SYNTHETIC APERTURE RADAR IMAGES SEGMENTATION USING MINIMUM CROSS-ENTROPY WITH GAMMA DISTRIBUTION

Ali El-Zaart

Department of Mathematics and Computer Science, Faculty of Science, Beirut Arab University, Lebanon

ABSTRACT

Computer apparition plays the most important role in human perception, which is limited to only the visual band of the electromagnetic spectrum. The need for Radar imaging systems, to recover some sources that are not within human visual band, is raised. This paper present new algorithm for Synthetic Aperture Radar (SAR) images segmentation based on thresholding technique. Entropy based image thresholding has received sustainable interest in recent years. It is an important concept in the area of image processing. Pal (1996) proposed a cross entropy thresholding method based on Gaussian distribution for bi-modal images. Our method is derived from Pal method that segment images using cross entropy thresholding based on Gamma distribution and can handle bi-modal and multimodal images. Our method is tested using Synthetic Aperture Radar (SAR) images and it gave good results for bi-modal and multimodal images. The results obtained are encouraging.

KEYWORDS

SAR images, Segmentation, Gamma distribution, Cross-Entropy.

1. INTRODUCTION

In recent years, SAR imaging has been rapidly gaining fame in applications such as high resolution remote sensing for mapping, surface surveillance, search-and-rescue, mine detection, navigation position location, sensor platform pose control technology, and Automatic Target Recognition (ATR) [12]. For applications such as these, segmentation can play a key role in the subsequent analysis for target detection and recognition. The most important SAR attribute leading to its expanding in popularity is the ability to image large areas of land at fine resolutions and in all weather conditions. A major issue in SAR image is that basic textures are generally affected by multiplicative speckle noise [12]. As we note, in many applications, image segmentation is one of the most difficult and challenging problems. Thresholding is a well-known and most effective technique for image segmentation according to their simplicity and its speed during processing [3]. It is a technique for converting a grayscale or color image to a binary image based upon a threshold value. The threshold can be chosen manually or by using automated techniques [1]. Threshold methods can be categorized into two groups: global and local methods. Global thresholding technique thresholds the complete image with a single
threshold value. Local thresholding methods segment the image into a number of sub-images and use a different threshold value for each sub-image. Global thresholding methods are easy to implement and faster due to its less need for computations as proved in [1] and [2]. In [9], Sezgin and Sankur categorize thresholding methods in six groups according to the information they are exploiting. Among these methods, entropy-based methods have drawn the attentions of many researchers. Since it has been proven to be successful and reasonably robust [3]. Minimum Cross Entropy Thresholding (MCET) is a method that adopted in this paper. It selects optimal threshold value that minimizes cross entropy between original and resultant images. Li and Lee in [2] developed a sequential method to search for optimal threshold value using MCET that based on Gaussian distribution. In [2], Li and Tam proposed an iterative method that derived for MCET using Gaussian distribution. In [4], Al-Attas and El-Zaart proposed a sequential method for finding optimal threshold value by minimizing Cross Entropy using Gamma distribution to describe data in image. In this paper, we improve the previous works by developing an iterative algorithm for MCET that used for estimating optimal threshold value based on the Gamma distribution. Pal(1996) has derived a new method, which used minimum cross-entropy thresholding method for estimating optimal threshold value based on Gaussian distribution. However, our method process bi-modal and multimodal images, whereas previous method was based on Gamma distribution for the process of bi-modal images. The paper is organized as follows: Section 2 explains the statistical Gamma Distribution. In section 3, we describe image segmentation, section 4 describes image thresholding based on cross Entropy Section 5, presents the Pal cross entropy which is based on Gaussian distribution. Section 6 presents the proposed improvement method of Pal which is based on Gamma distribution. Section 7 presents the experimental results. We conclude in section 8.

2. STATISTICAL GAMMA DISTRIBUTION

Gamma Distribution is a two parameter family of continuous probability distributions in probability theory and statistics. It is a general type of statistical distribution. The probability density function of the Gamma distribution in homogenous area is given by [3][4]:

\[
f(x, \mu, N) = \frac{2q N^N}{\mu \tau(N)} \left(\frac{qx}{\mu}\right)^{2N-1} e^{-N(qx/\mu)^2}
\]

\[q = \frac{\tau(N + 0.5)}{\tau(N)\sqrt{N}},\]  \(x\) is the intensity of the pixel, \(\mu\) is the mean value of the distribution and \(N\) represents the parameter shape of the distribution.

Gamma distribution is having better aptitude than Gaussian as it endow with symmetric and non-symmetric histograms whereas Gaussian distribution works only with symmetric histograms. For a symmetric histogram, we set \(N\) to a high value and to get a histogram skewed to the right set \(N\) to a small value (See figure 1). In this figure, if the value of parameter shape \(N\) is equal to 1 then the histogram is skewed to the right and if the value of parameter shape \(N\) is equal to 10 then the histogram is almost symmetric.
3. IMAGE SEGMENTATION

The goal of segmentation is to simplify the representation of an image into something that is more meaningful and easier to analyze. Image thresholding, one of the most important techniques for image segmentation, the objective of this operation is that the objects and background are separated into non-overlapping sets [13,14,15, 17,18,19]. Image thresholding is necessary step in many image analysis applications. Image thresholding can be handled globally by determining a single threshold value for an entire image or locally apply different threshold value in different spatial regions [6,13]. Thresholding techniques can be divided into bi-level and multilevel category depending on number of image segments. Image Thresholding classifies pixels into two categories – Those to which some property measured from the image falls below a threshold, and those at which the property equals or exceeds a threshold. Thresholding creates a binary image (Binarization) using the following formula:

\[
s(x, y) = \begin{cases} 
1 & I(x, y) > T \\
0 & I(x, y) \leq T 
\end{cases}
\]

Where \( s(x, y) \) is a segmented/thresholded image, \( I(x, y) \) is the original image, \( T \) is the Threshold \([1]\). Many different methods have been accessible to get the threshold valued. Surveys were conducted and have been proposed to binarize an image \([7]\). Thresholding methods have been classified into six categories \([\text{ref}]\). Entropy based Method is wieldy used in image threshholding. In next section, we will explain the entropy based thresholding.

4. IMAGE THRESHOLDING BASED ON ENTROPY

Entropy is a measure of the number of random ways in which a system may be arranged. Entropy measure the information content in probability distribution. It separates the information into two regions. Entropy based thresholding considers an image histogram as a probability distribution. There are two techniques developed for entropy based thresholding -the local entropy (LE) and joint entropy (JE). Entropy based thresholding usually considers and image histogram as a probability distribution, and then selects an optimal threshold value \([8]\). Many entropy based thresholding methods have been proposed in the literature. These methods are classified into...
three groups: entropic thresholding, fuzzy entropic thresholding and cross entropic thresholding. Entropic thresholding “considers the image foreground and background as two different signal sources, so that when the sum of the two class entropies reaches its maximum, the image is said to be optimally thresholded” [9]. Fuzzy entropic thresholding “considers the fuzzy memberships as indicators of how strongly a grey value belongs to the background or to the foreground” [9]. Cross entropic thresholding “considers the thresholding as the minimization of an information theoretic distance” [9]. This paper is focused on the cross entropy thresholding method. Many algorithms have been proposed for cross entropy thresholding, among them are Li and Lee (1993) have introduced the minimum cross entropy thresholding algorithm that selects the threshold, which minimizes the cross entropy between the segmented image and the original image [10]. Li and Lee have proposed a method that is based on Gaussian distribution in sequential manner [3]. They used the kullback’s information theoretic distance \( D \) between two probability distributions [3], \( D \) is also known as directed divergence or cross entropy. Let \( I=\{f_1, f_2, \ldots, f_n\} \) be the distribution of original image and \( It=\{g_1, g_2, \ldots, g_n\} \) the distribution of thresholded image then the cross entropy is defined as follows:

\[
D(I, It) = \sum_{i=0}^{n} f_i \log(f_i / g_i)
\]

\( D(I, It) \) is non-symmetric.


\[
D(I, It) = \sum_{i=0}^{n} f_i \log(f_i / g_i) + \sum_{i=0}^{n} g_i \log(g_i / f_i)
\]

Brink and Pendock in [16] redefine the information theoretic distance as:

\[
D(I, It) = \sum_{i=1}^{L} f(i) \log \left( \frac{f(i)}{g(i)} \right) + \sum_{i=1}^{L} g(i) \log \left( \frac{g(i)}{f(i)} \right)
\]

El-Zaart worked on Brink cross entropy which is an improvement of Lin and Lee cross entropy [13,14,15,16]. Pal[11] is developed a cross entropy method that is more general than Li and Brink. Next section we will explain the Pal method.

5. PAL THRESHOLDING METHOD BASED ON GAUSSIAN DISTRIBUTION

A cross entropy thresholding method where image histogram is modeled by a mixture of Poisson distributions was proposed by Pal [11]. In general any thresholding method segments the image into object and background regions based on the threshold \( t \). The gray level values of object is defined from \([0-t] \) and the gray level values of background is defined from \([t+1, L] \), where \( L \) is the possible number of gray levels. Then the distribution of gray level in the object and background regions can be defined as:
\[ f_O = \{ f_1^O, f_2^O, \ldots, f_t^O \} \quad \text{and} \quad f_B = \{ f_1^B, f_2^B, \ldots, f_L^B \} \]

Where: \( f_{i}^{O} = \frac{h_i}{\text{sum}} \quad i = 1, 2, 3, \ldots, t \) and \( f_{i}^{B} = \frac{h_i}{M \times N - \text{sum}} \quad i = t + 1, t + 2, \ldots, L \)

Where \( \text{sum} = \sum_{i=1}^{t} h_i \)

Note that: \( \sum_{i=1}^{t} f_{i}^{O} = 1 \) and \( \sum_{i=t+1}^{L} f_{i}^{B} = 1 \), \( M \times N \) is the width and the length of the image, and \( h_i \) is the histogram of gray level \( i \).

We consider two Poisson probability distributions \( G_O \) and \( G_B \) for object and background respectively,

\[ G_O = \{ g_1^O, g_2^O, \ldots, g_t^O \} \quad G_B = \{ g_1^B, g_2^B, \ldots, g_L^B \} \]

\[ g_{i}^{O} = \frac{e^{-\mu_O} \mu_O^i}{i!} \quad i = 1, 2, \ldots, t \quad \text{and} \quad g_{i}^{B} = \frac{e^{-\mu_B} \mu_B^i}{i!} \quad i = t + 1, t + 2, \ldots, L \]

Where \( \mu_O \) and \( \mu_B \) are the mean value of object and background regions and they estimated using Gaussian distribution as follows:

\[ \mu_O = \left( \sum_{i=1}^{t} ih_i \right) / \sum_{i=1}^{t} h_i \quad \text{and} \quad \mu_B = \left( \sum_{i=t+1}^{L} ih_i \right) / \sum_{i=t+1}^{L} h_i \]

The cross entropy for the object and background regions are:

\[ D_O(t) = D_t(f_O, G_O) = \sum_{i=1}^{t} f_{i}^{O} \log \left( f_{i}^{O} / g_{i}^{O} \right) + \sum_{i=1}^{t} g_{i}^{O} \log \left( g_{i}^{O} / f_{i}^{O} \right) \]

\[ D_B(t) = D_t(f_B, G_B) = \sum_{i=t+1}^{L} f_{i}^{B} \log \left( f_{i}^{B} / g_{i}^{B} \right) + \sum_{i=t+1}^{L} g_{i}^{B} \log \left( g_{i}^{B} / f_{i}^{B} \right) \]

The total cross entropy is:

\[ D(t) = D_O(t) + D_B(t) \]

**6. IMPROVEMENT OF PAL METHOD**

Pal (1996) developed a thresholding method for images containing only two classes (bi-modal thresholding) by assuming that the data in image is modelled by Gaussian distribution. There are two problems in the Pal method: (i) if the image contains more than two classes that needs more than one threshold and (ii) if the data in image is not symmetric which means that Gaussian
distribution is not suitable. In our improvement of Pal method we solved these two problems by first extending the method to multimodal thresholding and second by using a more general distribution than Gaussian. We used Gamma distribution [4,12] which can model symmetric and non-symmetric data. Thus, the image is composed of two Gamma distributions; the first is for the object region and the second for the background region. The estimated means for these two regions are [4]:

\[
\mu_O(t) = \sqrt{\left(\frac{\sum_{i=0}^{L} h(i) i^2 q^2}{\sum_{i=0}^{L} h(i)}\right) / \sum_{i=0}^{L} h(i)} \quad \text{and} \quad \mu_B(t) = \sqrt{\left(\frac{\sum_{i=L+1}^{L} h(i) i^2 q^2}{\sum_{i=L+1}^{L} h(i)}\right) / \sum_{i=L+1}^{L} h(i)}
\]

Where \(h(i)\) is the histogram defined on the gray level [0,L] and \(q = \frac{\tau(N + 0.5)}{\tau(N)\sqrt{N}}\).

The improvement of Pal cross entropy thresholding algorithm using Gamma distribution is as follows:

\[
D(t) = \sum_{i=0}^{L} f_i^O \log\left(\frac{f_i^O}{g_i^O}\right) + \sum_{i=1}^{L} f_i^O \log\left(\frac{g_i^O}{f_i^O}\right) + \\
\sum_{i=t+1}^{L} f_i^B \log\left(\frac{f_i^B}{g_i^B}\right) + \sum_{i=t+1}^{L} g_i^B \log\left(\frac{g_i^B}{f_i^B}\right)
\]

The optimal value of \(t^*\) is the value which minimizes the objective function \(D(t)\) [11].

**6.1 Improvement Bi-modal Algorithm of Pal:**

Begin

Set \(\text{min} = \text{max value of Int as initial value}\)

for \(t = 1..255\)

Begin

Compute \(\mu_O\) using Gamma distribution

Compute \(\mu_B\) using Gamma distribution

Compute \(f_i^O, i = 0, \ldots, t\)

Compute \(f_i^B, i = t + 1, \ldots, 255\)

Compute \(g_i^O, i = 0, \ldots, t\)

Compute \(g_i^B, i = t + 1, \ldots, 255\)

Compute \(D(t)\) using equation

\(\text{if} (\min > D(T))\)

Begin

\(\min = D(t)\)

\(\text{threshold} = t\)

end

end

end
6.2 New Multimodal Thresholding Algorithm

We extended the bi-model thresholding method to multimodal using Gamma distribution. Assume that we have \( m \) classes of pixels in an image. Then we have a threshold vector of \( m-1 \) thresholds \( T = \{ t_1, t_2, t_3, \ldots, t_{m-1} \} \), where \( t_0 < t_1 < t_2 < t_3 < \ldots < t_{m-1} < t_m \), \( t_0 = 0 \) and \( t_m = 255 \). The multimodal cross-entropy is defined as follows:

\[
D(t) = \left( \sum_{i=0}^{t_1} f_i^O \log \left( \frac{f_i^O}{g_i^O} \right) + \sum_{i=t_0}^{t_3} g_i^O \log \left( \frac{g_i^O}{f_i^O} \right) \right) + \left( \sum_{i=t_{l_1+1}}^{t_3} f_i^B \log \left( \frac{f_i^B}{g_i^B} \right) + \sum_{i=t_{l_1+1}}^{t_3} g_i^B \log \left( \frac{g_i^B}{f_i^B} \right) \right) + \ldots + \left( \sum_{i=t_{l_m-1}}^{t_m} f_i^B \log \left( \frac{f_i^B}{g_i^B} \right) + \sum_{i=t_{l_m-1}}^{t_m} g_i^B \log \left( \frac{g_i^B}{f_i^B} \right) \right)
\]

The mean value of \( k^{th} \) class, will be estimated using Gamma distribution:

\[
\mu_k(t) = \frac{\left( \sum_{i=t_{l_{k-1}}}^{t_{l_k}} \frac{h(i)}{i^2 q^2} \right) / \sum_{i=t_{l_{k-1}}}^{t_{l_k}} h(i)}
\]

The proposed new Multimodal algorithm is as follows:

1- Initial values of thresholds can be estimated using k-mean algorithm
\( T_0 = \{ t_1, t_2, t_3, \ldots, t_{m-1} \} \), \( t_0 = 0 \) and \( t_m = 255 \)

2- On each two consecutive modes in the multimodal histogram, apply bi-modal thresholding algorithm, in order to obtain the new value of threshold \( T_{new}(k) \). These two modes starting from \( t_{k-1} \) and ending at \( t_k \). For \( k = 1, 2, \ldots, m-1 \).

3- We compare the two threshold vectors \( T_0 \) and \( T_{new} \)
   If \( |T_0 - T_{new}| < \varepsilon \) then optimal threshold values \( T^* = \{ t_1^*, t_2^*, t_3^*, \ldots, t_{m-1}^* \} \) are reached
   else Assign \( T_0 \leftarrow T_{new} \) and go to step 2

7. EXPERIMENTAL RESULTS

The improvement bi-modal Pal method is implemented using Gamma distribution and applied on several real SAR images, and then the Multimodal thresholding is applied on multimodal image.

7.1. SAR Images with Two classes

In this section, we will apply the improved bimodal thresholding method on three real SAR images, where the parameter \( N \) is given by the user.

- Figure 2a presented a real SAR image, we applied the bimodal algorithm on this image with value of \( N=10 \). The segmented image presented in figure 2b with estimated threshold \( T=30 \). Figure 2c, presented the histogram of the real SAR image with the estimated threshold in green line.
Figure 2: (a) Real SAR image. (b) Segmented image with N=10 and T=30.

(c) Image histogram with estimated threshold.

- Figure 3a presented a real SAR image, we applied the bimodal algorithm on this image with value of N=8. The segmented image presented in figure 3b with estimated threshold T=29. Figure 3c, presented the histogram of the real SAR image with the estimated threshold in green line.
Figure 3: (a) Real SAR image. (b) Segmented image with N=8 and t=29. (c) Image histogram with estimated threshold.

- Figure 4a presented a real SAR image, we applied the bimodal algorithm on this image with value of N=12. The segmented image presented in figure 4b with estimated threshold T=27. Figure 4c, presented the histogram of the real SAR image with the estimated threshold in green line.
Figure 4: (a) Real SAR image. (b) Segmented image with N=12 and T=27.

(c) Image histogram with estimated threshold

7.2 Real SAR Image with Three Classes

Figure 5a presented a real SAR image with three classes of pixels, the proposed multimodal thresholding method is applied to this image with N=12. The estimated thresholds are T1=14 and T2=42. The segmented image is presented in figure 5b. Figure 5c presented the histogram of real SAR image with two estimated thresholds.
Figure 5: (a) Real SAR image. (b) Segmented image with N=12, T1=13 and T2=42. (c) Image histogram two estimated thresholds

8. CONCLUSION AND FUTURE WORK

In this paper, we proposed an improvement of the Pal cross entropy thresholding. Pal (1996) developed a thresholding method for images containing only two classes (bi-modal thresholding) by assuming that the data in image is modelled by Gaussian distribution. In our improvement of Pal method we extended his work for multimodal thresholding and using a more general distribution than Gaussian. We used Gamma distribution which can model symmetric and non-symmetric data. Thus, the image is composed of two Gamma distributions; the first is for the object region and the second for the background region. The experimental results showed that the improvement method segmented well the SAR images having two and three classes. The estimated threshold is done in sequential manner, as future work we will develop method that can find the optimal threshold using iterative method.
REFERENCES

AUTHOR

Ali El-Zaart was a senior software developer at Department of Research and Development, Semiconductor Insight, Ottawa, Canada during 2000-2001. From 2001 to 2004, he was an assistant professor at the Department of Biomedical Technology, College of Applied Medical Sciences. From 2004-2011, he was an assistant professor and then associate professor at the Department of Computer Science, College of computer and information Sciences. Since 2011 he is an associate professor at the department of Mathematics and computer science, Faculty of Science, Beirut Arab University, Lebanon. He published more than 110 articles and proceedings in the areas of image processing, remote sensing, and computer vision. He received a B.Sc. in computer science from the University of Lebanon; Beirut, Lebanon in 1990, M.Sc. degree in computer science (image processing) from the University of Sherbrooke, Sherbrooke, Canada in 1996, and Ph.D. degree in computer science (image processing) from the University of Sherbrooke, Sherbrooke, Canada in 2000. His research interests include image processing, pattern recognition, remote sensing, and computer vision, Data Mining, e-government, smart cities.