ON THE COVERING RADIUS OF CODES OVER Z_6

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ABSTRACT

In this correspondence, we give lower and upper bounds on the covering radius of codes over the finite ring Z_6 with respect to different distances such as Hamming, Lee, Euclidean and Chinese Euclidean. We also determine the covering radius of various Block Repetition Codes over Z_6.

KEYWORDS

*Covering radius, Codes over finite rings, Hamming codes.**(2010) Mathematical Subject Classification:** 94B25, 94B05, 11H31, 11H71.*

1. INTRODUCTION

In the last decade, there are many researchers doing research on code over finite rings. There has much interest in codes over finite rings in recent years, especially the rings Z_{2k} where 2k denotes the ring of integers modulo 2k. In particular, codes over Z_4 have been widely studied [1, 2, 3, 4, 5]. Good binary linear and non-linear codes can be obtained from codes over Z_4 via the gray map. The covering radius of binary linear codes was studied [6, 7]. Recently the covering radius of codes over Z_4 has been investigated with respect to Lee, Euclidean distances [1, 10] and Chinese Euclidean distance [8]. In 1999, Sole et al gave many upper and lower bounds on the covering radius of a code over Z_4 with different distances. In the recent paper, the covering radius of some codes over Z_6 have been investigated. In this correspondence, we consider the finite ring is the set Z_6 of integers modulo 6.

A linear code C of length n over Z_6 is an additive subgroup of Z_6^n. An element of C is called a codeword of C and a generator matrix of C is a matrix whose rows generate C. The Hamming weight $w_H(x)$ of a vector x in Z_6^n is the number of non-zero coordinates.

In [11], the Lee weight for a codeword $x=(x_1,x_2,...,x_n) \in Z_6^n$ is defined by

$$w_L(x)= \sum_{i=1}^{n} \{| x_i || 6 - x_i |\}.$$

The Lee distance between the codeword's x and $y \in Z_6^n$ is defined as $d_L(x,y)=w_L(x-y)$.

The Euclidean weight is given by the relation

$$w_E(x)=\sum_{i=1}^{n}\left\{| x_i |^2 | 6 - x_i |^2\right\}$$

and Euclidean distance between the codeword's x and y$\in Z_6^n$ is defined as $d_E(x,y)=w_E(x-y)$.

The Chinese Euclidean weight $w_{CE}(x)$ of a vector x$\in Z_6^n$ is

$$\sum_{i=1}^{n}\left\{| 2 - 2\cos(\frac{2\pi x_i}{6}) |\right\}$$

and the Chinese Euclidean distance between the code words x and y $\in Z_6^n$ is defined as $d_{CE}(x,y)=w_{CE}(x-y)$.

The Hamming, Lee, Euclidean and Chinese Euclidean distances $d_H(x,y)$,$d_L(x,y)$, $d_E(x,y)$ and $d_{CE}(x,y)$ between two vectors x and y are $w_H(x-y)$, $w_L(x-y)$,$w_E(x-y)$ and $w_{CE}(x-y)$ respectively. The minimum Hamming, Lee, Euclidean and Chinese Euclidean weights d_H, d_L, d_E and d_{CE} of C are the smallest Hamming, Lee, Euclidean and Chinese Euclidean weights among all non-zero codewords of C respectively.

A linear Gray map ϕ from $Z_6^n \to Z_2^n \times Z_3^n$ is the coordinates-wise extension of the function from Z_6^n to $Z_2^n \times Z_3^n$ defined by $0\rightarrow(0, 0)$, $1\rightarrow(1,1)$, $2\rightarrow(0,2)$,

$3\rightarrow(1,0)$,$4\rightarrow(0,1)$ and $5\rightarrow(1,2)$. The image ϕ , of a linear code C over $\in Z_6^n$ of length n by the Gray map, is a mixed binary/ternary code of length 2n[11].

Two codes are said to be equivalent if one can be obtained from the other by permuting the coordinates or changing the signs of certain coordinates or multiplying non-zero element in a fixed column. Codes differing by only a permutation of coordinates are called permutation-equivalent.

Any linear code C over Z_6 is permutation-equivalent to a code with generator matrix G (the rows of G generate C) of the form

$$G=\begin{bmatrix} I_{k_1} & A_{1,2} & A_{1,3} & A_{1,4} \\ 0 & 2I_{k_2} & 2A_{2,3} & 2A_{2,4} \\ 0 & 0 & 3I_{k_3} & 3A_{3,4} \end{bmatrix}$$

Where $A_{\{i,j\}}$ are matrices with entries 0 or 1 for i > 1 and I_k is the identity matrix of order k. Such a code is said to have rank $\{1^{k1}, 2^{k2}, 3^{k3}\}$ or simply rank { k_1, k_2, k_3 } and $|C| = 6^{k1}3^{k2}2^{k3}$.
If $k_2 = k_3 = 0$, then the rank of C is $\{k_1, 0, 0\}$ or simply $k_1 = k$.

In this paper, we define the covering radius of codes over Z_6 with respect to different distances. Section 2 contains basic results for the covering radius of codes over Z_6.

Determines the covering radius of different types of block repetition codes are given in Section 3.

2. Covering Radius of Codes

Let d be the general distance out of various possible distances such as Hamming, Lee, Euclidean and Chinese Euclidean. The covering radius of a code C over Z_6 with respect to a general distances d is given by $r_d(C) = \max_{u \in z_6^n} \{\min_{c \in C} \{d(u,c)\}\}$

The following result of Mattson [6] is useful for computing covering radius of codes over rings generalized easily from codes over finite fields.

Proposition 2. 1. If C_0 and C_1 are codes over Z_6 generated by matrices G_0 and G_1 respectively and if C is the code generated by $G = \begin{pmatrix} 0 & G_1 \\ G_0 & A \end{pmatrix}$ then $r_d(C) \le r_d(C_0) + r_d(C_1)$ and the covering radius of D (concatenation of C_0 and C_1) satisfy the following $r_d(D) \ge r_d(C_0) + r_d(C_1)$, for all distances d over Z_6.

3. Covering Radius of Repetition Codes

A q-ary repetition code C over a finite field $F_q = \{\alpha_0 = 0, \alpha_1 = 1, \alpha_2, \alpha_3, \ldots, \alpha_{q-1}\}$ is an [n, 1, n] code $C = \{\overline{\alpha} | \alpha \in F_q\}$, where $\overline{\alpha} = (\alpha, \alpha, \cdots, \alpha)$. The covering radius is $\left\lceil \frac{n(q-1)}{q} \right\rceil$ [9]. Using this, it can be seen easily that the covering radius of block of size n repetition code [n(q-1),1,n(q-1)] generated by

$G = \left[\overbrace{11\cdots1}^{n} \overbrace{\alpha_2\alpha_2\cdots\alpha_2}^{n} \overbrace{\alpha_3\alpha_3\cdots\alpha_3}^{n} \cdots \overbrace{\alpha_{q-1}\alpha_{q-1}\cdots\alpha_{q-1}}^{n} \right]$ is $\left\lceil \frac{n(q-1)^2}{q} \right\rceil$ since it will be equivalent to a repetition code of length (q − 1)n.

Consider the repetition code over Z_6. There are two types of repetition codes of length n viz.

1. unit repetition code C_β: [n, 1, n, n, n, n] generated by $G_\beta = [\overbrace{11\cdots1}^{n}]$.
2. zero repetition code C_α : (n, 2, n, 3n, 9n, 4n) generated by $G_\beta = [\overbrace{33\cdots3}^{n}]$ and

   C_γ : (n, 3, n, 2n, 4n, 3n) generated by $G_\beta = [\overbrace{2424\cdots24}^{n}]$ or $[\overbrace{4242\cdots42}^{n}]$.

The code generated by [22 ... 2] and [44... 4] are equivalent to the code C_γ.

The following result determines the covering radius with respect to the Lee distance, Euclidean distance and Chinese Euclidean distance.

Theorem 3. 1.

$$r_L(C_\alpha)=\frac{3n}{2},\quad n\le r_L(C_\gamma)\le\frac{5n}{3},\quad r_L(C_\beta)=\frac{3n}{2}.$$

Proof. By the definition $r_L(C_\alpha)=\max_{x\in z_6^n}\left\{\min_{c\in C}\{d(x,c)\}\right\}$. Let $x=\overbrace{33...3}^{\frac{n}{2}}\overbrace{00...0}^{\frac{n}{2}}\in Z_6^n$. The code $C_\alpha=\{\alpha(33....3)\mid\alpha\in Z_6^n\}$, that is C_α={00....0, 3 3.... 3}, generated by [3 3.... 3] is an (n, 2, n) code.

Then d_L(x, 00....0)= $wt_L(\overbrace{33...3}^{\frac{n}{2}}\overbrace{00...0}^{\frac{n}{2}}-00....0)=\frac{n}{2}$ wt_L (3) = $\frac{3n}{2}$, since the Lee weight of 3 is 3 and d_L(x,33.... 3) = $\frac{3n}{2}$. Therefore, d_L(x, C_α) = min{ $\frac{3n}{2}, \frac{3n}{2}$} = $\frac{3n}{2}$ and hence $r_L(C_\alpha)\ge\frac{3n}{2}$. Let x be any word in Z_6^n . Let us take x has ω_0 coordinates as 0's, ω_1 coordinates as 1's, ω_2 coordinates as 2's, ω_3 coordinates as 3's, ω_4 coordinates as 4's and ω_5 coordinates as 5's, then $\omega_0 + \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 = n$. Since C_α= { 00.... 0, 33…. 3} and Lee weight of 0 is 0, 1, 5 is 1, 2, 4 is 2 and 3 is 3, d_L(x,00…0) = n- $\omega_0 + \omega_2 + 2\omega_3 + \omega_4$ and d_L(x, 33…. 3) = n- $\omega_3 + 2\omega_0 + \omega_1 + \omega_5$.

Thus d_L(x, C_α)=min{ n- $\omega_0 + \omega_2 + 2\omega_3 + \omega_4$, n- $\omega_3 + 2\omega_0 + \omega_1 + \omega_5$ }. Since the minimum of { n- $\omega_0 + \omega_2 + 2\omega_3 + \omega_4$, n- $\omega_3 + 2\omega_0 + \omega_1 + \omega_5$ } is less than or equal to its average, implies d_L(x, C_α) ≤ n+$\frac{n}{2}$ = $\frac{3n}{2}$ and $r_L(C_\alpha)\le\frac{3n}{2}$. Hence, $r_L(C_\alpha)=\frac{3n}{2}$. The correspondent arguments of γ type, so, n≤ $r_L(C_\alpha)\le\frac{5n}{2}$. The covering radius $r_L(C_\beta)\le\frac{3n}{2}$. To show that $r_L(C_\beta)\ge\frac{3n}{2}$.

Let $x=\overbrace{00\cdots0}^{t}\overbrace{11\cdots1}^{t}\overbrace{22\cdots2}^{t}\overbrace{33\cdots3}^{t}\overbrace{44\cdots4}^{t}\overbrace{55\cdots5}^{n-5t}\in Z_6^n$, where $t=\left\lfloor\frac{n}{6}\right\rfloor$, then

d_L(x,00… 0) = n+3t, d_L(x,11...1) =2n-3t, d_L(x, 22… 2) =3n-9t, d_L(x, 33…. 3) = 2n-2t,

d_L(x,44…. 4) = n+3t and d_L(x,55…. 5) = 9t. Therefore $r_L(C_\beta)\ge\min\{n+3t,2n-3t,3n-9t,9t\}\ge\frac{3n}{2}$ and $r_L(C_\beta)=\frac{3n}{2}$.

The above similar arguments can be used to compute the covering radius of Euclidean weight and Chinese Euclidean weight for the α type, β type and γ type codes over Z_6 (Euclidean weight of Z_6 of 0 is 0, 1 and 5 are 1, 2 and 4 are 4 and 3 is 9 and Chinese Euclidean weight of Z_6 of 0 is 0, 1 and 5 are 1, 2 and 4 are 3 and 3 is 4). We have the following theorem

Theorem 3. 2.

$$r_E(C_\alpha)=\frac{9n}{2},\quad 2n\le r_E(C_\gamma)\le\frac{11n}{3}\ \text{ and }\ r_E(C_\beta)=\frac{19n}{6}.$$

Theorem 3. 3.

$$r_{CE}(C_\alpha) = 2n, \quad \frac{3n}{2} \le r_{CE}(C_\gamma) \le 2n \quad \text{and} \quad r_{CE}(C_\beta) = 2n..$$

In order to determine the covering radius of Z_6 two blocks each of size n repetition code $BRep^{2n}$: [2n, 1, n, 2n, 4n, n] generated by $G = [\overbrace{11\cdots1}^{n}\overbrace{33\cdots3}^{n}]$. We have following theorem

Theorem 3. 4.

Let C be a code over Z_6 generated by the matrix $G = [\overbrace{11\cdots1}^{n}\overbrace{33\cdots3}^{n}]$, then $r_L(BRep^{2n})$ = 3n, $r_E(\{BRep^{2n}) = \frac{46n}{6}$ and $r_{CE}(BRep^{2n})$=4n.

Proof.

By Theorem 3.1, the Proposition 2.1 and the given generator matrix G, we get

$$r_L(\{BRep^{2n}) \ge 3n \qquad (3.1)$$

For the reverse inequality, let x=(v|w) $\in Z_6^{2n}$ and let us take in v, 0 appears r_0 times, 1 appears r_1 times, 2 appears r_2 times, 3 appears r_3 times, 4 appears r_4 times and 5 appears $r_5$times and in w, 0 appears s_0 times, 1 appears s_1 times, 2 appears s_2 times, 3 appears s_3 times, 4 appears s_4 times and 5 appears s_5 times with $\sum_{i=0}^{5} r_i = n = \sum_{i=0}^{5} s_i$. Then $d_L(x,c_0)$= $2n-r_0+r_2+2r_3+r_4-s_0+s_2+2s_3+s_4$, $d_L(x, c_1)$= $2n-r_1+r_3+2r_4+r_5-s_3+2s_0+s_1+s_5$,$d_L(x,c_2)$= $2n-r_2+r_0+r_4+2r_5-s_0+s_2+2s_3+s_4$, $d_L(x,c_3)$=$2n-r_3+2r_0+r_1+r_5-s_3+2s_0+s_1+s_5$, $d_L(x,c_4)$= $2n-r_4+r_0+2r_1+r_2-s_0+s_2+2s_3+s_4$ and $d_L(x,c_5)$ = $2n-r_5+r_1+2r_2+r_3-s_3+2s_0+s_1+s_5$.

We get

$d_L(x, BRep^{2n})$ = min{$d_L(x,c_0)$, $d_L(x,c_1)$, $d_L(x,c_2)$, $d_L(x,c_3)$, $d_L(x,c_4)$, $d_L(x,c_5)$}

$$\le \{2n-r_0+r_2+2r_3+r_4-s_0+s_2+2s_3+s_4+ 2n-r_1+r_3+2r_4+r_5-s_3+2s_0+s_1+s_5+ 2n- r_2+r_0+r_4+2r_5-s_0+s_2+2s_3+s_4+ 2n-r_3+2r_0+r_1+r_5-s_3+2s_0+s_1+s_5+ 2n-r_4+r_0+2r_1+r_2-s_0+s_2+2s_3+s_4+2n-r_5+r_1+2r_2+r_3-s_3+2s_0+s_1+s_5\}/6.$$

Therefore, $d_L(x, BRep^{2n}) \le 3n$. Thus $r_L(\{BRep^{2n}) \le 3n$ (3.2)

By the Equations (3.1) and (3.2), so $r_L(BRep^{2n}) = 3n.$

Similarly, $r_E(BRep^{2n}) = \frac{46n}{6}$ and $r_{CE}(BRep^{2n})$ = 4n.

One can also define a Z_6 codes of three blocks each of size n repetition code $BRep^{3n}$:[3n, 1, 2n, 4n, 8n, 6n] generated by $G=[\overbrace{11\cdots1}^{n}\overbrace{22\cdots2}^{n}\overbrace{33\cdots3}^{n}]$. The proof of the theorem 3. 5 and 3. 6 is similar to the theorem 3. 4, we can state following

Theorem 3. 5.

Let C be a code over Z_6 generated by the matrix $G=[\overbrace{11\cdots1}^{n}\overbrace{22\cdots2}^{n}\overbrace{33\cdots3}^{n}]$. then

1 . $4n \le r_L(BRep^{3n}) \le \frac{9n}{2}$,

2. $\frac{29n}{3} \le r_E(BRep^{3n}) \le \frac{34n}{3}$ and

3. $\frac{11n}{2} \le r_{CE}(BRep^{3n}) \le \frac{37n}{6}$.

In Z_6, the four blocks each of size n repetition code $BRep^{4n}$: [4n, 1, 2n, 6n, 12n, 8n] generated by $G=[\overbrace{11\cdots1}^{n}\overbrace{22\cdots2}^{n}\overbrace{33\cdots3}^{n}\overbrace{44\cdots4}^{n}]$. We have following theorem

Theorem 3. 6.

Let C be a code over Z_6 generated by the matrix $G=[\overbrace{11\cdots1}^{n}\overbrace{22\cdots2}^{n}\overbrace{33\cdots3}^{n}\overbrace{44\cdots4}^{n}]$, then

1. $5n \le r_L(BRep^{4n}) \le \frac{19n}{3}$,

2. $\frac{38n}{3} \le r_E(BRep^{4n}) \le \frac{45n}{3}$ and

3. $7n \le r_{CE}(BRep^{4n}) \le \frac{49n}{6}$.

In order to determine the covering radius of Z_6 codes of the five blocks each of size n repetition code $BRep^{5n}$: [5n, 1, 3n, 8n, 16n, 12n] generated by

$$G=[\overbrace{11\cdots1}^{n}\overbrace{22\cdots2}^{n}\overbrace{33\cdots3}^{n}\overbrace{44\cdots4}^{n}\overbrace{55\cdots5}^{n}].$$

We have

Theorem 3. 7.

Let C be a code generated by the matrix $G=[\overbrace{11\cdots1}^{n}\overbrace{22\cdots2}^{n}\overbrace{33\cdots3}^{n}\overbrace{44\cdots4}^{n}\overbrace{55\cdots5}^{n}]$.

Then,

1. $\frac{13n}{2} \le r_L(B\mathrm{Re}\,p^{5n}) \le \frac{47n}{6}$,
2. $\frac{89n}{6} \le r_E(B\mathrm{Re}\,p^{5n}) \le \frac{109n}{6}$ and
3. $9n \le r_{CE}(BRep^{5n}) \le \frac{55n}{6}$.

The two different length of Block repetition code of size m and n is $BRep^{m+n}$: [m+n, 1, m, min {2m,m+2n}, min {4m, 4m+6n}, min{3m,3m+2n}] generated by

$G = [\overbrace{11\cdots1}^{m}\overbrace{33\cdots3}^{n}]$. We have the following theorem

Theorem 3. 8.

- $\frac{3m+3n}{2} \le r_L(BRep^{m+n}) \le \frac{3m+4n}{2}$
- $r_E(BRep^{m+n}) = \frac{19m+27n}{6}$ and
- $r_{CE}(BRep^{m+n}) = 2m+2n$.$

Proof.

By theorem 3.1 and by the above generator matrix

$$r_L(BRep^{m+n}) \ge \frac{3m}{2} + \frac{3n}{2} = \frac{3m+3n}{2} \qquad (3.3)$$

Let z = (x | y) $\in Z_6^{m+n}$ where x$\in Z_6^m$ and y$\in Z_6^n$. Let us take x has m_0 times 0 as coordinates, m_1 times 1 as coordinates, m_2 times 2 as coordinates m_3 times 3 as coordinates
m_4 times 4 as coordinates and m_5 times 5 as coordinates and y has n_0 times 0 as coordinates, $n_1$times 1 as coordinates, n_2 times 2 as coordinates n_3 times 3 as coordinates n_4 times 4 as coordinates and n_5 times 5 as coordinates such that $\sum_{i=0}^{5} m_i = m$ and $\sum_{i=0}^{5} n_i = n$. Then by the above Matrix, the code is C= {c_0= (00....0 00....0), c_1=(11....1 3 3.... 3),c_2=(2 2.... 2 0....0), c_3 = (3 3.... 3 3 3..... 3), c_4= (4 4 ..... 4 0 0 .... 0), c_5=(5 5.... 5 3 3 ... 3)}.

$d_L(z,c_0) = wt_L(z-c_0) = wt_L((x|y)-c_0) = wt_L(x-c_0)+wt_L(y-c_0)$
$= m_1+2m_2+3m_3+4m_4+5m_5+n_1+2n_2+3n_3+4n_4+5n_5$, since the Lee weight of 2, 4 is 2 and 3 is 3 and 1, 5 is 1.

Thus $d_L(z,c_0) = m+n-m_0+m_2+2m_3+m_4-n_0+n_2+2n_3+n_4$.

Similarly, $d_L(z,c_1) = m+n-m_1+m_3+2m_4+m_5-n_3+2n_0+n_1+2n_5$,

$d_L(z,c_2) = m+n-m_2+m_0+1m_4+2m_5-n_0+n_2+2n_3+n_4$,

$d_L(z,c_3) = m+n-m_3+2m_0+m_1+2m_5 -n_3+2n_0+n_1+2n_5$,
$d_L(z,c_4) = m+n-m_4+m_0+2m_1+m_2-n_0+n_2+2n_3+n_4$ and
$d_L(z,c_5) = m+n-m_5+m_1+2m_2+m_3 -n_3+2n_0+n_1+2n_5$.

Therefore,

$$\begin{aligned} d_L(z, BRep^{m+n}) \le & \{d_L(z,c_0)+d_L(z,c_1)+d_L(z,c_2)+d_L(z,c_3)+d_L(z,c_4\})+d_L(z,c_5)\}/6 \\ = & \{ m+n-m_0+m_2+2m_3+m_4-n_0+n_2+2n_3+n_4 + m+n-m_1+m_3+2m_4+ \\ & m_{5-} n_3+2n_0+n_1+2n_5 + m+n-m_2+m_0+1m_4+2m_5 -n_0+n_2+2n_3+ \\ & n_4 +m+n-m_3+2m_0+m_1+2m_5 -n_3+2n_0+n_1+2n_5 + m+n-m_4+m_0+ \\ & 2m_1+m_2- n_0+n_2+2n_3+n_4 +m+n-m_5+m_1+2m_2+m_3 -n_3+2n_0+n_1+2n_5\}/6 \end{aligned}$$

$$\begin{aligned} d_L(z, BRep^{m+n}) &= m+n+\{3m+3n+3n_5\}/6 \\ &= m+n+3m+3n+3n/6, \text{ since } n_5\le n \\ d_L(z, BRep^{m+n}) &= \{3m+4n\}/2. \end{aligned}$$

Thus $r_L(BRep^{m+n}) \le \{3m+4n\}/2$ (3.4)

From equation (3.3) and (3.4), $\{3m+3n\}/2\le r_L(\{BRep^{m+n}) \le\{3m+4n\}/ 2$. Similar arguments of above, we have $r_E(BRep^{m+n})= \{19m+27n\}/6$ and $r_{CE}(BRep^{m+n})=2m+2n$.

In a three different Block repetition code of length is m, n and o is $BRep^{m+n+o}$: [m+n+o, 1, 2m, min{4m, 2m+2n+2o}, min{8m, 8m+4n+4o}, min{6m, 6m+n+o}] generated by $G=[\overbrace{11\cdots1}^{m}\overbrace{22..2}^{n}\overbrace{33\cdots3}^{o}]$. We have the following theorem

Theorem 3.9.

- $\{m+2n+2o\}/2\le r_L(BRep^{m+n+o}) \le\{9m+10n+9o\}/ 6$,
- $\{13m+9n+12o\}/6\le r_E(BRep^{m+n+o}) \le\{13m+16n+15o\}/6$ and
- $2m+\{3n\}/2+2o \le r_{CE}(BRep^{m+n+o}) \le 2m+2n+2o$.

Four different Block repetition code of length are m, n, o and p: $BRep^{m+n+o+p}$: [m+n+o+p, 1, 2m, min{6m,2m+2n+2o+2p},min{12m, 12m+2n+2o+2p}, min {8m, 3m+2n+2o+2p}]generated by

$$G=[\overbrace{11\cdots1}^{m}\overbrace{22.\cdots2}^{n}\overbrace{33\cdots3}^{o}\overbrace{44\cdots4}^{p}]$$. We have the following

Theorem 3.10.

- $m+n+o+p \le r_L(BRep^{m+n+o+p}) \le\{9m+10n+9o+10p\}/6$,
- $\{13m+12n+9o+12p\}/6\le r_E(BRep^{m+n+o+p}) \le\{13m+16n+15o+16p\}/6$ and
- $2m+\{3n\}/2+2o+\{3p\}/2\le r_{CE}(BRep^{m+n+o+p}) \le 2m+\{13n\}/6+2o+2p$.

The five different Block repetition code of length of size are m, n, o, p and q, $BRep^{m+n+o+p+q}$: [m+n+o+p+q, 1, 3m, min{8m, 2m+2n+2o+2p+q}, min{16m, 4m+4n+4o +4p+3q}, min {12m, 3m+3n+3o+2p+1q}] generated by

$$G=[\overbrace{11\cdots1}^{m}\overbrace{22\cdots2}^{n}\overbrace{33\cdots3}^{o}\overbrace{44\cdots4}^{p}\overbrace{55\cdots5}^{q}]$$. We have

Theorem 3.11.

- $\{3m+2n+3o+2p+3q\}/2 \le r_L(\{BRep^{m+n+o+p+q}) \le \{9m+10n+9o+10p+9q\}/6$,
- $\{19m+12n+27o+12p+19q\}/6 \le r_E(BRep^{m+n+o+p+q}) \le \{19m+22n+27o+22p+19q\}/6$ and
- $\{4m+3n+4o+3p+4q\}/2 \le r_{CE}(BRep^{m+n+o+p+q}) \le 2m+2n+2o+2p+2q$.

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