SKEW CONSTACYCLIC CODES OVER THE RING
\( F_q[u; v]/ < u^2 - 1, v^2 - 1, uv - vu > \)

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ABSTRACT

Skew constacyclic codes are displaying vital role in the field of coding theory. This paper is mainly focused on skew \(-\alpha\) constacyclic codes over \( R = F_q + uF_q + vF_q + uvF_q, \) with \( u^2 = v^2 = 1, \) \( uv = vu, q = 3^m, \) and \( \alpha \) is an unit of \( R \) fixed by the automorphism \( \theta. \) It is briefly derived as well that the Gray map of skew cyclic code of length \( n \) over \( R \) is skew quasicyclic code of length \( 4n \) over \( F_q \) of index 4. Furthermore, the decomposition of a skew \( \alpha \) constacyclic codes over \( R \) are also determined. A complete analysis about the relationship between skew \( \alpha \) constacyclic code and its dual is also carried out.

KEYWORDS

Gray map, skew constacyclic codes, finite ring, negacyclic codes, skew polynomial ring.

1. INTRODUCTION

Skew cyclic codes over finite ring has gotten much attention of the researchers since after the first idea of skew cyclic codes which is a natural generalization of cyclic codes on skew polynomial ring \((1) \ (8)\) . L.F.T. Cuitio et al. determined some basic
properties of skew generalized cyclic (GC) codes over finite fields ([9]). Furthermore, T. Yao et al. also studied the decomposition of skew cyclic code over the finite chain ring \( H = F_q + uF_q + vF_q + uvF_q \) with \( u^2 = u, \ v^2 = v, \ uv = vu, \ q = p^m \), with \( p \) odd prime (according to Frobenius automorphism)([10]). Later on, H. Islam et al. introduced the Frobenius automorphism over \( H \) ([11]). Similarly, A. Dertli et al. defined the Gray map from \( F_q[u,v] \) \((u^2-1, v^2-1, uv-vu)\) to \( F_q^4 \), and determined the decomposition of skew cyclic code over the ring \( F_q[u,v] \) \((u^2-1, v^2-1, uv-vu)\) ([12]).

In this article we have described the skew \((a + a u + a v + a uv)\)-constacyclic code over the ring \( F_q[u,v] \) \((u^2-1, v^2-1, uv-vu)\), where \( q = 3^m \), and is also proved that the Gray image of skew cyclic code of length \( n \) over \( F_q[u,v] \) \((u^2-1, v^2-1, uv-vu)\) is skew quasi-cyclic code over \( F_q \) having length \( 4n \) and index 4. This paper is organized as follow:

In section 2, we will give some properties about the ring \( R = F_q[u,v] \) \((u^2-1, v^2-1, uv-vu)\), and recall some auxiliary results in coding theory. Section 3 define the Gray map, and describe the \( F_q \)-image of skew cyclic code over \( R \), and characterization of skew constacyclic codes over \( R \) is presented in section 4.

2 Preliminaries

Let \( q = 3^m \) and \( R = F_q[u,v] \) \((u^2-1, v^2-1, uv-vu)\) which is isomorphic to \( F_q + uF_q + vF_q + uvF_q \), with \( u^2 = v^2 = 1 \). The ring has four maximal ideals with index of stability 1, which are given in the following:

\( m_1 = < u + 1, v + 1 > \), \( m_2 = < u + 1, v - 1 > \),
\( m_3 = < u - 1, v + 1 > \), and \( m_4 = < u - 1, v - 1 > \).

And the map \( \psi \) defined by

\[
\psi : R \rightarrow \frac{R}{m_1} \times \frac{R}{m_2} \times \frac{R}{m_3} \times \frac{R}{m_4} \cong F_q^4 \quad x \mapsto (x + m_1, x + m_2, x + m_3, x + m_4)
\]
is a canonical isomorphism. According to A.Dertli et al. ([12]) we recall the following definitions

**Definition 2.1.** The Frobenius automorphism $\theta$ acting on $F_q$ defined by $\theta(y) = y^{p^i}$ for $y \in F_q$ induces on automorphism of the ring $R$ as: $\forall \alpha = r_1 + r_2 u + r_3 v + r_4 u v \in R, \theta(\alpha) = r_1^{p^i} + r_2^{p^i} u + r_3^{p^i} v + r_4^{p^i} u v$. Then by the help of this $R$-authomorphism, we can define a ring as:

$$R[y, \theta] = \{ r_0 + r_1 y + r_2 y^2 + \cdots + r_n y^n : r_i \in R, 1 \leq i \leq n \}$$

with the multiplication denoted by $\cdot$ such that $(ry^i) \cdot (ly^j) = r\theta^i(l^{i+j})$, and by the usual addition of polynomials ring, $R[y, \theta]$ is called skew polynomial ring which is non-commutative unless $\theta$ is identity.

**Definition 2.2.** The linear code $C$ of length $n$ over $R$ is called skew $\alpha$-constacyclic with the automorphism $\theta$, where $\alpha$ is unit of $R$ fixed by $\theta$ if $\forall r = (r_1, r_2, \cdots, r_n) \in C \Rightarrow \tau(r) = (\alpha \theta((r_n)), \theta(r_1), \cdots, \theta(r_{n-1})) \in C$. In particular, if $\alpha = 1$, then $C$ is called skew cyclic codes, and if $\alpha = -1$ then $C$ is said to be skew negacyclic code over $R$.

While on another side, if $\alpha = 1$, assuming $\tau(r) = (\theta((r_n)), \theta(r_1), \cdots, \theta(r_{n-1})) = \sigma(r)$.

**Definition 2.3.** Let $C$ be a linear code of length $ln$ over $F_q$. If $\pi_l(C) = C$, since $\pi_l$ defined by

$$\pi_l(r^1 | r^2 | \cdots | r^l) = (\sigma(r^1) | \sigma(r^2) \cdots | \sigma(r^l)), \text{ for } r^i \in F_q^n, 1 \leq i \leq l$$

, then $C$ is named by skew quasi-cyclic code.

**Definition 2.4.** Set $C^\perp = \{ \bar{y} = (y_1, \cdots, y_n) \in R^n \mid \sum_{i=1}^n z_i y_i = 0, \forall \bar{z} = (z_1, \cdots, z_n) \in C \}$, then $C^\perp$ the dual code of $C$. 


3 Gray map and Gray image of skew cyclic code over \( R \)

According to the fundamental result on the decomposition of modules, we have the following lemma:

**Lemma 3.1.** ([13]) Let \( R \) be a finite ring and \( I_1, I_2, \ldots, I_n \) be ideals of \( R \). The following statements are equivalent:

- \( R = I_1 \oplus I_2 \oplus \cdots \oplus I_n \).
- There exists a unique family of idempotents \((\beta_i)_{1 \leq i \leq n}\) such that \( \beta_i^2 = \beta_i \), \( \beta_i \beta_j = 0 \) for \( i \neq j \), \( \sum_{i=1}^{n} \beta_i = 1 \) and \( I_i = \beta_i R, \forall i, j \in \{1, \cdots n\} \).

By [12] we recall the following family idempotent is given by \( \beta_1, \beta_2, \beta_3, \beta_4 \) where,

\[
\begin{align*}
\beta_1 &= 1 + u + v + uv, \\
\beta_2 &= 1 - u - v - uv, \\
\beta_3 &= 1 - u + v - uv, \\
\beta_4 &= 1 - u - v + uv.
\end{align*}
\]

Any element of \( a \) of \( R \) can be written as

\[
a = a_1 + a_2 u + a_3 v + a_4 uv = \\
\beta_1(a_1 + a_2 + a_3 + a_4) + \beta_2(a_1 + a_2 - a_3 - a_4) + \beta_3(a_1 - a_2 + a_3 - a_4) + \beta_4(a_1 - a_2 - a_3 + a_4).
\]

Let \( C \) be a linear code of length \( n \) over \( R \), and:

\[
\begin{align*}
C_1 &= \{a_1 + a_2 + a_3 + a_4 \in F_q^n | a_1 + a_2 u + a_3 v + a_4 uv \in C\}, \\
C_2 &= \{a_1 + a_2 - a_3 - a_4 \in F_q^n | a_1 + a_2 u + a_3 v + a_4 uv \in C\}, \\
C_3 &= \{a_1 - a_2 + a_3 - a_4 \in F_q^n | a_1 + a_2 u + a_3 v + a_4 uv \in C\}, \\
C_4 &= \{a_1 - a_2 - a_3 + a_4 \in F_q^n | a_1 + a_2 u + a_3 v + a_4 uv \in C\}.
\end{align*}
\]

Assume that \( C = \beta_1 C_1 + \beta_2 C_2 + \beta_3 C_3 + \beta_4 C_4 \).

**Lemma 3.2.** if \( G_1, G_2, G_3, \) and \( G_4 \) are generator matrices of Linear codes \( C_1, C_2, C_3, \) and \( C_4 \) respectively, then the generator matrix of the linear code \( C \) which is defined above has the following generator matrix

\[
\begin{align*}
\end{align*}
\]
Let $\eta$ be the map:

$$\eta : \frac{R[y,\theta]}{(y^n-1)} \to \frac{R[y,\theta]}{(y+1)}$$

$$h(y) \mapsto h(-y)$$

**Theorem 3.3.** if $n$ is odd then $\eta$ is a left $R$-module isomorphism.

**Proof.** Firstly, we have to prove that $\eta$ is well-defined.

If $h_1(y) \equiv h_2(y) \pmod {(y^n-1)} \iff \exists p(y) \in R[y,\theta]$

$$\iff h_1(y) = h_2(y) + p(y)(y^n-1)$$

$$\iff h_1(-y) = h_2(-y) + p(-y)((-y)^n-1)$$

$$\iff \eta(h_1(y)) \equiv \eta(h_2(y)) \text{ (since $n$ is odd)}. \text{ Hence it can be concluded that } \eta \text{ is well-defined.}$$

Secondly, $\eta$ is $R$-module monomorphism injective, and $\frac{R[y,\theta]}{(y^n-1)}$, $\frac{R[y,\theta]}{(y^n+1)}$ are finite rings which implies that $\eta$ is a left $R$-module isomorphism.

By theorem 3.3 one can obtain the below corollary:

**Corollary 3.4.** suppose that $C$ is a linear code of odd length $n$. Then $C$ is skew-cyclic code with respect to the automorphism $\theta$ over $R$ if and only if $\eta(C)$ is skew-negacyclic code over $R$.

**Proof.** if $C$ is skew-cyclic code over $R \iff C$ is an ideal of $\frac{R[y,\theta]}{(y^n-1)}$ since $n$ is odd (by 3.3) $\eta(C)$ is also an ideal of $\frac{R[y,\theta]}{(y^n+1)} \iff \eta(C)$ is skew-negacyclic code over $R$.

**Lemma 3.5.** Let $f_\pi$ the permutation version of Gray map from $R^n$ to $F_q^{4n}$ given by

$$f_\pi(r = (r_0, r_1, \cdots, r_{n-1})) = (\tilde{a}, \tilde{b}, \tilde{c}, \tilde{d}) \text{ with}$$
\[ \vec{a} = (r_0^0 + r_0^1 + r_0^2 + r_0^3, r_1^0 + r_1^1 + r_1^2 + r_1^3, \ldots, r_{n-1}^0 + r_{n-1}^1 + r_{n-1}^2 + r_{n-1}^3) \]
\[ \vec{b} = (r_0^0 + r_0^1 - r_0^2 - r_0^3, r_1^0 + r_1^1 - r_1^2 - r_1^3, \ldots, r_{n-1}^0 - r_{n-1}^1 + r_{n-1}^2 - r_{n-1}^3) \]
\[ \vec{c} = (r_0^0 - r_0^1 + r_0^2 - r_0^3, r_1^0 - r_1^1 + r_1^2 - r_1^3, \ldots, r_{n-1}^0 - r_{n-1}^1 + r_{n-1}^2 - r_{n-1}^3) \]
\[ \vec{d} = (r_0^0 - r_0^1 - r_0^2 + r_0^3, r_1^0 - r_1^1 - r_1^2 + r_1^3, \ldots, r_{n-1}^0 - r_{n-1}^1 - r_{n-1}^2 + r_{n-1}^3) \]
where
\[ r_i = r_i^0 + r_i^1 u + r_i^2 v + r_i^3 w \in R \text{ for } 0 \leq i \leq n - 1. \]

Let \( \pi_4 \) be the operator shift of quasi-cyclic, and \( \sigma \) is the skew-cyclic shift which are defined in preliminary. Then we have \( f_\pi \sigma = \pi_4 f_\pi \)

**Proof.** Let \( r = (r_0, r_1, \ldots, r_{n-1}) \in R \) such that
\[ r_i = r_i^0 + r_i^1 u + r_i^2 v + r_i^3 w, \text{ for } 0 \leq i \leq n - 1 \]
\[ f_\pi \sigma(r) = f_\pi(\theta(r_{n-1}), \theta(r_0), \ldots, \theta(r_{n-2})), \text{ with } \theta(r_i) = r_i^0 \eta^t + r_i^1 \eta^t u + r_i^2 \eta^t v + r_i^3 \eta^t w \]
for \( 0 \leq i \leq n - 1. \)

On another side \( \pi_4 f_\pi(r) = \pi_4(r_0^0 + r_0^1 + r_1^2 + r_0^3, r_1^0 + r_1^1 + r_1^2 + r_1^3, \ldots, r_{n-1}^0 + r_{n-1}^1 + r_{n-1}^2 + r_{n-1}^3) \)

**Theorem 3.6.** A linear code \( C \) of length \( n \) is skew cyclic over \( R \) if and only if \( f_\pi(C) \) is a skew quasi-cyclic code of length \( 4n \) over \( F_q \) of index 4.

**Proof.** Let \( C \) be a linear code of length \( n \). Suppose that \( C \) is skew cyclic code i.e \( \sigma(C) = C \) then \( f_\pi(\sigma(C)) = f_\pi(C) \). By lemma 3.5 we have \( \pi_4(f_\pi(C)) = f_\pi(\sigma(C)) \), which implies that \( \pi_4(f_\pi(C)) = f_\pi(C) \), i.e, \( f_\pi(C) \) is skew quasi-cyclic code of length \( 4n \).

Conversely, let \( f_\pi(C) \) be a skew quasi-cyclic code of index 4. By applying lemma 3.5 which gives us \( \pi_4 f_\pi(C) = f_\pi(C) \), we can get \( f_\pi \sigma(C) = f_\pi(C) \).

As \( f_\pi \) is injective, then \( \sigma(C) = C \), which implies \( C \) is skew cyclic code. \( \square \)
4 Skew constacyclic codes over $R$

In this part we have described the decomposition of skew $\alpha$-constacyclic codes over $R$, since $\alpha$ is unit of $R$ fixed by $\theta$.

S. Jitman et al. ([14]) have proved that a linear code of length $n$ is skew $\lambda$-constacyclic code over finite ring $M$ if and only if the skew representation polynomial of $C$ is a left ideal of the ring $\frac{M[x,\theta]}{(x^n-\lambda)}$.

Any element $\alpha_0 + \alpha_1 u + \alpha_2 v + \alpha_3 uv \in R$ can be uniquely expressed as

$$\alpha_0 + \alpha_1 u + \alpha_2 v + \alpha_3 uv = (\alpha_0 + \alpha_1 + \alpha_2 + \alpha_3)\beta_1 + (\alpha_0 + \alpha_1 - \alpha_2 - \alpha_3)\beta_2 + (\alpha_0 - \alpha_1 + \alpha_2 - \alpha_3)\beta_3 + (\alpha_0 - \alpha_1 - \alpha_2 + \alpha_3)\beta_4.$$

Now the decomposition of skew $\alpha$-constacyclic codes over $R$ can be given as following

**Theorem 4.1.** Let $\alpha = \alpha = \alpha_0 + \alpha_1 u + \alpha_2 v + \alpha_3 uv$ be a unit of $R$ fixed by $\theta$. Suppose that $C$ is a linear code over $R$ can be written as a direct sum of linear codes as follow:

$$C = \beta_1 C_1 \oplus \beta_2 C_2 \oplus \beta_3 C_3 \oplus \beta_4 C_4,$$

where $C_i$, for $1 \leq i \leq 4$ are linear codes over $F_q$. Then $C$ is skew $\alpha$-constacyclic code with automorphism $\theta$ over $R$ if and only if $C_1$ is skew $[\alpha_0 + \alpha_1 + \alpha_2 + \alpha_3]$-constacyclic code over $F_q$, $C_2$ is skew $[\alpha_0 + \alpha_1 - \alpha_2 - \alpha_3]$-constacyclic code over $F_q$, $C_3$ is skew $[\alpha_0 - \alpha_1 + \alpha_2 - \alpha_3]$-constacyclic code over $F_q$, and $C_4$ is skew $[\alpha_0 - \alpha_1 - \alpha_2 + \alpha_3]$-constacyclic code over $F_q$.

**Proof.** Let $r = (r_0, r_1, \cdots, r_{n-1}) \in C$ such that $r_i = \beta_1 a_i + \beta_2 b_i + \beta_3 c_i + \beta_4 d_i$, for $0 \leq i \leq n - 1$.

Let $a = (a_0, a_2, \cdots, a_{n-1})$, $b = (b_0, b_2, \cdots, b_{n-1})$, $c = (c_0, c_2, \cdots, c_{n-1})$, and $d = (d_0, d_2, \cdots, d_{n-1})$ then we have $a \in C_1$, $b \in C_2$, $c \in C_3$, and $d \in C_4$.

Suppose $C_1$ is skew $[\alpha_0 + \alpha_1 + \alpha_2 + \alpha_3]$-constacyclic, $C_2$ is skew $[\alpha_0 + \alpha_1 - \alpha_2 - \alpha_3]$-constacyclic, $C_3$ is skew $[\alpha_0 - \alpha_1 + \alpha_2 - \alpha_3]$-constacyclic, and $C_4$ is skew $[\alpha_0 - \alpha_1 - \alpha_2 + \alpha_3]$-constacyclic codes over $F_q$. 


Then \( \tau(r) = ([\alpha_0 + \alpha_1 u + \alpha_2 v + \alpha_3 wv] \theta(r_{n-1}), \theta(r_0), \cdots, \theta(r_{n-2})) \)
\[= ([\alpha_0 + \alpha_1 + \alpha_2 + \alpha_3] \theta(a_{n-1}), \theta(a_0), \cdots, (a_{n-2})) \beta_1 + [\alpha_0 + \alpha_1 - \alpha_2 - \alpha_3](\theta(b_{n-1}), \theta(b_0), \cdots, \theta(b_{n-2})) \beta_2 + ([\alpha_0 - \alpha_1 + \alpha_2 - \alpha_3] \theta(c_{n-1}), \theta(c_0), \cdots, \theta(c_{n-2})) \beta_3 + ([\alpha_0 - \alpha_1 - \alpha_2 + \alpha_3] \theta(d_{n-1}), \theta(d_0), \cdots, \theta(d_{n-2})) \beta_4). \]

So \( \tau(r) \in C \), for \( r \in C \). Then \( C \) is skew \( \alpha \)-constacyclic code over \( R \). Conversely, let \( a = (a_0, a_2, \cdots, a_{n-1}) \in C_1 \), \( b = (b_0, b_2, \cdots, b_{n-1}) \in C_2 \), \( c = (c_0, c_2, \cdots, c_{n-1}) \in C_3 \), and \( d = (d_0, d_2, \cdots, d_{n-1}) \in C_4 \).

Suppose that \( r_i = \beta_1 a_i + \beta_2 b_i + \beta_3 c_i + \beta_4 d_i \), for \( 0 \leq i \leq n - 1 \). Since \( C \) is skew \( \alpha \)-constacyclic, then
\[
\tau(r) \in C \implies ([\alpha_0 + \alpha_1 + \alpha_2 + \alpha_3] \theta(a_{n-1}), \theta(a_0), \cdots, (a_{n-2})) \in C_1,
\]
\[
[\alpha_0 + \alpha_1 - \alpha_2 - \alpha_3](\theta(b_{n-1}), \theta(b_0), \cdots, \theta(b_{n-2})) \in C_2,
\]
\[
([\alpha_0 - \alpha_1 + \alpha_2 - \alpha_3] \theta(c_{n-1}), \theta(c_0), \cdots, \theta(c_{n-2})) \in C_3, \text{ and}
\]
\[
([\alpha_0 - \alpha_1 - \alpha_2 + \alpha_3] \theta(d_{n-1}), \theta(d_0), \cdots, \theta(d_{n-2})) \beta_4) \in C_4.
\]

We can conclude that \( C_1 \) is skew \( ([\alpha_0 + \alpha_1 + \alpha_2 + \alpha_3] \)-constacyclic code, \( C_2 \) is skew \( ([\alpha_0 + \alpha_1 - \alpha_2 - \alpha_3] \)-constacyclic code, \( C_3 \) is skew \( ([\alpha_0 - \alpha_1 + \alpha_2 - \alpha_3] \)-constacyclic code, and \( C_4 \) is skew \( ([\alpha_0 - \alpha_1 - \alpha_2 + \alpha_3] \)-constacyclic code over \( F_q \). \( \Box \)

**Lemma 4.2.** ([15]) Let \( R \) be a finite ring and \( R^\times \) is its group of units. If \( R \) can be written as a direct sum as following \( R = R_1 \oplus R_2 \oplus \cdots \oplus R_m \) of rings \( R_i \) for \( i \leq m \), then \( R^\times \) will be written as \( R^\times = R_1^\times \oplus R_2^\times \oplus \cdots \oplus R_m^\times \).

Then by Lemma 4.2 we have concluded the following corollary:

**Corollary 4.3.** Let \( \alpha = \alpha_0 + \alpha_1 u + \alpha_2 v + \alpha_3 wv \) be an element of \( R \). Then \( \alpha \) can be a unit in \( R \) if and only if \( (\alpha_0 + \alpha_1 + \alpha_2 + \alpha_3), (\alpha_0 + \alpha_1 - \alpha_2 - \alpha_3), (\alpha_0 - \alpha_1 + \alpha_2 - \alpha_3), \) and \( (\alpha_0 - \alpha_1 - \alpha_2 + \alpha_3) \) are non zero elements of \( F_q \).

By Lemma [3.1] in [14]
Corollary 4.4. Let \( C = \beta_1 C_1 \oplus \beta_2 C_2 \oplus \beta_3 C_3 \oplus \beta_4 C_4 \) where \( C_i \), \( 1 \leq i \leq 4 \) are linear codes over \( F_q \). Then \( C \) is skew \( \alpha \)-constacyclic code with automorphism \( \theta \). Then its dual \( C^\perp = \beta_1 C_1^\perp \oplus \beta_2 C_2^\perp \oplus \beta_3 C_3^\perp \oplus \beta_4 C_4^\perp \) is skew \( \alpha^{-1} \)-constacyclic codes over \( R \), where \( C_1^\perp \), \( C_2^\perp \), \( C_3^\perp \), and \( C_4^\perp \) are skew \( [\alpha_0 + \alpha_1 + \alpha_2 + \alpha_3]^{-1} \)-constacyclic, skew \( [\alpha_0 + \alpha_1 - \alpha_2 - \alpha_3]^{-1} \)-constacyclic, skew \( [\alpha_0 - \alpha_1 + \alpha_2 - \alpha_3]^{-1} \)-constacyclic, and skew \( [\alpha_0 - \alpha_1 - \alpha_2 + \alpha_3]^{-1} \)-constacyclic codes over \( F_q \) of length \( n \).

Proof. we have \( \alpha \) is an element of \( R \) then \( a = a_1 + a_2 u + a_3 v + a_4 w = \beta_1 (a_1 + a_2 + a_3 + a_4) + \beta_2 (a_1 + a_2 - a_3 - a_4) + \beta_3 (a_1 - a_2 + a_3 + a_4) + \beta_4 (a_1 - a_2 - a_3 + a_4) \). Since \( \alpha \) is an unit of \( R \) which equivalent to \((a_1 + a_2 + a_3 + a_4),(a_1 + a_2 - a_3 - a_4),(a_1 - a_2 + a_3 + a_4)\), and \((a_1 - a_2 - a_3 + a_4)\) are also units.

we have \( C \) is skew \( \alpha \)-constacyclic code \iff \( (\text{by theorem 4.1}) C_1 \) is skew \( [\alpha_0 + \alpha_1 + \alpha_2 + \alpha_3] \)-constacyclic code, \( C_2 \) is skew \( [\alpha_0 + \alpha_1 - \alpha_2 - \alpha_3] \)-constacyclic code, \( C_3 \) is skew \( [\alpha_0 - \alpha_1 + \alpha_2 - \alpha_3] \)-constacyclic code, and \( C_4 \) is skew \( [\alpha_0 - \alpha_1 - \alpha_2 + \alpha_3] \)-constacyclic code over \( F_q \). And by Lemma [3.1] in [14] \iff \( C_1^\perp \) is skew \( [\alpha_0 + \alpha_1 + \alpha_2 + \alpha_3]^{-1} \)-constacyclic, and \( C_2^\perp \) is skew \( [\alpha_0 + \alpha_1 - \alpha_2 - \alpha_3]^{-1} \)-constacyclic, \( C_3^\perp \) skew \( [\alpha_0 - \alpha_1 + \alpha_2 - \alpha_3]^{-1} \)-constacyclic, and \( C_4^\perp \) is skew \( [\alpha_0 - \alpha_1 - \alpha_2 + \alpha_3]^{-1} \)-constacyclic codes over \( F_q \) of length \( n \). By applying again theorem 4.1 we have \( C^\perp \) is skew \( \alpha^{-1} \)-constacyclic code over \( R \).

5 Conclusion

In this paper, we have introduced the linear codes of length \( n \) over \( R = F_q + u F_q + v F_q + uvF_q \), where \( u^2 = v^2 = 1 \), and \( q = 3^n \). It has also been proved that the Gray image of skew cyclic code of length \( n \) over \( R \) is skew quasi-cyclic code of length \( 4n \) over \( F_q \).

It has also shown in detail that each linear code over \( R \) can be defined by \( \beta_1 C_1 \oplus \beta_2 C_2 \oplus \beta_3 C_3 \oplus \beta_4 C_4 \) (where \( C_i \), \( 1 \leq i \leq 4 \) are linear codes of length \( n \) over \( F_q \) ) is skew \( \alpha \)-constacyclic code if and only if \( C_1 \) is skew \( [\alpha_0 + \alpha_1 + \alpha_2 + \alpha_3] \)-constacyclic, \( C_2 \) is
skew $[\alpha_0 + \alpha_1 - \alpha_2 - \alpha_3]$-constacyclic, $C_3$ is skew $[\alpha_0 - \alpha_1 + \alpha_2 - \alpha_3]$-constacyclic, $F_q$ and $C_4$ is skew $[\alpha_0 - \alpha_1 - \alpha_2 + \alpha_3]$-constacyclic codes over $F_q$.

References


