# TIME OF ARRIVAL BASED LOCALIZATION IN WIRELESS SENSOR NETWORKS: A NON-LINEAR APPROACH

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## ABSTRACT

*In this paper, we aim to obtain the location information of a sensor node deployed in a Wireless Sensor Network (WSN). Here, Time of Arrival based localization technique is considered. We calculate the position information of an unknown sensor node using the non- linear techniques. The performances of the techniques are compared with the Cramer Rao Lower bound (CRLB). Non-linear Least Squares and the Maximum Likelihood are the non-linear techniques that have been used to estimate the position of the unknown sensor node. Each of these non-linear techniques are iterative approaches, namely, Newton Raphson estimate, Gauss Newton Estimate and the Steepest Descent estimate for comparison. Based on the results of the simulation, the approaches have been compared. From the simulation study, Localization based on Maximum Likelihood approach is having higher localization accuracy.*

## KEYWORDS

*Node Localization, Time of Arrival, Maximum Likelihood approach, Non-linear Least Squares approach and CRLB.*

## 1. INTRODUCTION

Location awareness has gained great interest in many wireless communication systems such as mobile cellular networks, wireless local area networks and wireless sensor networks because of its wide range of applications and add-ons [1]. Location information based services such as position based social networking, location based advertisement and E-911 emergency services have become more important in order to enhance the future life style [2]. Wireless sensor networks are a group of large number of small, low cost, low energy and multi-task sensors capable of many functions such as sensing, computing and communication between these wireless devices deployed in a very large geographic area [3][4][5][6].

Because of the developments in wireless systems, communication in wireless sensor networks has become a potential area of research over the last decade [7][8][9]. Many applications in sensor networks need the location information of these tiny sensors, data collected from these sensors are of limited use if the information is without the location information of the sensor. Despite the huge research, still a reliable or well accepted technique to obtain the location information is yet to be realized [10][11][12].

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Since the sensor nodes are cost effective and also when deployed, they are in a large number, it is not practical to have Global Positioning Systems (GPS) receivers equipped sensors to obtain the position information [13]. Moreover GPS receivers are too costly [14]. Many localization techniques have been proposed in literature, but there is not a single approach which is simple and distinct which gives a better solution for sensor networks.

The basic approaches for measuring the location information in wireless sensor networks are Time of Arrival (TOA), Angle of Arrival (AOA), Received Signal Strength (RSS) and Time Difference of Arrival (TDOA)[15][16][17][18]. Of these approaches, the TOA, TDOA and RSS gives the distance measurements while the AOA gives both distance and angle measurements. The approaches seem to be simple but calculating the angle and distance information is not simple because of the Non-linear relationships with the source node.

The main goal of this paper is calculating the location information based on the Time of Arrival technique. Further the performance of the various non-linear techniques such as Non linear least squares and Maximum likelihood techniques are compared with the Cramer Rao Lower bound. Both the nonlinear techniques employ iterative Newton Raphson, Gauss Newton and Steepest Descent methods. We assume a two dimensional rectangular area, where the sensors are deployed in Line of Sight (LOS) scenario [19].

The rest of the paper is organized as follows. In section 2, we present the mathematical measurement model of time of arrival based localization technique and its positioning principles. The Non-Linear approaches are discussed in section 3 and Simulation results are presented in Section 4. Finally Conclusions are drawn in section 5.

## 2. MATHEMATICAL MEASUREMENT MODEL.

The measurement model for Time of arrival based node localization approach is given by:

$$\mathbf{r} = \mathbf{f}(\mathbf{x}) + \mathbf{n} \tag{1}$$

Where '**r**' is the measurement vector, '**x**' is the position of the source to be estimated, '**n**' is the additive zero mean Gaussian vector and **f(x)** is a nonlinear function.

### 2.1. Time of Arrival

Localization is the mechanism of obtaining the exact location of an unknown sensor node from known nodes (anchors or beacons) by means of the intersection of three or more measurements from the known nodes. For the TOA based technique, the distance information is extracted from the propagation delay between the transmitter and the receiver. This technique can be further classified into two approaches namely, One- way ranging TOA and Two way ranging TOA[20].

The former approach requires perfect synchronization between the transmitter and the receiver, while the latter does not require synchronization between the transmitter and the receiver. We consider synchronous networks and the broadcast mode for anchor transmission in this paper.

The measured TOA represents a circle with its centre at the receiver and the source must lie on the circumference in a 2 dimensional space. A minimum of three or more circles are needed for two dimensional position estimate [21]. If the number of sensors is less than three, there is a possibility that there may not be any intersecting points and hence not a feasible solution. With the knowledge of the sensor array geometry the source position can be estimated based on the optimization criterion and these can be represented as a set of circular equations.

Let $\mathbf{x}=\begin{bmatrix} x & y \end{bmatrix}^T$ be the unknown sensor source position and $\mathbf{x}_k=\begin{bmatrix} x_k & y_k \end{bmatrix}^T$ be the known coordinates of the k$^{th}$ anchor, where k=1, 2,….., K. the number of receivers K must be greater than or equal to three. The distance between the source and the k$^{th}$ senor, denoted by $d_k$, is simply:

$$d_k = \left\|\mathbf{x}-\mathbf{x}_k\right\|_2 = \sqrt{\left(x-x_k\right)^2+\left(y-y_k\right)^2}, \qquad k=1,2,.....,K \tag{2}$$

The source transmits a signal at time 0 and the k$^{th}$ sensor receives it at time $t_{k.}$, i.e., there is a simple relationship between the measured TOAs and the distance, and is represented as

$$t_k = \frac{d_k}{c}, \qquad k=1,2,...K \tag{3}$$

Where '$c$' is the velocity of light. The measured TOAs have small errors; hence the range based measurement model is modelled as:

$$\begin{aligned} r_{TOA,k} &= d_k + n_{TOA,k} \\ &= \sqrt{\left(x-x_k\right)^2+\left(y-y_k\right)^2} + n_{TOA,k}, \qquad k=1,2,.....,K \end{aligned} \tag{4}$$

Where $n_{TOA,k}$ is the range error of the measured TOAs. Complete derivation of the TOA function is presented in our earlier communication [22].

The source position measurement problem based on the obtained TOA values is to calculate $\mathbf{x}$ given $r_{TOA,k}$. It is assumed that the range errors $\left\{n_{TOA,k}\right\}$ are uncorrelated Gaussian processes with variances $\left\{\sigma^2_{TOA,k}\right\}$ and zero mean. It is known that zero mean property indicates Line of sight transmission. The probability density function for each of the random variable $r_{TOA,k}$ denoted by $P\left(r_{TOA,k}\right)$ has the form

$$P\left(r_{TOA,k}\right) = \frac{1}{\sqrt{2\pi\ \sigma^2_{TOA,k}}} \exp\left(-\frac{1}{2\sigma^2_{TOA,k}}\left(r_{TOA,k}-d_k\right)^2\right) \tag{5}$$

In other words, we can write, $r_{TOA,k} \approx ¥\left(d_k, \sigma^2_{TOA,k}\right)$ which are the mean and variances. And the PDF is given by

$$P\left(r_{TOA}\right) = \frac{1}{\left(2\pi\right)^{K/2}\left|\mathbf{C}_{TOA}\right|^{1/2}} \exp\left(-\frac{1}{2}\left(\mathbf{r}_{TOA}-d\right)^T \mathbf{C}^{-1}_{TOA}\left(\mathbf{r}_{TOA}-d\right)\right) \tag{6}$$

Where $\mathbf{C}_{TOA} = diag\left(\sigma^2_{TOA,1}, \sigma^2_{TOA,2}, ......, \sigma^2_{TOA,K}\right)$.

### 2.2. Cramer Rao Lower Bound

The CRLB, in general can be defined as the theoretical lower bound on the variance of any unbiased estimator of an unknown parameter [23][24]. The factors affecting the TOA based position estimation are position of the anchors, unknown source node and the measurement noise variances [25]. The CRLB is calculated using the Fisher Information Matrix (FIM) whose elements are defined as

$$\mathbf{I}(\mathbf{x}) = E\left[\frac{\partial^2 \ln p(\mathbf{r})}{\partial x \partial x^T}\right] \tag{7}$$

Using the PDF in equation (6), the FIM can be calculated as

$$\mathbf{I}_{TOA}(\mathbf{X}) = \begin{bmatrix} \sum_{k=1}^{K} \frac{(x-x_k)}{\sigma_{TOA,k}^2 d_k^2} & \sum_{k=1}^{K} \frac{(x-x_k)(y-y_k)}{\sigma_{TOA,k}^2 d_k^2} \\ \sum_{k=1}^{K} \frac{(x-x_k)(y-y_k)}{\sigma_{TOA,k}^2 d_k^2} & \sum_{k=1}^{K} \frac{(y-y_k)}{\sigma_{TOA,k}^2 d_k^2} \end{bmatrix} \tag{8}$$

Where the lower bound for **x** and **y** are denoted by $\left[\mathbf{I}^{-1}(\mathbf{x})\right]_{1,1}$ and $\left[\mathbf{I}^{-1}(\mathbf{x})\right]_{2,2}$ respectively, and the $\mathrm{CRLB}_{TOA}(\mathbf{x})$, is

$$CRLB_{TOA}(\mathbf{x}) = \left[\mathbf{I}^{-1}(\mathbf{x})\right]_{1,1} + \left[\mathbf{I}^{-1}(\mathbf{x})\right]_{2,2} \tag{9}$$

## 3. NON LINEAR APPROACHES FOR SOURCE LOCALIZATION

The Non-Linear approach directly uses equation (1) to solve for **x** by minimising the least squares cost function obtained from the error function:

$$\mathbf{e}_{nonlinear} = \mathbf{r} - \mathbf{f}(\tilde{\mathbf{x}}) \tag{10}$$

Where $\tilde{\mathbf{x}} = [\tilde{x} \quad \tilde{y}]^T$ is the optimization variable for **x,** which represents the nonlinear least squares or the Maximum Likelihood estimators, respectively and the global convergence of these schemes is not assured since their optimization cost functions are multi-modal [26].

In this paper two Non-Linear positioning approaches, namely, Nonlinear Least Squares (NLS) and Maximum Likelihood are presented. Further these two Non-Linear approaches use three different iterative local search algorithms, namely, Newton Raphson algorithm, Gauss Newton algorithm and Steepest Descent Algorithm.

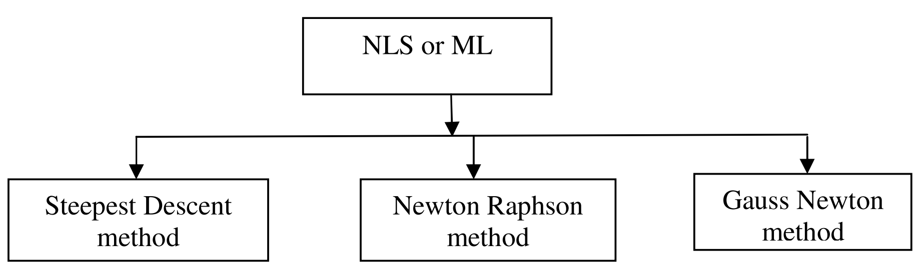



Figure 1: Various Iterative approaches in NLS and ML algorithms.

### 3.1. Nonlinear Least Squares (NLS) approach for TOA based positioning

The nonlinear least squares method is simple and is an obvious choice when the noise information is not available. The NLS approach directly tries to minimize the least square cost function obtained from equation (4) and is as follows:

Based on equation (4), the NLS cost function is represented as:

$$\begin{aligned} J_{NLS,TOA}(\tilde{\mathrm{x}}) &= \sum_{k=1}^{K}\left(r_{TOA,k}-\sqrt{(\tilde{x}-x_k)^2+(\tilde{y}-y_k)^2}\right)^2 \\ &= \left(\mathbf{r}_{TOA}-\mathbf{f}_{TOA}(\tilde{\mathrm{x}})\right)^T\left(\mathbf{r}_{TOA}-\mathbf{f}_{TOA}(\tilde{\mathrm{x}})\right) \end{aligned} \tag{11}$$

The smallest value in the NLS cost function $J_{NLS,TOA}(\tilde{\mathrm{x}})$ is equal to the NLS position estimate and is given by:

$$\hat{\mathrm{x}} = \arg\min_{\tilde{\mathrm{x}}} J_{NLS,TOA}(\tilde{\mathrm{x}}) \tag{12}$$

And obtaining $\hat{\mathrm{x}}$ is not easy, since both the local and global minimum values are contained in 2 dimensional surface of $J_{NLS,TOA}(\tilde{\mathrm{x}})$. .

In order to obtain the value of $\hat{\mathrm{x}}$ there are two ways, the first method is to use iterative local search algorithms based on an initial position estimate $\hat{\mathrm{x}}^0$, where 0 refers to the iteration number. If the value obtained after $0^{th}$ iteration is approximately close to x, then $\hat{\mathrm{x}}$ can be iterated to the closest value of x. and second method is using the random search or the grid search techniques namely, the Genetic Algorithm (GA)[27] and Particle Swarm Optimization (PSO)[28][29]. In this paper, we limit our studies to the iterative search techniques.

The three iterative approaches, namely, Newton Raphson iterative procedure, Gauss Newton iterative procedure, and Steepest Descent iterative procedure, are presented in the following subsections.

#### 3.1.1. Newton Raphson Iterative procedure

The iterative Newton – Raphson procedure for $\hat{\mathrm{x}}$ is given by:

$$\hat{\mathbf{x}}^{m+1} = \hat{\mathbf{x}}^m - \mathbf{H}^{-1}\left(J_{NLS,TOA}\left(\hat{\mathbf{x}}^m\right)\right)\nabla\left(J_{NLS,TOA}\left(\hat{\mathbf{x}}^m\right)\right) \tag{13}$$

Where $\mathbf{H}\left(J_{NLS,TOA}\left(\hat{\mathbf{x}}^m\right)\right)$ is the Hessian Matrix and $\nabla\left(J_{NLS,TOA}\left(\hat{\mathbf{x}}^m\right)\right)$ is the gradient vector measured at the $m^{th}$ iteration estimate.

The algorithm for the Newton raphson procedure is summarized in Table 1.

#### 3.1.2. Gauss Newton Iterative procedure

The iterative Gauss - Newton procedure for $\hat{\mathbf{x}}$ is given by:

$$\hat{\mathbf{x}}^{m+1} = \hat{\mathbf{x}}^m + \left(\mathbf{G}^T\left(\mathbf{f}_{TOA}\left(\hat{\mathbf{x}}^m\right)\right)\right)^{-1}\mathbf{G}^T\left(\mathbf{f}_{TOA}\left(\hat{\mathbf{x}}^m\right)\right)\left(\mathbf{r}_{TOA}-\mathbf{f}_{TOA}\left(\hat{\mathbf{x}}^m\right)\right) \tag{14}$$

Where $\mathbf{G}\left(\mathbf{f}_{TOA}\left(\hat{\mathbf{x}}^m\right)\right)$ is the jacobian matrix of $\mathbf{f}_{TOA}\left(\hat{\mathbf{x}}^m\right)$ calculated at $\hat{\mathbf{x}}^m$.

The algorithm for the Gauss- Newton iterative procedure is summarized in Table 2:

Table 1: Newton Raphson Iterative Procedure algorithm

| **Algorithm 1:** Newton Raphson Method |
| --- |
| Input: K = number of anchor nodes<br>M = number of iterations<br>X = {set containing all anchor nodes};<br>Initialize μ<br>Output: estimated coordinates<br>X (M) = $[x_1, y_1]^T$<br>Initialization: a random point x (0)<br>For i=1: M do<br>For j=1: K do<br>$H = \begin{bmatrix} \frac{\partial^2 J_{NLS,TOA}(x)}{\partial x^2} & \frac{\partial^2 J_{NLS,TOA}(x)}{\partial x \partial y} \\ \frac{\partial^2 J_{NLS,TOA}(x)}{\partial x \partial y} & \frac{\partial^2 J_{NLS,TOA}(x)}{\partial y^2} \end{bmatrix}$<br>End<br>$G = \begin{bmatrix} \frac{\partial(J_{NLS,TOA}(x))}{\partial x} \\ \frac{\partial(J_{NLS,TOA}(x))}{\partial y} \end{bmatrix}$<br>$\mathbf{X} = \mathbf{x} - \mathbf{H}^{-1}\mathrm{G}$<br>End |

Table 2: Gauss Newton Iterative Procedure algorithm

| **Algorithm 2:** Gauss Newton method |
| --- |
| Input: K = number of anchor nodes<br>M = number of iterations<br>X = {set containing all anchor nodes};<br>Output: estimated coordinates<br>X (M) = $[x_1, y_1]^T$<br>Initialization: a random point x (0)<br>For i=1: M do<br>For j=1: K do<br>$G = \begin{bmatrix} \frac{\partial\sqrt{(x-x_1)^2+(y-y_1)^2}}{\partial x} & \frac{\partial\sqrt{(x-x_1)^2+(y-y_1)^2}}{\partial y} \\ \frac{\partial\sqrt{(x-x_2)^2+(y-y_2)^2}}{\partial x} & \frac{\partial\sqrt{(x-x_2)^2+(y-y_2)^2}}{\partial y} \\ \vdots & \vdots \\ \frac{\partial\sqrt{(x-x_L)^2+(y-y_L)^2}}{\partial x} & \frac{\partial\sqrt{(x-x_L)^2+(y-y_L)^2}}{\partial y} \end{bmatrix}$<br>End<br>$\mathbf{X} = \mathbf{x} + \left(\mathrm{G}^T\mathrm{G}\right)^{-1}\mathrm{G}^T\left(r - f(x)\right);$<br>End |

### 3.1.3. Steepest Descent Iterative procedure

The iterative procedure for the Steepest Descent iterative procedure is given by:

$$\hat{\mathbf{x}}^{m+1} = \hat{\mathbf{x}}^m - \mu\nabla\left(J_{NLS,TOA}\left(\hat{\mathbf{x}}^m\right)\right) \quad (15)$$

Where $\mu$ is a positive constant which commands the convergence rate and stability.

Table 3, summarizes the algorithm for Steepest Descent iterative procedure.

Table 3: Steepest Descent Iterative Procedure algorithm

| **Algorithm 3:** Steepest Descent method |
|---|
| Input: K = number of anchor nodes<br>M = number of iterations<br>X = {set containing all anchor nodes};<br>Initialize μ<br>Output: estimated coordinates<br>$X(m) = [x_1, y_1]^T$<br>Initialization: a random point x (0)<br>For i=1: M do<br>For j=1: K do<br>$G = \begin{bmatrix} \frac{\partial(J_{NLS,TOA}(x))}{\partial x} \\ \frac{\partial(J_{NLS,TOA}(x))}{\partial y} \end{bmatrix}$<br>End<br>$\mathbf{X} = \mathbf{x} - \mu G$<br>End |

## 3.2. Maximum Likelihood approach for TOA based positioning

The ML method maximises the probability density functions of the measured TOAs under the assumption that error distribution is known [30][31][32][33][34]. The maximization of the measured TOAs using ML method corresponds to the weighted version of the Non-linear least squares approach [35].

We consider the logarithmic version of the equation (6):

$$\ln\left(P\left(\mathbf{r}_{TOA}\right)\right) = \ln\left(\frac{1}{(2\pi)^{L/2}\left|\mathbf{C}_{TOA}\right|^{1/2}}\right) - \ln\left(\frac{1}{2}\left(\mathbf{r}_{TOA} - d\right)^T \mathbf{C}_{TOA}^{-1}\left(\mathbf{r}_{TOA} - d\right)\right) \qquad (16)$$

The first term in the RHS is independent of **x,** and maximising equation (16) is equal to minimising the second term, and hence the ML estimate can be obtained as:

$$\begin{aligned}\hat{\mathbf{x}} &= \arg\min_{\tilde{\mathbf{x}}} \ln\left(\left(\mathbf{r}_{TOA} - \mathbf{f}_{TOA}(\tilde{\mathbf{x}})\right)^T \mathbf{C}_{TOA}^{-1}\left(\mathbf{r}_{TOA} - \mathbf{f}_{TOA}(\tilde{\mathbf{x}})\right)\right) \\ &= \arg\min_{\tilde{\mathbf{x}}} \left(\mathbf{r}_{TOA} - \mathbf{f}_{TOA}(\tilde{\mathbf{x}})\right)^T \mathbf{C}_{TOA}^{-1}\left(\mathbf{r}_{TOA} - \mathbf{f}_{TOA}(\tilde{\mathbf{x}})\right)\end{aligned} \qquad (17)$$

Or we can write

$$\hat{\mathbf{x}} = \arg\min_{\tilde{\mathbf{x}}} J_{ML,TOA}(\tilde{\mathbf{x}}) \qquad (18)$$

Where $J_{ML.TOA}(\tilde{\mathbf{x}})$ is the ML cost function, which is of the form:

$$J_{ML.TOA}(\tilde{\mathrm{x}}) = \left(\mathbf{r}_{TOA} - \mathbf{f}_{TOA}(\tilde{\mathrm{x}})\right)^T \mathbf{C}_{TOA}^{-1}\left(\mathbf{r}_{TOA} - \mathbf{f}_{TOA}(\tilde{\mathrm{x}})\right)$$
$$= \sum_{k=1}^{K} \frac{\left(r_{TOA,k} - \sqrt{(\tilde{x} - x_k)^2 + (\tilde{y} - y)^2}\right)^2}{\sigma_{TOA,k}^2} \tag{19}$$

The ML estimator is generalized to NLS method because under the assumption of zero mean Gaussian noise, $\sigma_{TOA,k}^2$ is large which corresponds to a large noise in $r_{TOA,k}$, a small weight of $1/\sigma_{TOA,k}^2$ is employed in the squared term $\left(r_{TOA,k} - \sqrt{(\tilde{x} - x_k)^2 + (\tilde{y} - y)^2}\right)^2$, and vice versa. And also when $\mathbf{C}_{TOA}^{-1}$ is very near to the identity matrix or $\sigma_{TOA,k}^2 \quad k = 1, 2, ....., K$ are identical [35].

#### 3.2.1. Newton Raphson Iterative procedure

The iterative Newton – Raphson procedure for $\hat{\mathrm{x}}$ is given by:

$$\hat{\mathrm{x}}^{k+1} = \hat{\mathrm{x}}^k - \mathbf{H}^{-1}\left(J_{ML,TOA}\left(\hat{\mathrm{x}}^k\right)\right)\nabla\left(J_{ML,TOA}\left(\hat{\mathrm{x}}^k\right)\right) \tag{20}$$

Where $\mathbf{H}\left(J_{NLS,TOA}\left(\hat{\mathbf{x}}^m\right)\right)$ is the Hessian Matrix and $\nabla\left(J_{NLS,TOA}\left(\hat{\mathbf{x}}^m\right)\right)$ is the gradient vector measured at the $m^{\text{th}}$ iteration estimate.

#### 3.2.2. Gauss Newton Iterative procedure

The iterative Gauss - Newton procedure for $\hat{\mathbf{x}}$ is given by:

$$\hat{\mathrm{x}}^{k+1} = \hat{\mathrm{x}}^k + \left(\mathbf{G}^T\left(\mathbf{f}_{TOA}\left(\hat{\mathrm{x}}^k\right)\right)\mathbf{C}_{TOA}^{-1}\mathbf{G}\left(\mathbf{f}_{TOA}\left(\hat{\mathrm{x}}^k\right)\right)\right)^{-1}\mathbf{G}^T\left(\mathbf{f}_{TOA}\left(\hat{\mathrm{x}}^k\right)\right)\mathbf{C}_{TOA}^{-1}\left(\mathbf{r}_{TOA} - \mathbf{f}_{TOA}\left(\hat{\mathrm{x}}^k\right)\right) \tag{21}$$

Where $\mathbf{G}\left(\mathbf{f}_{TOA}\left(\hat{\mathbf{x}}^m\right)\right)$ is the jacobian matrix of $\mathbf{f}_{TOA}\left(\hat{\mathbf{x}}^m\right)$ calculated at $\hat{\mathbf{x}}^m$.

#### 3.2.3. Steepest Descent Iterative procedure

The iterative procedure for the Steepest Descent iterative procedure is given by:

$$\hat{\mathrm{x}}^{k+1} = \hat{\mathrm{x}}^k - \mu\nabla\left(J_{ML,TOA}\left(\hat{\mathrm{x}}^k\right)\right) \tag{22}$$

Where $\mu$ is a positive constant which commands the convergence rate and stability.

## 4. SIMULATION AND RESULTS

Computer simulations have been carried out to evaluate the performance of the localization algorithms. In this paper, we consider a 2D geometry of K=4 receivers with known coordinates at (0, 0), (0, 10), (10, 10) and (10, 0), while the unknown source position is assumed to be (x, y) = (2, 3), such that the source is located inside the square bounded by four receivers. The Signal- to - Noise - Ratio (SNR) = 30dB has been assumed. The results of the simulations are averaged over 50 iterations. And the step size parameter μ = 0.01 is assumed for the Steepest Descent method. All the simulations have been conducted using MATLAB [TM] Version 7.10.0.499 (R2010A)on Microsoft Windows XP, Professional Version 2002, Service pack 3, 32 bit operating system installed on Intel[R], Core [TM] 2 Duo CPU, E4500 @ 2.20GHz, 2.19GHz, 2.0GB of RAM.

We have estimated the X and Y position using Nonlinear Least Squares (NLS) approach. There are three main parts in the simulation, i.e., generating the range measurements, position estimation using NLS estimator which is realized by the Newton Raphson, Gauss Newton and Steepest Descent methods and displaying the results. Figure 2(a), 2(c), and 2(e) show the X- estimate using Newton Raphson, Gauss Newton and Steepest Descent methods whereas Figure. 2(b), 2(d), and 2(f) show the Y- estimate using Newton Raphson, Gauss Newton and Steepest Descent methods respectively. In the 2D geometry, the x-axis represents the number of iterations, i.e., 50 iterations at 30 dB, whereas the y-axis represents the position of the both X and Y estimate. The initial guess for estimating the position is assumed at (3, 2). All the schemes provide the same position estimate upon convergence. It can be seen that the Newton Raphson and Gauss Newton methods converge in about 3 iterations while the steepest descent algorithm needs approximately 15 iterations to converge.

We have estimated the X and Y position using Maximum Likelihood (ML) approach. And the structure of the simulation is same as in the first example where there are three main parts in the simulation i.e., generating the range measurements, position estimation using ML estimator which is realized by the Newton Raphson, Gauss Newton and Steepest Descent methods and displaying the results. Figure 3(a), 3(c), and 3(e) shows the X- estimate using Newton Raphson, Gauss Newton and Steepest Descent methods whereas Figure 3(b), 3(d), and 3(f) shows the Y- estimate in Newton Raphson, Gauss Newton and Steepest Descent methods respectively. In the 2D geometry the x-axis represents the number of iterations i.e., 50 iterations at 30 dB, whereas the y-axis represents the position of the both X and Y estimate. The initial guess for estimating the position is assumed at (3, 2). similar to the NLS approach, all the schemes provide the same position estimate upon convergence, it can be seen the Newton Raphson and Gauss Newton methods converge faster than the steepest Descent algorithm, Nevertheless, it is difficult to see that the Maximum Likelihood (ML) estimator is superior to the NLS approach in terms of positioning accuracy based on a single run.

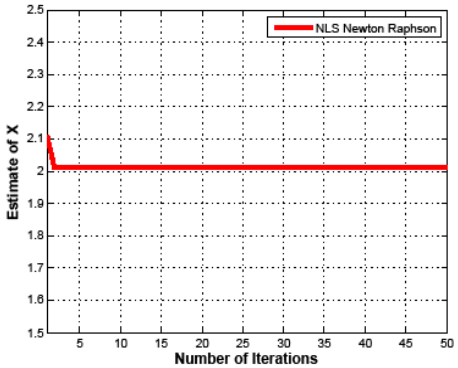
Table 4: Estimated TOA Position measurement for NLS method when the Number of Anchor Nodes, n=4

| Method : **NLS** | **Time of Arrival Estimate** | |
|---|---|---|
| | X- Position Estimate in Meters | Y- Position Estimate in Meters |
| Newton Raphson Approach | 1.97 | 2.96 |
| Gauss Newton Approach | 2.09 | 3.04 |
| Steepest Descent Approach | 2.07 | 2.99 |

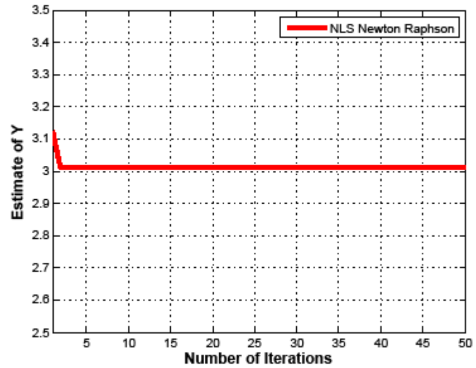
Table 5: Estimated TOA Position measurement for ML method when the Number of Anchor Nodes, n=4

| Method : **ML** | **Time of Arrival Estimate** | |
|---|---|---|
| | X- Position Estimate in Meters | Y- Position Estimate in Meters |
| Newton Raphson Approach | 1.98 | 2.97 |
| Gauss Newton Approach | 2.01 | 3.01 |
| Steepest Descent Approach | 2.05 | 3.07 |

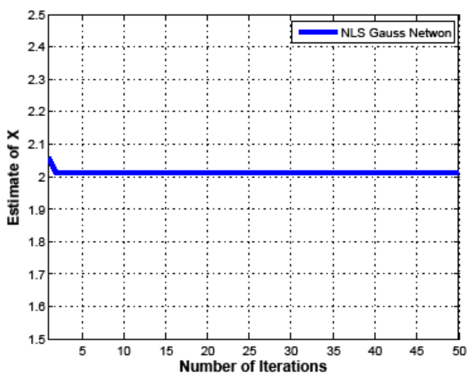
The result of the position measurement estimate of the both NLS and ML method is given in Table 4 and Table 5 respectively. From Table 4, it can be seen that all the approaches estimate the X and Y positions, and the results of the Gauss Newton approach is better when compared with the other approaches. From Table 5, it is clear that the Gauss Newton method is near to the true positions and is having higher localization accuracy and the other approaches have lower localization accuracy. Comparing Table 4 and 5, the iterative Gauss Newton method of the Maximum Likelihood approach is the better approach having higher localization accuracy.



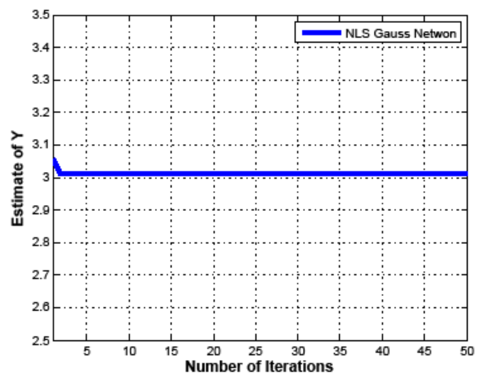

(a). Newton Raphson X estimate



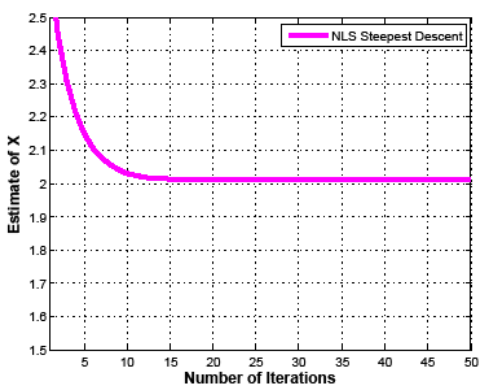

(b). Newton Raphson Y estimate



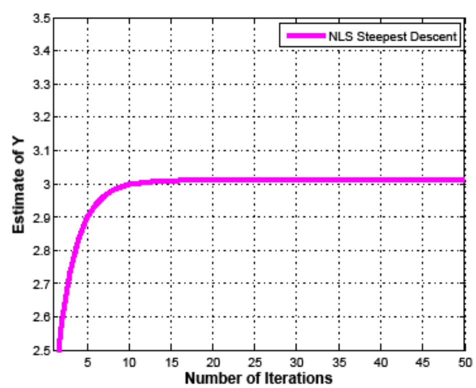

(c). Gauss Newton X estimate




(d). Gauss Newton Y estimate

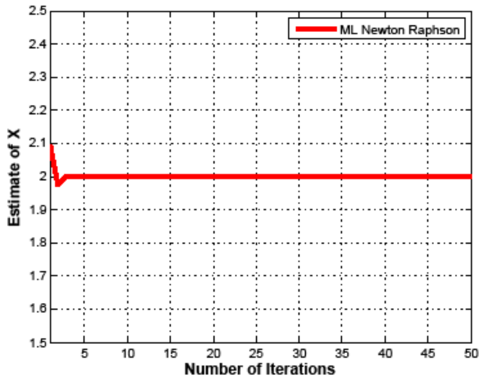



(e). Steepest Descent X estimate

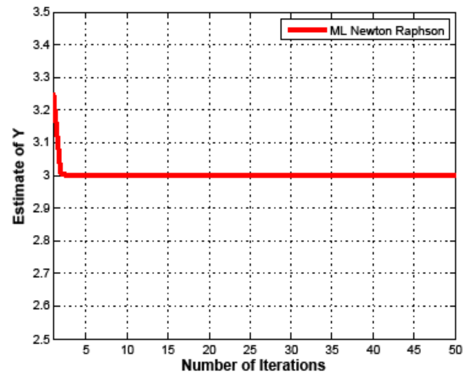



(f). Steepest Descent Y estimate

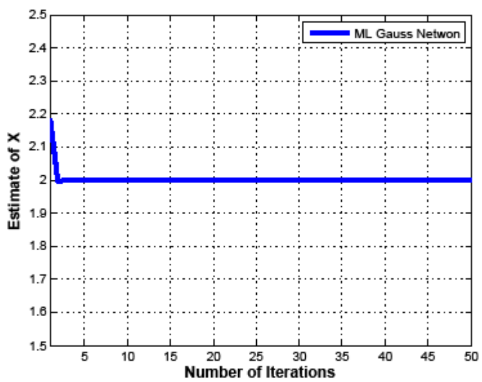
Figure 2: Estimate of X and Y position Versus Number of Iterations in NLS Approach



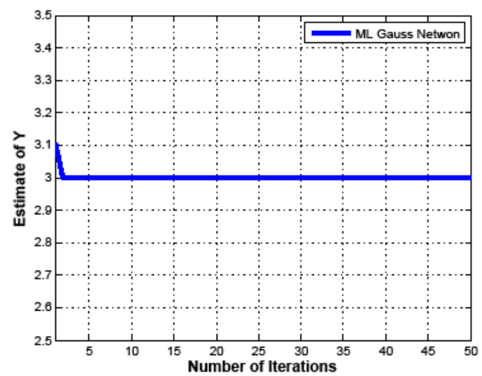

(a). Newton Raphson X estimate



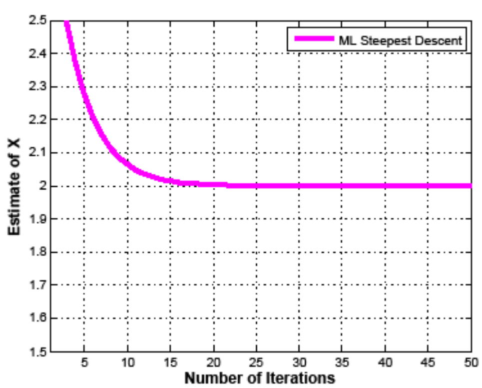

(b). Newton Raphson Y estimate



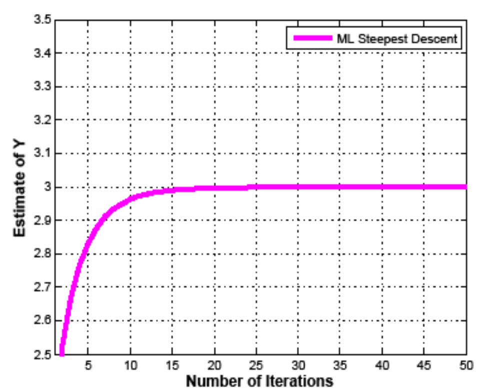

(c). Gauss Newton X estimate




(d). Gauss Newton Y estimate

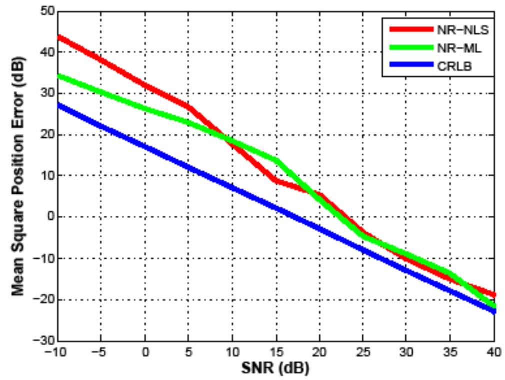



(e). Steepest Descent X estimate

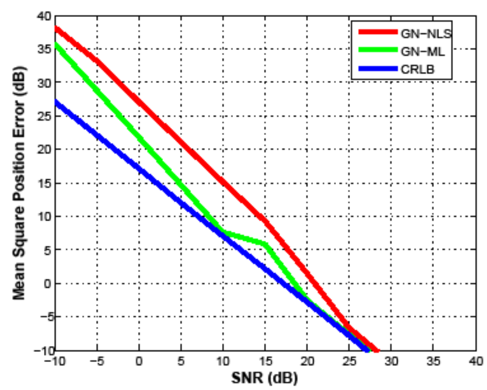



(f). Steepest Descent Y estimate

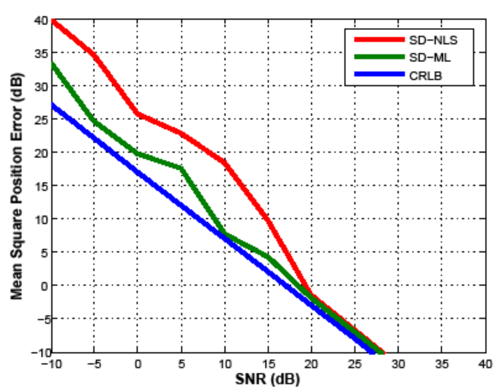
Figure 3: Estimate of X and Y position Versus Number of Iterations in ML Approach




(a). MSPE Newton Raphson approach




(b). MSPE Gauss Newton approach




(c). MSPE Steepest Descent approach

Figure 4: Mean Square Position Error Comparison of Non-Linear Approaches

We estimate the Mean Square Position Error (MSPE) comparison of NLS and ML approaches with Cramer Rao Lower Bound for Time of Arrival based positioning, the range error variance $\sigma^2_{TOA,k}$ is proportional to $d_k^2$ with Signal to Noise Ratio (SNR) = $d_k^2 / \sigma^2_{TOA,k}$. The MSPE is defined as $E\left\{\left(\hat{x}-x\right)^2+\left(\hat{y}-y\right)^2\right\}$. Here we compute the empirical MSPE based on 1000 independent runs, which is given as $\sum_{i=1}^{1000}\left[\left(\hat{x}-x\right)^2+\left(\hat{y}-y\right)^2\right]/1000$ where $\left(\hat{x}_i,\hat{y}_i\right)$, denotes the position estimate of the $i^{\text{th}}$ run. The simulation is run for both the NLS and ML approaches for the iterative Newton Raphson, Gauss Newton and the Steepest Descent Techniques and the function for the CRLB is also included, and the results are depicted in Figure 4(a), 4(b) and 4(c) for the various approaches in both NLS and ML method. Both the axes employ dB scale in the range $SNR \in \left[-10, 40\right]$ dB. From the figures we say that the performance of the Gauss Newton based ML estimator is superior and achieves optimal estimation performance, while the other approaches are suboptimal. Hence the ML estimator is superior to the NLS approach and its MSPE can attain CRLB.

## 5. CONCLUSIONS

The work presented addresses the problem of position estimation of a sensor node, using time of arrival measurements. The Cramer Rao Lower Bound for the position estimation problem has been presented and the two nonlinear approaches such as Nonlinear least squares and Maximum likelihood approaches have been presented. Further both the Nonlinear Least Squares and Maximum likelihood approaches use three iterative local search algorithms, namely, Newton Raphson, Gauss Newton and the steepest descent methods and the results of these techniques are compared. Extensive simulation reveals that the Maximum Likelihood approach is superior to Nonlinear least squares approach under line of sight measurements. And the Maximum likelihood approach based Gauss Newton method is having higher localization accuracy and its MSPE can attain CRLB at SNR = 20dB when compared to the other approaches.

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