A NEW APPROACH FOR RANKING SHADOWED FUZZY NUMBERS AND ITS APPLICATION

Mohamed A. H. El-Hawy

Department of Management Information Systems, Modern Academy for Computer Science and Management Technology, Cairo, Egypt

ABSTRACT

In many decision situations, decision-makers face a kind of complex problems. In these decision-making problems, different types of fuzzy numbers are defined and, have multiple types of membership functions. So, we need a standard form to formulate uncertain numbers in the problem. Shadowed fuzzy numbers are considered granule numbers which approximate different types and different forms of fuzzy numbers. In this paper, a new ranking approach for shadowed fuzzy numbers is developed using value, ambiguity and fuzziness for shadowed fuzzy numbers. The new ranking method has been compared with other existing approaches through numerical examples. Also, the new method is applied to a hybrid multi-attribute decision making problem in which the evaluations of alternatives are expressed with different types of uncertain numbers. The comparative study for the results of different examples illustrates the reliability of the new method.

KEYWORDS

Fuzzy numbers, Intuitionistic fuzzy numbers, Shadowed sets, Shadowed fuzzy numbers, Ranking, Fuzziness measure.

1. INTRODUCTION

Vague information is represented by many uncertain sets. Fuzzy set is one of the most important uncertain set which was proposed by Zadeh. Fuzzy set is determined by its membership function that represents vagueness and imprecision in linguistic term. fuzzy number is a special type of fuzzy set which is defined on real numbers scale.

Intuitionistic fuzzy sets (IFSs) are introduced by Atanassov which generalized the concept of fuzzy set. IFSs are characterized by two functions (membership and non-membership) [1]. These features provide more flexibility in representing uncertain numbers. The shadowed sets proposed by Pedrycz for approximation of fuzzy sets by three values {0, 1, [0,1]}[2]. Fuzzy membership values assign to 0, l or uncertain interval. Three areas are induced from fuzzy set to define shadowed set. The elements with membership grade 0 constitute the excluded area of the shadowed set. The core area consists of the elements that almost certainly belong to the fuzzy set. The shadow area relates to the elements that possibly belong to the fuzzy set.

The author proposed an improved form of shadowed fuzzy numbers (SFNs) which preserves two types of uncertainty (fuzziness and non-specificity) [3]. Also, we extended this idea to a higher type of fuzzy sets [4].

In the literature, numerous ranking approaches have been developed to rank fuzzy numbers. One category of these methods ranks fuzzy numbers based on the integration between fuzzy mean and

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spread. The mean of fuzzy number is represented generally as the centroid value of it. The spread of fuzzy numbers is used to support ranking methods, especially in the cases of embedded fuzzy numbers with different spreads [5]. Many researchers have dealt with the issue of ranking fuzzy numbers using centroid point and spread [6, 7, 8, 9]. Chen and Lu presented the ranking method for fuzzy numbers which consider the middle-point and spread of each α-cut of fuzzy numbers [10]. Abu Bakar et al. proposed ranking method using five distance-based components for ranking fuzzy numbers that include centroid point, height and spread of fuzzy numbers [11]. S.M. Chen and J.H. Chen presented a new ranking method based on the defuzzified values, the heights and the spreads for generalized fuzzy numbers [12]. Abu Bakar et al. proposed a ranking index which integrates centroid point and spread for fuzzy numbers [13]. R. Chutia and B. Chutia discussed the concept of parametric form of fuzzy number and proposed a new ranking method using the value and the ambiguity of it at different decision levels [14]. The same concept of the integration between value and ambiguity is applied on intuitionistic fuzzy numbers IFNs. Deng-Feng Li developed a new methodology for ranking triangular Intuitionistic fuzzy numbers TIFNs based on a ratio of the value index to the ambiguity index and applied to multi-attribute decision making problem [15]. P. K. De and D. Das proposed a new ranking approach for trapezoidal intuitionistic fuzzy numbers (TrIFN) using the value and the ambiguity indexes of them [16]. Some ranking methods provide ability to rank different types of fuzzy numbers using value and ambiguity. Also, other approaches are intended for one type of fuzzy numbers or one kind of membership functions. Some researchers proposed ranking method has related to the fuzziness of fuzzy numbers [17]. In this paper, a new method for ranking shadowed fuzzy numbers is proposed to order different types of fuzzy numbers and different membership functions. The value and ambiguity of a shadowed fuzzy number SFN will be defined. The proposed method uses values and ambiguities of SFNs to rank them. Also, the fuzziness values of SFNs are used to support ranking approach in the case of the equality of ranking values and ambiguities. The proposed ranking approach will be presented and applying to different fuzzy numbers ranking examples. Also, the new algorithm is proposed to solve a hybrid multi-attribute decision making problem that includes the new SFNs ranking approach. This MADM problem has different data types include interval numbers, type-l fuzzy numbers with two different membership function types and intuitionistic fuzzy numbers. The reset of this paper is organized as follows: section 2 introduces the basic definitions of FNs, IFNs and SFNs. Section 3 defines the concepts of the value, the ambiguity and the fuzziness of SFNs. we introduce proposed steps for new ranking method of SFNs. Section 4, numerical examples are provided, and a comparative study is presented with previous methods. Also, we present a hybrid multi-attribute decision making problem which is solved by using the proposed algorithm. Finally, conclusions and the main features of the proposed ranking approach are discussed in Section 5.

2. DEFINITIONS AND PRELIMINARIES

2.1. Fuzzy Sets

Fuzzy set provides excellent means to model the linguistic terms by introducing gradual memberships. The membership function of a fuzzy set A is defined as follows [18]

$$\mathrm{A}\colon \mathrm{X} \rightarrow [0, 1] \tag{1}$$

The membership function mapping elements of universe of discourse X to unit interval [0, 1]. The membership function is essential for describing fuzzy set.

2.1.1. Fuzzy Number

A fuzzy number (FN) $\widetilde{A}$ is a fuzzy set that is defined on the real numbers scale $\mathbb{R}$ with the following conditions [19, 20].

- $\widetilde{A}$ is normal, i.e. at least one element x_i such that $\mu(x_i) = 1$.
- $\widetilde{A}$ is a convex such that $\widetilde{A}(\delta x + (1-\delta)y) \geq \min\left(\widetilde{A}(x), (y)\right) \forall x, y \in U$ and $\delta \in [0,1]$ where U is a universe of discourse.
- The support of $\widetilde{A}$ is bounded.

A fuzzy number is important to approximate uncertainty concept about numbers or intervals. The membership function of the real fuzzy number $\widetilde{A}$ is defined by [19]

$$\mu_{\widetilde{A}}(x) = \begin{cases} l_{\widetilde{A}}(x) & \text{if } a \leq x \leq b \\ 1 & \text{if } b \leq x \leq c, \\ r_{\widetilde{A}}(x) & \text{if } c \leq x \leq d, \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

where $l_{\widetilde{A}}$ and $r_{\widetilde{A}}$ are two continuous increasing and decreasing functions for left and right side of fuzzy number, $x \in U$ and a, b, c, d are real numbers.

2.2. Intuitionistic Fuzzy Sets

The concept of intuitionistic fuzzy sets (IFS) is introduced in 1986 by Atanassov. This concept is defined with membership function and non-membership function [1, 21]. Let A is an intuitionistic fuzzy set in finite set X and is defined as

$$A = \{< x, \mu_A(x), v_A(x) > | x \in X\} \tag{3}$$

Where $\mu_A(x) : X \to [0, 1]$ is the membership function of A, $v_A(x) : X \to [0, 1]$ is the non-membership function of A, such that

$$0 \leq \mu_A(x) + v_A(x) \leq 1 \tag{4}$$

For each intuitionistic fuzzy set in X, the intuitionistic index of x in A or a hesitancy degree of x to A is defined as

$$\pi_A = 1 - \mu_A(x) - v_A(x) \tag{5}$$

Where $0 \leq \pi_A \leq 1$. For each $x \in X$

2.2.1. Intuitionistic Fuzzy Numbers (IFN)

An intuitionistic fuzzy subset is called an intuitionistic fuzzy number, if it's defined on real numbers domain as [22, 23]

$$A = \{< x, \mu_A(x), v_A(x) > | x \in \mathbb{R}\} \tag{6}$$

Where

1. A is normal, i.e. at least two points x_0, x_1 belong to A such that $\mu_A(x_0) = 1, v_A(x_1) = 1$.
2. A is convex, i.e. μ_A is fuzzy convex and υ_A is fuzzy concave.
3. μ_A is upper semicontinuous and υ_A is lower semicontinuous.
4. support(A) = $\{x \in X \mid \upsilon_A(x) < 1\}$ is bounded.

2.3. Shadowed Sets

In this section, we present some basic concepts of shadowed sets. The shadowed set S is defined by a mapping from a universal set X to the set of three values as [2, 24, 25]

$$S: X \rightarrow \{0, 1, [0,1]\} \tag{7}$$

Shadowed set is created by optimization of the threshold α that is calculated using the objective function as

$$v(r_1) + v(r_2) = v(r_3) \tag{8}$$

where v is uncertainty of regions r_1, r_2, r_3. The r_1 region is induced by reduce all membership values less than the threshold α to 0. The r_2 region is created by elevated membership values more than 1-α to 1. The r_3 is a shadow region for membership values around 0.5 as illustrated in Figure 1.

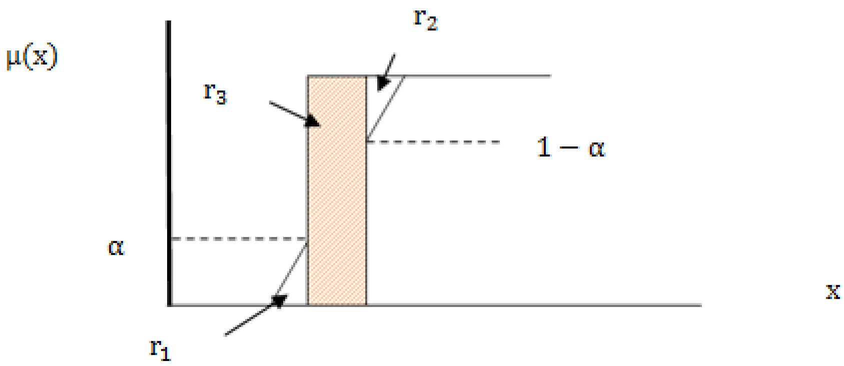

Figure 1: Regions that construct shadowed set

The optimal α can be derived by minimizing the objective function which achieves the balance of uncertainty with these regions as the following

$$V_\alpha = |v(r_1) + v(r_2) - v(r_3)| \tag{9}$$

where V_α is the performance index for the threshold α and $\alpha \in [0, 0.5)$. Pedrycz calculated optimum α for triangular, Gaussian and parabolic fuzzy sets to be 0.4142, 0.395 and 0.405 respectively.

2.3.1. Shadowed Fuzzy Numbers

Shadowed fuzzy numbers (SFNs) are induced from fuzzy numbers [26]. The author proposed an improved approach to create SFN that preserves uncertainty characteristics of fuzzy number and can be deduced from type-1 fuzzy numbers and higher type of them e.g., intuitionistic fuzzy sets (IFS) [4]. The author method is deduced SFN by building core interval and fuzziness intervals as in Figure 4 [23]. In the case of type-1 fuzzy numbers, the α –core can be derived using the following equation [3]

$$A_R(\alpha) - A_L(\alpha) + 1 = 2^{H_A} \qquad (10)$$

where H_A is the non-specificity value of fuzzy set A [27, 23]. $A_L(\alpha)$ and $A_R(\alpha)$ are left and right α-cut functions of fuzzy number A. The α-core interval illustrates in Figure 2.

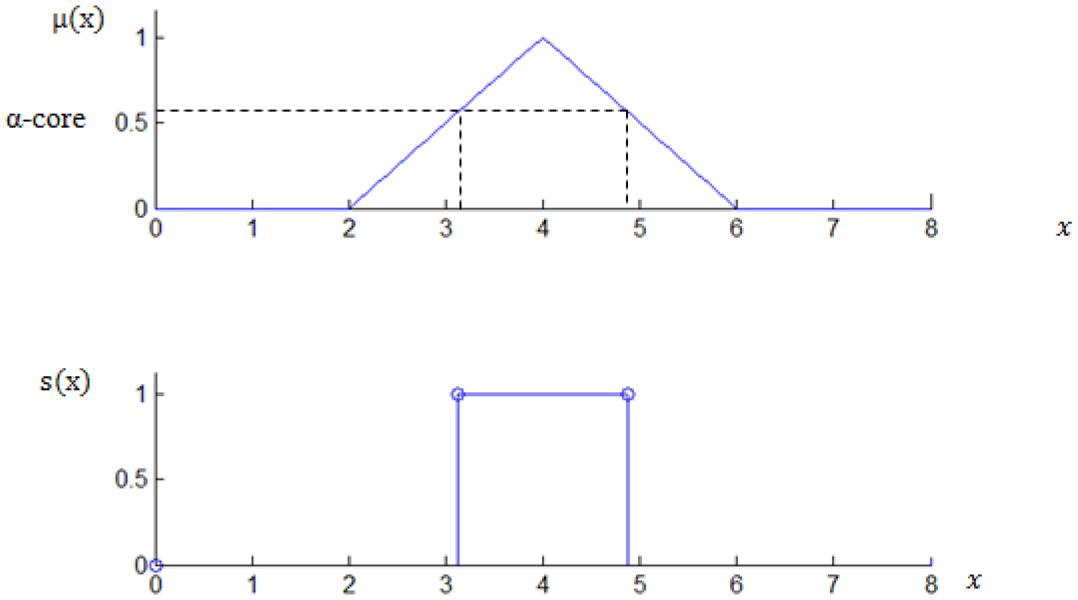

Figure 2: core interval for triangular fuzzy number

The shadow intervals represent fuzziness of fuzzy sets and are calculated for type-1 fuzzy number as

$$w_L = \sum_{x_L} f_A(x_L) \qquad (11)$$

$$w_R = \sum_{x_R} f_A(x_R) \qquad (12)$$

where x_L and x_R are respectively the left support and right support of fuzzy number A from core value. $w_L$andw_R, are the left and right fuzziness intervals. The fuzziness intervals represent uncertainty regions. f_A is the fuzziness set of fuzzy number A as in Figure 3. It is proposed by Tahayori and is defined as the following [28]

$$f_A = (x, \mathrm{fuzz}(x)), \qquad (13)$$

$$\mathrm{fuzz}(x) = 1 - |2\mu_A(x) - 1|. \qquad (14)$$

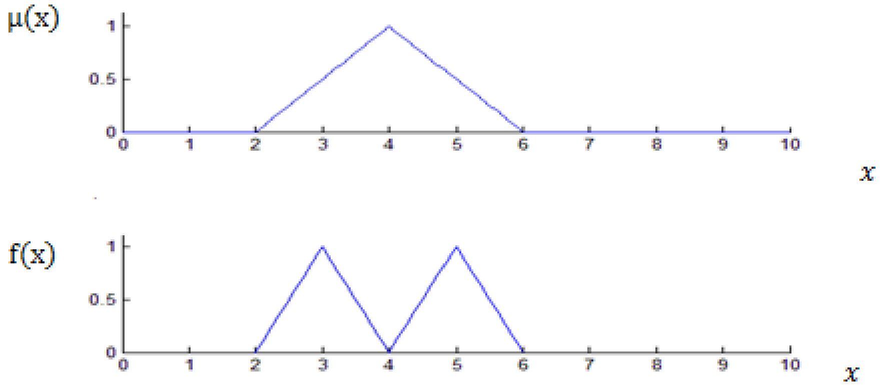

Figure 3: Fuzziness set for a triangular fuzzy set

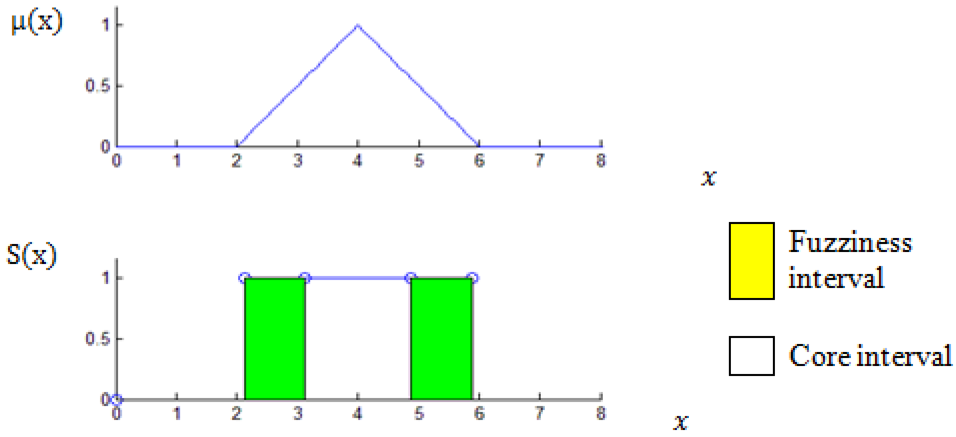

Figure 4: The SFN for triangular fuzzy number

3. THE PROPOSED METHOD FOR RANKING SFNS

In this section, we present a new approach for ranking shadowed fuzzy numbers (SFNs). This method is used to order fuzzy numbers that SFNs are induced from them. Our proposed method is based on new concepts of SFN namely the value and the ambiguity of SFN. These concepts will be proposed based on Delgado et al. definitions of the value and the ambiguity [29].

Definition 3.1:

Let us consider SFN S_A that parameterizes as the following

$S_A = (s_1^A, s_2^A, s_3^A, s_4^A)$then $C_A = (s_2^A, s_3^A)$ is the core interval of S_A. Also, let F_S^A is the fuzziness value ofS_A such that [18]

$$F_S^A = \left(s_2^A - s_1^A\right) + \left(s_4^A - s_3^A\right) \tag{15}$$

Definition 3.2:

Let V_S^A is the value of the SFN S_A and is defined as

$$\mathrm{V}_{\mathrm{S}}^{\mathrm{A}}=\frac{\left(\mathrm{s}_2^{\mathrm{A}}+\mathrm{s}_3^{\mathrm{A}}\right)}{2} \tag{16}$$

Definition 3.3:

Let S_{A} is the SFN which induced from fuzzy number A. Then the ambiguity value $\mathrm{u}_{\mathrm{S}}^{\mathrm{A}}$ of S_{A} is defined as

$$\mathrm{u}_{\mathrm{S}}^{\mathrm{A}}=\mathrm{s}_3^{\mathrm{A}}-\mathrm{s}_2^{\mathrm{A}} \tag{17}$$

Definition 3.4:

Let $\mathrm{S}_{\mathrm{A}}$and S_{B} are two SFNs which induced from fuzzy numbers A and B. Let $\mathrm{Amb}_{\mathrm{S}}^{\mathrm{A}}$ and $\mathrm{Amb}_{\mathrm{S}}^{\mathrm{B}}$ are the ambiguity indexes for S_{A} and S_{B} respectively and are defined as

$$\mathrm{Amb}_{\mathrm{S}}^{\mathrm{A}}=1-\frac{\mathrm{u}_{\mathrm{S}}^{\mathrm{A}}}{\mathrm{u}_{\mathrm{S}}^{\mathrm{A}}+\mathrm{u}_{\mathrm{S}}^{\mathrm{B}}} \tag{18}$$

and

$$\mathrm{Amb}_{\mathrm{S}}^{\mathrm{B}}=1-\frac{\mathrm{u}_{\mathrm{S}}^{\mathrm{B}}}{\mathrm{u}_{\mathrm{S}}^{\mathrm{A}}+\mathrm{u}_{\mathrm{S}}^{\mathrm{B}}} \tag{19}$$

where $\mathrm{u}_{\mathrm{S}}^{\mathrm{A}}$ and $\mathrm{u}_{\mathrm{S}}^{\mathrm{B}}$ are ambiguity values for S_{A} and S_{B} respectively

Definition 3.5:

Let S_{A} is the SFN which induced from fuzzy number A. Then the rank value $\mathrm{R}_{\mathrm{S}}^{\mathrm{A}}$ of $\mathrm{S}_{\mathrm{A}}$is defined as

$$\mathrm{R}_{\mathrm{S}}^{\mathrm{A}}=\mathrm{V}_{\mathrm{S}}^{\mathrm{A}}+\mathrm{Amb}_{\mathrm{S}}^{\mathrm{A}}\times\lambda \tag{20}$$

where $\mathrm{V}_{\mathrm{S}}^{\mathrm{A}}$ is the value of S_{A} , $\mathrm{Amb}_{\mathrm{S}}^{\mathrm{A}}$ is the ambiguity index of S_{A} and λ is an attitude value (AV) where $\lambda \in [0.5, 1]$

An attitude value (AV) represents the attitude of the decision maker against ambiguity. In the case of $\lambda = 1$, this value indicates a decision maker's optimistic attitude towards ambiguity. If $\lambda \in]0.5, 1[$ then AV refers to decision maker's neutral attitude towards ambiguity. When $\lambda = 0.5$, this value indicates decision maker's pessimistic attitude towards ambiguity. In rank examples, we prefer to $\lambda = 0.5$ which more reasonable.

A ranking procedure of two SFNs S_A and S_B, as the following steps:

Step 1: Calculate $\mathrm{V}_{\mathrm{S}}^{\mathrm{A}}$, $\mathrm{V}_{\mathrm{S}}^{\mathrm{B}}$, $\mathrm{Amb}_{\mathrm{S}}^{\mathrm{A}}$ and $\mathrm{Amb}_{\mathrm{S}}^{\mathrm{B}}$ for S_{A} and S_{B} using (16), (17), (18) and (19).

Step 2: Calculate rank values for S_{A} and S_{B} with an attitude value (AV) λ and $\lambda \in [0.5, 1]$ using (20).

Step 3: the ranking of two SFNs is according to

If $R_S^A > R_S^B$ then $S_A > S_B$ and $A > B$.
If $R_S^A < R_S^B$ then $S_A < S_B$ and $A < B$.
If $R_S^A = R_S^B$ then calculate F_S^A and F_S^B of two SFNs S_A and S_B using (15) and
If $F_S^A < F_S^B$ then $S_A > S_B$ and $A > B$
If $F_S^A > F_S^B$ then $S_A < S_B$ and $A < B$
If $F_S^A = F_S^B$ then $S_A = S_B$ and $A = B$

4. NUMERICAL EXAMPLES

In this section, five numerical examples are used to demonstrate the new proposed method reliability for ranking the fuzzy numbers. The proposed method is compared with other existing ranking methods that integrate the centroid point and the spread of fuzzy numbers. Also, the comparison extends to other methods that have ordered fuzzy numbers using their values and ambiguities. This comparative study is summarized in Table 1. A new additional example will be presented in this section to illustrate the characteristic of the new method for ranking different types of IFNs. Also, a hybrid multi-attribute decision making problem will be solved using the new approach.

Example 1: Let A and B are two triangular fuzzy numbers TFNs such that A = (0.1, 0.4, 0.7) and B = (0.3, 0.4, 0.5) as in Figure 5. Using our method, Two SFNs S_A and S_B are induced from two fuzzy numbers A and B such that S_A = (0.1, 0.25, 0.55, 0.7) and S_B = (0.3, 0.35, 0.45, 0.5)[23]. We use (20) to obtain the rank values of S_A and S_B using $\lambda = 0.5$ as the following:

R_A = 0.525 and R_B= 0.775. Based on these results, the ranking of fuzzy numbers is A<B.

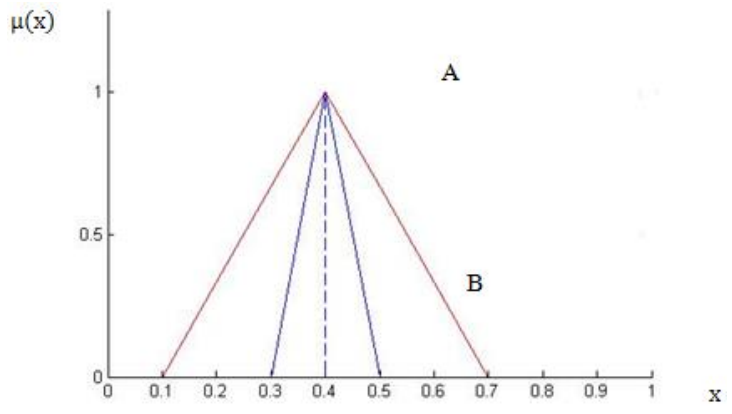

Figure 5: Fuzzy numbers A and B of Example 1

Example 2:

Let A is a trapezoidal fuzzy number TrFN and B is a triangular fuzzy numbers TFN such thatA= (0.1, 0.2, 0.4, 0.5) and B = (0.1, 0.3, 0.5) as in Figure 6.

Using the author method, two SFNs S_A and S_B are obtained from two FNs A and B such that S_A = (0.1, 0.15, 0.45, 0.5) and S_B = (0.1, 0.2, 0.4, 0.5) [3]. The rank values for S_A and S_B are calculated with $\lambda = 0.5$ and the results are R_A = 0.5 and R_B= 0.6. The order of fuzzy numbers is A<B.

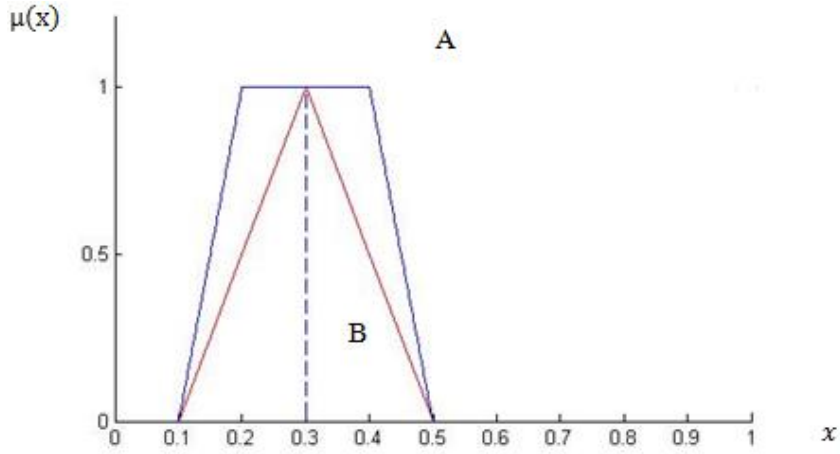

Figure 6: Fuzzy numbers A and B of Example 2

Example 3:

Consider the following two fuzzy numbers with different heights as shown in Figure 7.
A = (0.1, 0.4, 0.7) and height(A) = 0.8, B = (0.1, 0.4, 0.7) and height(B) = 1

Using the same calculation steps as the previous examples, we getS_A = (0.08 0.2538 0.5462 0.72); S_B = (0.1038, 0.2538, 0.5462, 0.6962); R_A = 0.65 **;**R_B= 0.65.

Based on these results, the two ranking values are equal so, we use (15) to obtain fuzziness values for each SFN. The fuzziness values are F_S^A = 0.345 and F_S^B = 0.3 then the ranking of fuzzy numbers is A<B.

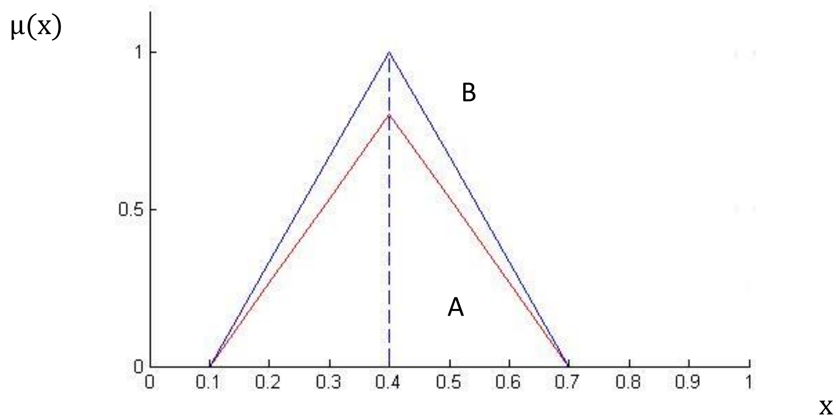

Figure 7: Fuzzy numbers A and B of Example 3

Example 4:

Let A and C are TrFNs and B is TFN such that A= (0, 0.4, 0.6, 0.8), B = (0.2, 0.5, 0.9) and C= (0.1, 0.6, 0.7, 0.8), as in Figure 8.

Using the author method, we induce three SFNs where S_A = (0.0034, 0.2034, 0.6983, 0.7983) , S_B = (0.2052, 0.3552, 0.6931, 0.8931) and S_C = (0.1124, 0.3624, 0.7475, 0.7975)[3]. We use (20) to obtain the rank values for S_A , S_B and S_C using $\lambda = 0.5$ as the following:

R_A = 0.7477, R_B= 0.8854 and R_C = 0.8968. According to these results, the ranking of fuzzy numbers is $A < B < C$.

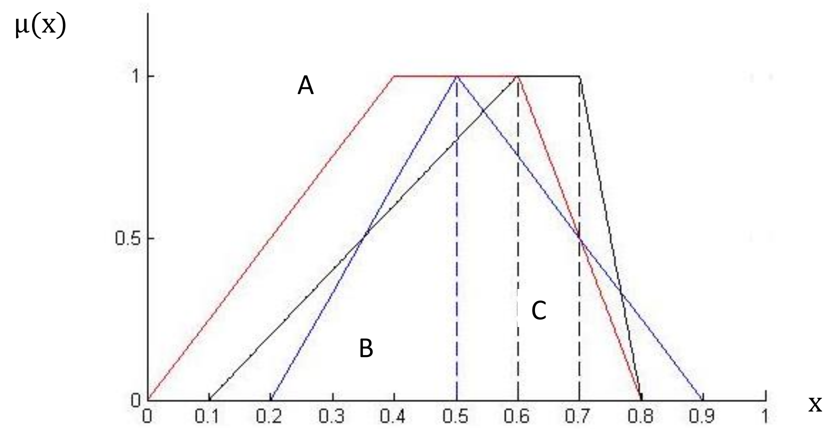

Figure 8: Fuzzy numbers A , B and C of Example 4

Table 1: Comparative results of the proposed ranking method with the existing ranking methods

Examples	Chen and Sanguansat [5]	Abu Bakar and Gegov [13]	R.Chutia and B.Chutia [14]	Proposed method
1	$A \approx B$	$A > B$	$A < B$	A<B
2	$A \approx B$	$A > B$	$A < B$	A<B
3	A < B	$A < B$	A < B	A<B
4	$A < B < C$	$A < B < C$	$A < B < C$	$A < B < C$

Example 5:

Let A_1, A_2 and A_3 are three trapezoidal intuitionistic fuzzy numbers (TrIFNs),where A_1= [(0.1,0.3,0.5,0.8), (0.1,0.3,0.5,0.8) 0.5,0.2] , A_2= [(0.2 ,0.3,0.6 ,0.9), (0.2 ,0.3,0.6 ,0.9) 0.6 ,0.4] and A_3= [(0.1, 0.5,0.7, 0.9), (0.1, 0.5, 0.7, 0.9) 0.5,0.3] [16]. We induce three SFNs using author method such that S_{A_1} = (0.15, 0.27, 0.54, 0.725), S_{A_2} = (0.23, 0.29, 0.64, 0.81) and S_{A_3} = (0.23, 0.47, 0.71, 0.83)[4]. The rank values of S_{A_1} , S_{A_2} and S_{A_3} are calculated by using (20) with$\lambda =$ 0.5 and the results are R_{A_1} = 0.748, R_{A_2}= 0.7615 and R_{A_3} = 0.9505. According to these results, the order of three TrIFNs is $A_1 < A_2 < A_3$.

Example 6:

In this example, we apply the proposed method on the case of ranking three different types of intuitionistic fuzzy numbers.

Let B_1, B_2 and B_3 are three different types of intuitionistic fuzzy numbers(IFNs),where B_1= [(2,3,5,6), (1,3,5,7) 1,0] is trapezoidal intuitionistic fuzzy number(TrIFN), B_2 = [(m= 4, σ = 0.5) ,(m= 4, σ = 1) 1,0] is Gaussian intuitionistic fuzzy number(GIFN), and B_3= [(3,6,9) , (4,6,8) 1,0] is triangular intuitionistic fuzzy numbers (TIFN) as in Figure 9. Three SFNs are induced using authors method as S_1= [1.615, 2.31, 5.7, 6.386] , S_2= [2.12, 3.17, 4.83, 5.88] and S_3= [3.653, 4.93, 7.07, 8.35][4]. Using the new ranking method, The rank values of S_1 , S_2 and S_3 are R_1 = 4.2693 , R_2= 4.3846 and R_3 = 6.3512 with $\lambda = 0.5$ then the order of SFNs is $S_3 > S_2 > S_1$. According to this result, the ranking for three IFNs is $B_3 > B_2 > B_1$.

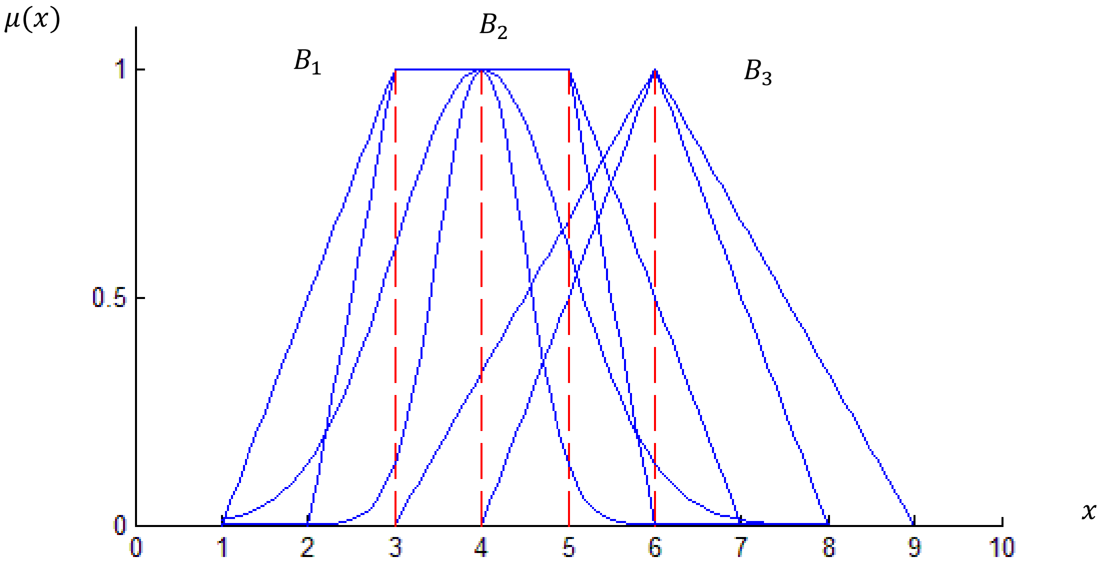

Figure 9: Three IFNs B_1, B_2 and B_3

4.1. Discussion of the Results of Examples

1. For the fuzzy numbers A and B as in the example 1, Chen and Sanguansat method fail to discriminate between two fuzzy numbers which they have the same value as in Table 1. Abu Baker and Gegov method prefers fuzzy number A to B but the ambiguity of B is less than A. R.Chutia and B.Chutia method and the proposed method get the same ranking order, which is consistent with intuition. The two fuzzy numbers have the same value, but they are different in ambiguities.
2. In the example 2, the correct ranking order of fuzzy numbers for this case should be A < B due to the same reasons mentioned in the previous example.
3. In both examples 3 and 4, the order by the proposed method is consistent with other methods as illustrated in Table 1.
4. The ranking result for example 5 like the ranking order from D. Das method [16]. This result explains that the proposed approach works well.
5. Example 6 is presented to explain the case of ranking three different types of intuitionistic fuzzy numbers.

4.2. Application of the Proposed Method in MADM Problem

The multiple attribute decision-making problems under fuzzy environment have been studied extensively by many authors. In this section, we will focus on the personnel selection problem, which were presented by both Mahdavi and Deng-Feng Li [15] [30] [31]. Also, the problem was solved using value and ambiguity indexes in [31]. We develop a new algorithm to solve the personnel selection problem in the case of hybrid data types such as interval numbers, type-1 fuzzy numbers and IFNs.

Suppose that a software company desires to hire a system analyst [31]. After preliminary screening, three candidates' alternatives A_1, A_2 and A_3 remain for further evaluation. The decisionmaking committee assesses the three candidates. The decision makers consider five benefit criteria to evaluate these candidates, including, emotional steadiness (C_1), oral communication skill (C_2), personality (C_3), experience (C_4), and self-confidence (C_5).

The evaluations for these criteria vary between interval numbers, type-1 fuzzy numbers and intuitionistic fuzzy numbers. The assessment for C_1 is represented by triangular fuzzy numbers TFNs. The criterion C_2 is evaluated using intervals numbers INs. The assessment for the criterion C_3 can be represented by Gaussian fuzzy numbers GFNs. The triangular intuitionistic fuzzy numbers TIFNs are used for C_4 and C_5 criteria. The evaluation values are given in Table (2) by the decision makers. The crisp weight w_j is assigned to each criterion such that $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$.

Table 2: The evaluation values of the three candidates under all criteria

	C_1	C_2	C_3	C_4	C_5
A_1	(5.7, 7.7, 9.3)	[5, 9]	(7.7; 0.7)	[(8.33,9.67,10);0.6,0.4]	[(3,5,7);0.6,0.3]
A_2	(6.3, 8.3, 9.7)	[9, 10]	(9.7; 0.12)	[(8,9,10);0.6,0.3]	[(7,9,10);0.6,0.2]
A_3	(6.3, 8, 9)	[7, 10]	(9; 0.46)	[(6,8,9);0.6,0.2]	[(6.3,8.3,9.7);0.7,0.2]

We propose the following steps to solve this problem as follows:

Step 1: The SFNs are obtained using author approach for the type-l fuzzy numbers as in Table 2 for criteria (C_1) and (C_3) [3]. Also, the author method is applied to transform IFNs to SFNs for criteria (C_4) and (C_5) [4]. The new decision matrix is obtained as in Table 3.

Table 3: The evaluation values of decision table using SFNs

	C_1	C_2	C_3	C_4	C_5
A_1	(5.82, 6.82, 8.4, 9.2)	[5, 9]	(6.194, 6.9, 8.5, 9.122)	(8.591, 9.35, 9.75, 9.937)	(3.344, 4.53, 5.47, 6.656)
A_2	(6.42,7.42, 8.92, 9.62)	[9, 10]	(9.424, 9.55, 9.85, 9.972)	(8.147, 8.74, 9.26, 9.853)	(7.239, 8.46, 9.27, 9.88)
A_3	(6.39, 7.24, 8.45, 8.95)	[7, 10]	(7.979, 8.46, 9.54, 9.987)	(6.239, 7.46, 8.27, 8.88)	(6.525,7.72, 8.71, 9.546)

Step 2: The normalized SFNs are calculated using (21) and normalized interval numbers are obtained using (22). The results are displayed in Table 4.

Definition 4.1:

Let r_{ij} is the normalized evaluation value for the j^{th} benefit criteria and two cases can be defined for it. In the case of SFN $s_{ij} = (s_{ij1}, s_{ij2}, s_{ij3}, s_{ij4})$ is defined as

$$r_{ij} = [\frac{s_{ij1}}{\bar{s}_{j4}}, \frac{s_{ij2}}{\bar{s}_{j4}}, \frac{s_{ij3}}{\bar{s}_{j4}}, \frac{s_{ij4}}{\bar{s}_{j4}}] \qquad (21)$$

where $\bar{s}_{j4} = \max_i\{s_{ij4} | i = 1,2, \ldots\ldots, m\}$, $j = 1,2, \ldots\ldots, n$. In the case of interval number IN $t_{ij} = [t_{ij}^{L}, t_{ij}^{R}]$ is defined as the following

$$r_{ij} = [\frac{t_{ij}^{L}}{\bar{t}_{j}^{R}}, \frac{t_{ij}^{R}}{\bar{t}_{j}^{R}}] \qquad (22)$$

Where $\bar{t}_{j}^{R} = \max_i\{t_{ij}^{R} | i = 1,2, \ldots\ldots, m\}$, $j = 1,2, \ldots\ldots, n$.

Table 4: The normalized evaluation values of decision table

	C_1	C_2	C_3	C_4	C_5
A_1	(0.6, 0.71, 0.87, 0.96)	[0.5, 0.9]	(0.62, 0.69, 0.85, 0.91)	(0.86, 0.94, 0.98, 1)	(0.34, 0.46, 0.55, 0.67)
A_2	(0.67, 0.77, 0.93, 1)	[0.9, 1]	(0.94, 0.96, 0.99, 1)	(0.82, 0.88, 0.93, 0.99)	(0.73, 0.86, 0.94, 1)
A_3	(0.66,0.75, 0.88, 0.93)	[0.7, 1]	(0.8, 0.85, 0.96, 1)	(0.63, 0.75, 0.83, 0.89)	(0.66, 0.78, 0.88, 0.97)

Step 3: Let w_1= 0.14, w_2= 0.3, w_3= 0.12, w_4= 0.3 and w_5= 0.14 are weights of attributes. The weighted normalized SFNs are obtained using (23) and the weighted normalized interval numbers are calculated using (24). The results shown in Table 5.

Definition 4.2:

Let $\mathrm{wr_{ij}}$ is the weighted normalized evaluation value for the j^{th} benefit criteria and we define two types of it. In the case of SFN is defined as

$$\mathrm{wr_{ij}} = [\bar{s}_{ij1} \times w_j, \bar{s}_{ij2} \times w_j, \bar{s}_{ij3} \times w_j, \bar{s}_{ij4} \times w_j] \qquad (23)$$

Where w_j is the weight value for the j^{th} criteria such that j = 1,2,…,n. and $\bar{s}_{ij} = [\bar{s}_{ij1}, \bar{s}_{ij2}, \bar{s}_{ij3}, \bar{s}_{ij4}]$ is a normalized SFN.

In the case of interval number IN, the wr_{ij} is defined as

$$\mathrm{wr_{ij}} = [\bar{t}^L_{ij} \times w_j, \bar{t}^R_{ij} \times w_j]. \qquad (24)$$

Where w_j is the weight value for thej^{th} criteria and$\bar{t}_{ij} = [\bar{t}^L_{ij}, \bar{t}^R_{ij}]$ is a normalized IN.

Table 5: The weighted normalized evaluation values of decision table

	C_1	C_2	C_3	C_4	C_5
A_1	(0.084,0.099, 0.122, 0.134)	[0.15, 0.27]	(0.074, 0.0828, 0.102, 0.109)	(0.258, 0.282, 0.294, 0.3)	(0.048, 0.064, 0.077, 0.094)
A_2	(0.094,0.108, 0.13, 0.14)	[0.27, 0.3]	(0.113, 0.115, 0.119, 0.12)	(0.246, 0.264, 0.279, 0.297)	(0.102, 0.12, 0.132, 0.14)
A_3	(0.092,0.105, 0.123, 0.13)	[0.21, 0.3]	(0.096, 0.102, 0.115, 0.12)	(0.189, 0.225, 0.249, 0.267)	(0.092, 0.109, 0.123, 0.136)

Step 4: we use (25) and (26) to create the aggregation values of weighted normalized SFNs for three alternatives and the results shown in Table 6.

Definition 4.3:

Let $\mathrm{ds_i}$ is an aggregation decision values of alternatives A_i where $i = 1, 2, \dots\dots, m$ are obtained as

$$\mathrm{ds_i} = \sum_{j=1}^{n} \mathrm{wr_{ij}} \qquad (25)$$

where ds_i $(i = 1,2, \ldots\ldots, m)$ are SFNs and wr_{ij} is the weighted normalized evaluation value for the j^{th} benefit criteria. This operation is defined between SFNs and INs as the following

$$T + S = (\bar{\bar{s}}_1, \bar{\bar{s}}_2, \bar{\bar{s}}_3, \bar{\bar{s}}_4) \quad (26)$$

such that S is a shadowed fuzzy number SFN, $\bar{\bar{s}}_1 = (s_1 + t_L)$, $\bar{\bar{s}}_2 = (s_2 + t_L)$, $\bar{\bar{s}}_3 = (s_3 + t_R)$, $\bar{\bar{s}}_4 = (s_4 + t_R)$ and $T = [t_L, t_R]$ is an IN.

Table 6: The aggregation values of weighted normalized SFNs

A_1	(0.614, 0.679, 0.865, 0.907)
A_2	(0.825, 0.877, 0.96, 0.997)
A_3	(0.68, 0.751, 0.911, 0.953)

Step 5: finally, we use the proposed approach to rank SFNs resulting from step 4. The rank values for alternatives $A_1, A_2$and A_3 are R_{A_1} = 1.0552, R_{A_2}= 1.3218 and R_{A_3}= 1.1445 with $\lambda = 0.5$. The order of the three alternatives is $A_2 > A_3 > A_1$.

4.3. Comparison Analysis of the Result

Based on the previous results of the MADM problem, the following remarks are found:

In Mahdavi et al. approach, the ranking of three alternatives is $A_2 > A_3 > A_1$ which it is the same result of the new approach [30]. In Mahdavi method, the MADM problem is presented with the type-1 triangular fuzzy numbers and orders alternatives using similarity values to ideal solution.

In Deng-Feng Li et al. method, the ranking of three alternatives is $A_3 > A_1 > A_2$ with the weight $\lambda \in [0,0.793]$ [31]. This weight represents the decision maker's preference information. In the case of $\lambda \in (0.793, 1]$, the order of three alternatives is $A_1 > A_3 > A_2$.

In Deng-Feng Li approach, the data inputs of MADM problem are represented by IFNs and the value-index and the ambiguity-index of IFNs are used to rank IFNs. According to Deng-Feng Li et al. method, if ratings of the alternatives on the attributes are reduced to type-1 triangular fuzzy numbers then the ranking order is $A_2 > A_3 > A_1$ [15] [31]. This result like the ranking order from the proposed method.

4.4. Discussion

Previous ranking approaches of fuzzy numbers with only one type of them (type-1 or higher type). These techniques were difficult to apply to complex decision-making problems that contain different types of fuzzy numbers. In the new approach, we unified the different types of fuzzy numbers using shadowed fuzzy numbers and preserve uncertainty characteristics of fuzzy numbers at the same time.

The new approach is more flexible to rank fuzzy numbers with different membership functions than previous methods.

5. CONCLUSION

This paper is proposed a new approach to rank SFNs. This method is applied to order fuzzy numbers from type-1 and higher type which transforms different types of fuzzy numbers to SFNs. The new ranking approach induces the rank values which integrates the value and the ambiguity of SFN. Also, it weighted an ambiguity value using the decision maker's attitude value. In the case of equal rank values, the fuzziness values are used to rank SFNs. The new algorithm is applied for different examples of ranking type-1 fuzzy numbers FNs and intuitionistic fuzzy numbers IFNs. The ranking results of the proposed method are compared with previous ranking approaches for type-1 FNs and IFNs. Also, the new algorithm is applied to solve a hybrid multi-attribute decision making problem where SFNs are used to unify the uncertain types of linguistic terms. The new method of ranking is applied to rank alternatives.

The new algorithm is more efficient and more flexible than previous methods which solved the same problem with one type of linguistic terms.

The future work will focus on verifying the usefulness of the new approach with other multi-criteria techniques. Also, we can study more applications of the new method with more decision-making problems.

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