ON FINDING MINIMUM AND MAXIMUM PATH LENGTH IN GRID-BASED WIRELESS NETWORKS

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ABSTRACT

In this paper, we obtain the minimum and maximum hop counts between any pair of cells in the 3D grid-based wireless networks. We start by determining the minimum path length between any two points in a 2D grid coordinate system. We establish that the minimum path length is the maximum difference between the corresponding coordinates of the two points. We then extend the result to derive the minimum and maximum hop counts for the 3D grid-based wireless networks. We establish that the maximum path length is the sum of the differences between the corresponding coordinates of the two cells. Whilst the minimum path length depends on the positions of the two cells; it does not exceed the maximum difference between the corresponding coordinates of the two cells.

KEYWORDS

Communication, wireless networks, 3D grid, hop count & path length

1. INTRODUCTION

The path length or hop count in a wireless network is the number of communication steps needed to send a packet from a source node to a destination node in the network. Factors such as transmission range, network topology, nodes' mobility and routing protocols have a strong impact on the path length [1]. The path length can affect the performance of the network. Specifically, it has a direct impact on the packet delivery ratio, packet delay and energy consumption. The studies in [2] [3] and [4] conducted mathematical analysis based on the path lengths to derive upper bounds for the average packet delivery probability in wireless networks.

In [5], an algorithm for finding k shortest paths that link two points in a graph is proposed. The algorithm can be used in finding paths that are of lengths less than a certain value. K. Day et al. [6] considered finding a maximum number of node-disjoint paths of minimum or near minimum lengths between any two points of the k-ary n-cube interconnection network. These paths are then used to derive the fault diameter of the network. S. Basagni et al. [7] proposed a Channel Aware Routing Protocol (CARP) for Underwater Wireless Sensor Networks (UWSNs). CARP uses a hop count to decide the next forwarder. In [8], a dynamic programming model was developed to compute the average path length of large scale-free networks, where the number of paths from a given node exhibits a low power distribution.

The contribution of this paper is twofold. First, we derive the minimum path length between any two points in a 2D grid coordinate system. We establish that the minimum path length between any two points is the maximum difference between the corresponding coordinates of the two points. Second, we use a similar strategy to derive the minimum and maximum path length between any source-destination pair of cells for the 3D grid-based wireless networks. We establish that the maximum path length is the sum of the differences between the corresponding coordinates of the two cells. The minimum path length depends on the positions of the two cells;

thus, we consider all possible cases and calculate the minimum path length for each case. The minimum path length does not exceed the maximum difference between the corresponding coordinates of the two cells.

The rest of this paper is organized as follows. In section 2, we derive formulas for determining the distance between any pair of points in a 2D grid coordinate system. Section 3 briefly presents the 3D grid-based topology for the wireless networks. A strategy similar to that used in section 2 is used in section 4 to find the minimum and maximum path lengths of the 3D grid-based wireless networks. Section 5 concludes the paper.

2. DETERMINING PATH LENGTH IN A 2D GRID COORDINATE SYSTEM

In this section, we demonstrate how to determine the minimum number of cells needed to move from one point to another in a 2D grid coordinate system. Such a path is called a minimal path. Consider the 2D grid coordinate system shown in Figure 1. There are up to 8 possible moves (i.e. 4 moves along the two axes and 4 diagonal moves) to move from one point to another. Let us assume that the two points are $A(x_1, y_1)$ and $B(x_2, y_2)$. In general, we will refer to the number of cells crossed by the minimal path as H_{min}. Furthermore, the absolute difference between the two points along an i-axis is referred to by δi. In other words, we denote $\delta x = |x_2-x_1|$ and $\delta y = |y_2-y_1|$.
Lemma 1: Given two points $A(x1, y1)$ and $B(x2, y2)$ in a 2D grid coordinates system, for which $\delta x = \delta y$, the number of grid cells crossed by the minimal path is given by:

$$H_{\min} = \delta x,$$

Proof: This case is shown in Figure 1. Recall that there are eight possible moves to neighboring cells. A horizontal move or a vertical move reduces the Hamming distance (the Hamming distance is being the sum of the differences between the corresponding coordinates) between the two cells by one along the x-axis or the y-axis, respectively. Whereas a diagonal move reduces the difference along each of the two dimensions by one. Thus, none of the 8 moves to neighboring cells reduces the Hamming distance by more than two. Since the claimed minimum path length is half of the Hamming distance (i.e. $(\delta x + \delta y)/2 = \delta x$), it is of minimum length.

Proposition 1: Given two points $A(x1, y1)$ and $B(x2, y2)$ in a 2D grid coordinates system, in which $\delta x \neq \delta y$. The number of grid cells crossed by the minimum path is given by:

$$H_{min} = Maximum\ (\delta x, \delta y)$$

Proof: Let us assume that $\delta x > \delta y$ as illustrated in Figure 2. Since $\delta x > \delta y,$ we have δy (i.e. *Minimum* $(\delta x, \delta y)$) diagonal moves that can give the highest reduction in the difference between the two cells. After δy diagonal moves, δy will be zero, while δx will be reduced to $(\delta x - \delta y)$. Thus, we need $(\delta x - \delta y)$ additional moves along the x-axis. Therefore, the maximum of the δx and δy is the length of the minimal path. It is worth mentioning that the order of the moves is not important, which means that different possible paths of minimal length can be obtained. Similar proof can be stated for the case $\delta y > \delta x$. Hence, in general

$$H_{min} = Maximum\ (\delta x, \delta y) \qquad \square$$
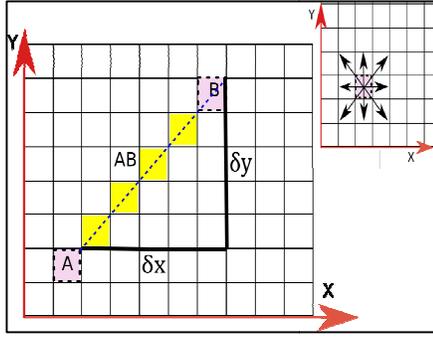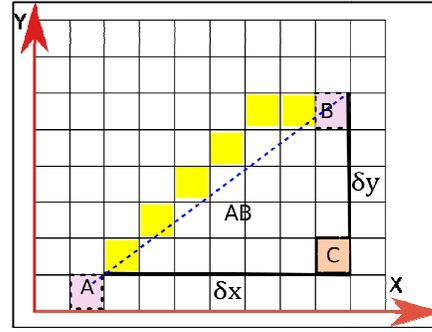
Figure 1: Case 1 ($\delta x = \delta y$ & $\delta z = 0$)   Figure 2: Cas2 ($\delta x > \delta y$ & $\delta z = 0$)

The same proving method will be used in section 4 to derive the number of minimum and maximum hops between any pair of source-destination cells in the 3D grid-based wireless networks.

3. OVERVIEW OF THE 3D GRID-BASED WIRELESS NETWORKS

In this section, we briefly present the structure of the 3D grid-based wireless networks. Such networks are viewed as 3D grids (see Figure 3 as an example). A cell is referred to by its (x, y, z) coordinates. The grid origin with (0, 0, 0) coordinates is assumed to be at the left forward corner of the top surface as illustrated in Figure 3. Two cells are neighbors of each other if any node in one cell can communicate directly with any node in the other cell. A cell can have up to 32 neighboring cells as shown in Figure 4. That is a cell is a neighbor of another cell if the difference in each of the three dimensions (x, y and z) is -1, 0 or +1 except when the difference is zero in all three dimensions. In addition, two cells are neighbors of each other if the difference along one dimension is -2 or +2 and 0 along the other two dimensions. Therefore, the number of neighboring cells of a cell is $3^3 - 1 + 6 = 32$. To attain this reachability, the length of the cell side d is selected based on the nodes' transmission range R such that $d = R/2\sqrt{3}$ [9].

We will use the notations x-1 or x+1 to refer to one move (one hop) to the left or to the right along the x-axis, respectively. Furthermore, we will use the notations x-2 or x+2 to refer to one move between two points which are two cells apart from each other to the left or to the right along the x-axis, respectively. Similar notations are used for the y-axis and z-axis. Accordingly, the 32 neighboring cells are: (x-1,y-1,z-1), (x-1,y-1,z), (x-1,y-1,z+1), (x-1,y,z-1), (x-1,y,z), (x-1,y,z+1), (x-1,y+1,z-1), (x-1,y+1,z), (x-1,y+1,z+1), (x,y-1,z-1), (x,y-1,z), (x,y-1,z+1), (x,y,z-1), (x,y,z+1), (x,y+1,z-1), (x,y+1,z), (x,y+1,z+1), (x+1,y-1,z-1), (x+1,y-1,z), (x+1,y-1,z+1), (x+1,y,z-1), (x+1,y,z), (x+1,y,z+1), (x+1,y+1,z-1), (x+1,y+1,z), (x+1,y+1,z+1), (x-2,y,z), (x+2,y,z), (x,y-2,z), (x,y+2,z), (x,y,z-2), (x,y,z+2).

A packet is forwarded on a cell-by-cell basis using cell neighboring moves until it reaches a sink node (i.e. final destination), which is assumed to be located at a surface cell. In this paper, we will establish the maximum and minimum path lengths that connect any pair of source-sink cells.
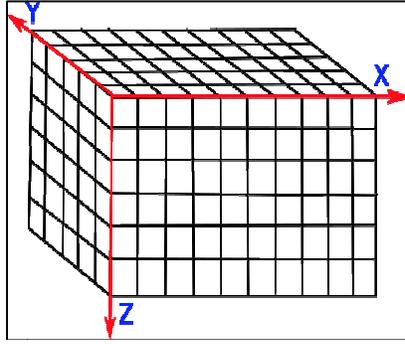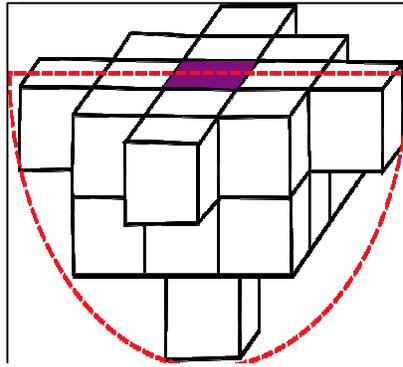
Figure 3: A view of the 3D grid [9]

Figure 4: A cross section of the 32 neighboring cells [9]

4. PATH LENGTHS FOR THE 3D GRID STRUCTURE

In this section, we will apply a similar strategy as in section 2 to calculate the maximum and the minimum number of hops between any two cells in a 3D grid structure. Recall that there are 32 possible moves, and the destination cells are always assumed to be at the grid surface level where the z-coordinate is equal to zero. Let us denote the xyz-coordinates of the source and destination cells by (sx, sy, sz) and (dx, dy, dz), respectively. The absolute value of the differences between the coordinates of the source and the destination in the three dimensions are denoted by $\delta x = |dx-sx|$, $\delta y = |dy-sy|$ and $\delta z = |dz|$.

The maximum number of hops H_{max} is easy to determine. It is basically the sum of the absolute differences in the three dimensions. This can be explained by selecting, in each hop, the neighbor that gives the smallest progress. In other words, selecting the neighbor that reduces the difference in one dimension by only one, while keeping the other dimensions unchanged. Therefore, the maximum path length is given by:

$$H_{max} = \delta x + \delta y + \delta z$$

The following is a possible path with maximum length:

$$< x + 1, y, z >^{\delta x} < x, y + 1, z >^{\delta y} < x, y, z + 1 >^{\delta z}$$

The length of the shortest path H_{min} depends mainly on the position of the destination cell relative to the source cell. The different possible positions are shown in Figure 5. From the cases shown in

this Figure, there are different subcases based on the differences in the three dimensions between the source and destination cells as described below.
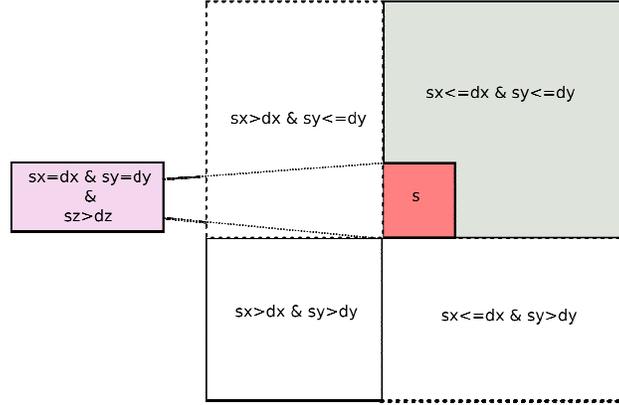

Figure 5: Different possible cases for the position of the destination cell relative to the source cell

4.1. Case 1: sx = dx & sy = dy & sz > 0

In this case, the source and the destination are in the same vertical column but in different horizontal planes. The minimal path length is:

$$H_{min} = \left\lceil \delta z / 2 \right\rceil$$

This length is achieved by moving the packet vertically $\left\lceil \delta z / 2 \right\rceil$ times along the z-axis via the neighbor (x, y, z-2), until reaching the target cell. Note that this is the best move (in terms of the maximum reduction in the difference between the coordinates of the two cells) among the available possible moves. Therefore, the minimal path (in terms of the minimum number of hops) for moving a packet is given by:

$$< x, y, z - 2 >^{\left\lceil \delta z / 2 \right\rceil}$$

4.2. Case 2: sx<=dx & sy<=dy

Case 2.1 δx = δy = δz:

$$H_{min} = \delta x$$

This is achieved by moving the packet diagonally in the xyz-space δx hops. In other words, the packet is moved along the following path:

$$< x + 1, y + 1, z - 1 >^{\delta x}$$

We can justify that this path is of minimum length by noticing that none of the 32 moves to neighboring cells reduces the Hamming distance by more than 3. Since the length of this path is equal to one third of the Hamming distance (i.e. (δx + δy + δz)/3 = δx), it is of minimum length.

Case 2.2 δx = δy & δz > δx:

$$H_{min} = \delta x + \left\lceil \frac{\delta z - \delta x}{2} \right\rceil$$

The best possible move (yielding a maximum reduction in the Hamming distance) among the available neighbors is a move to the diagonal neighbor in the xyz-space. This condition remains valid for a number of δx hops. After that, the move $\langle x, y, z-2 \rangle$ gives the best reduction, and we need $\left\lceil \frac{\delta z - \delta x}{2} \right\rceil$ such moves. Thus, we need at least $\delta x + \left\lceil \frac{\delta z - \delta x}{2} \right\rceil$ moves to reach the destination cell. Since the order of the moves is not important, one possible path will be:

$$< x+1, y+1, z-1 >^{\delta x}, < x, y, z-2 >^{\left\lceil \frac{\delta z - \delta x}{2} \right\rceil}$$

Case 2.3   δx = δy & δz < δx:

$$H_{min} = \delta z + (\delta x - \delta z) = \delta x$$

Initially, the best possible move among the available neighbors is a diagonal move in the xyz-space. This condition stays valid for a number of δz hops. After that, the move in the xy-plane $\langle x+1, y+1, z \rangle$ gives the maximum reduction in the Hamming distance, and we need $(\delta x - \delta z)$ such moves. Thus, we need at least δx moves to reach the destination cell. One possible path will be:

$$< x+1, y+1, z-1 >^{\delta z}, < x+1, y+1, z >^{\delta x - \delta z}$$

Case 2.4   δz = δy & δz > δx

$$H_{min} = \delta x + (\delta z - \delta x) = \delta z$$

This is achieved by moving the packet diagonally in the xyz-space δx hops followed by (δz - δx) moves in the yz-plane. Therefore, the path is as follows:

$$< x+1, y+1, z-1 >^{\delta x}, < x, y+1, z-1 >^{\delta z - \delta x}$$

The proof of the optimality of this path is similar to the proof of Case 2.3 with swapping of z and x.

Case 2.5   δz= δy & δz < δx

$$H_{min} = \delta z + \left\lceil \frac{\delta x - \delta z}{2} \right\rceil$$

This is achieved by moving the packet diagonally in the xyz-space δz hops followed by $\left\lceil \frac{\delta x - \delta z}{2} \right\rceil$ moves in the x-axis using the neighbor $\langle x+2, y, z \rangle$. Therefore, the path is as follows:

$$< x+1, y+1, z-1 >^{\delta z}, < x+2, y, z >^{\left\lceil \frac{\delta x - \delta z}{2} \right\rceil}$$

The proof of the optimality of this path is similar to the proof of Case 2.2 with swapping of z and x.

Case 2.6   δx = δz & δz > δy

$$H_{min} = (\delta z - \delta y) + \delta y = \delta z$$

This is achieved by moving the packet diagonally in the xyz-space δy hops, followed by (δz-δy) moves in the xz-plane. Therefore, the path is as follows:

$$< x + 1, y + 1, z - 1 >^{\delta y}, < x + 1, y, z - 1 >^{\delta z - \delta y}$$

Similar to the proof in the third subcase in Case 2, one can show that this path is of minimal path length.

Case 2.7 δx = δz & δx < δy

$$H_{min} = \left\lceil \frac{\delta y - \delta x}{2} \right\rceil + \delta x$$

This is achieved by moving the packet diagonally in the xyz-space δx hops. Then, moving it along the y-axis $\left\lceil \frac{\delta y - \delta x}{2} \right\rceil$ hops. Therefore, the path is as follows:

$$< x + 1, y + 1, z - 1 >^{\delta x}, < x, y + 2, z >^{\left\lceil \frac{\delta y - \delta x}{2} \right\rceil}$$

The proof of the optimality of this case is similar to the proof of Case 2.2 swapping the roles of y and z.

Case 2.8 δx ≠ δy, δy ≠ δz and δx ≠ δz:

To find the minimal path length in this sub case, we need to determine the maximum and the second maximum among δx, δy and δz. For example, if the order in their values is as follows:

$$\delta x > \delta y > \delta z; \text{ then:}$$

$$H_{min} = \delta z + (\delta y - \delta z) + \left\lceil \frac{\delta x - \delta y}{2} \right\rceil = \delta y + \left\lceil \frac{\delta x - \delta y}{2} \right\rceil$$

This is achieved by moving the packet diagonally in the xyz-space δz hops, which is the best move from the available moves. Then, moving in the xy-plane (δy-δz) hops, which is also the best among the available moves. Finally, moving along the x-axis $\left\lceil \frac{\delta x - \delta y}{2} \right\rceil$ hops. Thus, the total number of hops is $\delta y + \left\lceil \frac{\delta x - \delta y}{2} \right\rceil$. Therefore, the path is as follows:

$$< x + 1, y + 1, z - 1 >^{\delta z}, < x + 1, y + 1, z >^{\delta y - \delta z}, < x + 2, y, z >^{\left\lceil \frac{\delta x - \delta y}{2} \right\rceil}$$

4.3. Case 3: sx<=dx & sy>dy

The subcases are similar to those in case 2 except case 2.1when all deltas are equal, which is not part of this case. Thus, the path lengths of similar subcases are equal. However, the plus sign of the y coordinate is changed to negative sign.

4.4. Case 4: sx>dx & sy<=dy

The subcases are similar to those in case 2 except case 2.1when all deltas are equal, which is not part of this case. The path lengths of similar subcases are equal. However, the plus sign of the x coordinate is changed to negative sign.

4.5. Case 5: sx>dx & sy>dy

The subcases are similar to those in case 2 except case 2.1when all deltas are equal, which is not part of this case. The path lengths of similar subcases are equal. However, the plus sign of the x and y coordinates are changed to negative signs.

5. CONCLUSION

In this paper, we have first derived the length of the shortest paths connecting any two points in a 2D grid coordinate system. The obtained result shows that the path length is equal to the maximum difference between the corresponding coordinates of the two points. We have then used a similar approach to determine the maximum and minimum path lengths between any pair of cells with a characterization of the corresponding paths for the 3D grid-based wireless networks. Each cell has up to 32 neighboring cells as described earlier. We have proved that the maximum path length is the sum of the differences between the corresponding coordinates of the two cells. The minimum path length depends on the positions of the two cells; though, not exceeding the maximum difference between the corresponding coordinates of the two cells. These results can be used to design routing protocols based on the 3D grid structure.

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