<th>N</th>
<th>Z</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>P</td>
<td>N</td>
<td>Z</td>
</tr>
<tr>
<td>Z</td>
<td>N</td>
<td>Z</td>
<td>P</td>
</tr>
<tr>
<td>P</td>
<td>Z</td>
<td>P</td>
<td>N</td>
</tr>
</tbody>
</table>

The rules base are expressed as IF-THEN as follows:

- IF e is "N" AND Δe is "N" Then u is "P"
- IF e is "N" AND Δe is "Z" Then u is "N"
- IF e is "N" AND Δe is "P" Then u is "Z"
- IF e is "Z" AND Δe is "N" Then u is "N"
- IF e is "Z" AND Δe is "Z" Then u is "Z"
- IF e is "Z" AND Δe is "P" Then u is "P"
- IF e is "P" AND Δe is "N" Then u is "Z"
- IF e is "P" AND Δe is "Z" Then u is "P"
- IF e is "P" AND Δe is "P" Then u is "N"

3. OPTIMAL FUZZY LOGIC SYSTEMS

We develop an optimal fuzzy logic system, that is, it is optimal in the sense of matching all inputs-outputs pairs in the training set to any given accuracy [15,17-21]. For an arbitrary $\varepsilon > 0$, there exists, the difference between a real continuous function $g(x)$ and a fuzzy system $F(x)$ is in the form of:

$$|F(x) - g(x)| < \varepsilon$$  \hspace{1cm} (3)

Consider the set of fuzzy system with fuzzifier, product inference, defuzzifier, and isosceles triangle membership function consists of the functions of the form:

$$F(x) = \frac{\sum_{k=0}^{M} y^k \prod_{i=0}^{N} \mu_{i,F}(x)}{\sum_{k=0}^{M} \prod_{i=0}^{N} \mu_{i,F}(x)}$$  \hspace{1cm} (4)
Where \( x_i \) are the \( N \)-input variables to the fuzzy system, \( y \) is the \( M \)-output variable of the fuzzy system, and \( \mu_{Fi} (x) \) is the isosceles triangle membership function.

The first step in the approximation of such a function is to define the membership function for the input-output spaces, the membership function is expressed by a triangle whose center and width defined by \( a_i \) and \( b_i \) respectively \([14,15,17-21]\)

\[
\mu_i (x) = 1 - 2 \left( \frac{x-a_i}{b_i} \right) 
\]  

The function \( F(.) \) is the fuzzy logic system output. The number of rules in the optimal logic system equal to the number of input-output pairs in the training set with one rule responsible for matching one input-output pair \((x_i, y_i)\).

Many types of membership function exists \([2,3,14-20]\). In this paper, the triangular-shaped function \( \mu(e) \) is the membership function of the error signal, with positive label "P" and zero Label "Z" and negative labe "N". The triangular shaped function, \( \mu(\Delta e) \) is the membership function of the rate of change of the error signal with positive label “P”, zero label”ZE” and negative label “N” as shown in Fig.3,

![Figure 3. The Membership Function of error “e” and the error change “\( \Delta e \)”](image)

### 3.1 Fuzzy Basis Function

Fuzzy Logic control technique has represented an alternative method to solve the problems in control engineering in recent years. One of the most useful properties of fuzzy logic systems in control is their ability to approximate a certain desired behavior function up to a desired level of accuracy.

The Fuzzy Basis Function "FBF" is proposed for control multi-input-single output nonlinear function \([9-13]\). The fuzzy basis function (FBF) is defined as a series of algebraic superposition of fuzzy membership functions \([12]\).

\[
P_k(x) = \frac{\prod_{i=1}^{N} \mu_{Fi} (x)}{\sum_{k=0}^{N} \prod_{i=1}^{N} \mu_{Fi} (x)} 
\]

Then the fuzzy logic system is equivalent to an FBF expansion

\[
d(t) = \sum_j^M P_j (t) \ast \theta_j + e(t) 
\]
Where \(d(.)\) is the system output, \(\theta_j\) are real parameters, \(P_j(t)\) are fixed function of system inputs \(x(.)\). Suppose that, we are given \(N\)-input-output pairs \([x(t_0, d(t)], t=1,2,\ldots,N\). The algorithm is to design an FBF expansion \(F(x)\), such that the error function between the \(F(x(t))\) and \(d(t)\) is minimized. The Fuzzy Basis Function” FBF” can be determined as follows:

- Calculate the product of all membership functions for the linguistic terms in the IF part of rule
- Generate these product on a numerical input-output pair

It is an intuitive idea that FBF’s play an important role in the determining structure and property of fuzzy systems. We gave a systematic analysis of FBF’s and presented the following properties of fuzzitzity, and composition. The fuzzy logic system can be analyzed from two view points; First, if the parameters in the FBF are free design parameters. Second, the FBF expansion is nonlinear in the parameters.

In order to specify, such FBF expansion, it must use nonlinear optimization techniques [9-12]. The objective is to design an FBF expansion \(F(x)\), such that the error between \(F(x)\) and \(g(x)\) is minimized. In order to describethat, consider the Orthogonal Least Squares "OLS" learning algorithm works. In matrix form:

\[
d = P * \theta + e
\]  

Where

\[
d = [d_1, d_2, \ldots d_N]^T
\]  

\[
P = [P_1, P_2, \ldots, P_M]^T
\]  

\[
P_i = [p_i(1), p_i(2), \ldots, p_i(N)]^T
\]  

\[
\theta = [\theta_1, \theta_2, \ldots, \theta_M]^T
\]  

\[
e = [e(1), e(2), \ldots, e(N)]^T
\]

The orthogonal algorithm transforms the set of \(p_i\) into a set of orthogonal basis vectors and uses the significant basis vectors to form the final FBF expansion.

4. Fuzzy Control Algorithm

In this section, a systematic method is given to help designer get the best of reasoning algorithm that works well with the application required. The proposed approach to develop a fuzzy logic system consists of five stages. For the given pairs \((X_1, X_2, Y)\) Where \(F(X_1,X_2) \rightarrow Y\)

**Step 1**
Assume that, the range of each variable is known \([x_1, x'_1], [x_2, x'_2]\) and \([y_1, y'_1]\).
Normalize each and find the corresponding scaling factors \(K_{x_1}, K_{x_2}\) , and \(K_y\).

**Step 2**
Generate the fuzzy rules from the data pairs
Determine the degree of each \(x'_1, x'_2\), and \(y'\)

**Step 3**
Assign a degree of each rules
The degree of this rule \(D\text{\text{rule}}\) is defined as
\[
D\text{\text{rule}} = \mu_\lambda (x'_1) * \mu_\theta (x'_2)
\]
Step 4
Create a combined fuzzy basis function by making a look-up table as shown in table 1

Step 5
Determine the following defuzzification strategy to determine the output y for given inputs \((x_1, x_2)\)

5. Magnetic Ball System

The magnetic ball suspension system is proposed as a nonlinear system is shown in fig.4. In this system, the electro- mechanical nonlinear model of magnetic ball system can be described in terms of the following set of differential equations[11-13, 21-22].

\[
\begin{align*}
\frac{dx_1}{dt} &= x_2(t) \\
\frac{dx_2}{dt} &= c_1 + c_2 \cdot f(x) \\
\frac{dx_3}{dt} &= c_3 \cdot x_3(t) + c_4 \cdot u(t)
\end{align*}
\]

Where
- \(c_1\): The gravity acceleration \(g=9.8\)
- \(c_2\): The magnetic force constant \(c_2 = -10\)
- \(c_3\): The unit conversion coefficient \(c_3=-100\)
- \(c_4\): The coil conductance \(c_4=2\)

Equation (13) indicates that \(f(x)\) is a nonlinear of ball position

\[f(x) = \frac{x_3^2(t)}{x_1(t)}\]

6. Computer Simulation

The Magnetic Suspension or magnetic ball is a type of suspension system where the shock absorbers reacts to the road and adjusts much faster than regular absorbers. In this system, a steel ball of mass can counteract the effect of gravity at any point. This relationship is a nonlinear function as shown in Fig. 5.
To illustrate the design process of a fuzzy system, consider the nonlinear function in Eq.(15). We compute the performance of each membership function used the same information. The results clearly show the applicability of the proposed scheme to the representation of a nonlinear function. From this result, the following points can be concluded: The fuzzy logic has the advantage of being able to handle the behavior of a nonlinear function. The fuzzy logic is simpler than the conventional system. It is clear that, the function performance with fuzzy logic is acceptable, and the result is faster response during transient compared to the conventional technique. On contrary to that, the fuzzy logic result in a smaller steady state error. The two opposing forces applied to the ball (magnetic force \( f(x) \) and the fuzzy basis function ) almost identical are shown in Fig. 6. The difference between the two forces are almost negligible which shows the good fuzzy basis function followed the trajectory desire.

7. CONCLUSION

The paper presents a fuzzy logic strategy to ensure excellent study and guarantees the operation of the magnetic ball suspension. Simulation results show that the response shows a significant improvement in the approximator performance.

It is clear that the responses show a close correspondence between the real function and the estimated one throughout the operation periods which demonstrates the validity of the proposed algorithm. The fuzzy basis function expansion provides a natural framework to combine both
numerical information in the form of input-output pairs and linguistic information in the form of fuzzy IF-THEN rules.
An example of how to combine the fuzzy basis functions generated from a numerical state-control and the fuzzy basis function generated form the common-sense linguistic fuzzy control rules is shown and represented in this paper.

REFERENCES


AUTHORS

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