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ON THE COVERING RADIUS OF CODES OVER Z_4 WITH CHINESE EUCLIDEAN WEIGHT

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ABSTRACT

In this paper, we give lower and upper bounds on the covering radius of codes over the ring Z_4 with respect to chinese euclidean distance. We also determine the covering radius of various Repetition codes, Simplex codes Type α and Type β and give bounds on the covering radius for MacDonald codes of both types over Z_4.

KEYWORDS

Covering radius, Codes over finite rings, Simplex codes, Hamming codes.
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1 INTRODUCTION

In the last decade, there are many researchers doing research on code over finite rings. In particular, codes over Z_4 received much attention [1, 2, 3, 7, 9, 13, 14]. The covering radius of binary linear codes were studied [4, 5]. Recently the covering radius of codes over Z_4 has been investigated with respect to chinese euclidean distances [10]. In 1999, Sole et al gave many upper and lower bounds on the covering radius of a code over Z_4 with chinese euclidean distances. In [12], the covering radius of some particular codes over Z_4 have been investigated. In this correspondence, we consider the ring Z_4. In this paper, we investigate the covering radius of the Simplex codes of both types and MacDonald codes and repetition codes over Z_4. We also generalized some of the known bounds in [1].

A linear code C of length n over Z_4 is an additive subgroup of Z_4^n. An element of C is called a codeword of C and a generator matrix of C is a matrix whose rows generate C. In [10], the chinese Euclidean weight $w_{CE}(x)$ of a vector x is $\sum_{i=1}^{n}\left\{2-2\cos(\frac{2\pi x_i}{4})\right\}$

A linear Gray map φ from $Z_4 \rightarrow Z_2^2$ is defined by $\varphi(x + 2y) = (y, x + y)$, for all $x + 2y \in Z_4$. The image φ(C), of a linear code C over Z_4 of length n by the Gray map, is a binary code of length 2n with same cardinality [13].

Any linear code C over Z_4 is equivalent to a code with generator matrix G of the form

$$G = \begin{bmatrix} I_{k_0} & A & B \\ 0 & 2I_{k_1} & 2D \end{bmatrix}, \qquad (1.1)$$

where A, B and D are matrices over Z_4. Then the code C contain all code words $[v_0, v_1]G$, where v_0 is a vector of length k_1 over Z_4 and v_1 is a vector of length k_2 over Z_2 . Thus C contains a total of $4^{k_1}2^{k_2}$ codewords. The parameters of C are given [n, $4^{k_1}2^{k_2}$, d] where d represents the minimum chinese Euclidean distance of C.

A linear code C over Z_4 of length n, 2-dimension k, minimum chinese euclidean distance d_{CE} is called an [n, k, d_{CE}] or simply an [n, k] code.

In this paper, we define the covering radius of codes over Z_4 with respect to chinese euclidean distance and in particular study the covering radius of Simplex codes of type α and type β namely, S_k^α and S_k^β and their MacDonald codes and repetition codes over Z_4. Section 2 contains basic results for the covering radius of codes over Z_4. Section 3 determines the covering radius of different types of repetition codes. Section 4 determines the covering radius of Simplex codes and finally section 5 determines the bounds on the covering radius of MacDonald codes.

2 Covering Radius of Codes

Let d be a chinese euclidean distance. The covering radius of a code C over Z_4 with respect to chinese euclidean distance d is given by

$$. \; r_d(C) = \max_{u \in z_4^n} \left\{ \min_{c \in C} \left\{ d(c,u) \right\} \right\}$$

The following result of Mattson [4] is useful for computing covering radius of codes over rings generalized easily from codes over finite fields.

Proposition 2. 5. (Mattson) If C_0 and C_1 are codes over Z_4 generated by matrices G_0 and G_1 respectively and if C is the code generated by $G = \left(\begin{array}{c|c} 0 & G_1 \\ \hline G_0 & A \end{array} \right)$ then $r_d(C) \le r_d(C_0) + r_d(C_1)$ and the covering radius of D (concatenation of C_0 and C_1) satisfy the following $r_d(D) \ge r_d(C_0) + r_d(C_1)$, for all distances d over Z_4.

3 COVERING RADIUS OF REPETITION CODES

A q-ary repetition code C over a finite field $F_q = \{\alpha_0 = 0, \alpha_1 = 1, \alpha_2, \alpha_3, \ldots, \alpha_{q-1}\}$ is an [n, 1, n] code $C = \{\bar{\alpha} | \alpha \in F_q\}$, where $\bar{\alpha} = (\alpha, \alpha, \cdots, \alpha)$. The covering radius of C is $\left\lceil \frac{n(q-1)}{q} \right\rceil$ [11]. Using this, it can be seen easily that the covering radius of block of size n repetition code [n(q-1),1,n(q-1)] generated by $G = \left[\overbrace{11\cdots1}^{n} \overbrace{\alpha_2\alpha_2\cdots\alpha_2}^{n} \overbrace{\alpha_3\alpha_3\cdots\alpha_3}^{n} \cdots \overbrace{\alpha_{q-1}\alpha_{q-1}\cdots\alpha_{q-1}}^{n} \right]$ is

$\left\lceil \frac{n(q-1)^2}{q} \right\rceil$ since it will be equivalent to a repetition code of length (q − 1)n.

Consider the repetition code over Z_4. There are two types of them of length n viz. unit repetition code C_β : [n, 1, 2n] generated by $G_\beta = [\overbrace{11\cdots1}^{n}]$ and zero divisor repetition code C_α : (n, 2, 4n) generated by $G_\alpha = [\overbrace{22\cdots2}^{n}]$. The following result determines the covering radius with respect to chinese euclidean distance.

Theorem 3. 1. $4\left\lfloor \frac{n}{2} \right\rfloor \le r_{CE}(C_\alpha) \le 2n$ and $r_{CE}(C_\beta) = 2n$.

Proof. Let x = $\overbrace{22\cdots2}^{\lfloor n/2 \rfloor}\overbrace{00\cdots0}^{\lceil n/2 \rceil} \in Z_4^n$. The code $C_\alpha = \{\alpha(22\cdots2) \mid \alpha \in Z_4^n\}$, that is C_α = {00….0, 22…..2}, generated by [22….2] is an [n,1,2n] code. Then, d_{CE}(x,00….0) = $wt_{CE}(\overbrace{22\cdots2}^{\lfloor n/2 \rfloor}\overbrace{00\cdots0}^{\lceil n/2 \rceil} - 00\cdots0) = 4\left\lceil \frac{n}{2} \right\rceil$ and d_{CE}(x,22….2) =

$wt_{CE}(\overbrace{22\cdots2}^{\lfloor n/2 \rfloor}\overbrace{00\cdots0}^{\lceil n/2 \rceil} - 22\cdots2) = 4\left\lfloor \frac{n}{2} \right\rfloor$. Therefore $d_{CE}(x, C_\alpha)$ = min { $4\left\lceil \frac{n}{2} \right\rceil, 4\left\lfloor \frac{n}{2} \right\rfloor$ }.

Thus, by definition of covering radius $r_{CE}(C_\alpha) \ge 4\left\lfloor \frac{n}{2} \right\rfloor$ (3.1)

Let x be any word in Z_4^n . Let us take x has ω_0 coordinates as 0's, $\omega_1$coordinates as 1's, $\omega_2$coordinates as 2's, ω_3 coordinates as 3's, then $\omega_0 + \omega_1 +, \omega_2 + \omega_3 = n$. Since C_α ={00….0,22…2} and chinese euclidean weight of Z_4: 0 is 0, 1 and 3 is 2 and 2 is 4, we have $d_{CE}(x,00\ldots0) = n - \omega_0 + \omega_1 + 3\,\omega_2 + \omega_3$ and $d_{CE}(x,22\ldots2) = n - \omega_2 + \omega_1 + 3\,\omega_0 + \omega_3$.
Thus $d_{CE}(x, C_\alpha) = \min\{\, n - \omega_0 + \omega_1 + 3\,\omega_2 + \omega_3 ,\; n - \omega_2 + \omega_1 + 3\,\omega_0 + \omega_3 \,\}$.

$$d_{CE}(x, C_\alpha) \le n + + n = 2n \qquad (3.2)$$

Hence, from the Equation (3.1) and (3.2), we get $4\left\lfloor \frac{n}{2} \right\rfloor \le r_{CE}(C_\alpha) \le 2n$. Now, we find the covering radius of C_β covering with respect to the chinese euclidean weight. We have $d_{CE}(x,00\ldots0) = n - \omega_0 + \omega_1 + 3\,\omega_2 + \omega_3$, $d_{CE}(x,11\ldots1) = n - \omega_1 + \omega_0 + \omega_2 + 3\,\omega_3$, $d_{CE}(x,22\ldots2) = n - \omega_2 + 3\,\omega_1 + \omega_3$ and $d_{CE}(x, 33\ldots3) = n - \omega_3 + 3\omega_1 + \omega_0 + \omega_2$ for any $x \in Z_4^n$. This implies $d_{CE}(x, C_\beta) = \min\{\, n - \omega_0 + \omega_1 + 3\,\omega_2 + \omega_3,\ n - \omega_1 + \omega_0 + \omega_2 + 3\,\omega_3,\ n - \omega_2 + 3\,\omega_1 + \omega_3 ,\ n - \omega_3 + 3\omega_1 + \omega_0 + \omega_2 \} \le 2n$ and hence $r_{CE}(C_\beta) \le 2n$. To show that $r_{CE}(C_\beta) \ge 2n$, let

$x = \overbrace{00\cdots0}^{t}\overbrace{11\cdots1}^{t}\overbrace{22\cdots2}^{t}\overbrace{33\cdots3}^{n-3t} \in Z_4^n$, where t $= \left\lfloor \frac{n}{4} \right\rfloor$, then d_{CE}(x,00…0)=2n, d_{CE}(x,11…1)=4n-8t, d_{CE}(x,22…2)=2n and d_{CE}(x,33….3)=8t.

Therefore $r_{CE}(C_\beta) \geq \min\{2n,4n-8t,8t\} \geq 2n$.

To determine the covering radius of Z_4 three blocks each of size n repetition code BRep ^{3n} : [3n, 1, 8n] generated by $G = [\ \overbrace{11\cdots1}^{n}\overbrace{22\cdots2}^{n}\overbrace{33\cdots3}^{n}\ ]$ note that the code has constant chinese euclidean weight 8n and the block repetition code BRep ^{3n} :{c_0=(0…0 0….0 0…..0), c_1=(1….12….2 3….3), c_2=(2…2 0…0 2….2), c_3=(3….3 2….2 1…..1)}. Thus x = 11 $\cdots 1 \in Z_4^{3n}$, we have d_{CE} (x, BRep ^{3n}) = $4\left\lfloor \frac{n}{2} \right\rfloor + 4n.$ Hence by definition, r_{CE} (BRep ^{3n}) ≥ $4\left\lfloor \frac{n}{2} \right\rfloor + 4n.$ To find the upper Let x = (u|v|w) $\in Z_4^{3n}$ with u, v and w have compositions (r_0 , r_1 , r_2 , r_3), (s_0 , s_1 , s_2 , s_3) and (t_0 , t_1 , t_2 , t_3) respectively such that $\sum_{i=0}^{3} r_i = n, \sum_{i=0}^{3} s_i = n$ and $\sum_{i=0}^{3} t_i = n$, then d_{CE} (x, c_0) = 3n − r_0 +r_1+ 3 r_2 +r_3− s_0 + s_1+3 s_2 +s_3− t_0 +t_1+ 3 t_2+t_3 , d_{CE} (x, c_1) = 3n − r_1 +r0+r2+ 3 r_3 − s_2 +3s0+s1+$s3_0$ − t_3 +t_0+ 3t_1+t_2 , d_{CE} (x, c_2) = 3n− r_2 +r_1+3r_0+r_3 − s_0 +s_1+3 s_2+s_3 − t_2 +3t_0+t_1+t_3 and d_{CE} (x, c_3) = 3n− r_3 +3 r1+r0+r2 −s_2+3s_0+s_1+s_3 −t_1+3t_3+t_0+t_2 . Thus d_{CE} (x, BRep ^{3n}) = min{3n − r_0 +r_1+ 3 r_2 +r_3− s_0 + s_1+3 s_2 +s_3− t_0 +t_1+ 3 t_2+t_3 , 3n − r_1 +r0+r2+ 3 r_3 − s_2 +3s0+s1+$s3_0$ − t_3 +t_0+ 3t_1+$t_{2,}$ 3n− r_2 +r_1+3r_0+r_3 − s_0 +s_1+3 s_2+s_3 − t_2 +3t_0+t_1+$t_{3,}$ 3n− r_3 +3 r1+r0+r2 −s_2+3s_0+s_1+s_3−t_1+3t_3+t_0+t_2 . Thus we have the following theorem

Theorem 3. 2. $4\left\lfloor \frac{n}{2} \right\rfloor + 4n \leq$ r_{CE} (BRep ^{3n}) ≤ 6n.

One can also define a Z_4 block (two blocks each of size n) repetition code Brep ^{2n} : [2n, 1, 4n] generated by $G = \left[\overbrace{11\cdots1}^{n}\overbrace{22\cdots2}^{n} \right]$. We have following theorem

Theorem 3. 3. $4\left\lfloor \frac{n}{2} \right\rfloor + 2n \leq$ r_{CE} (BRep ^{3n}) ≤ 4n

Block code BRep $^{m+n}$ can be generalized to a block repetition code (two blocks of size m and n respectively) BRep $^{m+n}$: [m + n, 1, min{4m, 3m + 3n}] generated by $G = \left[\overbrace{11\cdots1}^{m}\overbrace{22\cdots2}^{n} \right]$.

Theorem 3.3 can be easily generalized for different length using similar arguments to the following.

Theorem 3. 4. 2m+ $4\left\lfloor \frac{n}{2} \right\rfloor \leq$ r_{CE} (BRep $^{m+n}$) ≤ 2m+2n

4 SIMPLEX CODES OF TYPE Α AND TYPE β OVER Z_4

Quaternary simplex codes of type α and type β have been recently studied in [2]. Type α Simplex code S_k^{α} is a linear code over Z_4 with parameters $[4^k,k]$ and an inductive generator matrix given by

$$G_k^{\alpha} = \left[\begin{array}{c|c|c|c} 00\ldots0 & 11\ldots1 & 22\ldots2 & 33..3 \\ \hline G_{k-1}^{\alpha} & G_{k-1}^{\alpha} & G_{k-1}^{\alpha} & G_{k-1}^{\alpha} \end{array}\right] \tag{4.1}$$

with G_1^{α} =[0 1 2 3]. Type simplex code S_k^{β} is a punctured version of S_k^{α} with parameters $[2^{k-1}(2^k - 1), k]$ and an inductive generator matrix given by

$$G_2^{\beta} = \left[\begin{array}{cccc|c|c} 1 & 1 & 1 & 1 & 0 & 2 \\ \hline 0 & 1 & 2 & 3 & 1 & 1 \end{array}\right] \tag{4.2}$$

and for $k > 2$

$$G_k^{\beta} = \left[\begin{array}{c|c|c} 11\cdots1 & 00\cdots0 & 22\cdots2 \\ \hline G_{k-1}^{\alpha} & G_{k-1}^{\beta} & G_{k-1}^{\beta} \end{array}\right] \tag{4.3}$$

where G_{k-1}^{α} is the generator matrix of S_{k-1}^{α}. For details the reader is refered to [2]. Type α code with minimum chinese euclidean weight is 8.

Theorem 4. 1. $r_{CE}(S_k^{\alpha}) \le 2^{2k+1}-3$.

Proof. Let $x = 11\ldots.. 1 \in Z_4^n$. By equation 4.1, the result of Mattson for finite rings and using Theorem 3. 2, we get

$$r_{CE}(S_k^{\alpha}) \le r_{CE}(S_{k-1}^{\alpha}) + r_{CE}(<\overbrace{11\cdots1}^{2^{2(k-1)}}\ \overbrace{22\cdots2}^{2^{2(k-1)}}\ \overbrace{33\cdots3}^{2^{2(k-1)}}>)$$

$$= r_{CE}(S_{k-1}^{\alpha}) + 6.2^{2(k-1)}$$

$$= 6.2^{2(k-1)} + 6.2^{2(k-2)} + 6.2^{2(k-3)} + \ldots.. + 6.2^{2.1} + r_{CE}(S_1^{\alpha})$$

$$r_{CE}(S_k^{\alpha}) \le 2^{2k+1}-3 (\text{since } r_{CE}(S_1^{\alpha}) = 5)$$

Theorem 4. 2. $r_{CE}(S_k^{\beta}) \le 2^k(2^k - 1) - 7$

Proof. By equation 4. 3, Proposition 2. 5 and Theorem 3. 4, we get

$$r_{CE}(S_k^{\beta}) \le r_{CE}(S_{k-1}^{\beta}) + r_{CE}(<\overbrace{11\cdots1}^{4^{(k-1)}}\ \overbrace{22\cdots2}^{2^{(2k-3)}-2^{(k-2)}}>)$$

$$= r_{CE}(S_{k-1}^{\beta}) + 2^{(2k-2)} + 2^{(2k-3)} - 2^{(k-2)}$$

$\leq 2(2^{(2k-2)} + 2^{(2k-4)} + \ldots + 2^4) + 2(2^{(2k-3)} + 2^{(2k-5)} + \ldots + 2^3) -$

$2(2^{(k-2)} + 2^{(k-3)} + \ldots + 2) + r_{CE}(S_2^{\beta})$

$r_{CE}(S_k^{\beta}) \leq 2^{k-1}(2^k - 1) - 7$(since $r_{CE}(S_2^{\beta}) = 5$).

5 MACDONALD CODES CODES OF TYPE A *A*ND β OVER Z_4

The q-ary MacDonald code $M_{k,t}(q)$ over the finite field F_q is a unique $\left[\frac{q^k - q^t}{q-1}, k, q^{k-1} - q^{t-1}\right]$ code in which every non-zero codeword has weight either q^{k-1} or $q^{k-1} - q^{t-1}$ [8]. In [11], he studied the covering radius of MacDonald codes over a finite field. In fact, he has given many exact values for smaller dimension. In [6], authors have defined the MacDonald codes over a ring using the generator matrices of simplex codes. For $2 \leq t \leq k-1$, let $G_{k,t}^{\alpha}$ be the matrix obtained from $G_{k,}^{\alpha}$ by deleting columns corresponding to the columns of G_t^{α}. That is,

$$G_{k,t}^{\alpha} = \left[\ G_k^{\alpha} \setminus \frac{0}{G_t^{\alpha}}\ \right] \tag{5.1}$$

and let $G_{k,t}^{\beta}$ be the matrix obtained from G_k^{β} by deleting columns corresponding to the columns of G_t^{β}. That is,

$$G_{k,t}^{\beta} = \left[\ G_k^{\beta} \setminus \frac{0}{G_t^{\beta}}\ \right] \tag{5.2}$$

where $[A \setminus B]$ denotes the matrix obtained from the matrix A by deleting the columns of the matrix B and **0** is a$(k - t)$ x 2^{2t} (($k - t$) x $2^{t-1}(2^t - 1)$). The code generated by the matrix $G_{k,t}^{\alpha}$ is called code of type α and the code generated by the matrix $G_{k,t}^{\beta}$ is called Macdonald code of type β. The type α code is denoted by $M_{k,t}^{\alpha}$ and the type β code is denoted by $M_{k,t}^{\beta}$. The $M_{k,t}^{\alpha}$ code is $[4^k-4^t,k]$ code over Z_4 and $M_{k,t}^{\beta}$ is a $[(2^{k-1}-2^{t-1})(2^k+2^t-1),k]$ code over Z_4. In fact, these codes are punctured code of S_k^{α} and S_k^{β} respectively.

Next Theorem gives a basic bound on the covering radius of above Macdonald codes.

Theorem 5. 1.

$$r_{CE}(M_{k,t}^{\alpha}) \leq 2^{2k+1} - 2^{2r+1} + r_{CE}(M_{r,t}^{\alpha}) \text{ for } t < r \leq k,$$

Proof. By Proposition 2.1 and Theorem 3.2,

$$r_{CE}(M_{k,t}^{\alpha}) \leq r_{CE}(<\overbrace{11\cdots1}^{2^{2(k-1)}}\ \overbrace{22\cdots2}^{2^{2(k-1)}}\ \overbrace{33\cdots3}^{2^{2(k-1)}}>) + r_{CE}(M_{r,t}^{\alpha})$$

$= 6.4^{k-1} + r_{CE}(M^{\alpha}_{k-1,t})$, for k ≥ r > t.

$\leq 6.4^{k-1} + 6.4^{k-2} + ... + 6.4^{r} + r_{CE}(M^{\alpha}_{r,t})$ for k ≥ r > t.

$r_{CE}(M^{\alpha}_{k,t}) \leq 2^{2k+1} - 2^{2r+1} + r_{CE}(M^{\alpha}_{r,t})$, for k ≥ r > t. .

Theorem 5. 2.

$$r_{CE}(M^{\beta}_{k,t}) \leq 2^{k}(2^{k}-1) + 2^{r}(1-2^{r}) + r_{CE}(M^{\beta}_{r,t}) \text{ for } t < r \leq k.$$

Proof. Using Proposition 2.1 and Theorem 3.4, we have

$$r_{CE}(M^{\beta}_{k,t}) \leq r_{CE}(< \overbrace{11\cdots1}^{2^{2(k-1)}} \quad \overbrace{22\cdots2}^{2^{2(k-1)-1}-2^{(k-1)-1}} >) + r_{CE}(M^{\beta}_{k-1,t})$$

$\leq 2.22^{(k-1)+}2.2^{2(k-1)-1}-2^{(k-1)-1} + r_{CE}(M^{\beta}_{k-1,t})$

$= 2.2^{2(k-1)+}2.2^{2(k-1)-1}-2^{2(k-1)-1}+2.2^{2(k-2)}+2.2^{2(k-2)-1}-2.2^{2(k-2)-1} + r_{CE}(M^{\beta}_{k-2,t})$

$\leq 2.2^{2(k-1)+}2.2^{2(k-1)-1}-2^{2(k-1)-1}+2.2^{2(k-2)}+2.2^{2(k-2)-1}-2.2^{2(k-2)-1}+\ldots+$

$2.2^{2r}+2.2^{2r-1}+2.2^{r-1} + r_{CE}(M^{\beta}_{r,t})$

$= 2^{2k}-2^{2r}-2^{k}+2^{r} + r_{CE}(M^{\beta}_{r,t})$, t < r ≤ k.

$r_{CE}(M^{\beta}_{k,t}) \leq 2^{k}(2^{k}-1)+2^{r}(1-2^{r}) + r_{CE}(M^{\beta}_{r,t})$, t < r ≤ k.

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