Colour-Texture Image Segmentation using Hypercomplex Gabor Analysis

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Abstract

Texture analysis such as segmentation and classification plays a vital role in computer vision and pattern recognition and is widely applied to many areas such as industrial automation, bio-medical image processing and remote sensing. In this paper, we first extend the well-known Gabor filters to color images using a specific form of hypercomplex numbers known as quaternions. These filters are constructed as windowed basis functions of the quaternion Fourier transform also known as hypercomplex Fourier transform. Based on this extension this paper presents the use of these new quaternionic Gabor filters in colour texture image segmentation. Experimental results on two colour texture images are presented. We tested the robustness of this technique for segmentation by adding Gaussian noise to the texture images. Experimental results indicate that the proposed method gives better segmentation results even in the presence of strongest noise.

Keywords

Colour texture image segmentation, Gabor filters, hypercomplex numbers, quaternions, quaternion Fourier transform.

1. Introduction

Texture is a basic cue for human beings to recognize objects. Research on texture is a very important task in computer vision and its applications. It has been a very active topic in the past three decades. There are several research focuses in the field of texture analysis, mainly including texture classification, texture segmentation, texture synthesis, shape from texture, etc. Texture segmentation deals with textured image. The task is to segment a given image into uniformly textured regions. This texture segmentation problem is one branch of the general problem of image segmentation which is one important step in many computer vision tasks. Regarding global variations of gray values or mean gray values over some neighborhood is in most cases not sufficient for a correct segmentation.

The posed problem is rather vague since the term texture is not well defined and there is no unique way of characterizing mathematically the local gray-value variations perceived as texture by human observers. For this reason very different approaches to texture segmentation have been taken. As local measure for the characterization of texture local statistical properties and local geometric building blocks have been used. Another whole branch in texture segmentation research is based on the local spatial frequency for characterizing texture. Gabor filters play a special role in the analysis of local frequency. On the one hand the Gabor filter based approaches to texture analysis are motivated by psychophysical research since 2D Gabor filters have proven to be a good model for the cortical receptive field profiles while on the other hand they are supported by the observation that a whole class of textures give rise to periodic gray value structures[1].

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In this paper we will restrict ourselves to the Gabor filter based approaches and introduce quaternionic Gabor filter based approach to colour texture segmentation. Experimental results have demonstrated the performance of this method. The remainder of this paper is organized as follows. Section 2 briefly discusses the major related work. Section 3 presents the mathematics of quaternions and Section 4 discusses representation of color pixels using quaternions. Section 5 speaks about quaternion Fourier transforms. Section 6 deals with quaternionic Gabor filters and proposed texture segmentation algorithm. Section 7 presents the experimental results of segmentation algorithm on colour texture images. Section 8 concludes the paper.

2. Related Work

There has been a lot of activity in texture segmentation using Gabor filters. We will comment on some of these approaches. In [2, 3] Dunn introduces a method for finding an optimal 2D Gabor filter for the discrimination of a two-texture image. This filter is designed such that the discontinuity in the magnitude of the filtered image is most significant at the texture boundary. In [4] Teuner et al. point out that the dominant frequencies do not necessarily coincide with the ones that are important for segmentation. A dominant frequency is only helpful for segmentation when it does not occur everywhere in the textured image. The authors provide a measure called the spectral feature contrast which indicates the discriminatory power of Gabor feature.

Bülow in [1] formulates quaternionic Gabor filters (QGF) by applying a Gaussian window to a quaternionic Fourier transform and propose a method for texture segmentation of grayscale images based on quaternionic Gabor filters. However, Bülow does not apply the quaternionic Gabor filters to color or vector-valued images. Lilong Shi and Brian Funt in [5] propose a colour texture segmentation method that makes possible encoding the structural and colour feature as a whole unit by using the quaternion represented colour and use quaternion principal component analysis(QPCA) with colours encoded as quaternions to calculate a basis for colour texture. Our approach to colour texture segmentation is directly based on Bülow’s method. Here we extend the Gabor filters to color images using quaternions and use these new quaternionic Gabor filters in colour texture image segmentation.

3. Quaternions

The concept of the quaternion was introduced by Hamilton in 1843 [6]. It is the generalization of a complex number. A complex number has two components: the real and the imaginary part. The quaternion, however, has four components, i.e., one real part and three imaginary parts and can be represented in Cartesian form as:

\[ q = w + xi + yj + zk \]  

(1)

where \( w, x, y \) and \( z \) are real numbers and \( i, j \) and \( k \) are complex operators which obey the following rules.

\[
egin{align*}
ij &= k, \quad jk = i, \quad ki = j, \\
ji &= -k, \quad kj = -i, \quad ik = -j
\end{align*}
\]

and also satisfies \( i^2 = j^2 = k^2 = ijk = -1 \). From these rules, it is clear that multiplication is not commutative. The quaternion conjugate is \( \bar{q} = w - xi - yj - zk \) and the modulus of a quaternion is given by

\[ |q| = \sqrt{w^2 + x^2 + y^2 + z^2} \]  

(2)
A quaternion with zero real part is called a pure quaternion and a quaternion with unit modulus is called a unit quaternion. The imaginary part of a quaternion has three components and may be associated with a 3-space vector. For this reason, it is sometimes useful to consider the quaternion as composed of a vector part and a scalar part. Thus $q$ can be expressed as

$$q = S(q) + V(q),$$

where the scalar part $S(q)$ is the real part i.e., $S(q) = w$ and the vector part is a composite of three imaginary components,

$$V(q) = xi + yj + zk$$

Euler’s formula for the complex exponential generalizes to hypercomplex form

$$e^{\mu \beta} = \cos \beta + \mu \sin \beta$$

where $\mu$ is a unit pure quaternion. Any quaternion may be represented in polar form as

$$q = |q| e^{\mu \beta}$$

where $\mu$ and $\beta$ are referred to as the eigenaxis and eigenangle of the quaternion, respectively. $\mu$ identifies the direction in 3-space of the vector part and may be regarded as a true generalization of the complex operator $i$, since $\mu^2 = -1$. $\beta$ is analogous to the argument of a complex number, but is unique only in the range $[0, \pi]$ because a value greater than $\pi$ can be reduced to this range by negating or reversing the eigenaxis.

We can visualize the eigenaxis as the imaginary axis of an Argand diagram, the real axis of which is aligned with the scalar axis of the quaternion 4-space. The eigenaxis is perpendicular to the real axis, but need not be aligned with any of the three imaginary axes defined by the imaginary operators $i, j$ and $k$.

### 4. Quaternion Representation of Color Image Pixels

Color image pixels have three components, and they can be represented in quaternion form using pure quaternions [7]. For images in RGB colour space, the three imaginary parts of a pure quaternion can be used to represent the red, green and blue colour components. For example, a pixel at image coordinates $(x, y)$ in an RGB image can be represented as

$$f(x, y) = r(x, y)i + g(x, y)j + b(x, y)k$$

where $r(x, y), g(x, y)$ and $b(x, y)$ are the red, green and blue components of the pixel, respectively. Fig.1 illustrates the approach adopted to represent RGB colour image using the quaternion form.

Using quaternions to represent the RGB color space, the three color channels are processed equally in operations such as multiplication. The advantage of using quaternion based operations to manipulate color information in an image is that we do not have to process each color channel independently, but rather, treat each color triple as a whole unit. We believe, by using quaternion operations, higher color information accuracy can be achieved because a color is treated as an entity.
Figure 1. Quaternion representation of a colour image.

5. Quaternion Fourier Transform

Based on the concept of quaternion multiplication and exponential, the Quaternion Fourier Transform (QFT) has been introduced. Due to the non-commutative multiplication rule of quaternion algebra, there are several forms of quaternion Fourier transforms. We adopted the form presented in the work of [7][8] which divides the discrete QFT into two categories, namely the right-side form and the left-side form.

Discrete version of the right-side and left-side quaternion Fourier transforms can be represented as

$$F^R(u, v) = \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) e^{-\mu 2\pi \left( \frac{xy}{M} + \frac{vy}{N} \right)}$$

$$F^L(u, v) = \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} e^{-\mu 2\pi \left( \frac{xy}{M} + \frac{vy}{N} \right)} f(x, y)$$

Similarly, the inverse quaternion Fourier transforms can be denoted as:

$$f(x, y) = F^{-R} = \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} F(u, v) e^{\mu 2\pi \left( \frac{ux}{M} + \frac{vy}{N} \right)}$$

$$f(x, y) = F^{-L} = \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} e^{\mu 2\pi \left( \frac{ux}{M} + \frac{vy}{N} \right)} F(u, v)$$

In this transform, the hypercomplex operator was generalized: $\mu$ is any unit pure quaternion. $\mu$ determines a direction in color space and an obvious choice for color images is the direction corresponding to the luminance axis which connects all the points $r=g=b$. In RGB color space this is the “gray line” and $\mu$ would be $\frac{\sqrt{3}}{2}$.
6. Quaternionic Gabor Filters

6.1 Definition

Based on existing work in monochrome applications of Gabor operations for texture segmentation, we suggest that extending the Gabor techniques to color images will provide good performance in analyzing texture images. To define a Gabor filter for color images, we will include the concept of quaternions, a four-component hypercomplex number defined by Sir William Hamilton [6].

A two-dimensional complex-valued Gabor filter is a linear shift-invariant filter with the impulse response

$$h(x, y) = g(x', y') \exp\left(2\pi i (u_0 x + v_0 y)\right)$$

with

$$g(x, y) = K \exp\left\{-\frac{1}{2} \left[ \left(\frac{x}{\sigma_1}\right)^2 + \left(\frac{y}{\sigma_2}\right)^2 \right]\right\}$$

where the coordinates $(x', y')$ are derived from $(x, y)$ by a rotation about the origin through the angle $\alpha$.

$$
\begin{pmatrix}
  x' \\
  y'
\end{pmatrix}
= 
\begin{pmatrix}
  \cos \alpha & \sin \alpha \\
  -\sin \alpha & \cos \alpha
\end{pmatrix}
\begin{pmatrix}
  x \\
  y
\end{pmatrix}
$$

and we will choose normalization constant $K$ such that $K = \frac{1}{2\pi \sigma_1 \sigma_2}$. The impulse response of this Gabor filter is a Gaussian-windowed basis functions of the 2-D Fourier transform.

Bülow and Sommer in [1][9] formulate quaternionic Gabor filters (QGF) by applying a Gaussian window to a quaternionic Fourier transform. However, they do not apply the quaternionic Gabor filters to color or vector-valued images. To extend this form to quaternions, we first define a pure unit quaternion $\mu (\mu_1 i + \mu_2 j + \mu_3 k$ with $\mu_0 = 0$) and use it in place of the simple imaginary root $i$. We then have

$$h(x, y) = g(x, y) \exp(\mu(2\pi u_0 x + 2\pi v_0 y))$$

where $g(x, y) = K \exp\left\{-\frac{1}{2} \left[ \left(\frac{x}{\sigma_1}\right)^2 + \left(\frac{y}{\sigma_2}\right)^2 \right]\right\}$ and $K = \frac{1}{2\pi \sigma_1 \sigma_2}$ with the result now a quaternion.

For a unit pure quaternion, Euler’s identity yields the relation $e^{i\mu \theta} = \cos \theta + i \sin \theta$. Therefore, the impulse response of a quaternionic Gabor filter is given by

$$h(x, y) = g(x, y)(\cos(2\pi u_0 x + 2\pi v_0 y) + \mu \sin(2\pi u_0 x + 2\pi v_0 y))$$

and is a Gaussian windowed basis functions of the QFT. This is the filter that we will apply to the colour texture segmentation. In the literature [7] the unit pure quaternion $\mu$ is stated as arbitrary, and is set to the “gray axis” in RGB space – the resulting quaternion is $(i + j + k)/\sqrt{3}$. A typical quaternionic Gabor filter is shown in Fig.2.

When performing a quaternionic Gabor filtering on the computer we have to use discrete quaternionic Gabor filter masks of the form: $h = [h_{m,n}]_{m,n \in \{1, \ldots, M\}}$ with

$$h_{m,n} = \exp\left\{-\frac{(m-M/2)^2}{2\sigma_1^2} - \frac{(n-M/2)^2}{2\sigma_2^2}\right\} \times \exp\left[\frac{2\pi}{M} \left( u (m - M/2) + v (n - M/2) \right)\right]$$

(15)
Using this convention the Gabor filter mask is a $M \times M$ quaternion matrix. The origin is located at the center of the matrix, therefore it is advantageous to choose $M$ odd, in order to have a center pixel in the filter mask. The frequencies $u$ and $v$ count how many periods fit into the filter mask in horizontal and vertical direction respectively. Fig. 3 shows the magnitude of each component of a typical quaternionic Gabor filter of size $21 \times 21$ for a single angle and scale combination. The components show the usual Gabor profile of a sine wave attenuated by a negative exponential. The different components have magnitudes related by the components of the color axis vector $\mu$.

Figure 2. The four components of a quaternionic Gabor filter with parameters $\sigma_1=3, \sigma_2=3$, $u_0=0.1$, $v_0=0.1$ and $\mu=(i+j+k)/\sqrt{3}$. The size of the filter mask is $21 \times 21$. (a) real component (b) i-component (c) j-component (d) k-component.
Figure 3. Mesh plots of Quaternionic Gabor Filter components with parameters $\sigma_1=3, \sigma_2=3, u_0=0.1, v_0=0.1$ and $\mu=(i+j+k)/\sqrt{3}$. The size of the filter mask is 21x21. (a) real component (b) i-component (c) j-component (d) k-component.

6.2 Local Quaternionic Phase

The local quaternionic phase of an image can be defined as the angular phase of the response to a quaternionic Gabor filter. Each quaternion $q$ given in Cartesian representation $q=w+xi+yj+zk$ can be represented in the form:

$$q = |q|e^{i\phi}e^{k\varphi}e^{j\theta} \text{ with } (\phi, \theta, \varphi)\in[\pi, \pi]\times[\pi/2, \pi/2] \times [\pi/4, \pi/4]$$

(16)

The $\theta$ and $\varphi$ components of the quaternionic phase correspond to the horizontal and vertical Fourier phase while the $\phi$-component represents a new entity, which is not effected by a mere shift of the image. The quaternionic phase angle $\varphi$ can be evaluated uniquely within the interval $[\pi/4, \pi/4]$ as [1].

$$\varphi = -\arcsin(2(xy-wz))/2$$

(17)

with $|q|=1$. The additional phase value $\varphi$ resulting from the quaternionic Gabor filtering allows the distinction of the patterns in an image.

6.3 Texture Segmentation algorithm

Input: Texture image of size $N\times N$.
Output: Texture segmented image.

Step 1. Select values for $u_0, v_0, \sigma_1$ and $\sigma_2$.

Step 2. Convert the input image into a 2-D quaternion matrix, say $i(x, y)$

Step 3. Obtain the impulse response of the filter, (which is also a 2-D quaternion matrix) by the following formula.

$$h(x, y) = g(x, y)\exp(\mu(2\pi u_0 x + 2\pi v_0 y))$$

where $g(x, y) = K \exp\left(-\frac{1}{2} \left(\frac{x}{\sigma_1}\right)^2 + \left(\frac{y}{\sigma_2}\right)^2\right)$ and $K = \frac{1}{2\pi\sigma_1\sigma_2}$

Step 4. Compute the convolution of $i(x, y)$ with $h(x, y)$, say it is $m(x, y)$, which is the 2-D quaternion matrix corresponding to the output image

$$m(x, y) = i(x, y) \otimes h(x, y)$$

Step 5. Obtain the output image. If it cleanly discriminates the textures then go to step 6, else go to step 1 to select other values of $u_0, v_0, \sigma_1, \sigma_2$ and repeat the procedure.

Step 6. Transform the filtered image into amplitude / phase representation.

Step 7. Extract the $\varphi$-component of the local phase by applying the following formula to each quaternion valued pixel $q = w+xi+yj+zk$ in the filtered image

$$\varphi = -\arcsin(2(xy-wz))/2 \text{ with } |q|=1$$

Step 8. Obtain the output segmented texture image by applying thresholding to the smoothed $\varphi$-component of the local quaternionic phase.

Step 9. Obtain the edges of the segmented image by applying a Sobel filter.

The steps involved in texture segmentation are shown in Fig. 4.
7. Experimental Results

We demonstrate the segmentation process on two texture images as shown in Figs. 5 (a) and 6(a). The input image is convolved with an optimally tuned quaternionic Gabor filter. We apply one QGF whose central frequencies have been tuned to the main peak in the power spectrum (QFT) of the image. The filtered image is transformed into amplitude/phase representation and the component of the local phase is extracted. The component of the quaternionic phase distinguishes not only local frequency and orientation but also local structure. The smoothed component of the local quaternionic phase is thresholded. The edges of the thresholded component are found by a Sobel filter. The results of segmentation are shown in Figs. 5 and 6. We tested the robustness of the for segmentation by adding Gaussian noise to the textured image shown in Fig. 5 (a). We added noise with zero mean and variance 0.2 and 0.4, respectively. The results of segmentation with noise are shown in Figs. 7 and 8. Although it is almost impossible for a human observer to segment the image with the strongest noise, by means of component most of the pixels are correctly classified.
Figure 6. (a) The textured image  (b) response of quaternionic Gabor filter mask of size $3 \times 3$ with parameters $\sigma_1=3, \sigma_2=3, u_0=0.1, v_0=0.1$ and $\mu=(i+j+k)/\sqrt{3}$  (c) the $\varphi$ –component of the local quaternionic phase (d) the segmentation result after applying a threshold (e) the edges of the $\varphi$ –component found by a Sobel filter.
Figure 7. (a) The texture from figure 5(a) with added Gaussian noise of zero mean and variance 0.2 (b) the $\varphi$ -component of the local quaternionic phase (c) the median filtered $\varphi$ -component (d) the segmented texture (e) the edges of the median filtered $\varphi$ -component found by a Sobel filter showing the wrongly classified pixels.

Figure 8. (a) The texture from figure 5(a) with added Gaussian noise of zero mean and variance 0.4 (b) the $\varphi$ -component of the local quaternionic phase (c) the median filtered $\varphi$ -component (d) the segmented texture (e) the edges of the median filtered $\varphi$ -component found by a Sobel filter showing the wrongly classified pixels.

8. Conclusion

Image segmentation is an essential step in pictorial pattern recognition and analysis applications. Segmentation accuracy determines the success or failure of analysis procedures. Texture segmentation is based on partitioning an image into different regions of similar textures based on a specified criterion. In this paper, we presented a new colour texture image segmentation method using quaternionic Gabor filters. On the basis of the QFT, we introduced quaternionic Gabor filters for colour images. Application of these filters in colour texture segmentation has been performed. In texture segmentation tasks the local quaternionic phase shows high discriminatory power. It is an important fact that this additional feature comes without any additional computational cost. We showed that the proposed texture segmentation method is more robust against image noise. Two different textures, collected from standard album, are used for experimentation. Future research will be concerned with extension of this application.

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