

# AN AHP(ANALYTIC HIERARCHY PROCESS)-BASED INVESTMENT STRATEGY FOR CHARITABLE ORGANIZATIONS OF GOODGRANT FOUNDATION

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## ABSTRACT

*This paper provides the optimal investment strategy for the Goodgrant Foundation. To determine the schools to be invested, firstly we find the factors about improving students' educational performance, including urgency of student's needs, school's demonstrate potential for effective use of private funding, the reputation of school, and return on investment etc; secondly, we utilize AHP (Analytic Hierarchy Process) to determine the weight of every factor and rank the schools in the list of candidate schools according to the composite index of every school calculated from the weight. To confirm the investment per school and obtains the investment duration time, we use DEA (Data Envelope Analyse) to get changes of scale efficiency; and then according to the change trend, we determine time duration of investment and effective utilization of private funding, which is the factor together with student population affecting the investment amount for per school. It is helpful to make a better decision on investing universities, We are convinced that our research is promising to benefit all sides of students, schools and Goodgrant.*

## KEYWORDS

*AHP, DEA, Educational Performance, Investment Strategy*

## 1. INTRODUCTION

Universities and colleges are places where young students gain valuable knowledge, resources and opportunities before they step into the society. That is why many foundations are willing to invest on undergraduates to help improve their educational performance. With so many universities and colleges in American, it is necessary for us to carry out a method about how to determine an optimal investment strategy to identify the schools, the investment amount per school efficiently and objectively. How to distribute the fund is exactly the key. It is a multi-aspect evaluate task including the urgency of students' needs, school's demonstrate potential for effective use of private funding, the reputation of school, return on investment etc. Mean while, the time duration that the organization's money have the highest likelihood of producing a strong positive effect on student performance should also be a primary consideration. Other large and known grant organizations such as Gates Foundation and Lumina Foundation show the current way to investigate the qualification which mainly concentrates on the low income of students' families and potential of universities. These models will take the ability of using the funding and rate of return into account, and obtain approximate investment duration time and return of investments oth at the Good grant Foundation can provide the assistance best to not only students but also schools and foundation itself.

The analytic hierarchy process (AHP)[2-4] is a structured technique for organizing and analyzing complex situation. It is based on mathematics and psychology. Rather than prescribe a "correct" decision, the AHP helps decision makers find one that best suits their goal and their understanding of the problem[5-6]. It provides a comprehensive and rational framework for

structuring a decision problem, for representing and quantifying its elements, for relating those elements to overall goals, and for evaluating alternative solutions.

This paper provides the optimal investment strategy for the Goodgrant Foundation by AHP method[2]. It helps improv educational performance of undergraduates in United States and make them graduate successfully, live a good life in the future. The rest of this paper is organized as follows: in Section 2, we present our model approach in detail, including the analytical hierarchy process model and data envelope analysis model. Conclusions are provided in the last Section.

## 2. MODEL DESIGN

### 2.1 THE ANALYTICAL HIERARCHY PROCESS MODEL

By using cluster analysis, We group all the colleges corecard data into urgency of students' needs, school's demonstrate potential for effective use of private funding, the reputation of school, return on investment 4 groups. Meanwhile urgency of students' needs contains share of part-time undergraduates, median debt of completers and average net price; School's demonstrate potential for effective use of private funding contains percentage of undergraduates who have received a Pell Grant, percent of all federal undergraduate students receiving a federal student loan and 3-year repayment rate; The reputation of school includes predominant degree awarded, discipline distribution and structure and whether it is operating with other institutions; Return on investment includes Median earnings of students 10 years after entry, share of students earning over 25,000dollar/year 6 years after entry. The dates of 2936 universities are in Table 1.

Table 1: Dates of 2936 Universities

university	p1	p2	p3	p4	p5	p6	p7	p8	p9	p10	p11
Alabama A M University	0.052	33611.5	14887.8	0.712	0.820	0.445	3	0.244	1	31400	0.462
University of Alabama at Birmingham	0.258	23117	15436.6	0.351	0.540	0.756	3	0.251	1	40300	0.660
Amridge University	0.3727	19492.7	14051	0.684	0.763	0.547	3	0.204	1	38100	0.647
University of Alabama in Huntsville	0.240	24738	18114.2	0.328	0.473	0.782	3	0.268	1	46600	0.661
Alabama State University	0.091	33452	10108.4	0.827	0.874	0.331	3	0.24646	1	27800	0.342225611
The University of Alabama	0.0852	2400	21078.4	0.211	0.415	3	0.258	1	42400	0.661	
Central Alabama Community College	0.456	19492.7	10190.27	0.652	0.478	0.438	2	0.241	1	27100	0.446
Auburn University at Montgomery	0.306	21791	13147.8	0.401	0.648	0.529	3	0.248	1	34800	0.555
...											
Central Georgia Technical College	0.555	5348	5088.6	0.674	0.2910.719	1	0.2451	1.000	37034	0.571	
Arizona State University-SkySong	0.4846	320375	14772	0.429	0.663	0.803	3.	0.239	1	37034	0.571
Louisiana Delta Community College	0.404	19492.7	18156	0.682	0	0.719	1	0.262	1	37034	0.571

#### 2.1.1 THE ESTABLISHMENT OF A HIERARCHY

The problem of the case can be divided into three layers in order.

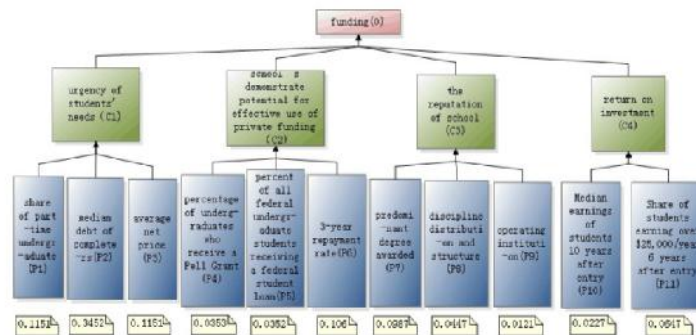


Figure 1: The three-tier funding distribution system

Intensity of Value	Interpretation
1	Requirements <i>i</i> and <i>j</i> have equal value.
3	Requirement <i>i</i> has a slightly higher value than <i>j</i> .
5	Requirement <i>i</i> has a strongly higher value than <i>j</i> .
7	Requirement <i>i</i> has a very strongly higher value than <i>j</i> .
9	Requirement <i>i</i> has an absolutely higher value than <i>j</i> .
2, 4, 6, 8	Intermediate scales between two adjacent judgments.
Reciprocals	Requirement <i>i</i> has a <i>lower</i> value than <i>j</i> .

Figure 2: The meaning of measure

### 2.1.2 THE WEIGHTS OF LAYER C IN LAYER O

Considering the relative importance of  $C_1$  compared with  $C_2, C_3, C_4$ ; we might arrive at the following pair wise comparison matrix.

$$A = \begin{pmatrix} 1 & 3 & 5 & 5 \\ \frac{1}{3} & 1 & 1 & 2 \\ \frac{1}{5} & 1 & 1 & 2 \\ \frac{1}{5} & \frac{1}{2} & \frac{1}{2} & 1 \end{pmatrix}$$

The maximum eigenvalue  $\lambda_1 = 4.05$ , and we can get the corresponding normalized eigenvector  $w_1 = (0.58, 0.17, 0.16, 0.09)$ .

### 2.1.3 CONSISTENCY TEST

Consistency Index

$$CI = \frac{\lambda_{max} - n}{n - 1}$$

when  $n = 4, \lambda_1 = 4.05, CI = 0.016$ , Random Consistency Index  $RI = 0.9$ , it can be get from the table below (Table 2).

Table 2: Random Consistency Index

number of order	1	2	3	4	5	6	7	8	9
$R_n$	0	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45

Consistency Ratios

$$CR = \frac{CI}{RI}$$

When  $RI = 0.9, CR = 0.018 < 0.1$  meet the inspection.

**2.1.4 THE WEIGHTS OF LAYER P IN LAYER C**

Considering the relative importance of P<sub>1</sub> compared with P<sub>2</sub>,P<sub>3</sub>;P<sub>4</sub> compared with P<sub>5</sub>,P<sub>6</sub>;P<sub>7</sub> compared with P<sub>8</sub>,P<sub>9</sub>;P<sub>10</sub> compared with P<sub>11</sub>; We might arrive at the following pairwise comparison matrix:

$$B_1 = \begin{pmatrix} 1 & \frac{1}{3} & 1 \\ 3 & 1 & 3 \\ 1 & \frac{1}{3} & 1 \end{pmatrix} \quad B_2 = \begin{pmatrix} 1 & 1 & \frac{1}{3} \\ 1 & 1 & \frac{1}{3} \\ 3 & 3 & 1 \end{pmatrix} \quad B_3 = \begin{pmatrix} 1 & 3 & 6 \\ \frac{1}{3} & 1 & 5 \\ \frac{1}{6} & \frac{1}{5} & 1 \end{pmatrix} \quad B_4 = \begin{pmatrix} 1 & \frac{3}{7} \\ \frac{7}{3} & 1 \end{pmatrix}$$

The maximum eigen value  $\lambda_2 = 3, \lambda_3 = 3, \lambda_4 = 3.10, \lambda_5 = 2$  and we can get the corresponding normalized eigen vector

$$w_1 = (0.2,0.6,0.2)^T, w_2 = (0.2,0.2,0.6)^T, w_3 = (0.635,0.287,0.078)^T, w_4 = (0.3,0.7)^T$$

After normalization, we can weight vector

$$CR_2 = \sum_{i=1}^4 CR_2^{(k)} = 0.08$$

**2.1.5 THE WEIGHTS OF LAYER P IN LAYER O**

$$W^{(k)} = (\xi_1^{(k)}, \xi_2^{(k)}, \dots, \xi_n^{(k)}) \quad (k = 1, 2, 3, 4)$$

$$W_2 = [W^{(1)}, W^{(2)}, W^{(3)}, W^{(4)}]_{1 \times 4}$$

$$W = W_2 \cdot W = [W^{(1)}, W^{(2)}, W^{(3)}, W^{(4)}] \cdot W = (0.115, 0.345, 0.115, 0.035, 0.035, 0.106, 0.099, 0.045, 0.012, 0.03, 0.065)^T$$

$$CR = CR_1 + CR_2 = 0.098 < 0.1$$

**2.1.6 DATA NORMALIZATION METHOD**

In order to reconcile all kinds of indicator sin one assessment system, we apply Min Max Normalization to normalize the indicators that mentioned in the database. This helps us to process data in various dimension. Our process of data normalization is as following formula

$$x^* = \frac{x - \min}{\max - \min}$$

$$s = \begin{pmatrix} 0.0010 & 0.0007 & 0.0004 & 0.0004 & 0.0006 & 0.0003 & 0.0004 & 0.0000 & 0.0000 & 0.0004 & 0.0004 \\ 0.0007 & 0.0008 & 0.0003 & 0.0005 & 0.0005 & 0.0002 & 0.0004 & 0.0000 & 0.0000 & 0.0003 & 0.0003 \\ 0.0011 & 0.0007 & 0.0003 & 0.0006 & 0.0006 & 0.0002 & 0.0003 & 0.0000 & 0.0000 & 0.0003 & 0.0003 \\ 0.0001 & 0.0008 & 0.0007 & 0.0004 & 0.0006 & 0.0003 & 0.0004 & 0.0000 & 0.0000 & 0.0004 & 0.0004 \\ \vdots & & & & & & & & & & \\ 0.0010 & 0.0004 & 0.0006 & 0.0002 & 0.0000 & 0.0000 & 0.0003 & 0.0000 & 0.0000 & 0.0003 & 0.0002 \\ 0.0001 & 0.0005 & 0.0005 & 0.0005 & 0.0006 & 0.0003 & 0.0004 & 0.0000 & 0.0000 & 0.0004 & 0.0004 \end{pmatrix}$$



## 2.2 DATA ENVELOPE ANALYSE

### 2.2.1 BASIC CONCEPTION OF DEA

After assessing the educational performance of all post secondary colleges and universities, in accordance with the rank, we select 200 universities who are most deserving grants. Then it comes to the question of the amount of money distributed to per school. We consider the ability of effective using of private funding and the population as the main factors. To quantize the funding use capacity, we evaluate the relative efficiency of the same type of output and input Decision Making Unit (DMU) and employ the Data Envelopment Analysis (DEA) [8-9].

Parameter Assumption :  $X$ : input index,  $Y$ : output index, to a certain project we assume that there are  $s$  decision making units per unit of which has  $m$  kinds of inputs and  $n$  kinds of outputs, weight coefficient correspondingly  $V=(v_1, v_2, v_3, \dots, v_m)^T$ ,  $U=(u_1, u_2, u_3, \dots, u_n)^T$ , every unit has its own efficiency evaluation goals ( $h$ ),  $h_j = uY_j/vX_j$ , we can always choose proper weight coefficient which satisfy  $h_j, j = 1, 2, \dots, s$ .

In order to estimate the efficiency of DMU, weight coefficient correspondingly  $v, u$  as variable, aim at the efficiency index of DMU, restraint by all efficiency indexes, develop the fractional programming model as followed:

$$\text{Max } h_k = \frac{\sum_{r=1}^n u_r y_{rk}}{\sum_{i=1}^m v_i x_{ik}}$$

$$\text{st. } \frac{\sum_{r=1}^n u_r y_{rk}}{\sum_{i=1}^m v_i x_{ik}} \leq 1$$

In which

$$u_r \geq v > 0, v_i \geq v > 0$$

Let  $U_r = t \times u_r, V_i = t \times v_i, U_r \geq v > 0, V_i \geq v > 0$  after Charnes-Cooper transform we get  $C^2R$  linear programming model

$$\text{Max } h_k = \frac{\sum_{r=1}^n U_r y_{rk}}{\sum_{i=1}^m V_i x_{ik}} \text{ st. } \sum_{i=1}^m V_i x_{ik} = 1, \sum_{r=1}^n U_r y_{rk} - \sum_{i=1}^m V_i x_{ik} \leq 0$$

Introduces slack variables and surplus variables and get the CCR Duality linear layout model

$$\text{Max } [ \theta - v(e^{AT} s^- + e^T s^+) ]$$

$$\text{st. } \sum_{r=1}^n x_j \lambda_j + s^- = \theta x_{j0}$$

$$\sum_{j=1}^n y_j \} - s^+ = y_{j0}$$

In which:  $k$  presents the potential quota of all inputs possibility of equal proportion reduction.  $e^T, e^T$  present  $m$  dimension unit column vector and  $n$  dimension unit column vector.  $\theta$  presents The Archimedes dimensionless, which is smaller than any number bigger than zero.

**2.2.2 THE SOURCE OF SAMPLE DATA**

After disposal data we choose the amount of Pell Grant and federal student loan as input indexes, the amount of earnings of students working and not enrolled 10 years after entry and high in come students 6years after entry as output indexes. The processed data was showed in the following table(Fig4).

Institutions	Inputs		Outputs	
	Federal Student Loan	Pell Grant	earnings of students for 10years	high income students salary
1. Arizona College	673566.51	540277.15	1940100.00	3,42026.41
2. Bialak Heights University	116032.3	150077.07	1847200.00	703256.4
3. Iowa College of Des Moines	110720.06	126270.02	1376300.00	810760.02
4. Boston College of Business	1104796.3	137302.42	1376300.00	619462.25
5. Sonoma College	1104002.93	1073470.22	1042000.00	1017804.09
6. Western University	220748.87	201562.02	1075000.00	400029.50
7. Washington University	210860.23	201702.58	2057100.00	1403227.02
8. Art Institute of College of Design	403881.43	380209.04	5010100.00	2013480.70
9. Indiana College	210544.09	140277.52	1076000.00	1333003.42
10. Ottawa University/College	110640.79	108074.74	1040100.00	471745.49
11. Citrus University of Jacksonville	21220.70	24254.44	510000.00	217740.87
12. Ottawa University/College	10200.22	5024.22	1012200.00	44000.89
13. Ottawa University/College	11000.9	11000.00	1010100.00	41000.74
14. Ottawa University/College	10100.00	10100.00	1100000.00	81000.00
15. Washington University	40100.00	37747.22	640000.00	107000.00
16. Western State College	20000.00	20000.00	1010100.00	101000.00
17. Western State College	40079.59	40074.02	1010100.00	101000.00
18. National University	190000.00	190000.00	1010000.00	101000.00
19. Idaho State University/College	110000.00	110000.00	1010000.00	101000.00
20. Western State College	110000.00	110000.00	1010000.00	101000.00
21. Western State College	110000.00	110000.00	1010000.00	101000.00
22. Western State College	110000.00	110000.00	1010000.00	101000.00
23. Western State College	110000.00	110000.00	1010000.00	101000.00
24. Western State College	110000.00	110000.00	1010000.00	101000.00

Figure 4: Input and output index

**2.2.3 ESTABLISHMENT OF MODEL**

CCR Model

$$\begin{aligned}
 \text{Max } h &= U_1 y_1 + U_2 y_2 = 78403800u_1 + 31492914u_2 \\
 78403800u_1 + 31492914u_2 - 6810564.51v_1 - 5436272.18v_2 &\leq 0 \\
 14847200u_1 + 7072340.41u_2 - 1940932.15v_1 - 1983377.62v_2 &\leq 0 \\
 12593500u_1 + 5532753.92u_2 - 1943720.06v_1 - 1983273.33v_2 &\leq 0 \\
 279864000u_1 + 95493619.73u_2 - 891828.08v_1 - 13107856.32v_2 &\leq 0 \\
 118895000u_1 + 24518661.35u_2 - 4553961.46v_1 - 1364384.63v_2 &\leq 0 \\
 7433400u_1 + 28817188.48u_2 - 8144706.34v_1 - 7008249.55v_2 &\leq 0 \\
 u_1, u_2, v_1, v_2 &\geq 0
 \end{aligned}$$

BCC model

Min<sub>z</sub>

$$78403800\}_{1} + 14847200\}_{2} + \dots + 118895000\}_{199} + 74334000\}_{200} + s_1^- = 78403800,$$

$$31492914.41\}_{1} + 7072340.41\}_{2} + \dots + 2451866.35\}_{199} + 28817188.48\}_{200} + s_2^- = 31492914.41,$$

$$6810564.51\}_{1} + \dots + 8144706.34\}_{200} - s_1^+ = 6810564.51$$

$$5436272.18\}_{1} + \dots + 7008249.55\}_{200} - s_2^+ = 5436272.18$$

$$\}_{1} + \}_{2} + \dots + \}_{200} = 1$$

$$\}_{1}, \}_{2}, \dots, \}_{200}, s^-, s^+ \geq 0$$

### 2.2.4 CALCULATING

We adopt calculating software Deap Version 2.1 developed by professor Coelli to process the sample data on the table and get the efficiency of DMU( ) and slack variable( $s^-, s^+$ ).

```

Results from DEAP Version 2.1
Instruction file = 520.ins
Data file      = 520.dta
Input orientated DEA
Scale assumption: VRS
Slacks calculated using multi-stage method

EFFICIENCY SUMMARY:
firm  crste  vrste  scale
1    0.208  0.216  0.965  ins
2    0.951  1.000  0.951  ins
3    0.954  1.000  0.954  ins
4    0.749  1.000  0.749  ins
5    0.672  0.123  0.555  ins
6    0.045  0.500  0.021  ins
7    0.379  0.500  0.759  ins
8    0.239  0.341  0.701  ins
9    1.000  1.000  1.000  -
10   0.248  1.000  0.248  ins
11   0.120  0.134  0.963  ins
12   0.107  0.202  0.118  drs
13   0.411  1.000  0.411  ins
14   0.343  1.000  0.343  drs
15   0.187  0.946  0.540  ins
16   0.168  0.500  0.336  ins
17   0.144  0.256  0.563  ins
18   0.137  0.146  0.938  drs
19   0.094  0.188  0.632  ins
20   0.099  0.101  0.988  drs
21   0.266  0.695  0.370  drs
22   0.103  0.105  0.985  drs
23   0.095  0.096  0.989  ins
24   0.052  0.162  0.567  ins
25   0.152  0.154  0.989  ins
26   0.183  0.116  0.682  ins
27   0.429  0.750  0.573  ins
28   0.500  0.650  0.709  drs
29   0.338  0.350  0.964  ins
30   0.230  0.500  0.461  ins
31   0.212  0.353  0.584  ins
32   0.218  0.357  0.778  ins
33   0.083  0.158  0.520  ins
34   0.152  0.209  0.727  drs
35   0.772  1.000  0.772  ins
36   0.052  0.148  0.354  ins
37   0.224  1.000  0.224  ins
38   0.093  0.219  0.494  drs
39   0.146  0.657  0.220  ins
40   0.075  0.254  0.283  ins
41   0.071  0.086  0.816  ins
42   0.125  0.130  0.958  ins
43   0.036  0.041  0.886  drs
    
```

Figure 5: computational results of Deap Version



**2.2.6 APPLICATION**

To balance the amount of money for per school. Whether the school can maximize the value of funding it receives is the major consideration factor plus the population effect. We divide the two factors as 7:3, and making the final decision according to each weighting. Part of the table has been showed on the below(Fig.6).

UNITID	INSTNM	POPULATION	EFFICIENCY OF DMU	Final score	money
1.00	Lewis College	10140.00	1.00	70.36	630674.58
2.00	Merrill College	2453.00	1.00	70.19	639626.15
3.00	La Salle University	4338.00	1.00	70.17	638603.97
4.00	Widener University Main Campus	2189.00	1.00	70.11	638230.90
5.00	Champlain College	2881.00	1.00	70.10	638120.83
6.00	Pennsylvania College of Health Sciences	1394.00	1.00	70.07	637610.26
7.00	Saint Joseph's College of Maine	1276.00	1.00	70.05	637369.02
8.00	Albany College of Pharmacy and Health Sciences	1075.00	1.00	70.04	637459.59
9.00	Sacrament Bible College	889.00	1.00	70.02	637433.43
10.00	Our Lady of Holy Cross College	842.00	1.00	70.03	637416.06
11.00	Avoyett University-Non-Traditional Programs	790.00	1.00	70.02	637298.74
12.00	Kettering College	783.00	1.00	70.03	637296.29
13.00	St Vincent's College	772.00	1.00	70.02	637292.44
14.00	Lakeland College	680.00	1.00	70.02	637260.20
15.00	Marjibast University	672.00	1.00	70.02	637257.40
16.00	Asian Pacific Online University	650.00	1.00	70.02	637249.89
17.00	Bennett College	612.00	1.00	70.02	637236.28
18.00	Bryan College of Health Sciences	590.00	1.00	70.02	637229.72
19.00	Wheaton College	568.00	1.00	70.02	637229.94
20.00	Virginia University of Lynchburg	493.00	1.00	70.02	637204.69
⋮					
189.00	Livingstone College	1172.00	0.47	33.68	324732.18
190.00	Mary St Mary's College	2017.00	0.41	31.16	305111.91
191.00	Williamva University	8528.00	0.41	30.90	302961.90
192.00	Berklee College of Music	4402.00	0.44	30.89	303188.81
193.00	Juazeiro University	2384.00	0.43	30.41	299116.25
194.00	Immaculate University	2452.00	0.43	29.51	303873.03
195.00	Golden Gate University-San Francisco	479.00	0.43	29.84	292676.62
196.00	Franklin University	2129.00	0.40	28.11	272658.64
197.00	Rose Hulman Institute of Technology	2165.00	0.40	27.66	272656.82
198.00	Otawa University-Jeffersonville	87.00	0.47	4.90	48129.02
199.00	Berry University	4284.00	0.65	3.79	37291.27
200.00	Fairleigh College	1734.00	0.64	2.31	24663.29

Figure 6: The calculation of investment money

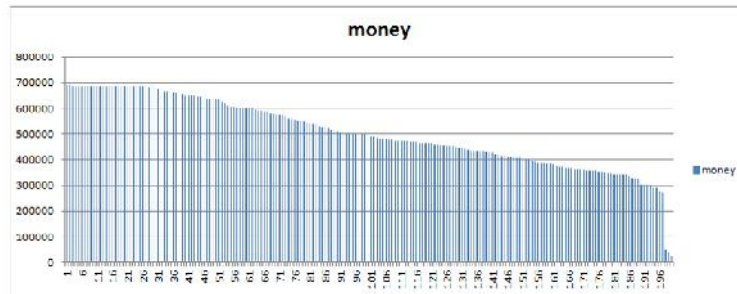


Figure 7: distribution of money

**2.2.7 FURTHER THINK**

To decide the duration that the organization’s money should be provided, the main aspect we consider is to stimulate the highest likelihood of producing a strong effect on student performance. So we consider the change condition of the return to scale in DMU:

If  $\frac{1}{n} \sum_{j=1}^n \lambda_j^0 = 1$  the return to scale is invariability.

If  $\frac{1}{n} \sum_{j=1}^n \lambda_j^0 > 1$  the return to scale is increase.

If  $\frac{1}{\theta} \sum_{j=1}^n \lambda_j^0 < 1$  the return to scale is decrease.

To encourage the institution as well as students better, the invested institution will be asked to submit its related information about return on scale once a year for offices in the organization to ponder whether it is necessary to invest that school next year. Supposed that one university's return to scale is drop dramatically, apparently means that the inputs is far beyond the outputs, it is more wise to stop and invest on another institution, for example the 2011th institution. Provided a university's return to scale goes rise for 5 years, definitely the investment duration for that school is 5 years.

### 3. CONCLUSIONS

The integration of AHP, DEA and MDU methodologies is a hybrid application of soft computing techniques. The aim of the hybrid application is to determine an optimal investment strategy to identify the schools, the investment amount per school efficiently and objectively. we utilize AHP (Analytic Hierarchy Process) to determine the weight of every factor and rank the schools in the list of candidate schools according to the composite index of every school calculated from the weight. So we could select schools in the top of the rank to invest suitable funds. Furthermore we use DEA (Data Envelope Analyse) to get changes of scale efficiency. With this model, we come up with a strategy on what will be both the most efficient and accurate way to invest the Goodgrant Foundation. Our future work will focus on refining the model to be more scientific and more believable. Besides, some factors which are neglected in this model can be further studied if there is more information available.

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