THE DESIGN OF REDUCED ORDER CONTROLLERS FOR THE STABILIZATION OF LARGE SCALE LINEAR DISCRETE-TIME CONTROL SYSTEMS

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ABSTRACT

This paper investigates the design of reduced order controllers for the stabilization of large scale linear discrete-time control systems. Sufficient conditions are derived for the design of reduced order controllers by obtaining a reduced order model of the original large scale linear system using the dominant state of the system. The reduced order controllers are assumed to use only the state of the reduced order model of the original plant.

KEYWORDS

Dominant state; model reduction; reduced-order controllers; stabilization; discrete-time systems.

1. INTRODUCTION

During the past four decades, a significant attention has been paid to the construction of reduced order observers and stabilization using reduced-order controllers for linear control systems [1-10]. In the recent decades, there has been a good attention paid to the control problem of large scale linear systems. This is due to the practical and technical issues like information transfer networks, data acquisition, sensing, computing facilities and the associated cost involved which stem from using full order controller design. Thus, there is a great demand for the control of large scale linear systems with the use of reduced-order controllers rather than full-order controllers.

In this paper, we present the design of reduced-order controllers for large scale linear discrete-time control systems. Our design is carried out by first deriving a reduced-order model of the large scale linear discrete-time plant retaining only the dominant state of the given system. The dominant state of a linear control system corresponds to the slow modes of the linear system, while the non-dominant state of the control system corresponds to the fast modes of the linear system [3-10].

As an application of our recent work [9-10], we first derive the reduced-order model of the given linear discrete-time control system. Using the reduced-order model obtained, we characterize the existence of a reduced-order controller that stabilizes the full linear system, using only the dominant state of the system.

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This paper is organized as follows. In Section 2, we derive sufficient conditions for the derivation of reduced order model for the original large scale linear system. In Section 3, we deploy the reduced order model obtained in Section 2 to derive conditions for the existence of reduced order controllers for the original system that uses only the state of the reduced-order model. In Section 4, a numerical example is shown to verify the result. Conclusions are contained in the final section.

2. REDUCED ORDER MODEL FOR LARGE SCALE LINEAR SYSTEMS

In this section, we consider a large scale linear discrete-time control system given by

$$x(k+1) = Ax(k) + Bu(k) \qquad (1)$$

where $x \in R^n$ is the state and $u \in R^m$ is the control input. Our goal is to derive a reduced-order model for the large scale linear plant (1).

We assume that A and B are constant matrices with real entries of dimensions $n \times n$ and $n \times m$ respectively.

First, we suppose that we have performed an identification of the dominant (slow) and non-dominant (fast) states of the original linear system (1) using the modal approach as described in [9].

Without loss of generality, we may assume that

$$x = \begin{bmatrix} x_s \\ x_f \end{bmatrix},$$

where $x_s \in R^r$ represents the *dominant* state and $x_f \in R^{n-r}$ represents the *non-dominant* state.

Then the system (1) takes the form

$$\begin{bmatrix} x_s(k+1) \\ x_f(k+1) \end{bmatrix} = \begin{bmatrix} A_{ss} & A_{sf} \\ A_{fs} & A_{ff} \end{bmatrix} \begin{bmatrix} x_s(k) \\ x_f(k) \end{bmatrix} + \begin{bmatrix} B_s \\ B_f \end{bmatrix} u(k) \qquad (2)$$

For the sake of simplicity, we shall assume that the matrix A has distinct eigenvalues. We note that this condition is usually satisfied in most practical situations. Then it follows that A is diagonalizable.

Thus, we can find a nonsingular (*modal*) matrix M consisting of the n linearly independent eigenvectors of A so that the linear transformation

$$x(k) = M \; z(k) \qquad (3)$$

results in the original system (2) being transformed into the following diagonal form

$$z(k+1) = \Lambda \; z(k) + \Gamma \; u(k) \qquad (4)$$

where

$$\Lambda = M^{-1}AM = \begin{bmatrix} \Lambda_s & 0 \\ 0 & \Lambda_f \end{bmatrix} \tag{5}$$

is a diagonal matrix consisting of the n eigenvalues of A and

$$\Gamma = M^{-1}B = \begin{bmatrix} \Gamma_s \\ \Gamma_f \end{bmatrix}. \tag{6}$$

Thus, the plant (4) can be written as

$$\begin{bmatrix} z_s(k+1) \\ z_f(k+1) \end{bmatrix} = \begin{bmatrix} \Lambda_s & 0 \\ 0 & \Lambda_f \end{bmatrix} \begin{bmatrix} z_s(k) \\ z_f(k) \end{bmatrix} + \begin{bmatrix} \Gamma_s \\ \Gamma_f \end{bmatrix} u(k) \tag{7}$$

Next, we make the following assumptions:

(H1) As $k \to \infty$, $z_f(k+1) \approx z_f(k)$, *i.e.* z_f takes a constant value in the steady-state.

(H2) The matrix $I - \Lambda_f$ is invertible.

Then it follows from (7) that for large values of k, we have

$$z_f(k) \approx \Lambda_f \; z_f(k) + \Gamma_f \; u(k) \tag{8}$$

i.e.

$$z_f(k) \approx (I - \Lambda_f)^{-1} \; \Gamma_f \; u(k) \tag{9}$$

The inverse of the linear transformation (3) is given by

$$z(k) = M^{-1}x(k) = G \; x(k) \tag{10}$$

i.e.

$$\begin{bmatrix} z_s(k) \\ z_f(k) \end{bmatrix} = \begin{bmatrix} G_{ss} & G_{sf} \\ G_{fs} & G_{ff} \end{bmatrix} \begin{bmatrix} x_s(k) \\ x_f(k) \end{bmatrix} \tag{11}$$

Next, we assume the following:

(H3) The matrix G_{ff} is invertible.

Using the assumption (H3), we derive the following equation from (11):

$$x_f(k) = -G_{ff}^{-1} G_{fs} \; x_s(k) + G_{ff}^{-1} \; z_f(k) \tag{12}$$

Substituting (8) into (12), we get

$$x_f(k) = P\ x_s(k) + Q\ u(k), \tag{13}$$

where

$$P = -G_{ff}^{-1} G_{fs} \quad \text{and} \quad Q = -G_{ff}^{-1} \left(I - \Lambda_f \right)^{-1} \Gamma_f \tag{14}$$

Substituting (13) into (2), we get the reduced order model of the original linear system as

$$x_s(k+1) = A_s^*\ x_s(k) + B_s^*\ u(k) \tag{15}$$

where

$$A_s^* = A_{ss} + A_{sf} P \quad \text{and} \quad B_s^* = B_s + A_{ff} Q \tag{16}$$

Thus, under the assumptions (H1)-(H3), the original linear system (1) can be expressed in a simplified form as

$$\begin{bmatrix} x_s(k+1) \\ x_f(k+1) \end{bmatrix} = \begin{bmatrix} A_s^* & 0 \\ A_f^* & 0 \end{bmatrix} \begin{bmatrix} x_s(k) \\ x_f(k) \end{bmatrix} + \begin{bmatrix} B_s^* \\ B_f^* \end{bmatrix} u(k) \tag{17}$$

3. Reduced Order Controller Design

In this section, we consider the design of reduced order controller for the linear system (17) using only the dominant state x_s of the system. Thus, for the linear system (17), we investigate the problem of finding a state feedback controller of the form

$$u(k) = -\begin{bmatrix} K_s^* & 0 \end{bmatrix} \begin{bmatrix} x_s(k) \\ x_f(k) \end{bmatrix} = -K_s^*\ x_s(k) \tag{18}$$

so that the resulting closed-loop system governed by the equations

$$\begin{aligned} x_s(k+1) &= \left(A_s^* - B_s^* K_s^* \right) x_s(k) \\ x_f(k+1) &= \left(A_f^* - B_f^* K_f^* \right) x_s(k) \end{aligned} \tag{19}$$

is exponentially stable. [Note that the stabilizing feedback control law (18), if it exists, will also stabilize the reduced-order linear system (15).]

From the first equation of (19), it follows that

$$x_s(k) = \left(A_s^* - B_s^* K_s^* \right)^k x_s(0) \tag{20}$$

which shows that $x_s(k) \to 0$ as $k \to \infty$ if and only if the system pair $\left(A_s^*, B_s^* \right)$ is stabilizable.

Especially, if the system pair $\left(A_s^*, B_s^*\right)$ is controllable, then the eigenvalues of the closed-loop system matrix $A_s^* - B_s^* K_s^*$ can be arbitrarily placed in the complex plane. In particular, we can always find a gain matrix F_s^* such that the closed-loop system matrix $A_s^* - B_s^* K_s^*$ is convergent. Hence, we obtain the following result

Theorem 1. Under the assumptions (H1)-(H3), the system (15) is a reduced-order model for the original linear system (1). Also, the original linear system (1) can be expressed by the equations (20). Next, the feedback control law (18) that uses only the dominant state of the linear system (1) stabilizes the system (17) if and only if it stabilizes the reduced-order linear system (15). Thus, the reduced order feedback controller problem for the given linear system (1) is solvable if and only if the system pair $\left(A_s^*, B_s^*\right)$ is stabilizable, i.e. there exists a feedback gain matrix K_s^* such that the closed-loop system matrix $A_s^* - B_s^* K_s^*$ is convergent. If the system pair $\left(A_s^*, B_s^*\right)$ is controllable, then we can always find eigenvalues inside the unit circle of the complex plane and hence we can construct the feedback control law (18) that stabilizes the full linear system (17). ■

4. NUMERICAL EXAMPLE

We consider the fourth order linear discrete-time control system described by

$$x(k+1) = A\, x(k) + B\, u(k) \tag{21}$$

where

$$A = \begin{bmatrix} 2.6966 & 0.8948 & 0.1310 & 0.2093 \\ 0.3557 & 1.3681 & 0.9408 & 0.4551 \\ 0.0490 & 0.2512 & 0.5124 & 0.0811 \\ 0.7553 & 0.9327 & 0.8477 & 0.8511 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0.9126 \\ 0.4523 \\ 0.6721 \\ 0.3895 \end{bmatrix}. \tag{22}$$

The eigenvalues of the matrix A are

$$\lambda_1 = 3.1402,\ \lambda_2 = 1.5550,\ \lambda_3 = 0.4280,\ \lambda_4 = 0.3050$$

Thus, we note that λ_1, λ_2 are unstable (slow) eigenvalues and λ_3, λ_4 are stable (fast) eigenvalues of the system matrix A.

The dominance measure of the eigenvalues is calculated as in [9] and obtained as

$$\Omega = \begin{bmatrix} -0.5232 & 0.6021 & 0.0018 & 0.1814 \\ 0.1922 & -0.6495 & -0.0243 & -0.4396 \\ -0.0363 & -0.1598 & -0.0247 & 0.7480 \\ -0.2644 & -0.4070 & -0.0999 & -0.6610 \end{bmatrix}$$

To determine the dominance of the k th eigenvalues in all the states, we use the measure

$$\Theta_k = \sum_{i=1}^{4} \Omega_{ik}$$

Thus, we obtain

$$\Theta = \begin{bmatrix} -1.0161 & -0.6143 & 0.0527 & -0.1713 \end{bmatrix}$$

Thus, it is clear that the first two states (x_1, x_2) are the dominant (slow) states, while the last two states (x_3, x_4) are the non-dominant (fast) states of the linear system (21).

Using the procedure described in Section 2, the reduced-order linear model for the given linear system (21) is obtained as

$$x_s(k+1) = A_s^* x_s(k) + B_s^* u(k) \tag{23}$$

where

$$A_s^* = \begin{bmatrix} 2.7375 & 1.0961 \\ 0.4343 & 1.9577 \end{bmatrix} \quad \text{and} \quad B_s^* = \begin{bmatrix} 0.2628 \\ -0.1301 \end{bmatrix}.$$

Clearly, the system pair (A_s^*, B_s^*) is completely controllable. Thus, the eigenvalues of the closed-loop system matrix $A_s^* - B_s^* K_s^*$ can be placed arbitrarily inside the unit circle.

In particular, we choose the control law

$$u = -K_s^* x_s \tag{24}$$

such that the closed-loop system matrix $A_s^* - B_s^* K_s^*$ has the eigenvalues $\lambda_1^* = 0.1, \lambda_2^* = 0.1$. A simple calculation in MATLAB (using Ackermann's formula) gives

$$K_s^* = \begin{bmatrix} 38.8383 & 43.9036 \end{bmatrix}$$

Upon substitution of the control law (24) into the reduced-order linear system (23), we obtain the closed-loop linear system

$$x_s(k+1) = \left(A_s^* - B_s^* K_s^* \right) x_s(k) \tag{25}$$

which has the stable eigenvalues $\lambda_1^* = 0.1$ and $\lambda_2^* = 0.1$.

Thus, the closed-loop system (25) is globally exponentially stable.

The general solution of the system (25) is given by the equation
$x_s(k) = \left(A_s^* - B_s^* K_s^*\right)^k x_s(0).$

The response $x_s(k)$ of the closed-loop system (25) for the initial state

$$x_s(0) = \begin{bmatrix} 10 \\ 10 \end{bmatrix}$$

is depicted in Figure 1.

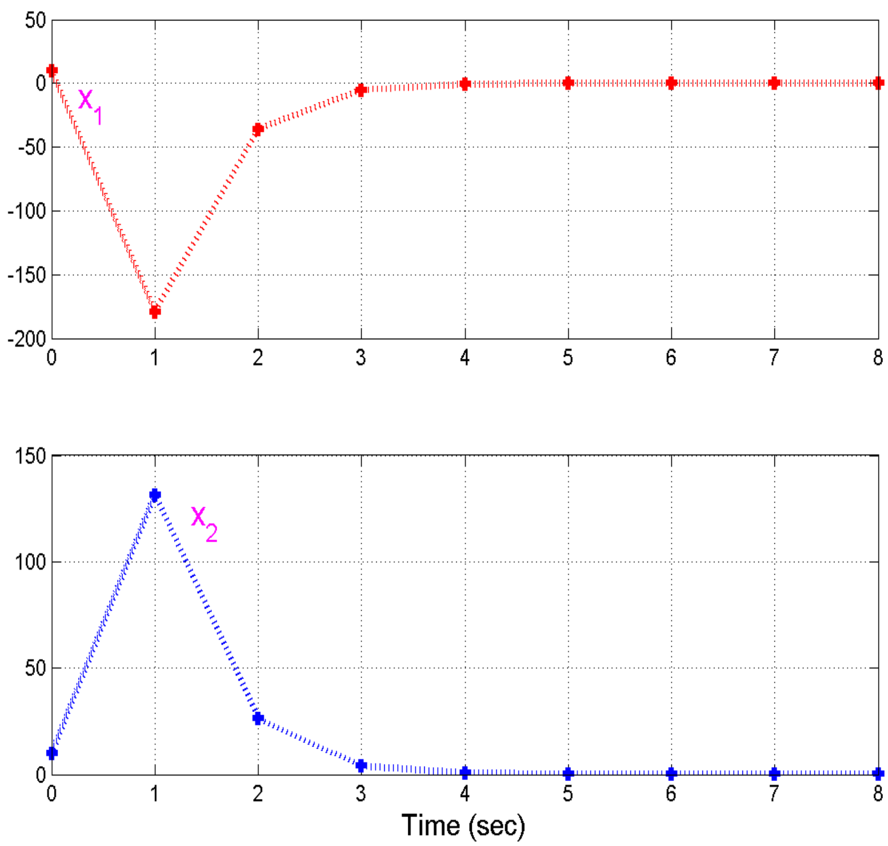

Figure 1. Time Responses of the Closed-Loop System (25)

4. CONCLUSIONS

In this paper, sufficient conditions are derived for the design of reduced order controllers by obtaining a reduced order model of the original plant using the dominant state of the system. The reduced order controllers are assumed to use only the state of the reduced order model of the original plant. An example has been presented to illustrate the effectiveness of the proposed design of reduced order controllers for a four-dimensional linear discrete-time control system.

REFERENCES

[1] Cumming, S.D. (1969) "Design of observers for reduced dynamics", *Electronics Letters*, Vol. 5, pp 213-214.

[2] Fortman, T.E. & Williamson, D. (1971) "Design of low-order observers for linear feedback control laws", *IEEE Trans. Automat. Control*, Vol. 17, pp 301-308.

[3] Litz, L. & Roth, H. (1981) "State decomposition for singular perturbation order reduction – a modal approach", *Internat. J. Control,* Vol. 34, pp. 937-954.

[4] Lastman, G. & Sinha, N. & Rozsa, P. (1984) "On the selection of states to be retained in reduced-order model", *IEE Proc. Part D,* Vol. 131, pp 15-22.

[5] Anderson, B.D.O. & Liu, Y. (1989) "Controller reduction: concepts and approaches", *IEEE Trans. Automat. Control*, Vol. 34, pp 802-812.

[6] Mustafa, D. & Glover, K. (1991) "Controller reduction by H_∞ balanced truncation", *IEEE Trans. Automat. Control*, Vol. 36, pp 668-692.

[7] Aldeen, M. (1991) "Interaction modelling approach to distributed control with application to interconnected dynamical systems", *Internat. J. Control*, Vol. 241, pp 303-310.

[8] Aldeen, M. & Trinh, H. (1994) "Observing a subset of the states of linear systems", *IEE Proc. Control Theory Appl.*, Vol. 141, pp 137-144.

[9] Sundarapandian, V. (2005) "Distributed control schemes for large scale interconnected discrete-time linear systems", *Math. Computer Modelling*, Vol. 5, pp 313-319.

[10] Sundarapandian, V. (2005) "Observer design and stabilization of the dominant state of discrete-time linear systems", *Math. Computer Modelling*, Vol. 41, pp 581-586.

[11] Yuste, A.J., Trivino, A., Trujillo, F.D. & Casilari, E. (2010) "Using fuzzy logic in hybrid multihop wireless networks", *Internat. J. Wireless and Mobile Networks*, Vol. 2, pp. 96-108.

[12] Cannady, J. (2010) "Dynamic neural networks in the detection of distributed attacks in mobile ad-hoc networks", *Internat. Journal of Network Security & Its Applications*, Vol. 2, pp. 1-7.