ALGEBRA OF THREE DIMENSIONAL GEOMETRIC FILTERS AND ITS RELEVANCE IN 3-D IMAGE PROCESSING

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ABSTRACT

*This paper gives a novel concept of what we call as Geometric Filters defined over a 3-D rectangular grid of pixels. These G-Filters have potential applications to processing of volumetric images. In addition, the paper describes in brief the algebra of G-Filters by formulating a lattice of convex polyhedrons (*structuring *elements) constructed in a 3X3X3 grid of pixels. The results are visualized as a distributive lattice. Mathematical morphology is an application of lattice theory .Many applications of mathematical morphology like erosion, dilation etc, use structuring elements. The work discussed in this paper is systematically generating the 3-D structuring elements by using geometric filters (G-Filters) for mathematical, morphological operations.*

KEYWORDS

*Geometric Filters, Lattice 2-D & 3-D, Mathematical morphology, 3-D structuring element*s, *3-D* image *processing*

1. INTRODUCTION

The processing of digital images based on well defined mathematical techniques has remained a subject of interest for many years. In particular mathematical theories associated with processing, enhancement analysis, and recognition [1] [8] of sensed imageries have received significance amount of attention and effort. At present there are methodologies for image processing using rigorous mathematical framework, for example, that of mathematical morphology [1], [5], [8]. In spite of these efforts,the wide variety of existing methodologies associated with image processing operations are yet to be consolidated under one rigorous unifying mathematical structure. The term mathematical structure refers to 3-tuple < X, O, R >, where X is a set of mathematical objects, O is the set of operations and R is a set of relations. A mathematical structure with binary operations alone would fall under the algebraic structures of monoids, groups, rings, integral domain and fields Alternatively, a mathematical structure with binary operations alone fall under the category of formal relational structures like that of lattices.

DOI : 10.5121/ijma.2012.4106

2. 2-D **Geometric Filters (2-D G-Filters)**

The theory of geometric filters has been formulated purely based on the extended topological filters [1], [2], [3], [4], [7]. Consider a 3x3 array of cells as shown in Figure 1a. The smallest convex polygon that would be found inside this cell array is shown in Figure 1b.

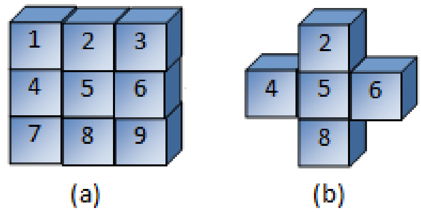


Figure 1. (a) 3x3 array of cell (b) Smallest Convex Polygon

Note that sixteen such different convex polygons can be formed by dropping indirect neighbour (corner) cells, which are pixels 1, 3, 7 and 9. Figure 2. shows all 16 convex polygons. The suffixes denote the cells that are dropped.

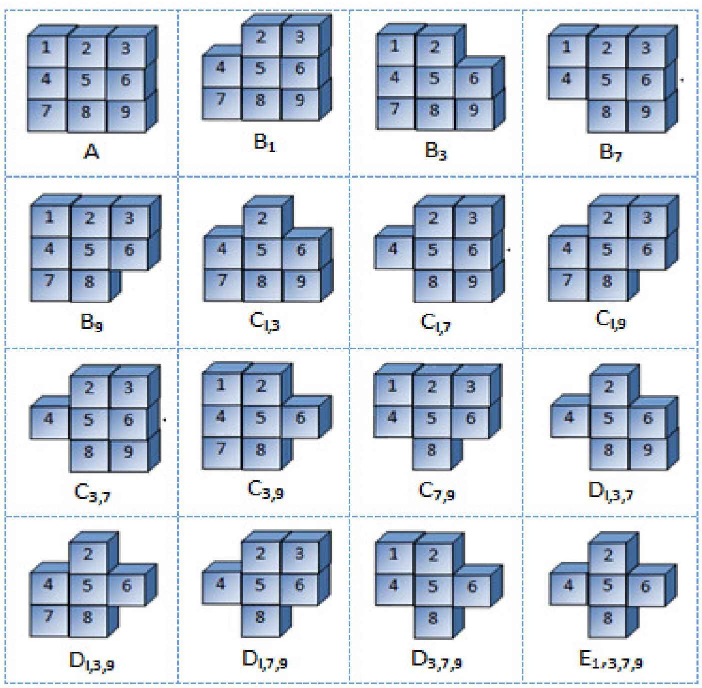


Figure 2. Sixteen Convex Polygons

2.1. Algebra of 2 -Dimensional G-Filters

The relational mathematical structure in which the G-filters are studied is a lattice $\phi < X \leq >$ where X is a set of G-filters and $\leq$ is the partial order relation 'finer than'. For example, the filter F1 = { A, B_1, B_7, B_9, $C_{1,7}$ } is finer than that of F2 = { A , B_1, B_7 $C_{1,7}$} [1].All the 2D convex polygons form a lattice with A as their supremum and E as their infimum. Note that there are 5 levels in the lattice. Figure 3. All the elements in a level constitute a group which is viewed as an immediate sub group of the group of elements in the just higher level. We can see from lattice that there are 24 linear chains consisting of proper envelopes starting from E .Each chain exhibits a linear hierarchy of generating A from E.

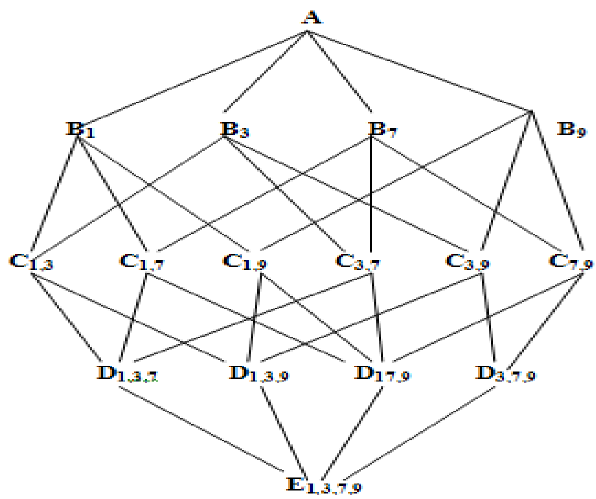


Figure 3. Lattice formed by 16 convex polygons

2.2. Theorem 1

One can construct a total of 95 hierarchical G-filters using 3x3 cell structure.
Proof:[1], [9].
We obtain the total number of hierarchical G-filters as

$$2+2\sum_{i=1}^{4} {}^{4}C_i + \sum_{i=1}^{6} {}^{6}C_i = 95 \qquad (1)$$

3. 3-D GEOMETRIC FILTERS (2-D G-FILTERS)

Consider a 3X3X3 array of 27 cells as shown in Figure 4a. The smallest convex polyhedron that could be formed inside this cell array is shown in Figure 4b.

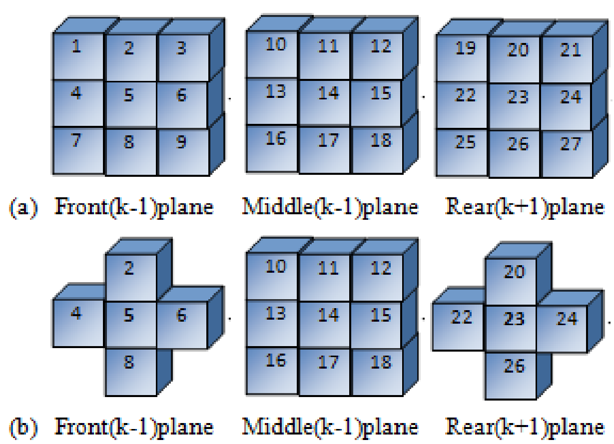


Figure 4. (a) Cubic array of size 3x3x3 (b) Smallest 3 D Convex Polyhedron $I_{1,3,7,9,19,21,25,27}$

The term convex polyhedron refers to a 3-D wire frame contour that could be drawn using certain neighbourhood pixels including a minimum of all 19 neighbourhood pixels shown in Figure 4, that is, cells 2, 4, 6, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 22, 24, 26. The cells 1, 3, 7, 9, 19, 21, 25 and 27 are the corner cells with respect to the central cell 14. For example the pixels (coner pixels) 1, 3, 7, 9,19, 21, 25 and 27 in Fig. 4 form a 3D contour (convex polyhedron) with respect to the central pixel 14. There are 8 corner cells in a 3x3x3 rectangular grid and the main idea is to construct various unique possible convex polyhedrons. 256 convex polyhedrons are obtained using the formula.

$$\sum_{i=0}^{8} {}^{8}C_i = {}^{8}C_0 + {}^{8}C_1 + {}^{8}C_2 + {}^{8}C_3 + {}^{8}C_4 + {}^{8}C_5 + {}^{8}C_6 + {}^{8}C_7 + {}^{8}C_8 = 2 \qquad (2)$$

Starting from, say A which has corner pixels 1, 3, 7, 9, 19, 21, 25 and 27 as in Figure 4, the process starts first by eliminating one corner pixel at a time and listing the different possibilities under group B, for set A we are able to find 8 B subsets ($B_1, B_3, B_7, B_9, B_{19}, B_{21}, B_{25}, B_{27}$) which is ${}^{8}C_1$. The process continues, explained in Theorem 2.The following are the combinations for different sets.

$$A = {}^{8}C_0 = 1 \quad B = {}^{8}C_1 = 8 \quad C = {}^{8}C_2 = 26 \quad D = {}^{8}C_3 = 56 \quad E = {}^{8}C_4 = 70$$

$$F = {}^{8}C_5 = 56 \quad G = {}^{8}C_6 = 28 \quad H = {}^{8}C_7 = 8 \quad I = {}^{8}C_8 = 1 \qquad (3)$$

256 convex polyhedrons are classified under nine groups as A, B, C, D, E, F, G, H, and I as shown in the Table 1

Table 1. *256* convex polyhedrons

A No pixel is eliminated we get 1 combination A={1,3,7,9,19,21,25,27}

Group B Eliminating one pixel we obtain 8 combinations

B1={3,7,9,19,21,25,27}	B3={1,7,9,19,21,25,27}	B7={1,3,9,19,21,25,27}	B9={1,3,7,19,21,25,27}
B19={1,3,7,9,21,25,27}	B21={1,3,7,9,19,25,27}	B25={1,3,7,9,19,21,27}	B27={1,3,7,9,19,21,25}

Group C Eliminating two pixels we obtain 28 combinations

C1,3={7,9,19,21,25,27}	C1,7={3,9,19,21,25,27}	C1,9={3,7,19,21,25,27}	C1,19={3,7,9,21,25,27}
C1,21={3,7,9,19,25,27}	C1,25={3,7,9,19,21,27}	C1,27={3,7,9,19,21,25}	C3,7={1,9,19,21,25,27}
C3,9={1,7,19,21,25,27}	C3,19={1,7,9,21,25,27}	C3,21={1,7,9,19,25,27}	C3,25={1,7,9,19,21,27}
C3,27={1,7,9,19,21,25}	C7,9={1,3,19,21,25,27}	C7,19={1,3,9,21,25,27}	C7,21={1,3,9,19,25,27}
C7,25={1,3,9,19,21,27}	C7,27={1,3,9,19,21,25}	C9,19={1,3,7,21,25,27}	C9,21={1,3,7,19,25,27}
C9,25={1,3,7,19,21,27}	C9,27={1,3,7,19,21,25}	C19,21={1,3,7,9,25,27}	C19,25={1,3,7,9,21,27}
C19,27={1,3,7,9,21,25}	C21,25={1,3,7,9,19,27}	C21,27={1,3,7,9,19,25}	C25,27={1,3,7,9,19,21}

Group D Eliminating three pixels we obtain 56 combinations

D1,3,7={9,19,21,25,27}	D1,3,9={7,19,21,25,27}	D1,3,19={7,9,21,25,27}	D1,3,21={7,9,19,25,27}
D1,3,25={7,9,19,21,27}	D1,3,27={7,9,19,21,25}	D1,7,9={3,19,21,25,27}	D1,7,19={3,9,21,25,27}
D1,7,21={3,9,19,25,27}	D1,7,25={3,9,19,21,27}	D1,7,27={3,9,19,21,25}	D1,9,19={3,7,21,25,27}
D1,9,21={3,7,19,25,27}	D1,9,25={3,7,19,21,27}	D1,9,27={3,7,19,21,25}	D1,19,21={3,7,9,25,27}
D1,19,25={3,7,9,21,27}	D1,19,27={3,7,9,21,25}	D1,21,25={3,7,9,19,27}	D1,21,27={3,7,9,19,25}
D1,25,27={3,7,9,19,21}	D3,7,9={1,19,21,25,27}	D3,7,19={1,9,21,25,27}	D3,7,21={1,9,19,25,27}
D3,7,25={1,9,19,21,27}	D3,7,27={1,9,19,21,25}	D3,9,19={1,7,21,25,27}	D3,9,21={1,7,19,25,27}
D3,9,25={1,7,19,21,27}	D3,9,27={1,7,19,21,25}	D3,19,21={1,7,9,25,27}	D3,19,25={1,7,9,21,27}
D3,19,27={1,7,9,21,25}	D3,21,25={1,7,9,19,27}	D3,21,27={1,7,9,19,25}	D3,25,27={1,7,9,19,21}
D7,9,19={1,3,21,25,27}	D7,9,21={1,3,19,25,27}	D7,9,25={1,3,19,21,27}	D7,9,27={1,3,19,21,25}
D7,19,21={1,3,9,25,27}	D7,19,25={1,3,9,21,27}	D7,19,27={1,3,9,21,25}	D7,21,25={1,3,9,19,27}
D7,21,27={1,3,9,19,25}	D7,25,27={1,3,9,19,21}	D9,19,21={1,3,7,25,27}	D9,19,25={1,3,7,21,27}
D9,19,27={1,3,7,21,25}	D9,21,25={1,3,7,19,27}	D9,21,27={1,3,7,19,25}	D9,25,27={1,3,7,19,21}
D19,21,25={1,3,7,9,27}	D19,21,27={1,3,7,9,25}	D19,25,27={1,3,7,9,21}	D21,25,27={1,3,7,9,19}

Group E Eliminating four pixels we obtain 70 combinations

E1,3,7,9={19,21,25,27}	E1,3,7,19={9,21,25,27}	E1,3,7,21={9,19,25,27}	E1,3,7,25={9,19,21,27}
E1,3,7,27={9,19,21,25}	E1,3,9,19={7,21,25,27}	E1,3,9,21={7,19,25,27}	E1,3,9,25={7,19,21,27}
E1,3,9,27={7,19,21,25}	E1,3,19,21={7,9,25,27}	E1,3,19,25={7,9,21,27}	E1,3,19,27={7,9,21,25}
E1,3,21,25={7,9,19,27}	E1,3,21,27={7,9,19,25}	E1,3,25,27={7,9,19,21}	E1,7,9,19={3,21,25,27}
E1,7,9,21={3,19,25,27}	E1,7,9,25={3,19,21,27}	E1,7,9,27={3,19,21,25}	E1,7,19,21={3,9,25,27}
E1,7,19,25={3,9,21,27}	E1,7,19,27={3,9,21,25}	E1,7,21,25={3,9,19,27}	E1,7,21,27={3,9,19,25}
E1,7,25,27={3,9,19,21}	E1,9,19,21={3,7,25,27}	E1,9,19,25={3,7,21,27}	E1,9,19,27={3,7,21,25}
E1,9,21,25={3,7,19,27}	E1,9,21,27={3,7,19,25}	E1,9,25,27={3,7,19,21}	E1,19,21,25={3,7,9,27}
E1,19,21,27={3,7,9,25}	E1,19,25,27={3,7,9,21}	E1,21,25,27={3,7,9,19}	E3,7,9,19={1,21,25,27}
E3,7,9,21={1,19,25,27}	E3,7,9,25={1,19,21,27}	E3,7,9,27={1,19,21,25}	E3,7,19,21={1,9,25,27}
E3,7,19,25={1,9,21,27}	E3,7,19,27={1,9,21,25}	E3,7,21,25={1,9,19,27}	E3,7,21,27={1,9,19,25}
E3,7,25,27={1,9,19,21}	E3,9,19,21={1,7,25,27}	E3,9,19,25={1,7,21,27}	E3,9,19,27={1,7,21,25}
E3,9,21,25={1,7,19,27}	E3,9,21,27={1,7,19,25}	E3,9,25,27={1,7,19,21}	E3,19,21,25={1,7,9,27}
E3,19,21,27={1,7,9,25}	E3,19,25,27={1,7,9,21}	E3,21,25,27={1,7,9,19}	E7,9,19,21={1,3,25,27}
E7,9,19,25={1,3,21,27}	E7,9,19,27={1,3,21,25}	E7,9,21,25={1,3,19,27}	E7,9,21,27={1,3,19,25}
E7,9,25,27={1,3,19,21}	E7,19,21,25={1,3,9,27}	E7,19,21,27={1,3,9,25}	E7,19,25,27={1,3,9,21}
E7,21,25,27={1,3,9,19}	E9,19,21,25={1,3,7,27}	E9,19,21,27={1,3,7,25}	E9,19,25,27={1,3,7,21}
E9,21,25,27={1,3,7,19}	E19,21,25,27={1,3,7,9}		

Group F Eliminating five pixels we obtain 56 combinations

F1,3,7,9,19={21,25,27}	F1,3,7,9,21={19,25,27}	F1,3,7,9,25={19,21,27}	F1,3,7,9,27={19,21,25}
F1,3,7,19,21={9,25,27}	F1,3,7,19,25={9,21,27}	F1,3,7,19,27={9,21,25}	F1,3,7,21,25={9,19,7}
F1,3,7,21,27={9,19,25}	F1,3,7,25,27={9,19,21}	F1,3,9,19,21={7,25,27}	F1,3,9,19,25={7,21,27}
F1,3,9,19,27={7,21,25}	F1,3,9,21,25={7,19,27}	F1,3,9,21,27={7,19,25}	F1,3,9,25,27={7,19,21}
F1,3,19,21,25={7,9,27}	F1,3,19,21,27={7,9,25}	F1,3,19,25,27={7,9,21}	F1,3,21,25,27={7,9,19}
F1,7,9,19,21={3,25,27}	F1,7,9,19,25={3,21,27}	F1,7,9,19,27={3,21,25}	F1,7,9,21,25={3,19,27}
F1,7,9,21,27={3,19,25}	F1,7,9,25,27={3,19,21}	F1,7,19,21,25={3,9,27}	F1,7,19,21,27={3,9,25}
F1,7,19,25,27={3,9,21}	F1,7,21,25,27={3,9,19}	F1,9,19,21,25={3,7,27}	F1,9,19,21,27={3,7,25}
F1,9,19,25,27={3,7,21}	F1,9,21,25,27={3,7,19}	F1,19,21,25,27={3,7,9}	F3,7,9,19,21={1,25,27}
F3,7,9,19,25={1,21,27}	F3,7,9,19,27={1,21,25}	F3,7,9,21,25={1,19,27}	F3,7,9,21,27={1,19,25}
F3,7,9,25,27={1,19,21}	F3,7,19,21,25={1,9,27}	F3,7,19,21,27={1,9,25}	F3,7,19,25,27={1,9,21}
F3,7,21,25,27={1,9,19}	F3,9,19,21,25={1,7,27}	F3,9,19,21,27={1,7,25}	F3,9,19,25,27={1,7,21}
F3,9,21,25,27={1,7,19}	F3,19,21,25,27={1,7,9}	F7,9,19,21,25={1,3,27}	F7,9,19,21,27={1,3,25}
F7,9,19,25,27={1,3,21}	F7,9,21,25,27={1,3,19}	F7,19,21,25,27={1,3,9}	F9,19,21,25,27={1,3,7}

Group G Eliminating six pixels we obtain 28 combinations

G1,3,7,9,19,21={25,27}	G1,3,7,9,19,25={21,27}	G1,3,7,9,19,27={21,25}	G1,3,7,9,21,25={19,27}
G1,3,7,9,21,27={19,25}	G1,3,7,9,25,27={19,21}	G1,3,7,19,21,25={9,27}	G1,3,7,19,21,27={9,25}
G1,3,7,19,25,27={9,21}	G1,3,7,21,25,27={9,19}	G1,3,9,19,21,25={7,27}	G1,3,9,19,21,27={7,25}
G1,3,9,19,25,27={7,21}	G1,3,9,21,25,27={7,19}	G1,3,19,21,25,27={7,9}	G1,7,9,19,21,25={3,27}
G1,7,9,19,21,27={3,25}	G1,7,9,19,25,27={3,21}	G1,7,9,21,25,27={3,19}	G1,7,19,21,25,27={3,9}
G1,9,19,21,25,27={3,7}	G3,7,9,19,21,25={1,27}	G3,7,9,19,21,27={1,25}	G3,7,9,19,25,27={1,21}
G3,7,9,21,25,27={1,19}	G3,7,19,21,25,27={1,9}	G3,9,19,21,25,27={1,7}	G7,9,19,21,25,27={1,3}

Group H Eliminating seven pixels we obtain 8 combinations

H1,3,7,9,19,21,25={27}	H1,3,7,9,19,21,27={25}	H1,3,7,9,19,25,27={21}	H1,3,7,9,21,25,27={19}
H1,3,7,19,21,25,27={9}	H1,3,9,19,21,25,27={7}	H1,7,9,19,21,25,27={3}	H3,7,9,19,21,25,27={1}

I Eliminating eight pixels we obtain 1 combination

I 1,3,7,9,19,21,25,27={ }

Few examples of convex polyhedrons constructed in a 3X3X3 grid of pixels as listed in table 1. is shown in Figure 5.An algorithm is developed for constructing the above 256 convex polyhedrons.The algorithm is not discussed here. Red represents front plane (k-1), Green represents central plane (k), Blue represents rear plane (k+1).[9]

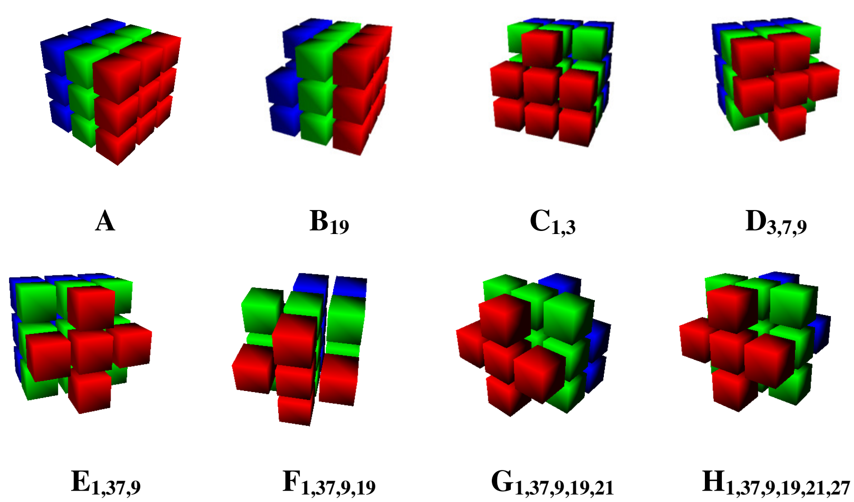


Figure 5. Convex Polyhedrons

3.2. Theorem 2

One can construct a total of

$$2+2\sum_{i=1}^{8}{}^{8}C_i+2\sum_{i=1}^{28}{}^{28}C_i+2+\sum_{i=1}^{56}{}^{56}C_i+\sum_{i=1}^{70}{}^{70}C_i \qquad (4)$$

hierarchical G-filters[2] in a 3 x3 x3 cell structure.

Proof:

Note that there are eight corners cells in a 3x3x3 cell structure and we construct $\sum {}^{8}C_{i}$ (i =0 to n) convex patterns in the following manner. The first group A contains $\mathbf{{}^{8}C_{0}}$ that is , one polyhedron which is 3x3 x3 cell structure itself. The group B contains $\mathbf{{}^{8}C_{1}}$ that is, 8 polyhedrons B_1 (without cell 1),B_2 (without cell 2), B_3 , B_7 , B_9 ,B_{19} $B_{21,}$ B_{25} and $B_{27.}$ Similarly the group C contains $\mathbf{{}^{8}C_{2}}$ that is, 28 polyhedrons $C_{1,3}$ (without cells 1 and 3), $C_{1,7,}$ $C_{1,9,}$ $C_{3,7,}$ $C_{3,9}$ and $C_{7,9}$ …… $C_{25,27}$ In this manner ,the elements of group D, E, F,G,H and I are identified.

Now the first G-filter F_1 contains the polyhedron I, in the set, and as per the definition of G-filter,F_1 should contain all of its 256 proper envelopes. It is easy to see the cardinality of the set to be 256. Now, the absence of I in the set would yeild a coarse G-filter F_2 with 255 remaining elements. Then by dropping one element at a time from the group H we can generate eight heirarchical G-filters with cardinality 254. Next, in order to get a filter with a cardinality 253,we leave out G and any two elements in group H. We see here ${}^{8}C_2$ such possibilities. subsequently we construct ${}^{8}C_3$ filters with cardinality 252 by dropping I and any three other elements of group D. Lastly we generate one filter with cardinality 251 by dropping I and any four other elements of H.This procedure is repeated for the remaining groups also. Ultimately we obtain the total number of heirarchical G-filters.

$$2+2\sum_{i=1}^{8}{}^{8}C_i+2\sum_{i=1}^{28}{}^{28}C_i+2+\sum_{i=1}^{56}{}^{56}C_i+\sum_{i=1}^{70}{}^{70}C_i$$

$$=1{,}180{,}735{,}735{,}908{,}220{,}000{,}000 \qquad (5)$$

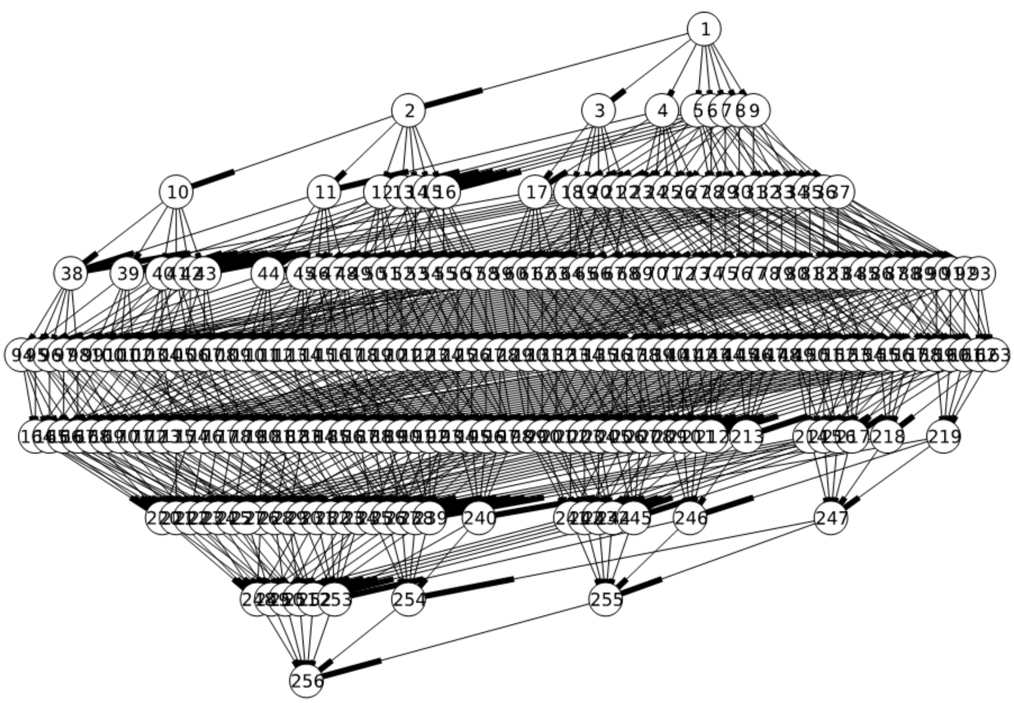


Figure 6. Lattice formed by 256 convex polyhedrons

4. USE OF G-FILTERS IN 3-D IMAGE PROCESSING

In sections 2 and 3, our attention has primarily been on discussing 2-D geometric filters and introducing 3-D geometric filters, respectively. The basic operations of mathematical morphology are the dual operations of dilation and erosion and these two operations and their combinations allow one to modify the form and shape of a digital image.The given digital image is processed by another image called structuring element whose shape decides the shape of the given image.The given image and the structuring element are viewed as sets and thus their union and intersection respectively are termed as dilation and erosion. A 3-D digital image is an array of volumetric pixel values, called voxels, arranged in a 3-D grid.Each voxel is modeled as a cube.

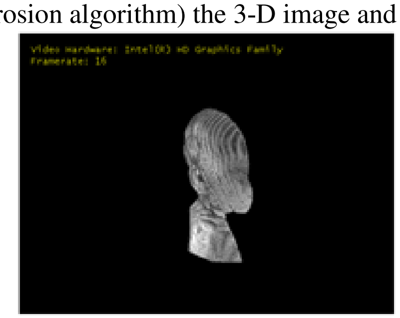
Intensive research has been carried out by a number of people in the morphological processing of two dimensional images.where as very little work has been found in the literature related to the morphological processing of three dimensional images.In this paper we present the processing of morphological operations, dilation and erosion.

Algorithm for Dilation and Erosion Dilation:

Dilation:
Repeat sliding the structuring element over the image
{
Add pixels of structuring element to the corresponding pixels of the image find the maximum, k, among all of them if at least one of the image pixels that are panned by structuring element is non-zero then replace the central pixels in the image with k; else replace it with 0
}
until the structuring element spans whole of the image.

Erosion:
Repeat sliding the structuring element over the image
{
subtract pixels of structuring element from the corresponding pixels of the image find the minimum, k, among all of them if all the structuring element pixels are less than the corresponding image pixels then replace the central pixels in the image with k; else replace it with 0
}
until the structuring element spans whole of the image.

Figure 7a. shows a 3-D image (MRI image) and 7b. shows a 27 nieghbourhood stucturing element A also called convex polyhedron.The 3-D image is dilated with the structuring element (dilation algorithm) and the result is shown in Figure 8a. The same structuring element has been used to erode (erosion algorithm) the 3-D image and the result is shown in Figure 8b.



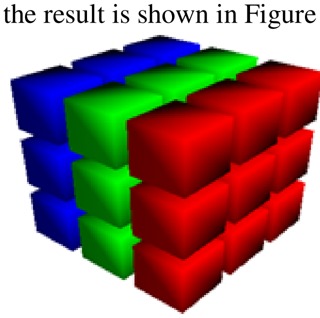


Figure 7. (a) Original 3-D image (b) Structuring element
Total number of voxels: 358389 A

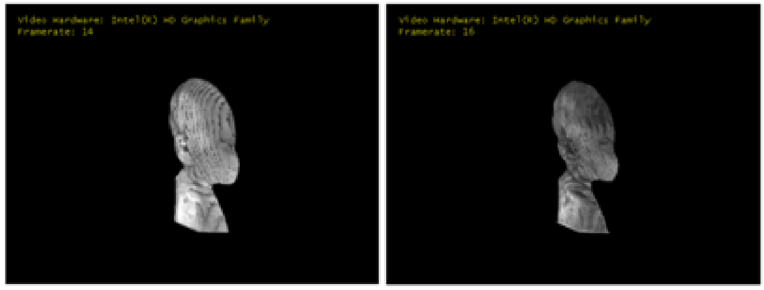


Figure 8. (a) Dilated image (b) Eroded image
Total number of voxels: 395506 Total number of voxels: 330926

5. RESULTS

The G-Filters for 3 Dimensional rectangular grid is identified and their corresponding convex polyhedrons are listed in the Table 1. All the 256 convex polyhedrons are arranged in 9 levels with A set as root node, I set as leaf node and sets B, C, D, E, F, G, H as intermediate nodes to form a complete distributive lattice, which is shown in the Fig 6.

In the constructed distributive lattice root node A is supremum and leaf node I is infimum. The number of subsets formed from their corresponding sets at each level is identified as 1, 8, 28, 56, 70, 56, 28, 8, 1 at level 0, level 1, level 2, level 3, level 4, level 5, level 6, level 7, level 8, level 9 respectively in the lattice. The hierarchical relationships among the sets and their corresponding sub sets is identified and visualized by using the *hierarchy construction algorithm* . Due to the complexity in visualisation the nodes are mentioned in numbers instead of set name. For example, node 1 in Fig 6 is A. Similarly, node 256 corresponds to $I_{1, 3, 7, 9, 19, 21, 25, 27.}$ The number of unique linear chains in the 2D and 3D lattices are identified and proved in the Theorem1 and Theorem 2 respectively. We also tested the use of 3-D convex polyhedrons in dilation and erosion of three dimensional images and got desired results.

6. CONCLUSIONS

In this paper, we have considered three dimensional polyhedrons developed in a 3X3X3 lattice. These 256 polyhedrons form a lattice with A as the supremum and $\mathbf{I_{1,3,7,9,19,21,25,27}}$ as infimum. We developed the notion of 3-D Geometric Filters and showed their potential applications to 3D image processing. With digital images represented in terms of certain basis convex patterns (sets) discussed above, the processing of such representations could be carried out using G-filters. In short, one could view Image Algebra as the Algebra of G-filters.

ACKNOWLEDGEMENTS

The authors would like to thank Mrs. G. Chitra, Managing Director of Pentagram Research centre for her continues support and encouragement and also thank the staff at PRC.

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