

ON MODIFICATION OF PROPERTIES OF P-N-JUNCTIONS DURING OVERGROWTH

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ABSTRACT

In this paper we consider influence of overgrowth of doped by diffusion and ion implantation areas of heterostructures on distributions of concentrations of dopants. Several conditions to increase sharpness of p-n-junctions (single and framework bipolar transistors), which were manufactured during considered technological process, have been determined. At the same time we analyzed influence of speed of overgrowth of doped areas and mechanical stress in the considered heterostructure on distribution of concentrations of dopants in the structure.

KEYWORDS

Diffusion-junction heterorectifier; implanted-junction heterorectifier; overgrowth of doped area; analytical approach for modeling

1. INTRODUCTION

In the present time they are several approaches could be used to manufacture *p-n*-junctions diffusion of dopants in a homogenous sample or an epitaxial layer of heterostructure, implantation of ions of dopants in the same situations or doping during epitaxial growth [1-7]. The same approaches could be used to manufacture systems of *p-n*-junctions: bipolar transistors and thyristors. The first and the second ways of doping are preferable in comparison with the third one because the approaches give us possibility to dope locally materials during manufacture integrated circuits easily in comparison with epitaxial growth. Using diffusion and ion implantation in homogenous sample to manufacturing *p-n*-junctions leads to production fluently varying and wide distributions of dopants. One of actual problems is increasing sharpness of *p-n*-junctions [5,7]. The increasing of sharpness gives us possibility to decrease switching time of *p-n*-junctions. Increasing of homogeneity of dopant distribution in enriched by the dopant area is also attracted an interest [5]. The increasing of homogeneity gives us possibility to decrease local overheat of the doped materials due to streaming of electrical current during operating of *p-n*-junction or to decrease depth of *p-n*-junction for fixed value of local overheats. One way to increase sharpness of *p-n*-junction based on using near-surficial (laser or microwave) types of annealing [8-15].

Framework the types of annealing one can obtain near-surficial heating of the doped materials. In this situation due to the Arrhenius law one can obtain increasing of dopant diffusion coefficient of near-surficial area in comparison with volumetric dopant diffusion coefficient. The increasing of dopant diffusion coefficient of near-surficial area leads to increasing of sharpness of *p-n*-junction. The second way to increase sharpness of *p-n*-junction based on using high doping of materials. In this case contribution of nonlinearity of diffusion process increases [4]. The third way to increase

sharpness of p - n -junction is using of inhomogeneity of heterostructure [16,17]. Framework the approach we consider simplest heterostructure, which consist of substrate and epitaxial layer. One can find increasing of sharpness of p - n -junction after annealing with appropriate annealing time in the case, when dopant diffusion coefficient in the substrate is smaller, than in the epitaxial layer. The fourth way to increase sharpness of p - n -junction is radiation processing of materials. The radiation processing leads to radiation-enhanced diffusion [18]. However using radiation processing of materials leads to necessity of annealing of radiation defects. Density of elements of integrated circuits could be increases by using mismatch-induced stress [19]. However one could obtain increased unsoundness of doped material (for example, to generation of dislocation of disagreement) by using the approach [7].

The considered approaches gives a possibility to increase sharpness of p - n -junction with increasing of homogeneity of dopant distribution in enriched by the dopant area. Using combination of the above approaches gives us possibility to increase sharpness of p - n -junction and increasing of homogeneity of dopant distribution in enriched area at one time.

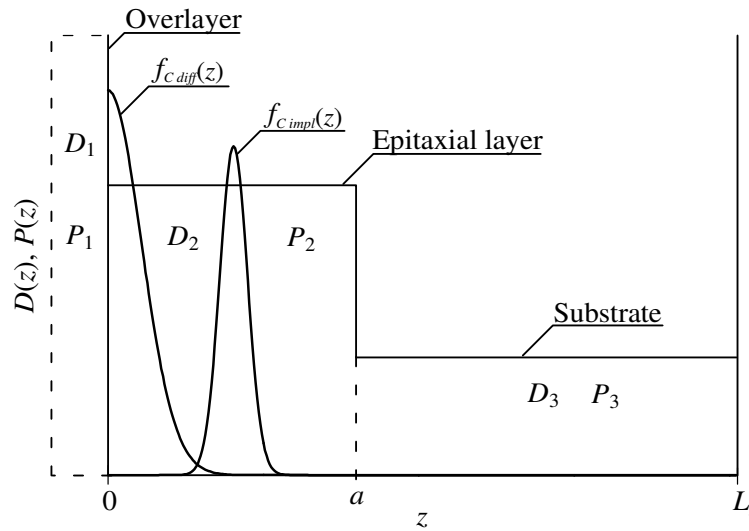


Fig. 1. Substrate, epitaxial and overlayers framework heterostructure

Framework this paper we consider a substrate with known type of conductivity (n or p) and an epitaxial layer, included into a heterostructure. The epitaxial layer have been doped by diffusion or by ion implantation to manufacture another type of conductivity (p or n). Farther we consider overgrowth of the epitaxial layer by an overlayer (see Fig. 1). The overlayer has type of conductivity, which coincide with type of conductivity of the substrate. In this paper we analyzed influence of overgrowth of the doped epitaxial layer on distribution of dopants in the considered heterostructure.

2. METHOD OF SOLUTION

In this section we calculate spatio-temporal distribution of concentration of dopant in the considered heterostructure to solve our aim. To calculate the distribution we solved the following boundary problem [1,20-22]

$$\frac{\partial C(x, y, z, t)}{\partial t} = \frac{\partial}{\partial x} \left[D \frac{\partial C(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[D \frac{\partial C(x, y, z, t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[D \frac{\partial C(x, y, z, t)}{\partial z} \right] + \quad (1)$$

$$+ \Omega \frac{\partial}{\partial x} \left[\frac{D_s}{kT} \nabla_s \mu(x, y, z, t) \int_{-vt}^{L_z} C(x, y, W, t) dW \right] + \Omega \frac{\partial}{\partial y} \left[\frac{D_s}{kT} \nabla_s \mu(x, y, z, t) \int_{-vt}^{L_z} C(x, y, W, t) dW \right].$$

Boundary and initial conditions for our case could be written as

$$\begin{aligned} \left. \frac{\partial C(x, y, z, t)}{\partial x} \right|_{x=0} = 0, \quad \left. \frac{\partial C(x, y, z, t)}{\partial x} \right|_{x=L_x} = 0, \quad \left. \frac{\partial C(x, y, z, t)}{\partial y} \right|_{y=0} = 0, \quad \left. \frac{\partial C(x, y, z, t)}{\partial y} \right|_{y=L_y} = 0, \\ \left. \frac{\partial C(x, y, z, t)}{\partial z} \right|_{z=-vt} = 0, \quad \left. \frac{\partial C(x, y, z, t)}{\partial z} \right|_{z=L_z} = 0, \quad C(x, y, z, 0) = f_C(x, y, z). \end{aligned}$$

In the above relations we used the function $C(x, y, z, t)$ as the spatio-temporal distribution of concentration of dopant; Ω is the atomic volume; surface concentration of dopant on interface between layers of heterostructure could be determined as the following integral $\int_{-vt}^{L_z} C(x, y, z, t) dz$

(we assume, that the interface between layers of heterostructure is perpendicular to the direction Oz); surface gradient we denote as symbol ∇_s ; $\mu(x, y, z, t)$ is the chemical potential (reason of accounting of the chemical potential is mismatch-induced stress); the parameters D and D_s are the coefficients of volumetric and surface diffusions. One can find the surface diffusions due to mismatch-induced stress. Diffusion coefficients depends on temperature and speed of heating and cooling of heterostructure, properties of materials of heterostructure, spatio-temporal distributions of concentrations of dopant and radiation defects after ion implantation. Approximations of these dependences could be approximated by the following functions [22,23]

$$D_s = D_{sL}(x, y, z, T) \left[1 + \xi_s \frac{C^\gamma(x, y, z, t)}{P^\gamma(x, y, z, T)} \right] \left[1 + \varsigma_1 \frac{V(x, y, z, t)}{V^*} + \varsigma_2 \frac{V^2(x, y, z, t)}{(V^*)^2} \right]. \quad (2)$$

Functions $D_L(x, y, z, T)$ and $D_{Ls}(x, y, z, T)$ described dependences of diffusion coefficients on coordinate (due to presents several layers in heterostructure, manufactured by using different materials) and temperature of annealing T (due to Arrhenius law); function $P(x, y, z, T)$ describes dependence of limit of solubility of dopant on coordinate and temperature; parameter γ could be integer and depends on properties of materials of heterostructure [23]; $V(x, y, z, t)$ is the spatio-temporal concentration of radiation vacancies; V^* is the equilibrium concentration of vacancies. Concentration-dependence of dopant diffusion coefficients has been described in details in [23].

We determine distributions of concentrations of point defects in space and time as solutions of the following system of equations [1,20-22]

$$\begin{aligned} \frac{\partial I(x, y, z, t)}{\partial t} = \frac{\partial}{\partial x} \left[D_I(x, y, z, T) \frac{\partial I(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[D_I(x, y, z, T) \frac{\partial I(x, y, z, t)}{\partial y} \right] - k_{I,J}(x, y, z, T) \times \\ \times I^2(x, y, z, t) + \frac{\partial}{\partial z} \left[D_I(x, y, z, T) \frac{\partial I(x, y, z, t)}{\partial z} \right] - k_{I,V}(x, y, z, T) I(x, y, z, t) V(x, y, z, t) \quad (3) \end{aligned}$$

$$\begin{aligned}
 & + \Omega \frac{\partial}{\partial x} \left[\frac{D_{IS}}{kT} \nabla_s \mu(x, y, z, t) \int_{-vt}^{L_z} I(x, y, W, t) dW \right] + \Omega \frac{\partial}{\partial y} \left[\frac{D_{IS}}{kT} \nabla_s \mu(x, y, z, t) \int_{-vt}^{L_z} I(x, y, W, t) dW \right] \\
 & \frac{\partial V(x, y, z, t)}{\partial t} = \frac{\partial}{\partial x} \left[D_V(x, y, z, T) \frac{\partial V(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[D_V(x, y, z, T) \frac{\partial V(x, y, z, t)}{\partial y} \right] - k_{V,V}(x, y, z, T) \times \\
 & \quad \times V^2(x, y, z, t) + \frac{\partial}{\partial z} \left[D_V(x, y, z, T) \frac{\partial V(x, y, z, t)}{\partial z} \right] - k_{I,V}(x, y, z, T) I(x, y, z, t) V(x, y, z, t) + \\
 & + \Omega \frac{\partial}{\partial x} \left[\frac{D_{VS}}{kT} \nabla_s \mu(x, y, z, t) \int_{-vt}^{L_z} V(x, y, W, t) dW \right] + \Omega \frac{\partial}{\partial y} \left[\frac{D_{VS}}{kT} \nabla_s \mu(x, y, z, t) \int_{-vt}^{L_z} V(x, y, W, t) dW \right].
 \end{aligned}$$

Boundary and initial conditions for these equations could be written as

$$\begin{aligned}
 \left. \frac{\partial I(x, y, z, t)}{\partial x} \right|_{x=0} &= 0, \quad \left. \frac{\partial I(x, y, z, t)}{\partial x} \right|_{x=L_x} &= 0, \quad \left. \frac{\partial I(x, y, z, t)}{\partial y} \right|_{y=0} &= 0, \quad \left. \frac{\partial I(x, y, z, t)}{\partial y} \right|_{y=L_y} &= 0, \\
 \left. \frac{\partial I(x, y, z, t)}{\partial z} \right|_{z=-vt} &= 0, \quad \left. \frac{\partial I(x, y, z, t)}{\partial z} \right|_{z=L_z} &= 0, \quad \left. \frac{\partial V(x, y, z, t)}{\partial x} \right|_{x=0} &= 0, \quad \left. \frac{\partial V(x, y, z, t)}{\partial x} \right|_{x=L_x} &= 0, \\
 \left. \frac{\partial V(x, y, z, t)}{\partial y} \right|_{y=0} &= 0, \quad \left. \frac{\partial V(x, y, z, t)}{\partial y} \right|_{y=L_y} &= 0, \quad \left. \frac{\partial V(x, y, z, t)}{\partial z} \right|_{z=-vt} &= 0, \quad \left. \frac{\partial V(x, y, z, t)}{\partial z} \right|_{z=L_z} &= 0, \\
 I(x, y, z, 0) &= f_I(x, y, z), \quad V(x, y, z, 0) = f_V(x, y, z).
 \end{aligned} \tag{4}$$

Here $I(x, y, z, t)$ is the distribution of concentration of radiation interstitials in space and time; I^* is the equilibrium concentration interstitials; $D_I(x, y, z, T)$, $D_V(x, y, z, T)$, $D_{IS}(x, y, z, T)$, $D_{VS}(x, y, z, T)$ are the coefficients of volumetric and surface diffusion; terms $V^2(x, y, z, t)$ and $I^2(x, y, z, t)$ corresponds to generation divacancies and analogous complexes of interstitials (see, for example, [22] and appropriate references in this work); the functions $k_{I,V}(x, y, z, T)$, $k_{I,I}(x, y, z, T)$ and $k_{V,V}(x, y, z, T)$ described dependences of parameters of recombination of point defects and generation their complexes on coordinate and temperature; k is the Boltzmann constant.

We calculate spatio-temporal distributions of concentrations of divacancies $\Phi_V(x, y, z, t)$ and diinterstitials $\Phi_I(x, y, z, t)$ by solution of the equations [20-22]

$$\begin{aligned}
 \frac{\partial \Phi_I(x, y, z, t)}{\partial t} &= \frac{\partial}{\partial x} \left[D_{\Phi_I}(x, y, z, T) \frac{\partial \Phi_I(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[D_{\Phi_I}(x, y, z, T) \frac{\partial \Phi_I(x, y, z, t)}{\partial y} \right] + k_I(x, y, z, T) \times \\
 & \times I(x, y, z, t) + \frac{\partial}{\partial z} \left[D_{\Phi_I}(x, y, z, T) \frac{\partial \Phi_I(x, y, z, t)}{\partial z} \right] + \Omega \frac{\partial}{\partial x} \left[\frac{D_{\Phi_I S}}{kT} \nabla_s \mu(x, y, z, t) \int_{-vt}^{L_z} \Phi_I(x, y, W, t) dW \right] + \\
 & + \Omega \frac{\partial}{\partial y} \left[\frac{D_{\Phi_I S}}{kT} \nabla_s \mu(x, y, z, t) \int_{-vt}^{L_z} \Phi_I(x, y, W, t) dW \right] + k_{I,I}(x, y, z, T) I^2(x, y, z, t)
 \end{aligned} \tag{5}$$

$$\begin{aligned}
 \frac{\partial \Phi_V(x, y, z, t)}{\partial t} &= \frac{\partial}{\partial x} \left[D_{\Phi_V}(x, y, z, T) \frac{\partial \Phi_V(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[D_{\Phi_V}(x, y, z, T) \frac{\partial \Phi_V(x, y, z, t)}{\partial y} \right] + k_V(x, y, z, T) \times \\
 & \times V(x, y, z, t) + \frac{\partial}{\partial z} \left[D_{\Phi_V}(x, y, z, T) \frac{\partial \Phi_V(x, y, z, t)}{\partial z} \right] + \Omega \frac{\partial}{\partial x} \left[\frac{D_{\Phi_V S}}{kT} \nabla_s \mu(x, y, z, t) \int_{-vt}^{L_z} \Phi_V(x, y, W, t) dW \right] +
 \end{aligned}$$

$$+ \Omega \frac{\partial}{\partial y} \left[\frac{D_{\Phi_V S}}{kT} \nabla_s \mu(x, y, z, t) \int_{-v_I}^{L_z} \Phi_V(x, y, W, t) dW \right] + k_{V,V}(x, y, z, T) V^2(x, y, z, t).$$

Boundary and initial conditions could be written as

$$\begin{aligned} \left. \frac{\partial \Phi_I(x, y, z, t)}{\partial x} \right|_{x=0} = 0, \quad \left. \frac{\partial \Phi_I(x, y, z, t)}{\partial x} \right|_{x=L_x} = 0, \quad \left. \frac{\partial \Phi_I(x, y, z, t)}{\partial y} \right|_{y=0} = 0, \quad \left. \frac{\partial \Phi_I(x, y, z, t)}{\partial y} \right|_{y=L_y} = 0, \\ \left. \frac{\partial \Phi_I(x, y, z, t)}{\partial z} \right|_{z=-v_I} = 0, \quad \left. \frac{\partial I(x, y, z, t)}{\partial z} \right|_{z=L_z} = 0, \quad \left. \frac{\partial \Phi_V(x, y, z, t)}{\partial x} \right|_{x=0} = 0, \quad \left. \frac{\partial V(x, y, z, t)}{\partial x} \right|_{x=L_x} = 0, \\ \left. \frac{\partial \Phi_V(x, y, z, t)}{\partial y} \right|_{y=0} = 0, \quad \left. \frac{\partial \Phi_V(x, y, z, t)}{\partial y} \right|_{y=L_y} = 0, \quad \left. \frac{\partial \Phi_V(x, y, z, t)}{\partial z} \right|_{z=-v_I} = 0, \quad \left. \frac{\partial \Phi_V(x, y, z, t)}{\partial z} \right|_{z=L_z} = 0, \\ \Phi_I(x, y, z, 0) = f_{\Phi I}(x, y, z), \quad \Phi_V(x, y, z, 0) = f_{\Phi V}(x, y, z). \end{aligned} \quad (6)$$

Here $D_{\Phi I}(x, y, z, T)$, $D_{\Phi V}(x, y, z, T)$, $D_{\Phi IS}(x, y, z, T)$ and $D_{\Phi VS}(x, y, z, T)$ are the coefficients of volumetric and surface diffusion; the functions $k_I(x, y, z, T)$ and $k_V(x, y, z, T)$ described dependences of parameters of decay of complexes of point defects on coordinate and temperature. One can determine chemical potential μ in the Eq.(1) by the following relation [20]

$$\mu = E(z) \Omega \sigma_{ij} [u_{ij}(x, y, z, t) + u_{ji}(x, y, z, t)] / 2. \quad (7)$$

Here E is the tension (Young) modulus; $u_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$ is the deformation tensor; σ_{ij} is the stress tensor; u_i, u_j are the components $u_x(x, y, z, t)$, $u_y(x, y, z, t)$ and $u_z(x, y, z, t)$ of the displacement tensor $\vec{u}(x, y, z, t)$; x_i, x_j are the coordinates x, y, z . The relation (3) could be transformed to the following form

$$\begin{aligned} \mu(x, y, z, t) = E(z) \frac{\Omega}{2} \left[\frac{\partial u_i(x, y, z, t)}{\partial x_j} + \frac{\partial u_j(x, y, z, t)}{\partial x_i} \right] \left\{ \frac{1}{2} \left[\frac{\partial u_i(x, y, z, t)}{\partial x_j} + \frac{\partial u_j(x, y, z, t)}{\partial x_i} \right] - \varepsilon_0 \delta_{ij} + \right. \\ \left. + \frac{\sigma(z) \delta_{ij}}{1 - 2\sigma(z)} \left[\frac{\partial u_k(x, y, z, t)}{\partial x_k} - 3\varepsilon_0 \right] - K(z) \beta(z) [T(x, y, z, t) - T_0] \delta_{ij} \right\}, \end{aligned}$$

where σ is the Poisson coefficient; the parameter $\varepsilon_0 = (a_s - a_{EL}) / a_{EL}$ describes the displacement parameter with lattice distances of the substrate and the epitaxial layer a_s, a_{EL} ; K is the modulus of uniform compression; the parameter b describes the thermal expansion; we assume, that the equilibrium temperature T_r coincides with the room temperature. Components of the displacement vector could be described by solving the following system of equations [24]

$$\begin{aligned} \rho(z) \frac{\partial^2 u_x(x, y, z, t)}{\partial t^2} &= \frac{\partial \sigma_{xx}(x, y, z, t)}{\partial x} + \frac{\partial \sigma_{xy}(x, y, z, t)}{\partial y} + \frac{\partial \sigma_{xz}(x, y, z, t)}{\partial z} \\ \rho(z) \frac{\partial^2 u_y(x, y, z, t)}{\partial t^2} &= \frac{\partial \sigma_{yx}(x, y, z, t)}{\partial x} + \frac{\partial \sigma_{yy}(x, y, z, t)}{\partial y} + \frac{\partial \sigma_{yz}(x, y, z, t)}{\partial z} \\ \rho(z) \frac{\partial^2 u_z(x, y, z, t)}{\partial t^2} &= \frac{\partial \sigma_{zx}(x, y, z, t)}{\partial x} + \frac{\partial \sigma_{zy}(x, y, z, t)}{\partial y} + \frac{\partial \sigma_{zz}(x, y, z, t)}{\partial z} \end{aligned}$$

where $\sigma_{ij} = \frac{E(z)}{2[1+\sigma(z)]} \left[\frac{\partial u_i(x, y, z, t)}{\partial x_j} + \frac{\partial u_j(x, y, z, t)}{\partial x_i} - \frac{\delta_{ij}}{3} \frac{\partial u_k(x, y, z, t)}{\partial x_k} \right] + K(z) \delta_{ij} \frac{\partial u_k(x, y, z, t)}{\partial x_k} - \beta(z) K(z) [T(x, y, z, t) - T_r]$, $\rho(z)$ describes the density of materials of heterostructure. The tensor δ_{ij} describes the Kronecker symbol. Accounting relation for σ_{ij} in the previous system of equations last system of equation could be written as

$$\begin{aligned} \rho(z) \frac{\partial^2 u_x(x, y, z, t)}{\partial t^2} &= \left\{ K(z) + \frac{5E(z)}{6[1+\sigma(z)]} \right\} \frac{\partial^2 u_x(x, y, z, t)}{\partial x^2} + \left\{ K(z) - \frac{E(z)}{3[1+\sigma(z)]} \right\} \frac{\partial^2 u_y(x, y, z, t)}{\partial x \partial y} + \\ &+ \frac{E(z)}{2[1+\sigma(z)]} \left[\frac{\partial^2 u_y(x, y, z, t)}{\partial y^2} + \frac{\partial^2 u_z(x, y, z, t)}{\partial z^2} \right] + \left\{ K(z) + \frac{E(z)}{3[1+\sigma(z)]} \right\} \frac{\partial^2 u_z(x, y, z, t)}{\partial x \partial z} - \\ \rho(z) \frac{\partial^2 u_y(x, y, z, t)}{\partial t^2} &= \frac{E(z)}{2[1+\sigma(z)]} \left[\frac{\partial^2 u_y(x, y, z, t)}{\partial x^2} + \frac{\partial^2 u_x(x, y, z, t)}{\partial x \partial y} \right] - \beta(z) K(z) \frac{\partial T(x, y, z, t)}{\partial y} + \\ &+ \frac{\partial}{\partial z} \left\{ \frac{E(z)}{2[1+\sigma(z)]} \left[\frac{\partial u_y(x, y, z, t)}{\partial z} + \frac{\partial u_z(x, y, z, t)}{\partial y} \right] \right\} + \left\{ \frac{5E(z)}{12[1+\sigma(z)]} + K(z) \right\} \frac{\partial^2 u_y(x, y, z, t)}{\partial y^2} + \\ &+ \left\{ K(z) - \frac{E(z)}{6[1+\sigma(z)]} \right\} \frac{\partial^2 u_y(x, y, z, t)}{\partial y \partial z} + K(z) \frac{\partial^2 u_y(x, y, z, t)}{\partial x \partial y} \end{aligned} \quad (8)$$

$$\begin{aligned} \rho(z) \frac{\partial^2 u_z(x, y, z, t)}{\partial t^2} &= \frac{E(z)}{2[1+\sigma(z)]} \left[\frac{\partial^2 u_z(x, y, z, t)}{\partial x^2} + \frac{\partial^2 u_z(x, y, z, t)}{\partial y^2} + \frac{\partial^2 u_x(x, y, z, t)}{\partial x \partial z} + \frac{\partial^2 u_y(x, y, z, t)}{\partial y \partial z} \right] + \\ &+ \frac{\partial}{\partial z} \left\{ K(z) \left[\frac{\partial u_x(x, y, z, t)}{\partial x} + \frac{\partial u_y(x, y, z, t)}{\partial y} + \frac{\partial u_x(x, y, z, t)}{\partial z} \right] \right\} - K(z) \beta(z) \frac{\partial T(x, y, z, t)}{\partial z} + \\ &+ \frac{1}{6} \frac{\partial}{\partial z} \left\{ \frac{E(z)}{1+\sigma(z)} \left[6 \frac{\partial u_z(x, y, z, t)}{\partial z} - \frac{\partial u_x(x, y, z, t)}{\partial x} - \frac{\partial u_y(x, y, z, t)}{\partial y} - \frac{\partial u_z(x, y, z, t)}{\partial z} \right] \right\}. \end{aligned}$$

Systems of conditions for these equations could be written as

$$\begin{aligned} \left. \frac{\partial \bar{u}(x, y, z, t)}{\partial x} \right|_{x=0} &= 0; \quad \left. \frac{\partial \bar{u}(x, y, z, t)}{\partial x} \right|_{x=L_x} &= 0; \quad \left. \frac{\partial \bar{u}(x, y, z, t)}{\partial y} \right|_{y=0} &= 0; \quad \left. \frac{\partial \bar{u}(x, y, z, t)}{\partial y} \right|_{y=L_y} &= 0; \\ \left. \frac{\partial \bar{u}(x, y, z, t)}{\partial z} \right|_{z=-vt} &= 0; \quad \left. \frac{\partial \bar{u}(x, y, z, t)}{\partial z} \right|_{z=L_z} &= 0; \quad \bar{u}(x, y, z, 0) &= \bar{u}_0; \quad \bar{u}(x, y, z, \infty) &= \bar{u}_0. \end{aligned}$$

We calculate distribution of concentration of dopant in space in time by method of averaging of function corrections [25-30]. To use the method we re-write Eqs. (1), (3) and (5) with account appropriate initial distributions (see Appendix). In future we replace the required concentrations in right sides of the obtained equations on their average values $\alpha_{i,\rho}$, which are not yet known. The equations modified after the replacement and solution of these equations are presented in the Appendix.

We determined average values of the first-order approximations of the considered concentrations by using the following standard relations [25-30]

$$\alpha_{1\rho} = \frac{1}{\Theta L_x L_y L_z} \int_0^{\Theta L_x} \int_0^{\Theta L_y} \int_0^{\Theta L_z} \rho_1(x, y, z, t) dz dy dx dt. \tag{9}$$

Substitution of the solutions of the modified equations into the relation (9) gives a possibility to obtain appropriate average values in the following form

$$\begin{aligned} \alpha_{1C} &= \frac{1}{L_x L_y L_z} \int_0^{\Theta L_x} \int_0^{\Theta L_y} \int_0^{\Theta L_z} f_C(x, y, z) dz dy dx dt, \quad \alpha_{1I} = \sqrt{\frac{(a_3 + A)^2}{4a_4^2} - 4 \left(B + \frac{\Theta a_3 B + \Theta^2 L_x L_y L_z a_1}{a_4} \right)} - \frac{a_3 + A}{4a_4}, \\ \alpha_{1V} &= \frac{1}{S_{IV00}} \left[\frac{\Theta}{\alpha_{1I}} \int_0^{\Theta L_x} \int_0^{\Theta L_y} \int_0^{\Theta L_z} f_I(x, y, z) dz dy dx dt - \alpha_{1I} S_{II00} - \Theta L_x L_y L_z \right], \\ \alpha_{1\Phi_I} &= \frac{R_{I1}}{\Theta L_x L_y L_z} + \frac{S_{II20}}{\Theta L_x L_y L_z} + \frac{1}{L_x L_y L_z} \int_0^{\Theta L_x} \int_0^{\Theta L_y} \int_0^{\Theta L_z} f_{\Phi_I}(x, y, z) dz dy dx dt, \\ \alpha_{1\Phi_V} &= \frac{R_{V1}}{\Theta L_x L_y L_z} + \frac{S_{VV20}}{\Theta L_x L_y L_z} + \frac{1}{L_x L_y L_z} \int_0^{\Theta L_x} \int_0^{\Theta L_y} \int_0^{\Theta L_z} f_{\Phi_V}(x, y, z) dz dy dx dt. \end{aligned}$$

Relations for calculations parameters S_{ppij} , a_i , A , B , q , p are presented in the Appendix.

We used standard iterative procedure of method of averaging of function corrections to calculate the second- and higher-order approximations of the considered concentrations [25-30]. Framework the procedure to calculate approximations of the n -th order of the above concentrations we replace the functions $C(x,y,z,t)$, $I(x,y,z,t)$, $V(x,y,z,t)$, $\Phi_I(x,y,z,t)$ and $\Phi_V(x,y,z,t)$ in the Eqs. (1), (3), (5) with account initial distributions (see Eqs. (1a), (3a), (5a) in the Appendix) on the sums of the not yet known average values of the considered approximations and approximations of the previous order, i.e. $\alpha_{n\rho} + \rho_{n-1}(x,y,z,t)$. These obtained equations and their solutions, which calculated are presented in the Appendix.

We calculate average values of the second-order approximations of required functions by using the following standard relation [25-30]

$$\alpha_{2\rho} = \frac{1}{\Theta L_x L_y L_z} \int_0^{\Theta L_x} \int_0^{\Theta L_y} \int_0^{\Theta L_z} [\rho_2(x, y, z, t) - \rho_1(x, y, z, t)] dz dy dx dt. \tag{10}$$

Substitution of the second-order approximations of concentrations of dopant into Eq.(10) leads to the considered average values $\alpha_{2\rho}$

$$\begin{aligned} \alpha_{2C}=0, \alpha_{2\Phi_I}=0, \alpha_{2\Phi_V}=0, \quad \alpha_{2V} &= \sqrt{\frac{(b_3 + E)^2}{4b_4^2} - 4 \left(F + \frac{\Theta a_3 F + \Theta^2 L_x L_y L_z b_1}{b_4} \right)} - \frac{b_3 + E}{4b_4}, \\ \alpha_{2I} &= \frac{C_V - \alpha_{2V}^2 S_{VV00} - \alpha_{2V} (2S_{VV01} + S_{IV10} + \Theta L_x L_y L_z) - S_{VV02} - S_{IV11}}{S_{IV01} + \alpha_{2V} S_{IV00}}. \end{aligned}$$

Relations for parameters b_i , C_ρ , F , r , s are presented in the Appendix.

Further we determine solutions of Eqs.(8). In this situation we determine approximation of displacement vector. To determine the first-order approximations of the considered components framework method of averaging of function corrections we replace the required values on their not yet known average values α_{1i} . The replacement leads to the following result

$$\begin{aligned} \rho(z) \frac{\partial^2 u_{1x}(x, y, z, t)}{\partial t^2} &= -K(z) \beta(z) \frac{\partial T(x, y, z, t)}{\partial x}, \\ \rho(z) \frac{\partial^2 u_{1y}(x, y, z, t)}{\partial t^2} &= -K(z) \beta(z) \frac{\partial T(x, y, z, t)}{\partial y}, \\ \rho(z) \frac{\partial^2 u_{1z}(x, y, z, t)}{\partial t^2} &= -K(z) \beta(z) \frac{\partial T(x, y, z, t)}{\partial z}. \end{aligned}$$

Integration of the left and right sides of the previous relations on time t gives a possibility to obtain the required components to the following result

$$\begin{aligned} u_{1x}(x, y, z, t) &= K(z) \frac{\beta(z)}{\rho(z)} \frac{\partial}{\partial x} \int_0^t \int_0^\vartheta T(x, y, z, \tau) d\tau d\vartheta - K(z) \frac{\beta(z)}{\rho(z)} \frac{\partial}{\partial x} \int_0^\infty \int_0^\vartheta T(x, y, z, \tau) d\tau d\vartheta + u_{0x}, \\ u_{1y}(x, y, z, t) &= K(z) \frac{\beta(z)}{\rho(z)} \frac{\partial}{\partial y} \int_0^t \int_0^\vartheta T(x, y, z, \tau) d\tau d\vartheta - K(z) \frac{\beta(z)}{\rho(z)} \frac{\partial}{\partial y} \int_0^\infty \int_0^\vartheta T(x, y, z, \tau) d\tau d\vartheta + u_{0y}, \\ u_{1z}(x, y, z, t) &= K(z) \frac{\beta(z)}{\rho(z)} \frac{\partial}{\partial z} \int_0^t \int_0^\vartheta T(x, y, z, \tau) d\tau d\vartheta - K(z) \frac{\beta(z)}{\rho(z)} \frac{\partial}{\partial z} \int_0^\infty \int_0^\vartheta T(x, y, z, \tau) d\tau d\vartheta + u_{0z}. \end{aligned}$$

We calculate the second-order approximations of components of the displacement vector by standard replacement of the required functions in the right sides of the Eqs.(8) on the standard sums $\alpha_{1,t} + u_i(x, y, z, t)$ [19,26]. Equations for components of the displacement vector are presented in the Appendix. Solutions of these equations are also presented in the Appendix.

Framework this paper all required concentrations (concentrations of dopant and radiation defects) and components of displacement vector have been calculated as the appropriate second-order approximations by using the method of averaging of function corrections. The second-order approximation gives usually enough information on quantitative behavior of spatio-temporal distributions of concentrations of dopant and radiation defects and also several quantitative results. We check all analytical results by using numerical approaches.

3. DISCUSSION

In this section we analyzed redistribution of dopant (for the ion doping of heterostructure) with account redistribution of radiation defects and their interaction with another defects. If growth rate is small ($v \ll D_1/v$), then overlayer will be fully doped by dopant, which was implanted in the epitaxial layer. Framework another limiting case it will be doped near-surface area of the overlayer only. If dopant diffusion coefficient in the overlayer and in the substrate are smaller, in comparison with the epitaxial layer, and type of conductivity of the overlayer and the substrate is different with type of conductivity of the epitaxial layer, than one can find a bipolar transistor. In this case one can find higher sharpness of $p-n$ -junctions framework the transistor in comparison with a bipolar transistor in homogenous sample with averaged diffusion coefficient of dopant. The increasing gives a possibility to increase switching time of $p-n$ -junctions (both single $p-n$ -junctions and $p-n$ -junctions framework their systems: bipolar transistors, thyristors et al). At the same time one can find increasing of homogeneity of concentration of dopant (see Fig. 2). In this situation one can decrease local overheats in doped areas during functioning of the considered devices or to decrease dimensions of these devices for fixed tolerance for local overheats. Qualitatively similar results could be obtained for diffusion type of doping. One can find smaller sharpness of left $p-n$ -junctions in the case, when dopant diffusion coefficient of the overlayer is larger, than in doped epitaxial layer. At the same time homogeneity of concentration of dopant in

the overlayer increases (see Fig. 3). Qualitatively similar results could be obtained for diffusion type of doping. Further we analyzed influence of mismatch-induced stress on distribution of concentration

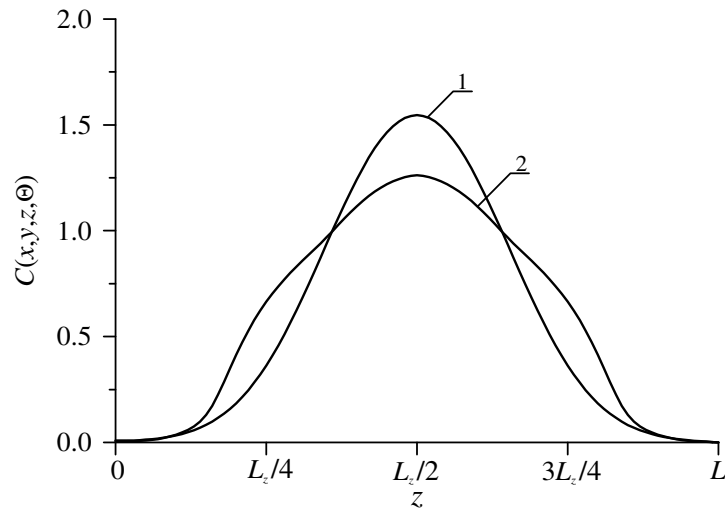


Fig. 2. Calculated spatial distributions of concentration of implanted dopant in homogenous sample (curve 1) and in heterostructure from Fig. 1 (curve 2) after annealing with the same continuance. Interfaces between layers of heterostructure are: $a_1=L_z/4$ and $a_2=3L_z/4$

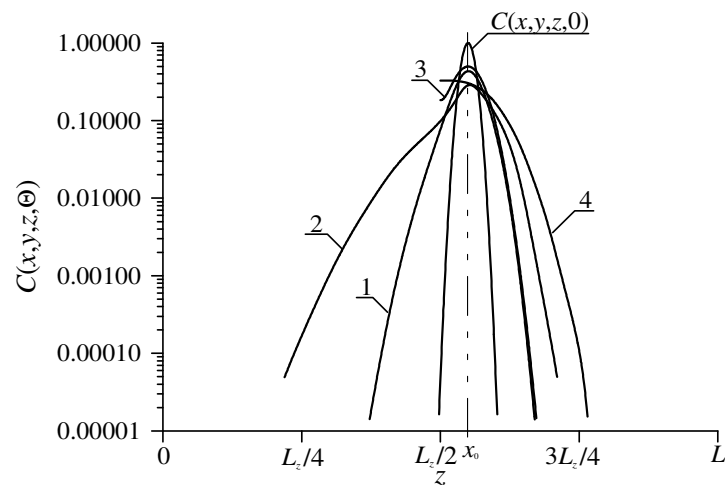


Fig. 3. Curves 1 and 2 are the calculated spatial distributions of concentration of implanted dopant in the system of two layers: overlayer and epitaxial layer. Curves 3 and 4 are the calculated spatial distributions of concentration of implanted dopant in the epitaxial layer only. Increasing of number of curves corresponds to increasing of value of relation D_1/D_2 . Coordinates of interfaces between layers of heterostructure are: $a_1=L_z/4$ (between overlayer and epitaxial layer) and $a_2=3L_z/4$ (between epitaxial layer and substrate)

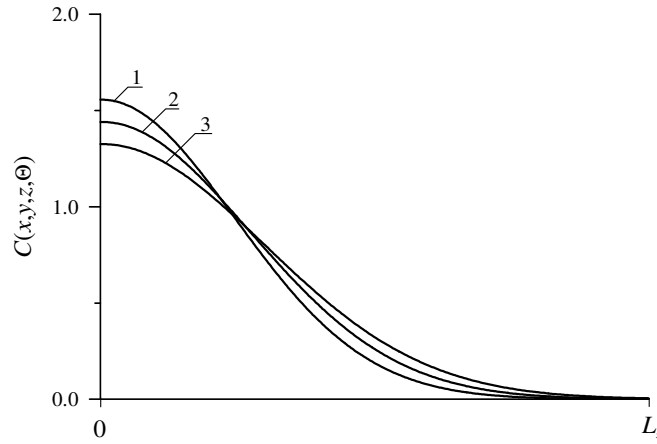


Fig. 4. Spatial distributions of concentration of dopant in diffusion-junction rectifier after annealing with equal continuance. Curve 1 corresponds to $\varepsilon_0 < 0$. Curve 2 corresponds to $\varepsilon_0 = 0$. Curve 3 corresponds to $\varepsilon_0 > 0$

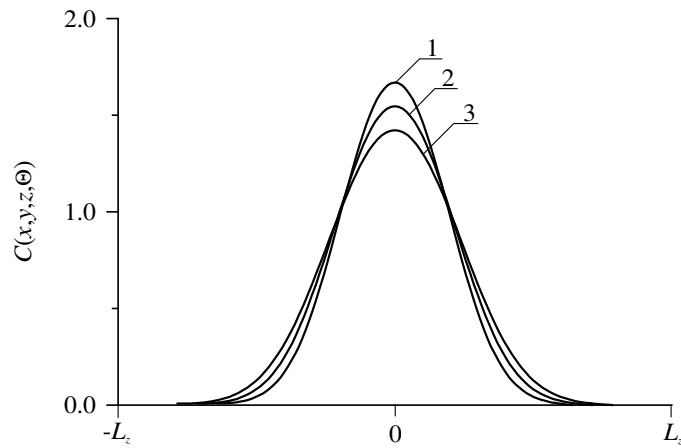


Fig. 5. Spatial distributions of concentration of dopant in implanted-junction rectifier after annealing with equal continuance. Curve 1 corresponds to $\varepsilon_0 < 0$. Curve 2 corresponds to $\varepsilon_0 = 0$. Curve 3 corresponds to $\varepsilon_0 > 0$

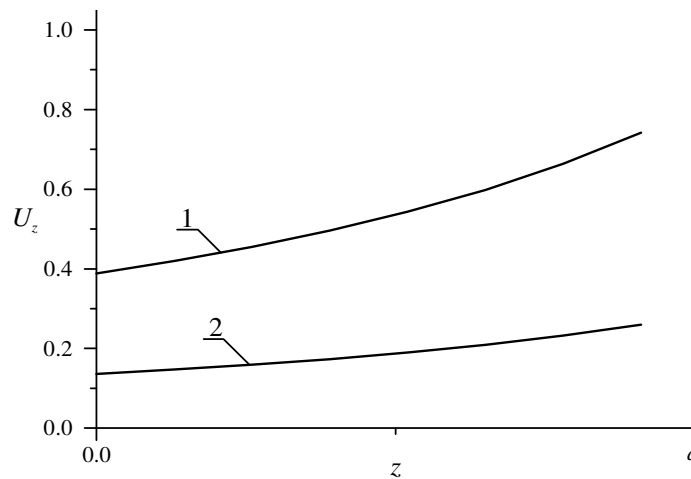


Fig.6. Normalized dependences of component u_z of displacement vector on coordinate z for epitaxial layers before radiation processing (curve 1) and after radiation processing (curve 2)

of dopant. We obtain during the analysis, that *p-n*-junctions, manufactured near interface between layers of heterostructures, have higher sharpness and higher homogeneity of concentration of dopant in enriched area. Existing mismatch-induced stress leads to changing of distribution of concentration of dopant in directions, which are parallel to the considered interface. For example, for $\varepsilon_0 < 0$ the above distribution in directions *x* and *y* became more compact (see Fig. 4). In this situation one can obtain increasing of density of elements of integrated circuits in this situation, when the circuits were fabricated in heterostructures. For $\varepsilon_0 > 0$ one can obtain opposite effect (see Fig. 5). It should be noted, that radiation processing of materials of heterostructure during ion doping of materials gives a possibility to decrease mismatch-induced stress (see Fig. 6).

Further we analyzed influence of mismatch-induced stress on distribution of concentration of dopant. We obtain during the analysis, that *p-n*-junctions, manufactured near interface between layers of heterostructures, have higher sharpness and higher homogeneity of concentration of dopant in enriched area. Existing mismatch-induced stress leads to changing of distribution of concentration of dopant in directions, which are parallel to the considered interface. For example, for $\varepsilon_0 < 0$ the above distribution in directions *x* and *y* became more compact (see Fig. 4). For $\varepsilon_0 > 0$ one can obtain opposite effect (see Fig. 5). It should be noted, that radiation processing of materials of heterostructure during ion doping of materials gives a possibility to decrease mismatch-induced stress (see Fig. 6). In this situation component of displacement vector perpendicular to interface between materials of heterostructure became smaller after radiation processing in comparison with analogous component of displacement vector in non processed heterostructure.

4. CONCLUSIONS

In this paper we analyzed influence of overgrowth of doped by diffusion or ion implantation areas of heterostructures on distributions of concentrations of dopants. We determine conditions to increase sharpness if implanted-junction and diffusion-junction rectifiers (single rectifiers and rectifiers framework bipolar transistors). At the same time we analyzed influence of overgrowth rate of doped areas and mismatch-induced stress in the considered heterostructure on distributions of concentrations of dopants.

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APPENDIX

Eqs. (1), (3) and (5) with account initial distributions could be written as

$$\frac{\partial C(x, y, z, t)}{\partial t} = \frac{\partial}{\partial x} \left[D \frac{\partial C(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[D \frac{\partial C(x, y, z, t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[D \frac{\partial C(x, y, z, t)}{\partial z} \right] + f_c(x, y, z) \delta(t) + \Omega \frac{\partial}{\partial x} \left[\frac{D_s}{kT} \nabla_s \mu(x, y, z, t) \int_{-v_t}^{L_s} C(x, y, W, t) dW \right] + \Omega \frac{\partial}{\partial y} \left[\frac{D_s}{kT} \nabla_s \mu(x, y, z, t) \int_{-v_t}^{L_s} C(x, y, W, t) dW \right] \quad (1a)$$

$$\begin{aligned} \frac{\partial I(x, y, z, t)}{\partial t} &= \frac{\partial}{\partial x} \left[D_I(x, y, z, T) \frac{\partial I(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[D_I(x, y, z, T) \frac{\partial I(x, y, z, t)}{\partial y} \right] + f_I(x, y, z) \delta(t) + \\ &+ \frac{\partial}{\partial z} \left[D_I(x, y, z, T) \frac{\partial I(x, y, z, t)}{\partial z} \right] - k_{I,V}(x, y, z, T) I(x, y, z, t) V(x, y, z, t) - k_{I,I}(x, y, z, T) I^2(x, y, z, t) + \\ &+ \Omega \frac{\partial}{\partial x} \left[\frac{D_{IS}}{kT} \nabla_s \mu(x, y, z, t) \int_{-v_I}^{L_z} I(x, y, W, t) dW \right] + \Omega \frac{\partial}{\partial y} \left[\frac{D_{IS}}{kT} \nabla_s \mu(x, y, z, t) \int_{-v_I}^{L_z} I(x, y, W, t) dW \right] \end{aligned} \quad (3a)$$

$$\begin{aligned} \frac{\partial V(x, y, z, t)}{\partial t} &= \frac{\partial}{\partial x} \left[D_V(x, y, z, T) \frac{\partial V(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[D_V(x, y, z, T) \frac{\partial V(x, y, z, t)}{\partial y} \right] + f_V(x, y, z) \delta(t) + \\ &+ \frac{\partial}{\partial z} \left[D_V(x, y, z, T) \frac{\partial V(x, y, z, t)}{\partial z} \right] - k_{I,V}(x, y, z, T) I(x, y, z, t) V(x, y, z, t) - k_{V,V}(x, y, z, T) V^2(x, y, z, t) + \\ &+ \Omega \frac{\partial}{\partial x} \left[\frac{D_{VS}}{kT} \nabla_s \mu(x, y, z, t) \int_{-v_I}^{L_z} V(x, y, W, t) dW \right] + \Omega \frac{\partial}{\partial y} \left[\frac{D_{VS}}{kT} \nabla_s \mu(x, y, z, t) \int_{-v_I}^{L_z} V(x, y, W, t) dW \right] \end{aligned}$$

$$\begin{aligned} \frac{\partial \Phi_I(x, y, z, t)}{\partial t} &= \frac{\partial}{\partial x} \left[D_{\Phi_I}(x, y, z, T) \frac{\partial \Phi_I(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[D_{\Phi_I}(x, y, z, T) \frac{\partial \Phi_I(x, y, z, t)}{\partial y} \right] + \delta(t) \times \\ &\times f_{\Phi_I}(x, y, z) + \frac{\partial}{\partial z} \left[D_{\Phi_I}(x, y, z, T) \frac{\partial \Phi_I(x, y, z, t)}{\partial z} \right] + \Omega \frac{\partial}{\partial x} \left[\frac{D_{\Phi_I S}}{kT} \nabla_s \mu(x, y, z, t) \int_{-v_I}^{L_z} \Phi_I(x, y, W, t) dW \right] + \\ &+ \Omega \frac{\partial}{\partial y} \left[\frac{D_{\Phi_I S}}{kT} \nabla_s \mu(x, y, z, t) \int_{-v_I}^{L_z} \Phi_I(x, y, W, t) dW \right] + k_I(x, y, z, T) I(x, y, z, t) + k_{I,I}(x, y, z, T) I^2(x, y, z, t) \\ \frac{\partial \Phi_V(x, y, z, t)}{\partial t} &= \frac{\partial}{\partial x} \left[D_{\Phi_V}(x, y, z, T) \frac{\partial \Phi_V(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[D_{\Phi_V}(x, y, z, T) \frac{\partial \Phi_V(x, y, z, t)}{\partial y} \right] + \delta(t) \times (5a) \\ &\times f_{\Phi_V}(x, y, z) + \frac{\partial}{\partial z} \left[D_{\Phi_V}(x, y, z, T) \frac{\partial \Phi_V(x, y, z, t)}{\partial z} \right] + \Omega \frac{\partial}{\partial x} \left[\frac{D_{\Phi_V S}}{kT} \nabla_s \mu(x, y, z, t) \int_{-v_I}^{L_z} \Phi_V(x, y, W, t) dW \right] + \\ &+ \Omega \frac{\partial}{\partial y} \left[\frac{D_{\Phi_V S}}{kT} \nabla_s \mu(x, y, z, t) \int_{-v_I}^{L_z} \Phi_V(x, y, W, t) dW \right] + k_V(x, y, z, T) V(x, y, z, t) + k_{V,V}(x, y, z, T) V^2(x, y, z, t). \end{aligned}$$

The first-order approximations of concentrations of dopant and radiation defects could be calculated by solution of the following equations

$$\begin{aligned} \frac{\partial C_1(x, y, z, t)}{\partial t} &= \alpha_{1c} \Omega \frac{\partial}{\partial x} \left[z \frac{D_S}{kT} \nabla_s \mu(x, y, z, t) \right] + \alpha_{1c} \Omega \frac{\partial}{\partial y} \left[z \frac{D_S}{kT} \nabla_s \mu(x, y, z, t) \right] + \\ &+ f_C(x, y, z) \delta(t) \end{aligned} \quad (1b)$$

$$\begin{aligned} \frac{\partial I_1(x, y, z, t)}{\partial t} &= \alpha_{1I} z \Omega \frac{\partial}{\partial x} \left[\frac{D_{IS}}{kT} \nabla_s \mu(x, y, z, t) \right] + \alpha_{1I} \Omega \frac{\partial}{\partial y} \left[z \frac{D_{IS}}{kT} \nabla_s \mu(x, y, z, t) \right] + f_I(x, y, z) \delta(t) - \\ &- \alpha_{1I}^2 k_{I,I}(x, y, z, T) - \alpha_{1I} \alpha_{1V} k_{I,V}(x, y, z, T) \end{aligned} \quad (3b)$$

$$\begin{aligned} \frac{\partial V_1(x, y, z, t)}{\partial t} &= \alpha_{1V} z \Omega \frac{\partial}{\partial x} \left[\frac{D_{VS}}{kT} \nabla_s \mu(x, y, z, t) \right] + \alpha_{1V} \Omega \frac{\partial}{\partial y} \left[z \frac{D_{VS}}{kT} \nabla_s \mu(x, y, z, t) \right] + f_V(x, y, z) \delta(t) - \\ &- \alpha_{1V}^2 k_{V,V}(x, y, z, T) - \alpha_{1I} \alpha_{1V} k_{I,V}(x, y, z, T) \end{aligned}$$

$$\begin{aligned} \frac{\partial \Phi_{I_l}(x, y, z, t)}{\partial t} &= k_l(x, y, z, T)I(x, y, z, t) + k_{l,l}(x, y, z, T)I^2(x, y, z, t) + f_{\Phi_l}(x, y, z)\delta(t) + \\ &+ \alpha_{1\Phi_l} z \Omega \frac{\partial}{\partial x} \left[\frac{D_{\Phi_l S}}{kT} \nabla_s \mu(x, y, z, t) \right] + \alpha_{1\Phi_l} z \Omega \frac{\partial}{\partial y} \left[\frac{D_{\Phi_l S}}{kT} \nabla_s \mu(x, y, z, t) \right] \end{aligned} \quad (5b)$$

$$\begin{aligned} \frac{\partial \Phi_{I_V}(x, y, z, t)}{\partial t} &= k_v(x, y, z, T)V(x, y, z, t) + k_{v,v}(x, y, z, T)V^2(x, y, z, t) + f_{\Phi_v}(x, y, z)\delta(t) + \\ &+ \alpha_{1\Phi_v} z \Omega \frac{\partial}{\partial x} \left[\frac{D_{\Phi_v S}}{kT} \nabla_s \mu(x, y, z, t) \right] + \alpha_{1\Phi_v} z \Omega \frac{\partial}{\partial y} \left[\frac{D_{\Phi_v S}}{kT} \nabla_s \mu(x, y, z, t) \right] \end{aligned}$$

The first-order approximations of the considered concentrations in the following form

$$\begin{aligned} C_1(x, y, z, t) &= \alpha_{1C} \frac{\partial}{\partial x_0} \int_0^t \nabla_s \mu_1(x, y, z, \tau) \left[1 + \frac{\xi_s \alpha_{1C}^\gamma}{P^\gamma(x, y, z, T)} \right] \left[1 + \zeta_1 \frac{V(x, y, z, \tau)}{V^*} + \zeta_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] \times \\ &\times \Omega D_{sL}(x, y, z, T) \frac{z}{kT} d\tau \left\} + \alpha_{1C} \Omega \frac{\partial}{\partial y_0} \int_0^t \nabla_s \mu_1(x, y, z, \tau) \left[1 + \zeta_1 \frac{V(x, y, z, \tau)}{V^*} + \zeta_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] \times \right. \\ &\left. \times D_{sL}(x, y, z, T) \frac{z}{kT} \left[1 + \frac{\xi_s \alpha_{1C}^\gamma}{P^\gamma(x, y, z, T)} \right] d\tau + f_c(x, y, z) \right. \end{aligned} \quad (1c)$$

$$\begin{aligned} I_1(x, y, z, t) &= \alpha_{1I} z \Omega \frac{\partial}{\partial x_0} \int_0^t \frac{D_{IS}}{kT} \nabla_s \mu(x, y, z, \tau) d\tau + \alpha_{1I} z \Omega \frac{\partial}{\partial y_0} \int_0^t \frac{D_{IS}}{kT} \nabla_s \mu(x, y, z, \tau) d\tau + f_I(x, y, z) - \\ &- \alpha_{1I}^2 \int_0^t k_{l,l}(x, y, z, T) d\tau - \alpha_{1I} \alpha_{1V} \int_0^t k_{l,v}(x, y, z, T) d\tau \end{aligned} \quad (3c)$$

$$\begin{aligned} V_1(x, y, z, t) &= \alpha_{1V} z \Omega \frac{\partial}{\partial x_0} \int_0^t \frac{D_{IS}}{kT} \nabla_s \mu_1(x, y, z, \tau) d\tau + \alpha_{1V} z \Omega \frac{\partial}{\partial y_0} \int_0^t \frac{D_{IS}}{kT} \nabla_s \mu_1(x, y, z, \tau) d\tau + f_v(x, y, z) - \\ &- \alpha_{1V}^2 \int_0^t k_{v,v}(x, y, z, T) d\tau - \alpha_{1I} \alpha_{1V} \int_0^t k_{l,v}(x, y, z, T) d\tau \end{aligned}$$

$$\begin{aligned} \Phi_{I_l}(x, y, z, t) &= \alpha_{1\Phi_l} z \Omega \frac{\partial}{\partial x_0} \int_0^t \frac{D_{\Phi_l S}}{kT} \nabla_s \mu(x, y, z, \tau) d\tau + \alpha_{1\Phi_l} z \Omega \frac{\partial}{\partial x_0} \int_0^t \frac{D_{\Phi_l S}}{kT} \nabla_s \mu(x, y, z, \tau) d\tau + \\ &+ f_{\Phi_l}(x, y, z) + \int_0^t k_l(x, y, z, T)I(x, y, z, \tau) d\tau + \int_0^t k_{l,l}(x, y, z, T)I^2(x, y, z, \tau) d\tau \end{aligned} \quad (5c)$$

$$\begin{aligned} \Phi_{I_V}(x, y, z, t) &= \alpha_{1\Phi_v} z \Omega \frac{\partial}{\partial x_0} \int_0^t \frac{D_{\Phi_v S}}{kT} \nabla_s \mu(x, y, z, \tau) d\tau + \alpha_{1\Phi_v} z \Omega \frac{\partial}{\partial x_0} \int_0^t \frac{D_{\Phi_v S}}{kT} \nabla_s \mu(x, y, z, \tau) d\tau + \\ &+ f_{\Phi_v}(x, y, z) + \int_0^t k_v(x, y, z, T)V(x, y, z, \tau) d\tau + \int_0^t k_{v,v}(x, y, z, T)V^2(x, y, z, \tau) d\tau. \end{aligned}$$

Relations for calculations parameters S_{ppij} , a_i , A , B , q , p could be written as

$$S_{ppij} = \int_0^\Theta (\Theta - t) \int_0^{L_x} \int_0^{L_y} \int_{-v_t}^{L_z} k_{\rho,\rho}(x, y, z, T) I_1^i(x, y, z, t) V_1^j(x, y, z, t) dz dy dx dt, \quad a_4 = (S_{IV00}^2 - S_{II00} S_{VV00}) \times$$

$$\begin{aligned}
 & \times S_{H00}, a_3 = S_{IV00}S_{H00} + S_{IV00}^2 - S_{H00}S_{VV00}, a_2 = S_{IV00} \Theta L_x^2 L_y^2 L_z^2 + 2 \int_0^{\Theta L_x L_y L_z} \int_0^{\Theta L_x L_y L_z} \int_0^{\Theta L_x L_y L_z} f_I(x, y, z) dz dy dx dt \times \\
 & \times S_{VV00}S_{H00} + S_{IV00}S_{IV00}^2 \int_0^{\Theta L_x L_y L_z} \int_0^{\Theta L_x L_y L_z} \int_0^{\Theta L_x L_y L_z} f_V(x, y, z) dz dy dx dt - S_{IV00}^2 \int_0^{\Theta L_x L_y L_z} \int_0^{\Theta L_x L_y L_z} \int_0^{\Theta L_x L_y L_z} f_I(x, y, z) dz dy dx dt - \Theta L_x^2 \times \\
 & \times L_y^2 L_z^2 S_{VV00}, a_1 = S_{IV00} \int_0^{\Theta L_x L_y L_z} \int_0^{\Theta L_x L_y L_z} \int_0^{\Theta L_x L_y L_z} f_I(x, y, z) dz dy dx dt, B = \sqrt[3]{\sqrt{q^2 + p^3} - q} - \sqrt[3]{\sqrt{q^2 + p^3} + q} + \\
 & + \frac{\Theta a_2}{6a_4}, a_0 = S_{VV00} \left[\int_0^{\Theta L_x L_y L_z} \int_0^{\Theta L_x L_y L_z} \int_0^{\Theta L_x L_y L_z} f_I(x, y, z) dz dy dx dt \right]^2, A = \left(\Theta^2 \frac{a_3^2}{a_4^2} - 4\Theta \frac{a_2}{a_4} + 8y \right)^{\frac{1}{2}}, q = \frac{\Theta^3 a_2}{24a_4^2} \times \\
 & \times \left(4a_0 - \Theta L_x L_y L_z \frac{a_1 a_3}{a_4} \right) - \Theta^2 \frac{a_0}{8a_4^2} \left(4\Theta a_2 - \Theta^2 \frac{a_3^2}{a_4} \right) - \frac{\Theta^3 a_2^3}{54a_4^3} - L_x^2 L_y^2 L_z^2 \frac{\Theta^4 a_1^2}{8a_4^2}, p = \Theta^2 \frac{4a_0 a_4}{12a_4^2} - \\
 & - \Theta \left(3\Theta^2 L_x L_y L_z a_1 a_3 + 2a_2 a_4 \right) / 36a_4^2, R_{\rho_i} = \int_0^{\Theta} \left(\Theta - t \right) \int_0^{\Theta L_x L_y L_z} \int_0^{\Theta L_x L_y L_z} \int_0^{\Theta L_x L_y L_z} k_I(x, y, z, T) T^i(x, y, z, t) dz dy dx dt.
 \end{aligned}$$

Equations for the second-order approximations of concentrations of dopant and radiation defects could be written as

$$\begin{aligned}
 \frac{\partial C_2(x, y, z, t)}{\partial t} &= \frac{\partial}{\partial x} \left(D_L(x, y, z, T) \left\{ 1 + \xi \frac{[\alpha_{2C} + C_1(x, y, z, t)]^\gamma}{P^\gamma(x, y, z, T)} \right\} \left[1 + \varsigma_1 \frac{V(x, y, z, t)}{V^*} + \varsigma_2 \frac{V^2(x, y, z, t)}{(V^*)^2} \right] \times \right. \\
 & \times \frac{\partial C_1(x, y, z, t)}{\partial x} \left. \right) + \frac{\partial}{\partial y} \left(\left[1 + \varsigma_1 \frac{V(x, y, z, t)}{V^*} + \varsigma_2 \frac{V^2(x, y, z, t)}{(V^*)^2} \right] \left\{ 1 + \xi \frac{[\alpha_{2C} + C_1(x, y, z, t)]^\gamma}{P^\gamma(x, y, z, T)} \right\} \frac{\partial C_1(x, y, z, t)}{\partial y} \times \right. \\
 & \times D_L(x, y, z, T) \left. \right) + \frac{\partial}{\partial z} \left(\left[1 + \varsigma_1 \frac{V(x, y, z, t)}{V^*} + \varsigma_2 \frac{V^2(x, y, z, t)}{(V^*)^2} \right] \left\{ 1 + \xi \frac{[\alpha_{2C} + C_1(x, y, z, t)]^\gamma}{P^\gamma(x, y, z, T)} \right\} \times \right. \\
 & \times D_L(x, y, z, T) \frac{\partial C_1(x, y, z, t)}{\partial z} \left. \right) + \Omega \frac{\partial}{\partial x} \left\{ \frac{D_s}{kT} \nabla_s \mu_2(x, y, z, t) \int_{-v_I}^{L_z} [\alpha_{2C} + C(x, y, W, t)] dW \right\} + \\
 & + \Omega \frac{\partial}{\partial y} \left\{ \frac{D_s}{kT} \nabla_s \mu_2(x, y, z, t) \int_{-v_I}^{L_z} [\alpha_{2C} + C(x, y, W, t)] dW \right\} + f_c(x, y, z) \delta(t) \quad (1d)
 \end{aligned}$$

$$\begin{aligned}
 \frac{\partial I_2(x, y, z, t)}{\partial t} &= \frac{\partial}{\partial x} \left[D_I(x, y, z, T) \frac{\partial I_1(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[D_I(x, y, z, T) \frac{\partial I_1(x, y, z, t)}{\partial y} \right] - k_{I,V}(x, y, z, T) \times \\
 & \times [\alpha_{I1} + I_1(x, y, z, t)] [\alpha_{IV} + V_1(x, y, z, t)] + \frac{\partial}{\partial z} \left[D_I(x, y, z, T) \frac{\partial I_1(x, y, z, t)}{\partial z} \right] - [\alpha_{I1} + I_1(x, y, z, t)]^2 \times \\
 & \times k_{I,I}(x, y, z, T) + \Omega \frac{\partial}{\partial x} \left\{ \frac{D_{IS}}{kT} \nabla_s \mu(x, y, z, t) \int_{-v_I}^{L_z} [\alpha_{2I} + I_1(x, y, W, t)] dW \right\} + \\
 & + \Omega \frac{\partial}{\partial y} \left\{ \frac{D_{IS}}{kT} \nabla_s \mu(x, y, z, t) \int_{-v_I}^{L_z} [\alpha_{2I} + I_1(x, y, W, t)] dW \right\} \quad (3d)
 \end{aligned}$$

$$\begin{aligned}
 \frac{\partial V_2(x, y, z, t)}{\partial t} &= \frac{\partial}{\partial x} \left[D_V(x, y, z, T) \frac{\partial V_1(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[D_V(x, y, z, T) \frac{\partial V_1(x, y, z, t)}{\partial y} \right] - k_{I,V}(x, y, z, T) \times \\
 & \times [\alpha_{IV} + V_1(x, y, z, t)] [\alpha_{IV} + V_1(x, y, z, t)] + \frac{\partial}{\partial z} \left[D_V(x, y, z, T) \frac{\partial V_1(x, y, z, t)}{\partial z} \right] - [\alpha_{IV} + V_1(x, y, z, t)]^2 \times
 \end{aligned}$$

$$\begin{aligned}
 & \times k_{v,v}(x, y, z, T) + \Omega \frac{\partial}{\partial x} \left\{ \frac{D_{vs}}{kT} \nabla_s \mu(x, y, z, t) \int_{-v\tau}^{L_z} [\alpha_{2v} + V_1(x, y, W, t)] dW \right\} + \\
 & \qquad \qquad \qquad + \Omega \frac{\partial}{\partial y} \left\{ \frac{D_{vs}}{kT} \nabla_s \mu(x, y, z, t) \int_{-v\tau}^{L_z} [\alpha_{2v} + V_1(x, y, W, t)] dW \right\} \\
 \frac{\partial \Phi_{2l}(x, y, z, t)}{\partial t} &= \frac{\partial}{\partial x} \left[D_{\Phi_l}(x, y, z, T) \frac{\partial \Phi_{1l}(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[D_{\Phi_l}(x, y, z, T) \frac{\partial \Phi_{1l}(x, y, z, t)}{\partial y} \right] + k_{l,l}(x, y, z, T) \times \\
 & \times I^2(x, y, z, t) + \Omega \frac{\partial}{\partial x} \left\{ \frac{D_{\Phi_l s}}{kT} \nabla_s \mu(x, y, z, t) \int_{-v\tau}^{L_z} [\alpha_{2\Phi_l} + \Phi_{1l}(x, y, W, t)] dW \right\} + f_{\Phi_l}(x, y, z) \delta(t) + \\
 & + \Omega \frac{\partial}{\partial y} \left\{ \frac{D_{\Phi_l s}}{kT} \nabla_s \mu(x, y, z, t) \int_{-v\tau}^{L_z} [\alpha_{2\Phi_l} + \Phi_{1l}(x, y, W, t)] dW \right\} + \frac{\partial}{\partial z} \left[D_{\Phi_l}(x, y, z, T) \frac{\partial \Phi_{1l}(x, y, z, t)}{\partial z} \right] + \\
 & \qquad \qquad \qquad + k_l(x, y, z, T) I(x, y, z, t) \tag{5d} \\
 \frac{\partial \Phi_{2v}(x, y, z, t)}{\partial t} &= \frac{\partial}{\partial x} \left[D_{\Phi_v}(x, y, z, T) \frac{\partial \Phi_{1v}(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[D_{\Phi_v}(x, y, z, T) \frac{\partial \Phi_{1v}(x, y, z, t)}{\partial y} \right] + k_{v,v}(x, y, z, T) \times \\
 & \times V^2(x, y, z, t) + \Omega \frac{\partial}{\partial x} \left\{ \frac{D_{\Phi_v s}}{kT} \nabla_s \mu(x, y, z, t) \int_{-v\tau}^{L_z} [\alpha_{2\Phi_v} + \Phi_{1v}(x, y, W, t)] dW \right\} + f_{\Phi_v}(x, y, z) \delta(t) + \\
 & + \Omega \frac{\partial}{\partial y} \left\{ \frac{D_{\Phi_v s}}{kT} \nabla_s \mu(x, y, z, t) \int_{-v\tau}^{L_z} [\alpha_{2\Phi_v} + \Phi_{1v}(x, y, W, t)] dW \right\} + \frac{\partial}{\partial z} \left[D_{\Phi_v}(x, y, z, T) \frac{\partial \Phi_{1v}(x, y, z, t)}{\partial z} \right] + \\
 & \qquad \qquad \qquad + k_v(x, y, z, T) V(x, y, z, t).
 \end{aligned}$$

The second-order approximations of concentrations of dopant and radiation defects could be written as

$$\begin{aligned}
 C_2(x, y, z, t) &= \frac{\partial}{\partial x_0} \int_0^t D_L(x, y, z, T) \left\{ 1 + \xi \frac{[\alpha_{2c} + C_1(x, y, z, \tau)]^\gamma}{P^\gamma(x, y, z, T)} \right\} \left[1 + \varsigma_1 \frac{V(x, y, z, \tau)}{V^*} + \varsigma_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] \times \\
 & \times \frac{\partial C_1(x, y, z, \tau)}{\partial x} d\tau + \frac{\partial}{\partial y_0} \int_0^t \left[1 + \varsigma_1 \frac{V(x, y, z, \tau)}{V^*} + \varsigma_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] \left\{ 1 + \xi \frac{[\alpha_{2c} + C_1(x, y, z, \tau)]^\gamma}{P^\gamma(x, y, z, T)} \right\} \times \\
 & \times D_L(x, y, z, T) \frac{\partial C_1(x, y, z, \tau)}{\partial y} d\tau + \frac{\partial}{\partial z_0} \int_0^t D_L(x, y, z, T) \left[1 + \varsigma_1 \frac{V(x, y, z, \tau)}{V^*} + \varsigma_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] \times \\
 & \times \frac{\partial C_1(x, y, z, \tau)}{\partial z} \left\{ 1 + \xi \frac{[\alpha_{2c} + C_1(x, y, z, \tau)]^\gamma}{P^\gamma(x, y, z, T)} \right\} d\tau + \Omega \frac{\partial}{\partial x_0} \int_0^t \frac{D_s}{kT} \int_{-v\tau}^{L_z} [\alpha_{2c} + C_1(x, y, W, \vartheta)] dW \times \\
 & \times \nabla_s \mu(x, y, z, \tau) d\vartheta d\tau + \Omega \frac{\partial}{\partial y_0} \int_0^t \frac{D_s}{kT} \nabla_s \mu(x, y, z, \tau) \int_{-v\tau}^{L_z} [\alpha_{2c} + C_1(x, y, W, \vartheta)] dW d\vartheta d\tau + \\
 & \qquad \qquad \qquad + f_c(x, y, z) \tag{1e}
 \end{aligned}$$

$$\begin{aligned}
 I_2(x, y, z, t) &= \frac{\partial}{\partial x_0} \int_0^t D_l(x, y, z, T) \frac{\partial I_1(x, y, z, \tau)}{\partial x} d\tau + \frac{\partial}{\partial y_0} \int_0^t D_l(x, y, z, T) \frac{\partial I_1(x, y, z, \tau)}{\partial y} d\tau + \\
 & + \frac{\partial}{\partial z_0} \int_0^t D_l(x, y, z, T) \frac{\partial I_1(x, y, z, \tau)}{\partial z} d\tau - \int_0^t k_{l,l}(x, y, z, T) [\alpha_{2l} + I_1(x, y, z, \tau)]^2 d\tau + f_l(x, y, z) + \\
 & + \Omega \frac{\partial}{\partial x_0} \int_0^t \frac{D_{ls}}{kT} \nabla_s \mu(x, y, z, \tau) \int_{-v\tau}^{L_z} [\alpha_{2l} + I_1(x, y, W, \vartheta)] dW d\vartheta d\tau + \Omega \frac{\partial}{\partial y_0} \int_0^t \frac{D_{ls}}{kT} \nabla_s \mu(x, y, z, \tau) \times \\
 & \times \int_{-v\tau}^{L_z} [\alpha_{2l} + I_1(x, y, W, \vartheta)] dW d\vartheta d\tau - \int_0^t k_{l,v}(x, y, z, T) [\alpha_{2l} + I_1(x, y, z, \tau)] [\alpha_{2v} + V_1(x, y, z, \tau)] d\tau \tag{3e}
 \end{aligned}$$

$$\begin{aligned}
 V_2(x, y, z, t) = & \frac{\partial}{\partial x_0} \int D_V(x, y, z, T) \frac{\partial V_1(x, y, z, \tau)}{\partial x} d\tau + \frac{\partial}{\partial y_0} \int D_V(x, y, z, T) \frac{\partial V_1(x, y, z, \tau)}{\partial y} d\tau + \\
 & + \frac{\partial}{\partial z_0} \int D_V(x, y, z, T) \frac{\partial V_1(x, y, z, \tau)}{\partial z} d\tau - \int_0^t k_{V,V}(x, y, z, T) [\alpha_{2V} + V_1(x, y, z, \tau)]^2 d\tau + f_V(x, y, z) + \\
 & + \Omega \frac{\partial}{\partial x_0} \int \frac{D_{VS}}{kT} \nabla_s \mu(x, y, z, \tau) \int_{-\nu\tau}^{L_z} [\alpha_{2V} + V_1(x, y, W, \vartheta)] dW d\vartheta d\tau + \Omega \frac{\partial}{\partial y_0} \int \frac{D_{VS}}{kT} \nabla_s \mu(x, y, z, \tau) \times \\
 & \times \int_{-\nu\tau}^{L_z} [\alpha_{2V} + V_1(x, y, W, \vartheta)] dW d\vartheta d\tau - \int_0^t k_{I,V}(x, y, z, T) [\alpha_{2I} + I_1(x, y, z, \tau)] [\alpha_{2V} + V_1(x, y, z, \tau)] d\tau \\
 \Phi_{2I}(x, y, z, t) = & \frac{\partial}{\partial x_0} \int D_{\Phi_I}(x, y, z, T) \frac{\partial \Phi_{II}(x, y, z, \tau)}{\partial x} d\tau + \frac{\partial}{\partial y_0} \int D_{\Phi_I}(x, y, z, T) \frac{\partial \Phi_{II}(x, y, z, \tau)}{\partial y} d\tau + \\
 & + \frac{\partial}{\partial z_0} \int D_{\Phi_I}(x, y, z, T) \frac{\partial \Phi_{II}(x, y, z, \tau)}{\partial z} d\tau + \Omega \frac{\partial}{\partial x_0} \int \nabla_s \mu(x, y, z, \tau) \int_{-\nu\tau}^{L_z} [\alpha_{2\Phi_I} + \Phi_{II}(x, y, W, \tau)] dW \times \\
 & \times \frac{D_{\Phi_I S}}{kT} d\tau + \Omega \frac{\partial}{\partial y_0} \int \frac{D_{\Phi_I S}}{kT} \nabla_s \mu(x, y, z, \tau) \int_{-\nu\tau}^{L_z} [\alpha_{2\Phi_I} + \Phi_{II}(x, y, W, \vartheta)] dW d\vartheta d\tau + f_{\Phi_I}(x, y, z) + \\
 & + \int_0^t k_{I,I}(x, y, z, T) I^2(x, y, z, \tau) d\tau + \int_0^t k_I(x, y, z, T) I(x, y, z, \tau) d\tau \quad (5e)
 \end{aligned}$$

$$\begin{aligned}
 \Phi_{2V}(x, y, z, t) = & \frac{\partial}{\partial x_0} \int D_{\Phi_V}(x, y, z, T) \frac{\partial \Phi_{IV}(x, y, z, \tau)}{\partial x} d\tau + \frac{\partial}{\partial y_0} \int D_{\Phi_V}(x, y, z, T) \frac{\partial \Phi_{IV}(x, y, z, \tau)}{\partial y} d\tau + \\
 & + \frac{\partial}{\partial z_0} \int D_{\Phi_V}(x, y, z, T) \frac{\partial \Phi_{IV}(x, y, z, \tau)}{\partial z} d\tau + \Omega \frac{\partial}{\partial x_0} \int \nabla_s \mu(x, y, z, \tau) \int_{-\nu\tau}^{L_z} [\alpha_{2\Phi_V} + \Phi_{IV}(x, y, W, \tau)] dW \times \\
 & \times \frac{D_{\Phi_V S}}{kT} d\tau + \Omega \frac{\partial}{\partial y_0} \int \frac{D_{\Phi_V S}}{kT} \nabla_s \mu(x, y, z, \tau) \int_{-\nu\tau}^{L_z} [\alpha_{2\Phi_V} + \Phi_{IV}(x, y, W, \vartheta)] dW d\vartheta d\tau + f_{\Phi_V}(x, y, z) + \\
 & + \int_0^t k_{V,V}(x, y, z, T) V^2(x, y, z, \tau) d\tau + \int_0^t k_V(x, y, z, T) V(x, y, z, \tau) d\tau.
 \end{aligned}$$

Parameters b_i, C_p, F, r, s have been determined by the following relations

$$\begin{aligned}
 b_4 = & \frac{1}{\Theta L_x L_y L_z} S_{IV00}^2 S_{VV00} - \frac{1}{\Theta L_x L_y L_z} S_{VV00}^2 S_{II00}, \quad b_3 = -(2S_{VV01} + S_{IV10} + \Theta L_x L_y L_z) \frac{S_{II00} S_{VV00}}{\Theta L_x L_y L_z} + \\
 & + \frac{S_{IV00} S_{VV00}}{\Theta L_x L_y L_z} (S_{IV01} + 2S_{II10} + S_{IV01} + \Theta L_x L_y L_z) + \frac{S_{IV00}^2}{\Theta L_x L_y L_z} (2S_{VV01} + S_{IV10} + \Theta L_x L_y L_z) - \frac{S_{IV00}^2 S_{IV10}}{\Theta^3 L_x^3 L_y^3 L_z^3}, \\
 b_2 = & \frac{S_{II00} S_{VV00}}{\Theta L_x L_y L_z} (S_{VV02} + S_{IV11} + C_V) - (\Theta L_x L_y L_z - 2S_{VV01} + S_{IV10})^2 + \frac{S_{IV01}}{\Theta L_x} (\Theta L_x L_y L_z + 2S_{II10} + S_{IV01}) \times \\
 & \times \frac{S_{VV00}}{L_y L_z} + \frac{S_{IV00}}{\Theta L_x L_y L_z} (\Theta L_x L_y L_z + S_{IV01} + 2S_{II10} + 2S_{IV01}) (2S_{VV01} + \Theta L_x L_y L_z + S_{IV10}) - \frac{S_{IV00}^2}{\Theta L_x L_y L_z} \times \\
 & \times (C_V - S_{VV02} - S_{IV11}) + \frac{C_I S_{IV00}^2}{\Theta^2 L_x^2 L_y^2 L_z^2} - \frac{2S_{IV10} S_{IV00}}{\Theta L_x L_y L_z} S_{IV01}, \quad b_1 = S_{II00} \frac{S_{IV11} + S_{VV02} + C_V}{\Theta L_x L_y L_z} (S_{IV10} + \Theta L_x \times \\
 & \times L_y L_z + 2S_{VV01}) + S_{IV01} \frac{\Theta L_x L_y L_z + 2S_{II10} + S_{IV01}}{\Theta L_x L_y L_z} (2S_{VV01} + S_{IV10} + \Theta L_x L_y L_z) - \frac{C_V - S_{VV02} - S_{IV11}}{\Theta L_x L_y L_z} \times \\
 & \times S_{IV00} (3S_{IV01} + 2S_{II10} + \Theta L_x L_y L_z) + 2C_I S_{IV00} S_{IV01} - \frac{S_{IV10} S_{IV01}^2}{\Theta L_x L_y L_z}, \quad b_0 = S_{II00} \frac{(S_{IV00} + S_{VV02})^2}{\Theta L_x L_y L_z} - S_{IV01} \times
 \end{aligned}$$

$$\begin{aligned} & \times \frac{C_V - S_{VV02} - S_{IV11}}{\Theta L_x L_y L_z} (\Theta L_x L_y L_z + 2S_{II10} + S_{IV01}) - \frac{C_V - S_{VV02} - S_{IV11}}{\Theta L_x L_y L_z} (\Theta L_x L_y L_z + 2S_{II10} + S_{IV01}) \times \\ & \times S_{IV01} + 2C_I S_{IV01}^2 - S_{IV01} \frac{C_V - S_{VV02} - S_{IV11}}{\Theta L_x L_y L_z} (\Theta L_x L_y L_z + 2S_{II10} + S_{IV01}), C_I = S_{IV00} \frac{\alpha_{II} \alpha_{IV}}{\Theta L_x L_y L_z} + \\ & + \frac{\alpha_{IV}^2 S_{II00}}{\Theta L_x L_y L_z} - \frac{S_{II20} S_{II20}}{\Theta L_x L_y L_z} - \frac{S_{IV11}}{\Theta L_x L_y L_z}, C_V = \alpha_{IV} \alpha_{IV} S_{IV00} + \alpha_{IV}^2 S_{VV00} - S_{VV02} - S_{IV11}, E = \sqrt{8y + \Theta^2 \frac{a_3^2}{a_4^2} - 4\Theta \frac{a_2}{a_4}}, \\ & r = \frac{\Theta^3 b_2}{24b_4^2} \left(4b_0 - \Theta L_x L_y L_z \frac{b_1 b_3}{b_4} \right) - b_0 \frac{\Theta^2}{8b_4^2} \left(4\Theta b_2 - \Theta^2 \frac{b_3}{b_4} \right) - \frac{\Theta^3 b_2^3}{54b_4^3} - L_x^2 L_y^2 L_z^2 \frac{\Theta^4 b_1^2}{8b_4^2}, s = \frac{\Theta^2}{12b_4^2} (4b_0 b_4 - \\ & - \Theta L_x L_y L_z b_1 b_3) - \Theta b_2 / 18b_4, F = \frac{\Theta a_2}{6a_4} + \sqrt[3]{\sqrt{r^2 + s^3} - r} - \sqrt[3]{\sqrt{r^2 + s^3} + r}. \end{aligned}$$

Equations for components of the displacement vector could be presented in the form

$$\begin{aligned} \rho(z) \frac{\partial^2 u_{2x}(x, y, z, t)}{\partial t^2} &= \left\{ K(z) + \frac{5E(z)}{6[1 + \sigma(z)]} \right\} \frac{\partial^2 u_{1x}(x, y, z, t)}{\partial x^2} + \left\{ K(z) - \frac{E(z)}{3[1 + \sigma(z)]} \right\} \frac{\partial^2 u_{1y}(x, y, z, t)}{\partial x \partial y} + \\ &+ \frac{E(z)}{2[1 + \sigma(z)]} \left[\frac{\partial^2 u_{1y}(x, y, z, t)}{\partial y^2} + \frac{\partial^2 u_{1z}(x, y, z, t)}{\partial z^2} \right] + \left\{ K(z) + \frac{E(z)}{3[1 + \sigma(z)]} \right\} \frac{\partial^2 u_{1z}(x, y, z, t)}{\partial x \partial z} - \\ &\quad - K(z) \beta(z) \frac{\partial T(x, y, z, t)}{\partial x} \\ \rho(z) \frac{\partial^2 u_{2y}(x, y, z, t)}{\partial t^2} &= \frac{E(z)}{2[1 + \sigma(z)]} \left[\frac{\partial^2 u_{1y}(x, y, z, t)}{\partial x^2} + \frac{\partial^2 u_{1x}(x, y, z, t)}{\partial x \partial y} \right] - K(z) \beta(z) \frac{\partial T(x, y, z, t)}{\partial y} + \\ &+ \frac{\partial}{\partial z} \left\{ \frac{E(z)}{2[1 + \sigma(z)]} \left[\frac{\partial u_{1y}(x, y, z, t)}{\partial z} + \frac{\partial u_{1z}(x, y, z, t)}{\partial y} \right] \right\} + \frac{\partial^2 u_{1y}(x, y, z, t)}{\partial y^2} \left\{ \frac{5E(z)}{12[1 + \sigma(z)]} + K(z) \right\} + \\ &\quad + \left\{ K(z) - \frac{E(z)}{6[1 + \sigma(z)]} \right\} \frac{\partial^2 u_{1y}(x, y, z, t)}{\partial y \partial z} + K(z) \frac{\partial^2 u_{1y}(x, y, z, t)}{\partial x \partial y} \\ \rho(z) \frac{\partial^2 u_{2z}(x, y, z, t)}{\partial t^2} &= \left[\frac{\partial^2 u_{1z}(x, y, z, t)}{\partial x^2} + \frac{\partial^2 u_{1z}(x, y, z, t)}{\partial y^2} + \frac{\partial^2 u_{1x}(x, y, z, t)}{\partial x \partial z} + \frac{\partial^2 u_{1y}(x, y, z, t)}{\partial y \partial z} \right] \times \\ &\times \frac{E(z)}{2[1 + \sigma(z)]} + \frac{\partial}{\partial z} \left\{ K(z) \left[\frac{\partial u_{1x}(x, y, z, t)}{\partial x} + \frac{\partial u_{1y}(x, y, z, t)}{\partial y} + \frac{\partial u_{1z}(x, y, z, t)}{\partial z} \right] \right\} + \frac{\partial}{\partial z} \left\{ \frac{E(z)}{6[1 + \sigma(z)]} \times \right. \\ &\times \left. \left[6 \frac{\partial u_{1z}(x, y, z, t)}{\partial z} - \frac{\partial u_{1x}(x, y, z, t)}{\partial x} - \frac{\partial u_{1y}(x, y, z, t)}{\partial y} - \frac{\partial u_{1z}(x, y, z, t)}{\partial z} \right] \right\} - K(z) \beta(z) \frac{\partial T(x, y, z, t)}{\partial z}. \end{aligned}$$

Integration of the left and the right sides of the above equations on time t leads to final relations for components of displacement vector

$$\begin{aligned} u_{2x}(x, y, z, t) &= \frac{1}{\rho(z)} \left\{ K(z) + \frac{5E(z)}{6[1 + \sigma(z)]} \right\} \frac{\partial^2}{\partial x^2} \int_0^t \int_0^\vartheta u_{1x}(x, y, z, \tau) d\tau d\vartheta + \frac{1}{\rho(z)} \left\{ K(z) - \frac{E(z)}{3[1 + \sigma(z)]} \right\} \times \\ &\times \frac{\partial^2}{\partial x \partial y} \int_0^t \int_0^\vartheta u_{1y}(x, y, z, \tau) d\tau d\vartheta + \left[\frac{\partial^2}{\partial y^2} \int_0^t \int_0^\vartheta u_{1y}(x, y, z, \tau) d\tau d\vartheta + \frac{\partial^2}{\partial z^2} \int_0^t \int_0^\vartheta u_{1z}(x, y, z, \tau) d\tau d\vartheta \right] \times \end{aligned}$$

$$\begin{aligned}
 & \times \frac{E(z)}{2\rho(z)[1+\sigma(z)]} + \frac{1}{\rho(z)} \frac{\partial^2}{\partial x \partial z} \int_0^\vartheta \int_0^\vartheta u_{1z}(x, y, z, \tau) d\tau d\vartheta \left\{ K(z) + \frac{E(z)}{3[1+\sigma(z)]} \right\} - K(z) \frac{\beta(z)}{\rho(z)} \times \\
 & \times \frac{\partial}{\partial x} \int_0^\vartheta \int_0^\vartheta T(x, y, z, \tau) d\tau d\vartheta - \frac{1}{\rho(z)} \left\{ K(z) + \frac{5E(z)}{6[1+\sigma(z)]} \right\} \frac{\partial^2}{\partial x^2} \int_0^\vartheta \int_0^\vartheta u_{1x}(x, y, z, \tau) d\tau d\vartheta - \frac{1}{\rho(z)} \times \\
 & \times \left\{ K(z) - \frac{E(z)}{3[1+\sigma(z)]} \right\} \frac{\partial^2}{\partial x \partial y} \int_0^\vartheta \int_0^\vartheta u_{1y}(x, y, z, \tau) d\tau d\vartheta - \frac{1}{2\rho(z)} \left[\frac{\partial^2}{\partial y^2} \int_0^\vartheta \int_0^\vartheta u_{1y}(x, y, z, \tau) d\tau d\vartheta + \right. \\
 & \left. + \frac{\partial^2}{\partial z^2} \int_0^\vartheta \int_0^\vartheta u_{1z}(x, y, z, \tau) d\tau d\vartheta \right] \frac{E(z)}{1+\sigma(z)} - \left\{ K(z) + \frac{E(z)}{3[1+\sigma(z)]} \right\} \frac{\partial^2}{\partial x \partial z} \int_0^\vartheta \int_0^\vartheta u_{1z}(x, y, z, \tau) d\tau d\vartheta \times \\
 & \quad \times \frac{1}{\rho(z)} + u_{0x} + K(z) \frac{\beta(z)}{\rho(z)} \frac{\partial}{\partial x} \int_0^\vartheta \int_0^\vartheta T(x, y, z, \tau) d\tau d\vartheta \\
 \\
 u_{2y}(x, y, z, t) = & \frac{E(z)}{2\rho(z)[1+\sigma(z)]} \left[\frac{\partial^2}{\partial x^2} \int_0^\vartheta \int_0^\vartheta u_{1x}(x, y, z, \tau) d\tau d\vartheta + \frac{\partial^2}{\partial x \partial y} \int_0^\vartheta \int_0^\vartheta u_{1x}(x, y, z, \tau) d\tau d\vartheta \right] + \\
 & + \frac{K(z)}{\rho(z)} \frac{\partial^2}{\partial x \partial y} \int_0^\vartheta \int_0^\vartheta u_{1y}(x, y, z, \tau) d\tau d\vartheta + \frac{1}{\rho(z)} \frac{\partial^2}{\partial y^2} \int_0^\vartheta \int_0^\vartheta u_{1x}(x, y, z, \tau) d\tau d\vartheta \left\{ K(z) + \frac{5E(z)}{12[1+\sigma(z)]} \right\} + \\
 & + \frac{5E(z)}{12[1+\sigma(z)]} \left\{ + \frac{1}{2\rho(z)} \frac{\partial}{\partial z} \left\{ \frac{E(z)}{1+\sigma(z)} \left[\frac{\partial}{\partial z} \int_0^\vartheta \int_0^\vartheta u_{1y}(x, y, z, \tau) d\tau d\vartheta + \frac{\partial}{\partial y} \int_0^\vartheta \int_0^\vartheta u_{1z}(x, y, z, \tau) d\tau d\vartheta \right] \right\} - \right. \\
 & - K(z) \frac{\beta(z)}{\rho(z)} \int_0^\vartheta \int_0^\vartheta T(x, y, z, \tau) d\tau d\vartheta + \frac{1}{\rho(z)} \frac{\partial^2}{\partial y \partial z} \int_0^\vartheta \int_0^\vartheta u_{1y}(x, y, z, \tau) d\tau d\vartheta \left\{ K(z) - \frac{E(z)}{6[1+\sigma(z)]} \right\} - \\
 & - \frac{E(z)}{2\rho(z)[1+\sigma(z)]} \left[\frac{\partial^2}{\partial x^2} \int_0^\vartheta \int_0^\vartheta u_{1x}(x, y, z, \tau) d\tau d\vartheta + \frac{\partial^2}{\partial x \partial y} \int_0^\vartheta \int_0^\vartheta u_{1x}(x, y, z, \tau) d\tau d\vartheta \right] - K(z) \frac{\beta(z)}{\rho(z)} \times \\
 & \times \int_0^\vartheta \int_0^\vartheta T(x, y, z, \tau) d\tau d\vartheta - \frac{K(z)}{\rho(z)} \frac{\partial^2}{\partial x \partial y} \int_0^\vartheta \int_0^\vartheta u_{1y}(x, y, z, \tau) d\tau d\vartheta - \frac{1}{\rho(z)} \frac{\partial^2}{\partial y^2} \int_0^\vartheta \int_0^\vartheta u_{1x}(x, y, z, \tau) d\tau d\vartheta \times \\
 & \times \left\{ K(z) + \frac{5E(z)}{12[1+\sigma(z)]} \right\} - \frac{\partial}{\partial z} \left\{ \frac{E(z)}{1+\sigma(z)} \left[\frac{\partial}{\partial z} \int_0^\vartheta \int_0^\vartheta u_{1y}(x, y, z, \tau) d\tau d\vartheta + \frac{\partial}{\partial y} \int_0^\vartheta \int_0^\vartheta u_{1z}(x, y, z, \tau) d\tau d\vartheta \right] \right\} \times \\
 & \quad \times \frac{1}{2\rho(z)} - \frac{1}{\rho(z)} \left\{ K(z) - \frac{E(z)}{6[1+\sigma(z)]} \right\} \frac{\partial^2}{\partial y \partial z} \int_0^\vartheta \int_0^\vartheta u_{1y}(x, y, z, \tau) d\tau d\vartheta + u_{0y} \\
 \\
 u_z(x, y, z, t) = & \frac{E(z)}{2\rho(z)[1+\sigma(z)]} \left[\frac{\partial^2}{\partial x^2} \int_0^\vartheta \int_0^\vartheta u_{1z}(x, y, z, \tau) d\tau d\vartheta + \frac{\partial^2}{\partial y^2} \int_0^\vartheta \int_0^\vartheta u_{1z}(x, y, z, \tau) d\tau d\vartheta + \right. \\
 & \left. + \frac{\partial^2}{\partial x \partial z} \int_0^\vartheta \int_0^\vartheta u_{1x}(x, y, z, \tau) d\tau d\vartheta + \frac{\partial^2}{\partial y \partial z} \int_0^\vartheta \int_0^\vartheta u_{1y}(x, y, z, \tau) d\tau d\vartheta \right] + \frac{\partial}{\partial z} \left\{ \left[\frac{\partial}{\partial x} \int_0^\vartheta \int_0^\vartheta u_{1x}(x, y, z, \tau) d\tau d\vartheta + \right. \right. \\
 & \left. \left. + \frac{\partial}{\partial y} \int_0^\vartheta \int_0^\vartheta u_{1x}(x, y, z, \tau) d\tau d\vartheta + \frac{\partial}{\partial z} \int_0^\vartheta \int_0^\vartheta u_{1x}(x, y, z, \tau) d\tau d\vartheta \right] K(z) \right\} \frac{1}{\rho(z)} + \frac{1}{6\rho(z)} \frac{\partial}{\partial z} \left\{ \frac{E(z)}{1+\sigma(z)} \right\} \times \\
 & \times \left[6 \frac{\partial}{\partial z} \int_0^\vartheta \int_0^\vartheta u_{1z}(x, y, z, \tau) d\tau d\vartheta - \frac{\partial}{\partial x} \int_0^\vartheta \int_0^\vartheta u_{1x}(x, y, z, \tau) d\tau d\vartheta - \frac{\partial}{\partial y} \int_0^\vartheta \int_0^\vartheta u_{1y}(x, y, z, \tau) d\tau d\vartheta - \right. \\
 & \left. - \frac{\partial}{\partial z} \int_0^\vartheta \int_0^\vartheta u_{1z}(x, y, z, \tau) d\tau d\vartheta \right] - K(z) \frac{\beta(z)}{\rho(z)} \frac{\partial}{\partial z} \int_0^\vartheta \int_0^\vartheta T(x, y, z, \tau) d\tau d\vartheta + u_{0z} .
 \end{aligned}$$

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