AN AHP(ANALYTIC HIERARCHY PROCESS)-BASED INVESTMENT STRATEGY FOR CHARITABLE ORGANIZATIONS OF GOODGRANT FOUNDATION

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ABSTRACT

This paper provides the optimal investment strategy for the Goodgrant Foundation. To determine the schools to be invested, firstly we find the factors about improving students’ educational performance, including urgency of student’s needs, school’s demonstrate potential for effective use of private funding, the reputation of school, and return on investment etc; secondly, we utilize AHP (Analytic Hierarchy Process) to determine the weight of every factor and rank the schools in the list of candidate schools according to the composite index of every school calculated from the weight. To confirm the investment per school and obtains the investment duration time, we use DEA (Data Envelope Analyse) to get changes of scale efficiency; and then according to the change trend, we determine time duration of investment and effective utilization of private funding, which is the factor together with student population affecting the investment amount for per school. It is helpful to make a better decision on investing universities. We are convinced that our research is promising to benefit all sides of students, schools and Goodgrant.

KEYWORDS

AHP, DEA, Educational Performance, Investment Strategy

1. INTRODUCTION

Universities and colleges are places where young students gain valuable knowledge, resources and opportunities before they step into the society. That is why many foundations are willing to invest on undergraduates to help improve their educational performance. With so many universities and colleges in American, it is necessary for us to carry out a method about how to determine an optimal investment strategy to identify the schools, the investment amount per school efficiently and objectively. How to distribute the fund is exactly the key. It is a multi-aspect evaluate task including the urgency of students’ needs, school’s demonstrate potential for effective use of private funding, the reputation of school, return on investment etc. Mean while, the time duration that the organization’s money have the highest likelihood of producing a strong positive effect on student performance should also be a primary consideration. Other large and known grant organizations such as Gates Foundation and Lumina Foundation show the current way to investigate the qualification which mainly concentrates on the low income of students’ families and potential of universities. These models will take the ability of using the funding and rate of return into account, and obtain approximate investment duration time and return of investments at the Good grant Foundation can provide the assistance best to not only students but also schools and foundation itself.

The analytic hierarchy process (AHP)[2-4] is a structured technique for organizing and analyzing complex situation. It is based on mathematics and psychology. Rather than prescribe a “correct” decision, the AHP helps decision makers find one that best suits their goal and their understanding of the problem[5-6]. It provides a comprehensive and rational framework for
structuring a decision problem, for representing and quantifying its elements, for relating those elements to overall goals, and for evaluating alternative solutions.

This paper provides the optimal investment strategy for the Goodgrant Foundation by AHP method[2]. It helps improve educational performance of undergraduates in United States and make them graduate successfully, live a good life in the future. The rest of this paper is organized as follows: in Section 2, we present our model approach in detail, including the analytical hierarchy process model and data envelope analysis model. Conclusions are provided in the last Section.

2. MODEL DESIGN

2.1 THE ANALYTICAL HIERARCHY PROCESS MODEL

By using cluster analysis, we group all the colleges corecard data into urgency of students’ needs, school’s demonstrate potential for effective use of private funding, the reputation of school, return on investment 4 groups. Meanwhile urgency of students’ needs contains share of part-time undergraduates, median debt of completers and average net price; School’s demonstrate potential for effective use of private funding contains percentage of undergraduates who have received a PellGrant, percent of all federal under graduate students receiving a federal student loan and 3-year repayment rate; The reputation of school includes predominant degree awarded, discipline distribution and structure and whether it is operating with other institutions; Return on investment includes Median earnings of students 10 years after entry, share of students earning over 25,000 dollar/year after entry. The dates of 2936 universities are in Table 1.

<table>
<thead>
<tr>
<th>university</th>
<th>p1</th>
<th>p2</th>
<th>p3</th>
<th>p4</th>
<th>p5</th>
<th>p6</th>
<th>p7</th>
<th>p8</th>
<th>p9</th>
<th>p10</th>
<th>p11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alabama A&amp;M University</td>
<td>0.682</td>
<td>38.115</td>
<td>58.872</td>
<td>0.712</td>
<td>0.520</td>
<td>0.445</td>
<td>3</td>
<td>2.241</td>
<td>1</td>
<td>38.401</td>
<td>0.462</td>
</tr>
<tr>
<td>University of Alabama-Birmingham</td>
<td>0.238</td>
<td>2.3117</td>
<td>65.436</td>
<td>0.731</td>
<td>0.580</td>
<td>0.756</td>
<td>3</td>
<td>2.351</td>
<td>1</td>
<td>43.000</td>
<td>0.660</td>
</tr>
<tr>
<td>Amherst University</td>
<td>0.0722</td>
<td>19.892</td>
<td>74.091</td>
<td>0.764</td>
<td>0.764</td>
<td>0.647</td>
<td>3</td>
<td>2.336</td>
<td>1</td>
<td>38.000</td>
<td>0.467</td>
</tr>
<tr>
<td>University of Alabama at Birmingham</td>
<td>0.248</td>
<td>26.734</td>
<td>38.156</td>
<td>0.328</td>
<td>0.663</td>
<td>0.762</td>
<td>3</td>
<td>2.298</td>
<td>1</td>
<td>46.000</td>
<td>0.668</td>
</tr>
<tr>
<td>Alabama State University</td>
<td>0.49</td>
<td>35.542</td>
<td>31.816</td>
<td>0.927</td>
<td>0.974</td>
<td>0.811</td>
<td>3</td>
<td>2.384</td>
<td>1</td>
<td>27.000</td>
<td>0.525</td>
</tr>
<tr>
<td>The University of Alabama</td>
<td>0.081</td>
<td>26.400</td>
<td>27.874</td>
<td>0.201</td>
<td>0.973</td>
<td>0.811</td>
<td>3</td>
<td>2.220</td>
<td>1</td>
<td>24.000</td>
<td>0.661</td>
</tr>
<tr>
<td>Central Alabama Community College</td>
<td>0.416</td>
<td>19.892</td>
<td>35.942</td>
<td>0.632</td>
<td>0.404</td>
<td>0.458</td>
<td>3</td>
<td>2.341</td>
<td>1</td>
<td>27.000</td>
<td>0.466</td>
</tr>
<tr>
<td>Auburn University at Montgomery</td>
<td>0.306</td>
<td>21.791</td>
<td>32.137</td>
<td>0.451</td>
<td>0.663</td>
<td>0.629</td>
<td>3</td>
<td>2.348</td>
<td>1</td>
<td>34.000</td>
<td>0.355</td>
</tr>
<tr>
<td>...</td>
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<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

2.1.1 THE ESTABLISHMENT OF A HIERARCHY

The problem of the case can be divided into three layers in order.

![Figure 1: The three-tier funding distribution system](image)
2.1.2 The Weights of Layer C in Layer O

Considering the relative importance of $C_1$ compared with $C_2$, $C_3$, $C_4$; we might arrive at the following pairwise comparison matrix.

\[
A = \begin{pmatrix}
1 & 3 & 5 & 5 \\
1/3 & 1 & 1 & 2 \\
1/5 & 1/2 & 1 & 2 \\
1/5 & 1/2 & 1/2 & 1 \\
\end{pmatrix}
\]

The maximum eigenvalue $\lambda_1 = 4.05$, and we can get the corresponding normalized eigenvector $w_1 = (0.58, 0.17, 0.16, 0.09)$.

2.1.3 Consistency Test

Consistency Index

\[
\sigma = \frac{\lambda_{\max} - n}{n-1}
\]

when $n = 4, \lambda_1 = 4.05, CI = 0.016$, Random Consistency Index $RI = 0.9$, it can be get from the table below (Table 2).

<table>
<thead>
<tr>
<th>Number of Order</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_h$</td>
<td>0</td>
<td>0</td>
<td>0.58</td>
<td>0.90</td>
<td>1.12</td>
<td>1.24</td>
<td>1.32</td>
<td>1.41</td>
<td>1.45</td>
</tr>
</tbody>
</table>

Consistency Ratios

\[
CR = \frac{\sigma}{RI}
\]

When $RI = 0.9$, $CR = 0.018 < 0.1$ meet the inspection.
2.1.4 The weights of layer P in layer C

Considering the relative importance of \( P_1 \) compared with \( P_2, P_3, P_4 \); \( P_5, P_6, P_7 \) compared with \( P_8, P_9, P_{10} \); \( P_{11} \); We might arrive at the following pairwise comparison matrix:

\[
B_1 = \begin{pmatrix}
1 & \frac{1}{3} & 1 \\
\frac{1}{3} & 3 & 1 \\
1 & 1 & \frac{1}{3}
\end{pmatrix} \quad B_2 = \begin{pmatrix}
1 & \frac{1}{3} & 1 \\
1 & 1 & \frac{1}{3} \\
\frac{1}{3} & 3 & 1
\end{pmatrix} \quad B_3 = \begin{pmatrix}
1 & 3 & 6 \\
1 & 1 & 5 \\
\frac{1}{6} & \frac{1}{5} & 1
\end{pmatrix} \quad B_4 = \begin{pmatrix}
1 & \frac{3}{7} & 1 \\
1 & 1 & 1
\end{pmatrix}
\]

The maximum eigen value \( \lambda_2 = 3, \lambda_3 = 3, \lambda_4 = 3.10, \lambda_5 = 2 \) and we can get the corresponding normalized eigen vector

\[ w_1 = (0.2, 0.6, 0.2)^T \quad w_2 = (0.2, 0.2, 0.6)^T \quad w_3 = (0.635, 0.287, 0.078)^T \quad w_4 = (0.3, 0.7)^T \]

After normalization, we can weight vector

\[ CR_2 = \sum_{i=1}^{4} CR_i^{(k)} = 0.08 \]

2.1.5 The weights of layer P in layer O

\[ W^k = (\omega_1^{(k)}, \omega_2^{(k)}, \ldots, \omega_n^{(k)}) \ (k = 1, 2, 3, 4) \]

\[ W = [W_1, W_2, W_3, W_4]_{1 \times 4} \]

\[ W = \begin{pmatrix}
0.115 & 0.345 & 0.115 & 0.035 & 0.035 & 0.035 & 0.106 & 0.099 & 0.045 & 0.012 & 0.03 & 0.065
\end{pmatrix} \]

\[ CR = CR_1 + CR_2 = 0.098 < 0.1 \]

2.1.6 Data normalization method

In order to reconcile all kinds of indicators in one assessment system, we apply Min Max Normalization to normalize the indicators that mentioned in the database. This helps us to process data in various dimension. Our process of data normalization is as following formula

\[ x^* = \frac{x - \min}{\max - \min} \]

\[
\begin{pmatrix}
0.0010 & 0.0007 & 0.0004 & 0.0004 & 0.0006 & 0.0003 & 0.0004 & 0.0004 & 0.0000 & 0.0000 & 0.0000 & 0.0004 & 0.0004 \\
0.0007 & 0.0008 & 0.0003 & 0.0005 & 0.0005 & 0.0002 & 0.0004 & 0.0000 & 0.0000 & 0.0000 & 0.0003 & 0.0003 \\
0.0011 & 0.0007 & 0.0003 & 0.0006 & 0.0006 & 0.0002 & 0.0003 & 0.0000 & 0.0000 & 0.0000 & 0.0003 & 0.0003 \\
0.0001 & 0.0008 & 0.0007 & 0.0004 & 0.0006 & 0.0003 & 0.0004 & 0.0000 & 0.0000 & 0.0000 & 0.0004 & 0.0004 \\
\vdots \\
0.0010 & 0.0004 & 0.0006 & 0.0002 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0003 & 0.0002 \\
0.0001 & 0.0005 & 0.0005 & 0.0006 & 0.0003 & 0.0004 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0004 & 0.0004
\end{pmatrix}
\]
2.1.7 COMPREHENSIVE RANK

We can figure out the following formula to quantize the satisfaction of Goodgrant to the school.

\[ D = \sum_{j=1}^{n} b_j H_i = 1, \ldots, 2936 \]

Lastly, we utilize Analytic Hierarchy Process (AHP) to determine the weight of every factor and rank the schools in the list of candidate schools according to the composite index of every school calculated from the weight. Then we can select schools in the top of the rank to invest suitable funds.

![Figure 3: The rank of schools](image-url)
2.2 DATA ENVELOPE ANALYSIS

2.2.1 BASIC CONCEPTION OF DEA

After assessing the educational performance of all post secondary colleges and universities, in accordance with the rank, we select 200 universities who are most deserving grants. Then it comes to the question of the amount of money distributed to per school. We consider the ability of effective using of private funding and the population as the main factors. To quantize the funding use capacity, we evaluate the relative efficiency of the same type of output and input Decision Making Unit (DMU) and employ the Date Envelopment Analysis (DEA) [8-9].

Parameter Assumption: \( X \): input index, \( Y \): output index, to a certain project we assume that there are \( s \) decision making units per unit of which has \( m \) kinds of inputs and \( n \) kinds of inputs, weight coefficient correspondingly \( V = (v_1, v_2, ..., v_m)^T \), \( U = (u_1, u_2, ..., u_n)^T \). Every unit has its own efficiency evaluation goals \( h_j = u_j Y_j / v_j X_j \), we can always choose proper weight coefficient which satisfy \( h_{j_1}, j = 1, 2, ..., s \).

In order to estimate the efficiency of DMU, weight coefficient correspondingly \( v, u \) as variable, aim at the efficiency index of DMU, restraint by all efficiency indexes, develop the fractional programming model as followed:

\[
\begin{align*}
\text{Max} & \quad h_k = \frac{\sum_{r=1}^{n} u_r y_{rk}}{\sum_{i=1}^{m} v_i x_{ik}} \\
\text{st.} & \quad \frac{\sum_{r=1}^{n} u_r y_{rk}}{\sum_{i=1}^{m} m v_i x_{ik}} \leq 1
\end{align*}
\]

In which

\( u_r \geq \varepsilon > 0 \), \( v_i \geq \varepsilon > 0 \)

Let \( U_t = t \times u_r \), \( V_t = t \times v_i \), \( U_r \geq \varepsilon > 0 \), \( V_r \geq \varepsilon > 0 \) after Charnes-Cooper transform we get the \( C^2R \) linear programming model

\[
\begin{align*}
\text{Max} & \quad h_k = \sum_{r=1}^{n} U_t y_{rk} \quad \text{st.} \quad \sum_{i=1}^{m} v_i x_{ik} = 1 \sum_{r=1}^{n} U_t y_{rk} - \sum_{i=1}^{m} v_i x_{ik} \leq 0
\end{align*}
\]

Introduces slack variables and surplus variables, and get the CCR Duality linear layout model

\[
\begin{align*}
\text{Max} & \quad [\theta - \varepsilon (e^{T}s^- + e^T s^+)] \\
\text{st.} & \quad \sum_{r=1}^{n} x_j \lambda_j + s^- = \theta x_j 0
\end{align*}
\]
\[ \sum_{j=1}^{n} y_j \lambda_j - s^+ = y_{j0} \]

In which: \( \theta_k \) presents the potential quota of all inputs possibility of equal proportion reduction. \( e^{AT}, e^T \) present m dimension unit column vector and n dimension unit column vector. \( \varepsilon \) presents The Archimedes dimensionless, which is smaller than any number bigger than zero.

2.2.2 The Source Of Sample Data

After disposal data we choose the amount of Pell Grant and federal student loan as input indexes, the amount of earnings of students working and not enrolled 10 years after entry and high in come students 6 years after entry as output indexes. The processed data was showed in the following table(Fig4).

<table>
<thead>
<tr>
<th>Institutions</th>
<th>Inputs</th>
<th>Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

![Figure 4: Input and output index](image)

2.2.3 Establishment Of Model

CCR Model

\[ \text{Max } h = U_1y_1 + U_2y_2 = 78403800u_1 + 31492914u_2 \]

78403800u_1 + 31492914u_2 - 6810564.51 \( \lambda_1 \) - 5436272.18 \( \lambda_2 \) \leq 0

14847200u_1 + 7072340.41 \( \lambda_1 \) - 1940932.15 \( \lambda_2 \) \leq 0

12593500u_1 + 5532753.92 \( \lambda_1 \) - 1943720.06 \( \lambda_2 \) \leq 0

27986400u_1 + 95493619.73 \( \lambda_1 \) - 891828.06 \( \lambda_2 \) \leq 0

11889500u_1 + 24518661.35 \( \lambda_1 \) - 1364384.63 \( \lambda_2 \) \leq 0

7433400u_1 + 28817188.48 \( \lambda_1 \) - 8144706.32 \( \lambda_2 \) \leq 0

\( u_1, u_2, \lambda_1, \lambda_2 \geq 0 \)
BCC model

Min \( \theta \)

\[
78403800 \lambda_1 + 14847200 \lambda_2 + \ldots + 118895000 \lambda_{199} + 74334000 \lambda_{200} + s^-_1 = 78403800 \theta
\]

\[
31492914.41 \lambda_1 + 7072340.41 \lambda_2 + \ldots + 2451866.35 \lambda_{199} + 28817188.48 \lambda_{200} + s^-_2 = 31492914.41 \theta
\]

\[
6810564.51 \lambda_1 + \ldots + 8144706.34 \lambda_{200} - s^+_1 = 6810564.51
\]

\[
5436272.18 \lambda_1 + \ldots + 7008249.55 \lambda_{200} - s^+_2 = 5436272.18
\]

\[
\lambda_1 + \lambda_2 + \ldots + \lambda_{200} = 1
\]

\( \lambda_1, \lambda_2, \ldots, \lambda_{200}, s^-, s^+ \geq 0 \)

2.2.4 Calculating

We adopt calculating software Deap Version 2.1 developed by professor Coelli to process the sample data on the table and get the efficiency of DMU(\( \theta \)) and slack variable(s, s\(^+\)).

![Results from Deap Version 2.1](image)

Figure 5: computational results of Deap Version
2.2.6 APPLICATION

To balance the amount of money for per school. Whether the school can maximize the value of funding it receives is the major consideration factor plus the population effect. We divide the two factors as 7:3, and making the final decision according to each weighting. Part of the table has been showed on the below(Fig.6).

![Figure 6: The calculation of investment money](image)

<table>
<thead>
<tr>
<th>SCHOOL</th>
<th>POPULATION</th>
<th>RELATIVE WEIGHT</th>
<th>FINAL VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test1</td>
<td>112.00</td>
<td>0.67</td>
<td>30.48</td>
</tr>
<tr>
<td>Test2</td>
<td>76.80</td>
<td>0.33</td>
<td>25.34</td>
</tr>
<tr>
<td>Test3</td>
<td>44.80</td>
<td>0.67</td>
<td>29.64</td>
</tr>
<tr>
<td>Test4</td>
<td>22.40</td>
<td>0.33</td>
<td>17.46</td>
</tr>
<tr>
<td>Test5</td>
<td>13.20</td>
<td>0.67</td>
<td>10.72</td>
</tr>
<tr>
<td>Test6</td>
<td>7.60</td>
<td>0.33</td>
<td>5.78</td>
</tr>
</tbody>
</table>

2.2.7 FURTHER THINK

To decide the duration that the organization’s money should be provided, the main aspect we consider is to stimulate the highest likelihood of producing a strong effect on student performance. So we consider the change condition of the return to scale in DMU:

If \( \frac{1}{\Theta^0} \sum_{j=1}^{n} \lambda_j^0 = 1 \), the return to scale is invariability.

If \( \frac{1}{\Theta^0} \sum_{j=1}^{n} \lambda_j^0 > 1 \), the return to scale is increase.

![Figure 7: distribution of money](image)
If $1 - \frac{1}{\lambda} \sum_{i=1}^{c} \lambda_i < 1$, the return to scale is decrease.

To encourage the institution as well as students better, the invested institution will be asked to submit its related information about return on scale once a year for offices in the organization to ponder whether it is necessary to invest that school next year. Supposed that one university’s return to scale is drop dramatically, apparently means that the inputs is far beyond the outputs, it is more wise to stop and invest on another institution, for example the 201thinstitution. Provided a university’s return to scale goes rise for 5 years, definitely the investment duration for that school is 5 years.

3. CONCLUSIONS

The integration of AHP, DEA and MDU methodologies is a hybrid application of soft computing techniques. The aim of the hybrid application is to determine an optimal investment strategy to identify the schools, the investment amount per school efficiently and objectively. We utilize AHP (Analytic Hierarchy Process) to determine the weight of every factor and rank the schools in the list of candidate schools according to the composite index of every school calculated from the weight. So we could select schools in the top of the rank to invest suitable funds. Furthermore we use DEA (Data Envelope Analyse) to get changes of scale efficiency. With this model, we come up with a strategy on what will be both the most efficient and accurate way to invest the Goodgrant Foundation. Our future work will focus on refining the model to be more scientific and more believable. Besides, some factors which are neglected in this model can be further studied if there is more information available.

REFERENCES