DESIGN AND IMPLEMENTATION OF DIGITAL FILTER BANK TO REDUCE NOISE AND RECONSTRUCT THE INPUT SIGNALS

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ABSTRACT
The main theme of this paper is to reduce noise from the noisy composite signal and reconstruct the input signals from the composite signal by designing FIR digital filter bank. In this work, three sinusoidal signals of different frequencies and amplitudes are combined to get composite signal and a low frequency noise signal is added with the composite signal to get noisy composite signal. Finally, noisy composite signal is filtered by using FIR digital filter bank to reduce noise and reconstruct the input signals.

KEYWORDS
Digital Filter Bank, Noise, LMS Filter, LMS Algorithm, Composite Signal

1. INTRODUCTION
Over the past two decades, the research on efficient design of filter banks has received considerable attention in numerous fields such as speech coding, scrambling, image processing etc. [1-2]. At the same time design of filter banks has received attention on noise reduction. So noises in any digital signals significantly hamper the actual performance of signals at the desired output [3-4]. So filter banks are required to reduce these noises. A filter bank is nothing but a group of parallel low pass, band pass and high pass filters [5] that separate the input signal into multiple components, each one carrying a single frequency subband of the original signal [6]. The process of decomposition performed by the filter bank is called analysis. The reconstruction process is called synthesis, meaning reconstitution of a complete signal resulting from the filtering process [7]. Filter banks are generally classified as two types, analysis filter banks and synthesis filter banks [8]. An analysis filter bank comprises of filters, with system transfer functions \{H_k(z)\}, where k= 0, 1, …., M-1; arranged in a parallel bank as shown in Figure 1. On the contrary, a synthesis filter bank comprises of a set of filters with system transfer functions \{G_k(z)\} , where k= 0, 1, …., M-1; arranged as shown in Figure 2, with corresponding inputs \{y_k(m)\}. The outputs of the filters are summed to form the synthesized signal \(\hat{x}(n)\). The blocks with arrows pointing downwards in Figure 1 indicate down sampling by factor N, and the blocks with arrows pointing upwards in Figure 2 indicate up sampling by N.
Sub sampling by N means that only every N-th sample is taken. This operation serves to reduce or eliminate redundancies in the M subband signals. Up sampling by N means the insertion of N-1 consecutive zeros between the samples. This allows us to recover the original sampling rate. There are many applications of filter banks such as graphic equalizer, signal compression, bank of receiver, noise reduce etc.

2. EXAMPLE OF A FILTER BANK

In this section, a two channel filter bank adapted from ref. [9] has been described. Figure 3 shows such kind of two channel filter bank. It is convenient to analyse the filter bank in the Z-domain. Therefore, by using Z-transform the expressions for the various intermediate signals in figure 3 are given by

\[ R_l(z) = H_l(z)U(z) \]  \hspace{1cm} (1)

\[ S_l(z) = \frac{1}{2} \left\{ R_l \left( z^{\frac{1}{2}} \right) + R_l \left( -z^{\frac{1}{2}} \right) \right\} \]  \hspace{1cm} (2)
\[ W_l(z) = S_l(z^2) \quad (3) \]

for \( l=0,1 \). After performing some algebra, the following equation is obtained

\[ W_l(z) = \frac{1}{2} \{ R_l(z) + R_l(-z) \} = \frac{1}{2} \{ H_l(z) U(z) + H_l(-z) U(-z) \} \quad (4) \]

Now the reconstructed output of the filter bank is attained by

\[ V(z) = G_0(z)W_0(z) + G_1(z) W_1(z) \quad (5) \]

Substituting equation (4) in equation (5), the output of the filter bank is obtained by

\[ V(z) = \frac{1}{2} \{ H_0(z)G_0(z) + H_1(z)G_1(z) \} U(z) + \frac{1}{2} \{ H_0(-z)G_0(z) + H_1(-z)G_1(z) \} U(-z) \quad (6) \]

The first term in the above equation describes the transmission of the signal \( U(z) \) through the system, while the second term describes the aliasing component at the output. The above equation can be compactly represented as

\[ V(z) = D(z) U(z) + B(z) U(-z) \quad (7) \]

Where \( D(z) = \frac{1}{2} \{ H_0(z)G_0(z) + H_1(z)G_1(z) \} U(z) \quad (8) \)

is called the distortion transfer function, and

\[ B(z) = \frac{1}{2} \{ H_0(-z)G_0(z) + H_1(-z)G_1(z) \} U(-z) \quad (9) \]

![Figure 3. Two channel filter bank, adapted from Ref. [1]](image)

**3. METHODOLOGY**

At the first step, amplitude, frequency and sample time of three sinusoidal signals, denoted by A1, B1 and C1, are fixed to form composite signal and a random source is considered to get random signal. The random source acts as a noise source. The block parameters of three sinusoidal signals are shown in Figure 4.
Figure 4. Block parameters of three sinusoidal signals A1, B1 and C1

At the second step, an FIR digital filter is designed by using MATLAB to allow a low frequency noise signal obtained from random source to pass through it. The magnitude response of this filter is shown in Figure 5.
At the third step, three FIR digital filters are designed to reconstruct the three sinusoidal signals from the composite signal. These three FIR digital filters are designed by using MATLAB. The magnitude responses of these three filters are shown in Figure 6, Figure 7 and Figure 8. Among three filter designs, the filter design1 is designed to reconstruct the sinusoidal signal A1. The filter design2 is designed to reconstruct the sinusoidal signal B2. The filter design3 is designed to reconstruct the sinusoidal signal C1. All three FIR digital filters form the filter combination block. To design these three FIR digital filters, initially the filter response type is fixed (Low pass response). Then the design method is fixed (FIR window) and the window type is fixed (Hamming window). Finally the cut off frequency, the filter order and sampling frequency are fixed to get the desired magnitude response. From Figure 6, Figure 7 and Figure 8, it is seen that the attenuation at cut off frequencies is fixed at 6dB (half the pass band gain) and the vertical axis represents the magnitude (dB) and the horizontal axis represents the normalized frequency ($\times \pi$ radian/sample).
Figure 7. Magnitude response of digital filter design 2

Figure 8. Magnitude response of digital filter design 3
3.1. LMS FILTER

The LMS Filter block, shown in Figure 9, can implement an adaptive FIR filter using five different algorithms. The block estimates the filter weights, or coefficients, needed to minimize the error between the output signal and the desired signal [10]. Connect the signal which I want to filter to the Input port. This input signal can be a sample-based scalar or a single-channel frame-based signal. Connect the desired signal to the desired port. The desired signal must have the same data type, frame status, complexity, and dimensions as the input signal. The Output port outputs the filtered input signal, which is the estimate of the desired signal. The output of the Output port has the same frame status as the input signal. The Error port outputs the result of subtracting the output signal from the desired signal [11]. An important general form for adaptive filters is the Least Mean Square (LMS) filter. Consider the N-th order FIR filter,

\[ y_n = h(1)x_{n-1} + h(2)x_{n-2} + \ldots + h(n)x_{n-N} \]  

the filter error is,

\[ e(n) = x_n - y_n \]  

The LMS filter adjusts the values of the coefficients, \( h \), proportional to the error, \( e \)

\[ h_i(n) = h_i(n) + 2\mu x_{n-i} e_n \]  

where \( \mu \) is a learning factor which controls how strongly the error is weighted. This equation is the consequence of the requirement to minimize mean square value of \( e \), hence the name of the filter [12].
3.2 LMS ALGORITHM
The least mean squares (LMS) algorithms adjust the filter coefficients to minimize the cost function. So LMS algorithm is important because of its simplicity and ease of computation [13]. The standard LMS algorithm performs the following operations to update the coefficients of an adaptive filter:

- Calculates the output signal \( y(n) \) from the adaptive filter.
- Calculates the error signal \( e(n) \) by using the following equation [14]:
  \[
  e(n) = d(n) - y(n);
  \]
- Updates the filter coefficients by using the following equation [15]:
  \[
  \hat{w}(n+1) = \hat{w}(n) + \mu e(n) \hat{u}(n)
  \]

Where \( \mu \) is the step size of the adaptive filter, \( \hat{w}(n) \) is the filter coefficients vector, and \( \hat{u}(n) \) is the filter input vector. The normalized LMS (NLMS) algorithm is a modified form of the standard LMS algorithm [16]. The NLMS algorithm updates the coefficients of an adaptive filter by using the following equation:

\[
\hat{w}(n+1) = \hat{w}(n) + \mu e(n) \frac{\hat{u}(n)}{\|\hat{u}(n)\|^2}
\]

3.3 FILTER DESIGN AND ANALYSIS TOOL (FDA Tool)
The Filter Design and Analysis Tool (FDA Tool) is a powerful user interface for designing and analyzing filters swiftly. FDA Tool enables to design digital FIR or IIR filters by setting filter specifications, by importing filters from MATLAB workspace, or by adding, moving or deleting poles and zeros. FDA Tool also provides tools for analyzing filters, such as magnitude and phase response and pole-zero plots [17].

3.4 VECTOR SCOPE
The Vector Scope block is a comprehensive display tool similar to a digital oscilloscope. The block can display time-domain, frequency-domain, or user-defined signals. The input to the Vector Scope block can be any real-valued M-by-N matrix, column or row vector, or 1-D (Dimensional) vector, where 1-D vectors are treated as column vectors. Regardless of the input frame status, the block treats each column of an M-by-N input as an independent channel of data with M consecutive samples [18].

3.5 OPERATION OF SIMULATION BLOCK DIAGRAM
Consider three sinusoidal signals A1, B1 and C1 as shown in Figure6. Let the amplitudes of A1, B1 and C1 are 0.5, 2 and 5 respectively and the frequencies are 0.4Hz, 0.25Hz and 0.125Hz respectively. These three sinusoidal signals are combined applying superposition principle by adder block. The output of the adder block is a composite signal. The composite signal is now applied at the input of two input sum block and the vector scope input. A random source produces a random signal (noise signal) and the random signal is applied at the input of a LMS filter. At the same time the random signal is passed through a digital filter that produces a low frequency noise signal. The low frequency noise signal is then applied at the desired input of the LMS filter and at the input of the sum block. The output of the sum block is a noisy composite signal and applied at the input of other sum block and vector scope input. The output of the LMS filter is then applied at the sum block input. The output of the sum block is the noiseless composite signal. The three sinusoidal signals are reconstructed by applying digital filter bank that is shown in Figure 10.
Finally noiseless composite signal and reconstructed signals are obtained which are shown in Figure 12 and figure 13.

![Filter combinations block](image)

Figure 10. Filter combinations block (containing three digital FIR filters)

4. RESULTS AND DISCUSSION

From Figure 12, it is observed that the noiseless composite signal is perfectly obtained from noisy composite signal by using digital filter bank. So the error signal represents zero. It is also observed that the reconstructed signals are exactly same as the input sinusoidal signals (shown in figure 13). This means that the amplitudes, frequencies and shape of reconstructed signals are similar as the input sinusoidal signals. Therefore, perfect noise reduction and reconstruction are obtained. The comparison among input sinusoidal signals and reconstructed signals according to their amplitudes are shown in table 1 and table 2. From the two tables, it is observed that the amplitudes of input sinusoidal signals are exactly same as the output reconstructed signals.
Figure 11. Composite signal (contains three input sinusoidal signals) and input sine wave A1, B1 and C1 respectively
Figure 12. Composite signal, Noisy composite signal, Noiseless composite signal and Error respectively
Figure 13. Reconstructed sine wave A1, sine wave B1, sine wave C1 and zero error respectively.
5. CONCLUSION

Design and implementation of digital filter bank have been done in this paper to reduce noise and reconstruct the input signals. This new process represents a significant improvement over analog filters that also reduce noises. Among the different tasks of digital filter bank, only one task is shown in this paper which is noise removal. In future work, I will try to implement others work of digital filter bank and invent new work over digital filter bank.

REFERENCES


[11] www.mathworks.com › ... › DSP Modeling › Scopes and Data Logging

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