ESTIMATION OF SEPARATION AND LOCATION OF WAVE EMITTING SOURCES : A COMPARISON STUDY BETWEEN PARAMETRIC AND NON-PARAMETRIC METHODS

Mujahid F. AL-Azzo and Azzah T.Qaba

Electronic Engineering Department, Electronic Engineering collage,
Ninevah University, Mosul-IRAQ.

ABSTRACT

A mathematical model for localization of acoustical sources with separation between them is derived and presented. A classical (Fourier transform) method and a modern ,parametric , (Burg) method are used . The results show the capability of Burg method to resolve the adjacent sources when compared with Fourier transform method, as well as the localization of the sources . The performance is studies with varying some parameters relating to the problem .

KEY WORDS

Burg method, spectral estimation, location of sources, sidelobes, parametric, non- parametric methods, AR process.

1. INTRODUCTION

The magnitudes and frequencies estimation of signal's harmonics with their number from measurement represent a problem in some applications of signal processing such as radio communication, direction of arrival estimation, and so on .

A synthetic aperture scanning of antenna (acoustic receiver) is used to collect the data and then extract the information from it. The non-parametric _Fourier transform , (FT) ,technique , known also as "traditional spectral estimation " technique is efficient in computation .However , it has two main disadvantages : high sidelobes , and moreover low resolution [1]. A long data records (long aperture) is required to overcome such problem. Resolution refers to the ability of the technique to detect the least separation between the points that the object consists of . Hence the aim of the work is to find another method of better resolution and smaller sidelobes. The parametric, "Modern" methods play big role in solving such problems .one of such methods is a Burg technique .it is high resolution and stable method [2,3]. It uses a linear prediction model and minimizes the backward and forward errors of linear prediction to estimate frequency components of the signal. The received acoustic wave is recorded as amplitude and phase . The wave is sampled , i.e. is recorded as discrete values .

The auto –regressive process (AR) is used to model the recorded data for localization problem A Burg technique is used to calculate the coefficients of AR process .

DOI: 10.5121/sipij.2020.11302

The recording of data , and then ,the reconstruction of the sources parameters both represent the process of localization and separation of the problem under consideration .

The contribution given by this paper is to use Burg method, as high resolution technique, to model the object's localization and separations between the objects.

1.1. Related Work.

As given in [2,3], the Burg method has other applications in medicine, radar, and so on. This work is categorized within spectral estimation problem. Some other related spectral estimation work can be found in long wavelength imaging, as acoustical holographic imaging [4], direction of arrival estimation problems [5,6] .

2. Field Analysis at Fresnel Region [7,8,9].

The problem under consideration (the two acoustic sources, also we called them as object) has a field distribution $D_1(p_1)$. The propagation of this field $S_1(x)$ toward the axis of recording x_1 is represented by

$$S_1(x_1) = \frac{\beta}{z_{o1}\lambda} \int_{p1} D_1(p_1) \exp\left(j\gamma r_1(p_1, x_1)\right) dp_1 \tag{1}$$

This represents an approximate to Fresnel principle . γ is a wave number , and β is a constant. The distance between the recoding plane and the object's plane is Z_{o1} . The distance between a certain point on α recording axis x_1 and a certain point on the object 'p1' is r_1, and is given by

$$r_1(p_1, x_1) = \sqrt{z_{o1}^{\,2} + (x_1 - p_1)^2} \tag{2}$$

Using a paraxial approximation ,

$$((x_1 - p_1)^2 / z_{o1}^{\,2}) \ll 1 \tag{3}$$

The r1 (p_1, x_1) is given as

$$r_1(p_1, x_1) = z_{o1} + \frac{(x_1 - p_1)^2}{2\, z_{o1}} - \frac{(x_1 - p_1)^4}{8 z_{o1}^{\,3}} + - \,----- \tag{4}$$

r1 , in Fresnel region , could be approximated using only two terms of equation (4) , i.e.

$$r_1(p_1, x_1) = z_{o1} + \frac{x_1^{\,2}}{2z_{o1}} + \frac{p_1^{\,2}}{2z_{o1}} - \frac{p_1 x_1}{z_{o1}} \tag{5}$$

Substituting (5) in (1) yields

$$S_1(x_1) = \beta_1 \exp\left(\frac{j\gamma x_1^{\,2}}{2z_{o1}}\right) \int_{p1} D_1(p_1) \exp\left(\frac{j\gamma p_1^{\,2}}{2z_{o1}}\right) exp\left(\frac{-j\gamma x_1 p_1}{2z_{o1}}\right) dp_1 \tag{6}$$

$\beta 1$: complex constant resulting from substitution of (5) in (1)

$$\dot{D_1}(p_1) = D_1(p_1)\exp(\frac{jϒp_1{}^2}{2z_{o1}}) \tag{7}$$

substituting equation (7) in equation (6) this will lead to

$$S_1(x_1) = \beta_1 \exp\ (\frac{jϒx_1{}^2}{2z_{o1}}) \int_{p_1} \dot{D_1}(p_1)\ exp(\frac{-jϒx_1\, p_1}{2z_{o1}})dp_1 \tag{8}$$

In equation (8) , the integration exmplifies Fourier transformation of the whole object distribution function $\dot{D_1}(p_1)$.

It is clear from equation (8) that the object distribution and the recorded field is associated with a Fourier transform. However $S_1(x)$ is also influenced by the quadratic Phase Factor plus Fourier transform, i.e

$$S_1(x_1) = \beta_1 \exp\ (\frac{jϒx_1{}^2}{2z_{o1}})\ F|\dot{D_1}(p_1)| \tag{9}$$

Where F signified the Fourier transformation.

3. BURG METHOD PRINCIPLE

Estimating [1,10] the AR parameters is achieved by using Burg method. Based on minimization of backward and forward error of linear predictors, the above method can be regarded as order recursive least squares lattice technique. The auto regressive parameters should satisfy the levinson-Durbin recursion. It is supposed that the data $S_1(n_1)$, $n_1 = 0,1,\ldots,$ N-1, is given. In order the estimator to be derived, the backward and forward linear predicting estimation of order i, is given by

$$\hat{S_1}(n_1) = -\sum_{\ell=1}^{i} a_{1i}(\ell)S_1(n_1 - \ell) \tag{10}$$

$$\hat{S_1}(n_1 - i) = -\sum_{\ell=1}^{i} a^*{}_{1i}(\ell)S_1(n_1 + \ell - i) \tag{11}$$

with the corresponding backward and forward errors f1i(n_1) and $g_{1i}(n_1)$ defined as $f_{1i}(n_1) = S_1(n_1) - \hat{S}_1(n_1)$ and $g_{1i} = S_1(n_1 - i) - \hat{S}_1(n_1 - i)$ where $a_{1i}(\ell)\ ,0 \le \ell \le i-1\ , i = 1,2,\ldots\ldots, c,$ represent the coefficients of prediction. ε1i represents a least square error and is given as

$$\varepsilon_{1i} = \sum_{n_1=i}^{N_1-1} [|f_{1i}(n_1)|^2 + |g_{1i}(n_1)|^2] \tag{12}$$

The above error should be minimized by taking the coefficients of prediction process according to the limitation of the recursion of Levnsion-Durbin which is given by

$$a_{1i}(\ell) = a_{1(i-1)}(\ell) + K_{1i} a^{*}_{1(i-1)}(i-\ell) \quad 1 \le \ell \le i-1 \quad, 1 \le i \le c \tag{13}$$

with K_{1i}=$a_{1i}(\ell)$is the ith coefficient of reflection in the realization of lattice filter for the predictor. Substituting (13) in expression for f_{1i} (n1) and $g_{1i}(n_1)$ will result in the pair of order recursion equations for backward and forward prediction errors which is given by

$$\begin{aligned} f_{1i}(n_1) &= f_{1(i-1)}(n_1) + K_{1i}\, g_{1(i-1)}(n_1 - 1), \qquad i = 1,2,\dots,c \\ g_{1i}(n_1) &= K^{*}_{1i}\; f_{1(i-1)}(n_1) + g_{1(i-1)}(n_1 - 1) \quad i = 1,2,\dots,c \end{aligned} \tag{14}$$

substitution from (13) into (14) and minimizing ε_{1i} with respect to K_{1i} , the result will be

$$\widehat{K}_{1i} = \frac{-\sum_{n_1=i}^{N_1-1} f_{1(i-1)}(n_1) g^{*}_{1(i-1)}(n_1 - 1)}{\frac{1}{2}\sum_{n_1=i}^{N_1-1}\left[\left|f_{1(i-1)}(n_1)\right|^2 + \left|g_{1(i-1)}(n_1 - 1)\right|^2\right]}, \qquad i = 1,2,\dots,c \tag{15}$$

The cross correlation estimation between the backward and forward prediction errors is given in the numerator of equation 15 . From the normalized factors of the denominator in equation 15 , it is clear that $|K_{1i}|$ is less than one , hence the model of all – pole gained from the obtained data is stable. The denominator in (15) represents the least squares estimation of the backward and forward errors $E^{b}_{1(i-1)}$ and $E^{f}_{1(i-1)}$ respectively [1] . Hence $\widehat{K}_{1i}$ in (15) is given as

$$\widehat{K}_{1i} = \frac{-\sum_{n_1=i}^{N_1-1} f_{1(i-1)}(n_1) g^{*}_{1(i-1)}(n_1 - 1)}{\frac{1}{2}\sum_{n_1-i}^{N_1-1}[\widehat{E}^{f}_{1(i-1)} + \widehat{E}^{b}_{1(i-1)}]}, \qquad i = 1,2,\dots,c \tag{16}$$

where $\widehat{E}^{f}_{1(i-1)} + \widehat{E}^{b}_{1(i-1)}$represents the estimation of a least- squared error E_{1i} : The denominator in equation (16) can be calculated according to an order-recursion as given by relation [1]

$$\widehat{E}_{1i} = \left(1 - |\widehat{K}_{1i}|^2\right)\widehat{E}_{1(i-1)} - |f_{1(i-1)}(i-1)|^2 - |g_{1(i-1)}(i-2)|^2 \tag{17}$$

where $\widehat{E}_{1i} = \widehat{E}^{f}_{1i} + \widehat{E}^{b}_{1i}$ represents the total-squared error [1,10].

Using (16) and (17) , the Burg method determine (estimate) the parameters of auto regressive process. Than the power spectrum estimation is found, by using the AR parameters, as following [1,10]:

$$P^{Bu}_{1xx}(f) = \frac{\widehat{E}_{1c}}{\left|1 + \sum_{\ell=1}^{c} \hat{a}_{1c}(\ell) e^{-j2\pi f\ell}\right|^2} \tag{18}$$

The advantages of the Burg method are [2,3]:

(1) It is highly efficacious
(2) Resulting in high spectral resolution
(3) Resulting in stability AR model

The limitation of Burg method is that it may exhibit spectral line splitting in sometimes.

4. SOURCES LOCALIZATION

Multiplication of the recorded data $S_1(x_1)$ by the quadratic phase factor $(\frac{\gamma x_1^2}{2Z_{o1}})$ and then applying one of the spectral resolution methods : either a Fourier transform method , or a high resolution Burg method.

5. RESULTS

Two acoustic transmitting sources (regarded as two points)are used. The computer is used to simulate the problem under consideration . The first source is positioned at lateral distance p1=2cm from a supposed vertical reference axis at which the receiving acoustic transducer begins from . The second transmitting source is positioned at a lateral distance p2 from the vertical reference axis .The received waveform from the two sources at the receiving (recording) axis is sampled , according to synthetic aperture scanning , by N1 samples .

The axial distance between transmitting and receiving axis is z_{o1} . The wavelength is λ , the sampling interval is ΔX, and the position difference between the two sources is Δp, i.e. p2-p1. Different value of z_{o1} , ΔX, λ and p2 are used as given in the following results (figures). Two spectral estimation methods are used :Fourier transform (FT) and Burg methods .

The minimum separation (resolution) that the Fourier transform could resolve is given by the following formula [11]

$$\sigma_1 = \frac{\lambda z_{o1}}{a_2} \tag{19}$$

a_2: half of the aperture length.

$$a_2 = \frac{(N_1 - 1)}{2} \Delta X \tag{20}$$

A percentage errors (P.E) of Δp,i.e. p2 –p1 are shown in figures 1,2,3,4,5,6. We assumed that when P.E$<$ 20% , it is an acceptable value .

5.1. Results Without Noisy Data .

Figure (1) shown the results (performance) when both methods are used . it is clear from the parameter and equation (19) and (20) that $\sigma 1 > 5$cm .Hence (FT) could not resolve the separation , Δp , for acceptable P.E. , for values less than σ_1=5 cm , while Burg method could resolve less thanσ_1 ,and up to p2=3cm , i.e. Δp =1 cm . Moreover , the percentage error, P.E, is too much less than that for(FT)method .

Figure (2) shows also the reinforced performance of Burg method when compared with (FT) method , with $\sigma_1 > 2.1$ cm . P.E. for Burg method is less than 7% while for (FT) when Δp is small , the P.E. is between 20% to 40% .

Figure (3) shows that (FT) could not resolve the two sources until Δp = 3 cm since $\sigma_1 > 5$ cm while Burg method resolve the two sources with Δp=2 cm .

5.2. Results With Noisy Data

For more investigation of the problem, a white noise is added to the data, and its effect on the parameters of the problem is noticed in the following figures.

Figure (4) shows that (FT) incapable to find two separate sources until $\Delta p = 2$ cm since $\sigma_1 >$ 2.1 cm while Burg method resolve the two sources with Δp=1 cm .

Figure (5) shows also the reinforced performance of Burg method when compared with (FT) method , with $\sigma_1 >$ 2.5 cm . P.E. for Burg method is less than 2% while for FT when P.E. is small the P.E. of (FT) is large.

Figure (6) shows also the reinforced performance of Burg method when compared with (FT) method , with σ_1> 1.4 cm . P.E. for Burg method is less than P.E. of (FT) .

Figure (7) shows the two thin peaks that correspond to the two acoustic sources with Δp=2 cm when Burg method is used , while (FT) method failed to resolve the two peaks since $\sigma_1 >$ 3.4cm and the result seems to be a single source .

Figure (8) shows the two thin peaks that correspond to the two acoustic sources with Δp= 3 cm when Burg method is used , while (FT) method failed to resolve the two peaks since $\sigma_1 >$ 6.8 cm and the result seems to be a single source.

Figure (9) also shows the results of the two methods: Burg and (FT). The two thin peaks correspond to the two acoustic sources when Burg method is applied with Δp= 2 cm. When (FT) is applied, it resolved the two sources with Δp= 3 cm . which is less than that of Burg method . σ_1> 2.8 cm. However , the two peaks have greater widths than that Burg method. Moreover, the two peaks are not equal although the sources have the same transmitting power (field distribution).

Figure (10) also shows the results of the two methods: Burg and (FT). The two thin peaks correspond to the two acoustic sources when Burg method is applied with Δp= 3 cm. When (FT) is applied, it resolved the two sources with Δp= 3 cm . σ_1> 3.4 cm However , the two peaks have greater widths than that Burg method. Moreover, the two peaks are not equal although the sources have the same transmitting power (field distribution).

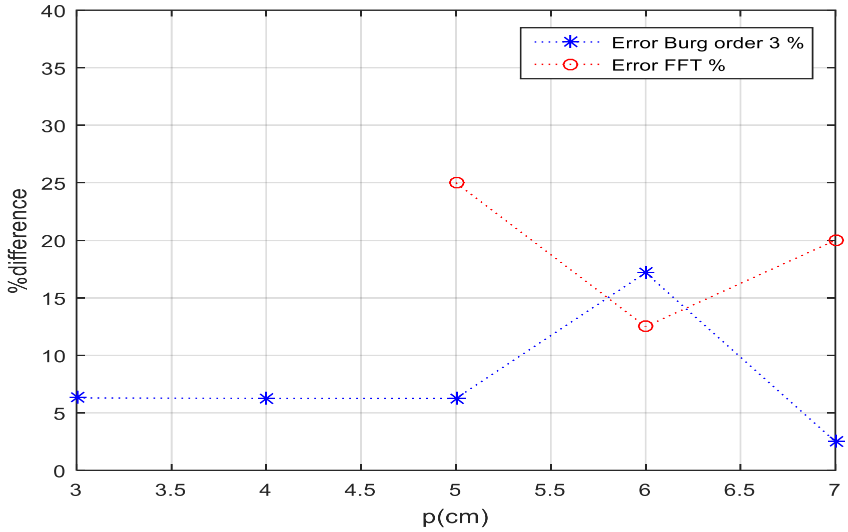

Figure 1. Two points without noise (z_{o1}=30, N_1=20 , ΔX=0.5, λ=0.8, p1=2)

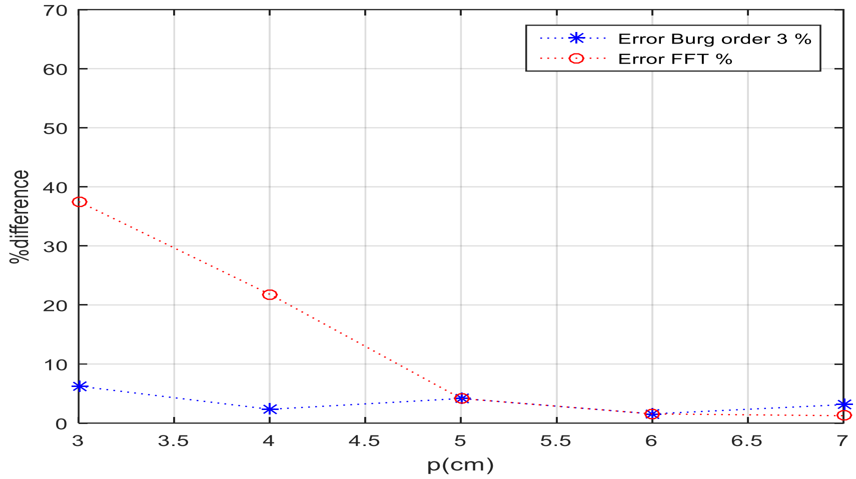

Figure 2. Two points without noise (z_{o1}=25, N_1=20 , ΔX=0.5, λ=0.4, p1=2)

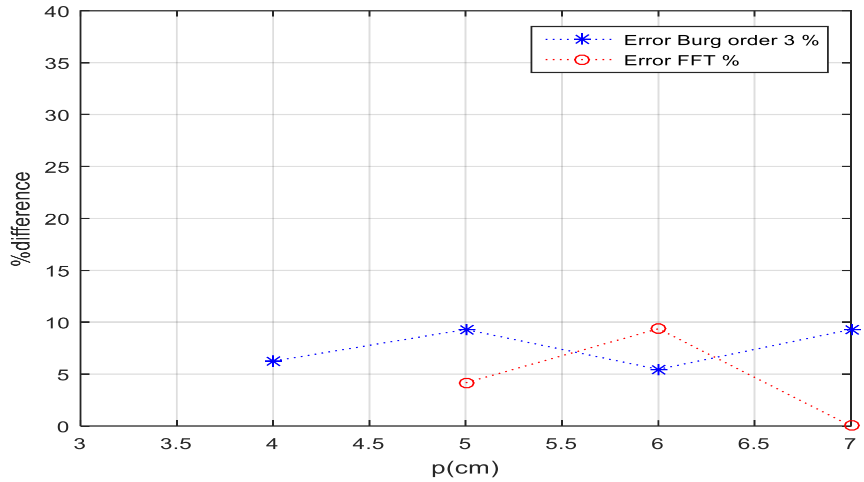

Figure 3. Two points without noise (z_{o1} =25, N_1=20 , ΔX =0.5, λ =0.8, p1=2)

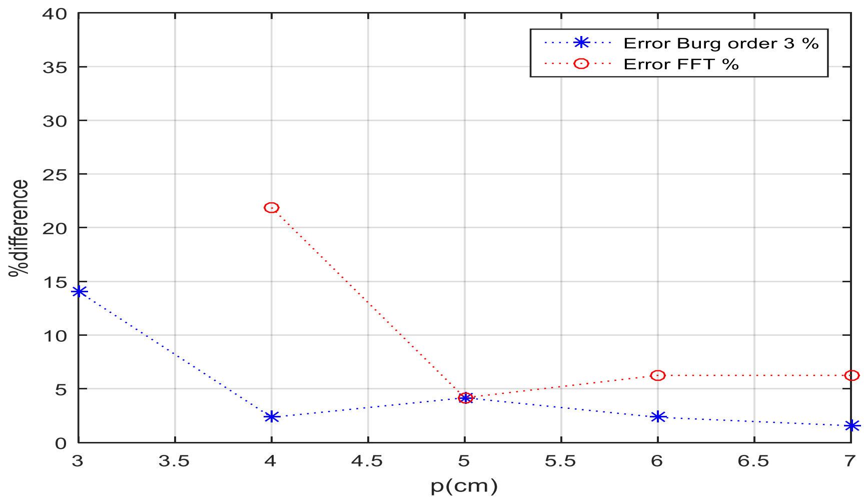

Figure 4. Two points with noise (z_{o1}=25, N_1=20 , ΔX =0.5, λ =0.4, p1=2)

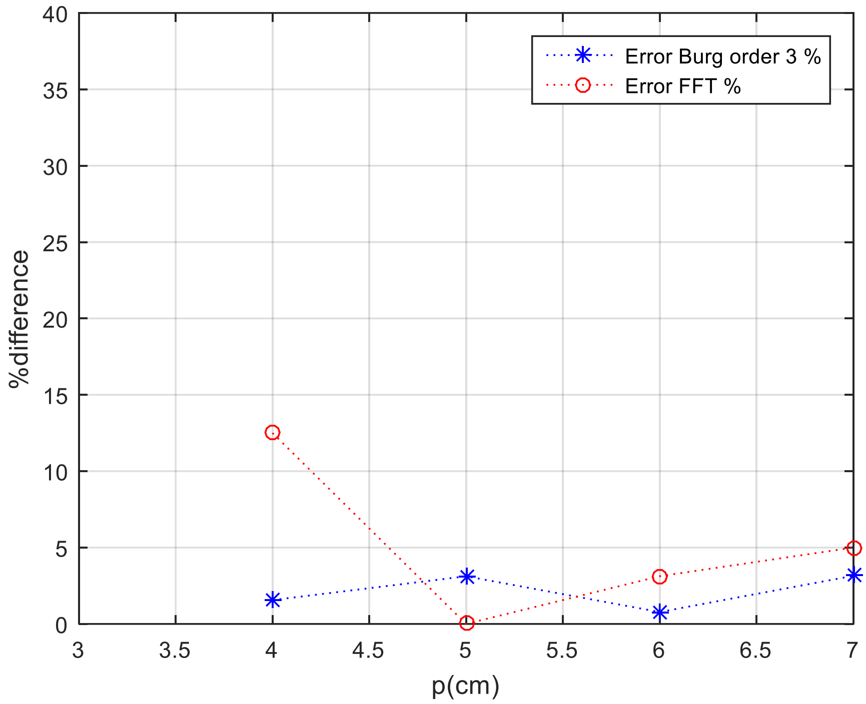

Figure 5. Two points with noise (z_{o1}=30, N_1=20 , ΔX =0.5, λ =0.4, p1=2)

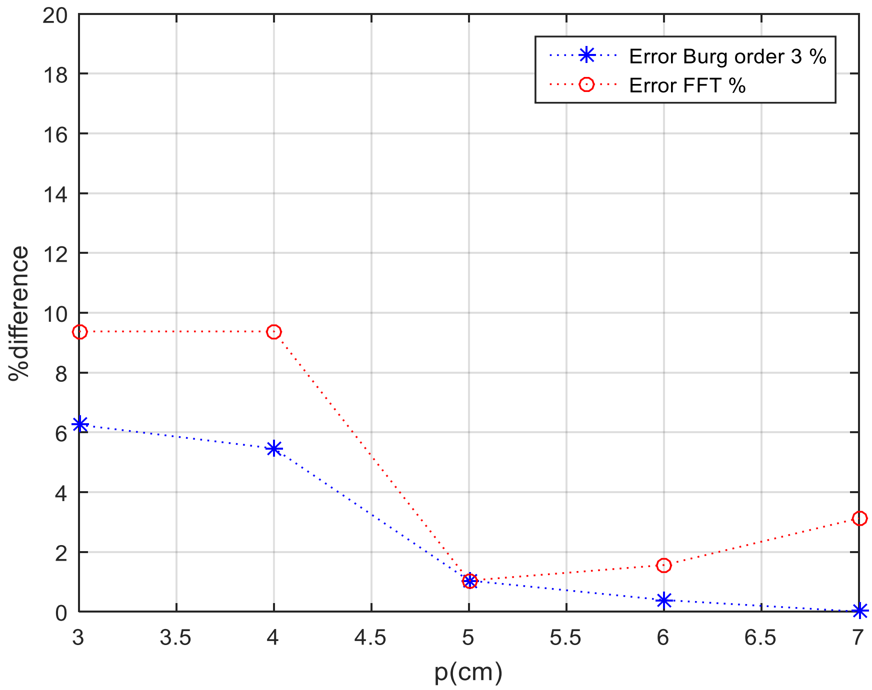

Figure 6. Two points with noise (z_{o1}=25, N_1=15 , ΔX =1, λ =0.4, p1=2)

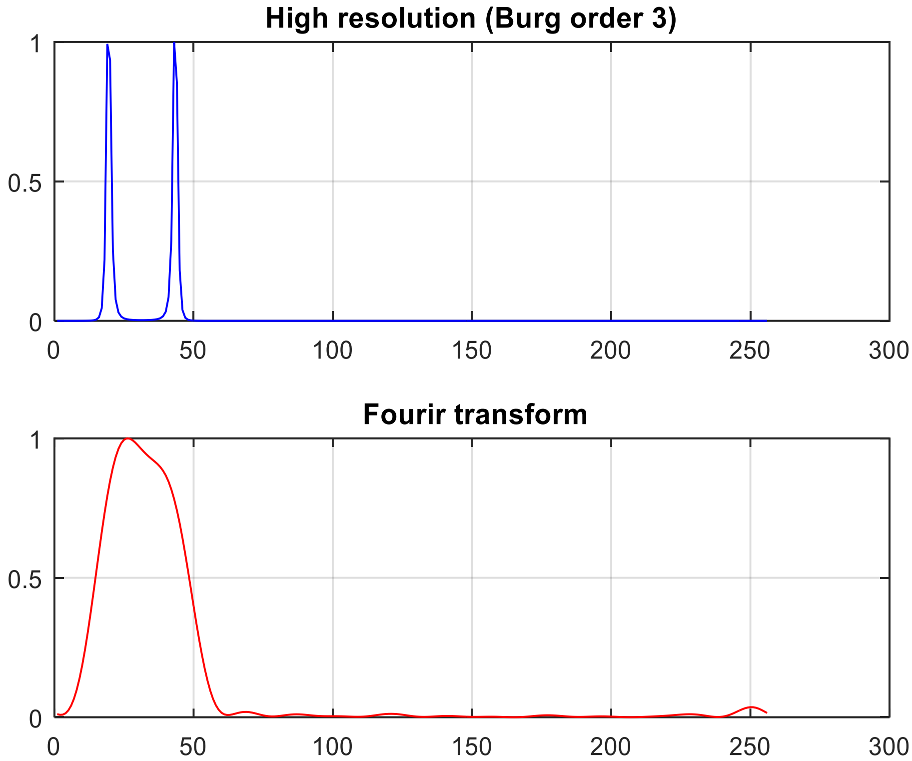

Figure 7. Two points with noise (z_{o1}=30, N_1=15 , ΔX =0.5, λ =0.4, p1=2,p2=4)

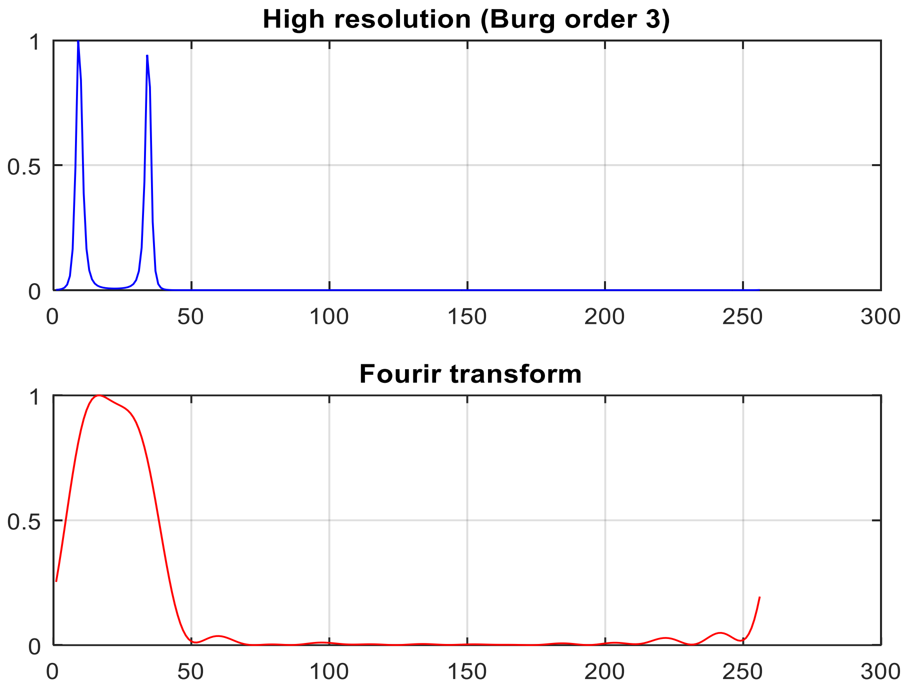

Figure 8. Two points with noise (z_{o1}=30, N_1=15 , ΔX =0.5, λ =0.8, p1=2,p2=5)

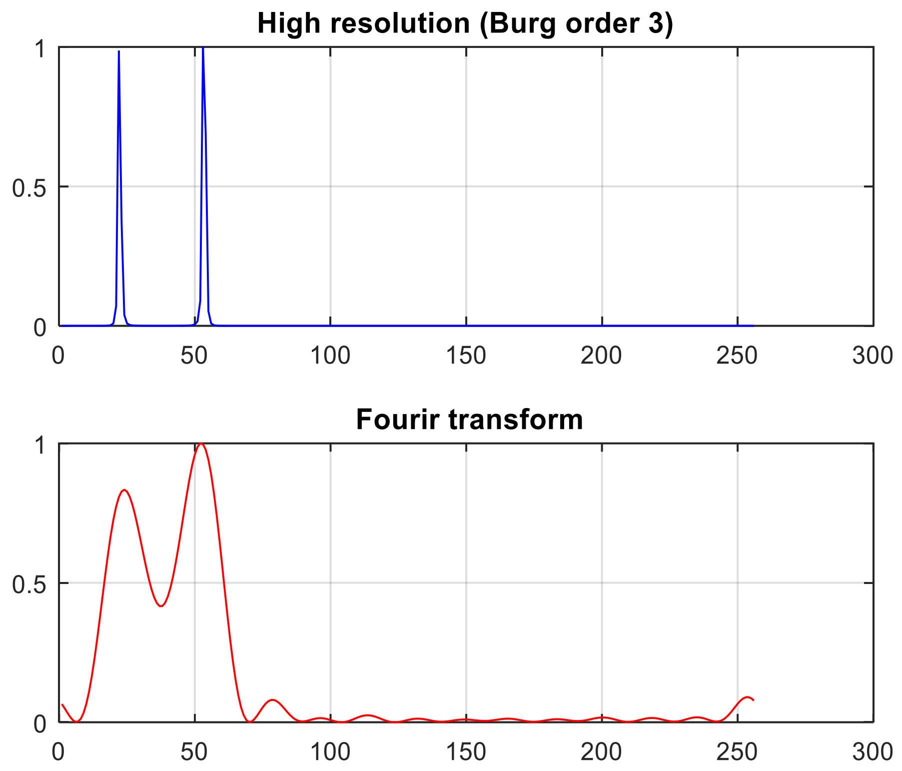

Figure 9. Two points with noise (z_{o1}=25, N_1=15 , ΔX =0.5, λ =0.4, p1=2,p2=4)

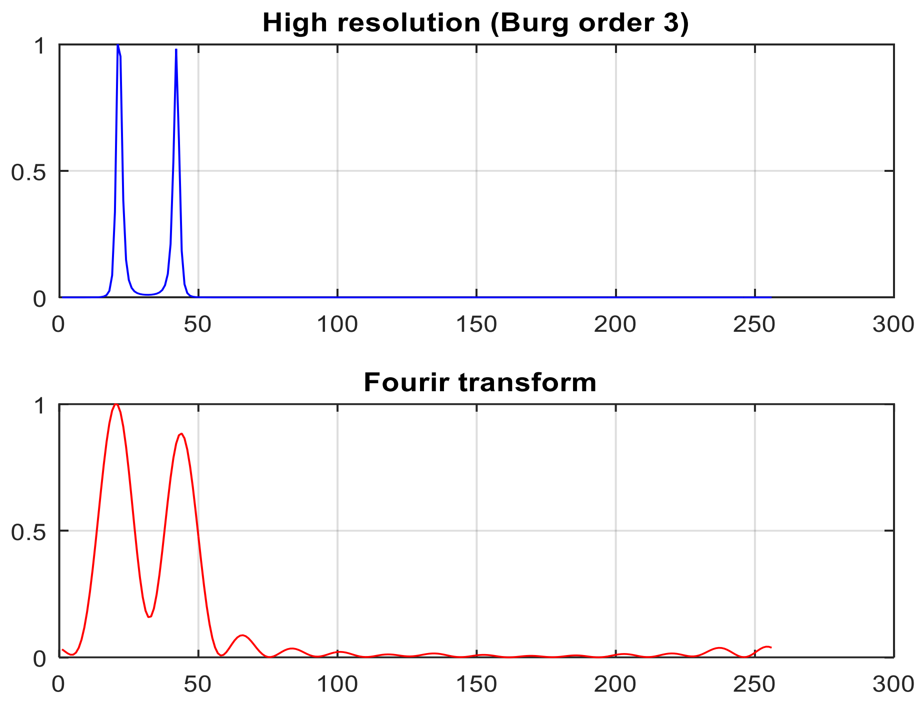

Figure 10. Two points with noise (z_{o1}=30, N_1=15 , ΔX =1, λ =0.8, p1=2,p2=5)

6. CONCLUSIONS

It is proved that Burg method can be used to find the locations of, and separation between the objects and to find other parameter, the amplitude, and to decrease the effect of sidelobes. So it has better performance than (FT) method since it has high resolution performance for small Δp and has less P.E. For a future work, a practical data is used and compared with simulation results.

REFERENCES

[1] Proakis, J. G. Dimitris. G, Manolakis. (2006). " Digital Signal Processing ", Prentice Hall Inc.

[2] Chen, W. L., Lin, C. H., Chen, T., Chen, P. J., & Kan, C. D. (2013). Stenosis detection using burg method with autoregressive model for hemodialysis patients. Journal of Medical and Biomedical Engineering, 33(4), 356-362.

[3] Decurninge, A., & Barbaresco, F. (2016). Robust Burg estimation of radar scatter matrix for autoregressive structured SIRV based on Fréchet medians. IET Radar, Sonar & Navigation, 11(1), 78-89.

[4] Al-azzo, M. (2019). Modeling of Holographic Imaging of Volume Object using Modified Covariance Method. International Journal of Signal Processing, 4, 1 -5.

[5] Chen,Z., Gokeda,G., & Yu,Y. (2010). “Introduction to Direction-of-Arrival Estimation”, Artech House.

[6] Al-azzo, M. F., & Al-Sabaawi, K. I. (2014). Comparison between classical and modern methods of direction of arrival (DOA) estimation. International Journal of Advances in Engineering & Technology, 7(3), 1082 - 1090

[7] Grilli, S., Ferraro, P., De Nicola, S., Finizio, A., Pierattini, G., & Meucci, R. (2001). Whole optical wavefields reconstruction by digital holography. Optics Express, 9(6), 294-302.

[8] Anderson, A.P. (1982, Aug). Microwave holography ,IEE Proc., vol.129,part H.

[9] Toal, V. (2011). Introduction to holography. CRC press.

[10] Byrne, C. L. (2014). Signal Processing: a mathematical approach. Chapman and Hall/CRC.

[11] Marie, K. H. S., Anderson, A. P., & Bennett, J. C. (1982, August). Digital in-line holographic techniques for long wavelength imaging. In IEE Proceedings H (Microwaves, Optics and Antennas) (Vol. 129, No. 4, pp. 211-220). IET Digital Library.

AUTHORS

Mujahid F. Al-Azzo, was born in Mosul/Iraq 1960. B.Sc. Electronics and communications Engineering , Mosul University 1982 M.Sc. Electronics and communications Engineering , Mosul University 1985 Ph.D. Communications Engineering , Mosul University, 199 9 , Professional Experience: 1985-1999 work Instructor Assistant Professor , Technical College /Mosul /Iraq , Technical Institute/ Dohuk /Iraq, Technical Institute / Basra /Iraq.2000, Assistant Professor, Industrial higher Institute Lybia. 2000-2001 work Lecturer Assistant Professor Engineering Faculty , Ajman University ,United Arabic Emirates. 2001-2007, Engineering Faculty ,Philadelphia University ,Jordan. 2008-2010 work Lecturer Associate Professor, Engineering Faculty, Philadelphia University ,Jordan. 2010-2011 work Lecturer Associate Professor, Faculty of Engineering Technology ,Zarqa University ,Jordan. 2011-2014 work Lecturer Associate Professor, Faculty of electronics Engineering ,Mosul university ,Iraq. 2014-Now work Lecturer , Associate Professor, Faculty of electronics Engineering , Ninevah university ,Iraq. Teaching Experience, Digital Signal Processing, Communication Engineering, Advanced Engineering Mathematics, Digital Communications, Electronics (1)&(2), Mobile Communication, Microwave Engineering, Acoustical Engineering, Biomedical Instrumentation, Instrumentation &Measurement, Laser & Optoelectronics, Biomedical Electronics .Teaching Experience for M.Sc. study, Digital Signal Processing.

Azzah T.Qaba , was born in Mosul/Iraq 1987 .B.Sc Electronics Engineering , Mosul University 2010 , H.D. Industrial Project Management, Mosul University 2012. Activities teaching