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Preface

The 3rd International Conference on Signal, Image Processing and Embedded Systems (SIGEM 2017) was held in Chennai, India, during July 29~30, 2017. The 3rd International Conference on Computer Science, Engineering and Applications (CSEA 2017), The 3rd International Conference on Fuzzy Logic Systems (Fuzzy 2017), and The 3rd International Conference on Natural Language Computing (NATL 2017) was collocated with The 3rd International Conference on Signal, Image Processing and Embedded Systems (SIGEM 2017). The conferences attracted many local and international delegates, presenting a balanced mixture of intellect from the East and from the West.

The goal of this conference series is to bring together researchers and practitioners from academia and industry to focus on understanding computer science and information technology and to establish new collaborations in these areas. Authors are invited to contribute to the conference by submitting articles that illustrate research results, projects, survey work and industrial experiences describing significant advances in all areas of computer science and information technology.

The SIGEM-2017, CSEA-2017, Fuzzy-2017, NATL-2017 Committees rigorously invited submissions for many months from researchers, scientists, engineers, students and practitioners related to the relevant themes and tracks of the workshop. This effort guaranteed submissions from an unparalleled number of internationally recognized top-level researchers. All the submissions underwent a strenuous peer review process which comprised expert reviewers. These reviewers were selected from a talented pool of Technical Committee members and external reviewers on the basis of their expertise. The papers were then reviewed based on their contributions, technical content, originality and clarity. The entire process, which includes the submission, review and acceptance processes, was done electronically. All these efforts undertaken by the Organizing and Technical Committees led to an exciting, rich and a high quality technical conference program, which featured high-impact presentations for all attendees to enjoy, appreciate and expand their expertise in the latest developments in computer network and communications research.

In closing, SIGEM-2017, CSEA-2017, Fuzzy-2017, NATL-2017 brought together researchers, scientists, engineers, students and practitioners to exchange and share their experiences, new ideas and research results in all aspects of the main workshop themes and tracks, and to discuss the practical challenges encountered and the solutions adopted. The book is organized as a collection of papers from the SIGEM-2017, CSEA-2017, Fuzzy-2017, NATL-2017.

We would like to thank the General and Program Chairs, organization staff, the members of the Technical Program Committees and external reviewers for their excellent and tireless work. We sincerely wish that all attendees benefited scientifically from the conference and wish them every success in their research. It is the humble wish of the conference organizers that the professional dialogue among the researchers, scientists, engineers, students and educators continues beyond the event and that the friendships and collaborations forged will linger and prosper for many years to come.

Dhinaharan Nagamalai
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IMAGE CONTENT DESCRIPTION USING LSTM APPROACH

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ABSTRACT

In this digital world, artificial intelligence has provided solutions to many problems, likewise to encounter problems related to digital images and operations related to the extensive set of images. We should learn how to analyze an image, and for that, we need feature extraction of the content of that image. Image description methods involve natural language processing and concepts of computer vision. The purpose of this work is to provide an efficient and accurate image description of an unknown image by using deep learning methods. We propose a novel generative robust model that trains a Deep Neural Network to learn about image features after extracting information about the content of images, for that we used the novel combination of CNN and LSTM. We trained our model on MSCOCO dataset, which provides set of annotations for a particular image, and after the model is fully automated, we tested it by providing raw images. And also several experiments are performed to check efficiency and robustness of the system, for that we have calculated BLUE Score.

KEYWORDS

Image Annotation, Feature Extraction, LSTM, Deep Learning, NLP.

1. INTRODUCTION

The image is an important entity of our digital system. It contains much useful information like an image of a receipt, an image taken from CCTV footage etc. We can surely say that an image tells a unique story in its way. In today's digital world, one can perform or gather a large information or facts just only after analyzing a digital image. When we are dealing with digital images, we have to gather what each and every part of it wants to contain. For that, we should extract each and every part with optimal care and extract information of that particular region and then gather the whole information to reach out to a conclusion. Here we need to get what the picture have like objects, boundaries, and colour etc. features. Here, we need an accurate description of an image and for digital images, we need an efficient and accurate model that could give accurate annotations of each and every region of that image and can provide a rich sentence form, so that we can understand what's happening in that image.

Image captioning[1] is used by several software giants companies like Google, Microsoft etc. It is used for many other specific tasks like explaining an image for a blind person by giving him a sentence generated form of annotations of an image. Such essential and significant functionalities make image captioning an important field to study and explore.

Image annotations generation of an image is very much close to scene understanding model. Computer vision involves the complete understanding of an image, so our model should not only just provide image annotations, but should be capable of expressing the scene and what exactly objects are doing in that image. In this way, computer vision and natural language processing go hand to hand for solving this problem of automatic image description by providing suitable generated sentence explaining the scene of an image. Likewise, a human can easily perceive the content of an image just by looking at it, and he can explain the scene in that image accurately. But, when it comes to the computer, it's a difficult task to generate and explain scene of an image by using machine learning algorithms. Human generated annotations have properties like rich, concise and accurate etc. Like, a human can generate a well fit sentence, and that sentence has only relevant things and accurate as it contains all essential region's information of an image.

Many researchers [1, 14] and others have explored this problem and provided few appropriate solutions. There are many advances in this field as large datasets like MSCOCO, Flickr30k etc. are available to train the model more efficiently. The basic model (as shown in figure 1.) for image captioning works like, first gather the features of an image and given captions in the dataset, and based on features provide a suitable annotation to that image. In figure 1, it shows a man lying on the table and a dog sitting near him. As this is a sample input image firstly its features based on colour, objects, boundaries, texture etc. is extracted and also features of given captions, then based on its attributes a common representation is produced. And from that common space embedding representation, an appropriate sentence is generated for this image like a man lying on the table with a dog.

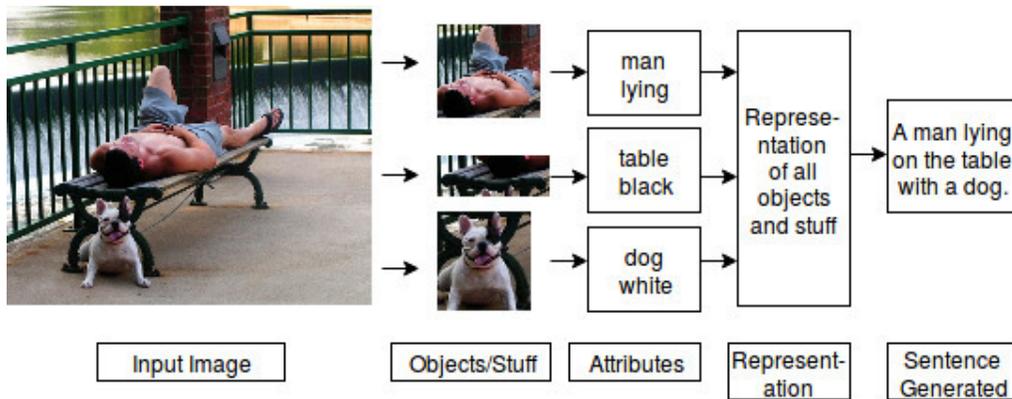


Figure 1. Basic working of Image Captioning model

Most of the works in this field are based on these two approaches: similarities approach and constructive approach. Similarities approach [1], [2], [5] and [6] means taking the model as a retrieval task, after extraction of features of image and captions, this approach provides an embedding representation of information and based on that most suitable annotation is selected as annotation for an image. This approach has few limitations like it doesn't provide good results when a raw image contain an unseen object or thing in it, as due to the limited size of its dictionary, it produce annotations based on previously gathered features and language models. In this way, this approach is not suitable in today's advance in this field. Constructive approach [3], [4], [7], [8], and others mean the generation of sentence based on firstly learning image features and after that sentence generation process occurs in many parts. Like a basic constructive approach based model consists of these basic steps: language modeling part, image features extraction and analysis part, and representation part which combines the previous both parts. In [4], they described an image by using the constructive approach as Convolutional Neural Network (CNN) is used for extraction of image features and after that Recurrent Neural Network

(RNN) is used to learn the representation of space embedding and generated a suitable sentence for an image. Here, language modeling means, it learns dense features information for every word related to the content of that particular image present in the dictionary and it gathers its semantic form in recurrent layers.

In [1], Pan et al. used similarities approach and found similarity measures between the captions keywords and the images. They described an image region in form of blob-tokens, it means a part of image region based on its features like colour, texture, size, position, boundaries, and shape. And [2], applied nearest neighbor approach along with similarities measures, as a set of keywords which are nearer to each other from the sentence. These models have some limitations like they are biased with training dataset, and doesn't give a good result for a new unseen image. In [3,4], they proposed a model based on CNN[3] with RNN[11] method, as this model is based on constructive approach and it produced some good results as compared to models defined with similarities approach. These models provide better scene understanding and able to express the content of the image semantically. But they has few limitations like over fitting of data and not able to give good results when attention is not given to similar type of images.

Our novel work involves most optimal technique with the constructive approach, as we used CNN method to extract image features and then the common representation of whole information and features of images and captions are made by using LSTM[15] model, which then produce appropriate sentence for any new image. Our model produces accurate annotations and also the sentence expresses the scene of the image, along with information about all objects and stuff in that image. Our contributions are as follows:

- Image feature extraction is done using robust CNN technique, which produces features based on colour, texture, and position of objects and stuff in the image.
- The common representation contains all gathered information in layers and cells of LSTM model and then based on input, hidden layers and then final output from output layers are obtained.
- For language modeling, we used n-gram model, so that accurate and rich form of a sentence is generated.
- Blue-4 metric is used for analysis of efficiency and robustness of our method. And the comparison of various previous models with our model.

A further portion of the paper is divided into sections as follows, Section 2 included related work portion which gives detailed information about previous works done in this field. Section 3 provides the complete overview of the technique of image captioning such as problem statement, input and output details, and also includes the motivation behind this work, and all relevant terminology and concepts are discussed in detail. After that our model is described in Section 4, then Section 5 give results and implementation details as dataset used are MSCOCO, Flickr30k, and Flickr8k, also the value of BLUE-4 metric score is provided as the efficiency measure of our model. Section 6 discuss the conclusion and future improvements in this work.

2. RELATED WORK

There are many advances in the field of automatic image captioning. As in earlier works, [1] Pan et al. proposed technique which work by annotating a particular part of image region, in this a word for each image region and then on combining we get the sentence, they discussed about considering the image regions as blobs-token which means an image regions based on its feature such as colour, texture, and position of object in image. But this technique has few limitations

such as it is effective only for a small dataset, this work include much manual work as they have to provide annotated words and blob-tokens with an image, and this approach sometimes give results based on a training set, that means it is biased on a training set.

Jacob et al. [2], provided technique that explores nearest neighbor images in training set to the query image and based on that appropriate k nearest neighbor images, their given captions are imported and based on that only caption is given for the query image. But there is a limitation of this approach as it performs better for highly similar images, but worse for highly dissimilar images. In [5], they used similarities approach as a candidate matching images with the query image are retrieved from the collection of captioned images, then after features are matched and based on the best rank obtained a caption is given to the query image. But this model has few limitations like re-ranking the given captions could create error for training images and related text, and also object and scene classifiers could give erroneous results, so the model could have given faulty results. In [6], Ali et al. proposed model based on computing of a score linking a sentence with an image. And worked based on the semantic distance between some words like two or three for a particular image, and SVM models are trained on it. This model lacks as dataset used is not much used and large, and not much emphasized for checking adjectives and other relevant potential good information from image regions.

In [3], [4], and [7,14], they used the constructive approach, but with different techniques for extracting images and then after that for sentence generation. Karpathy et al. [3], described the common intermodal representation between the visual information and the language models. They used the combination of CNN over image regions and Bi-directional Recurrent Neural Network (BRNN) for sentence generation approach by computing word representation. But this model also have certain limitations as this model didn't focus on attention concept for captioning.

In our model, CNN is used for extraction of features of an image, and it provided that information to the common representation. In [4], this paper focussed on tight connection between image objects and text related to that. As they acquired every detailed information of each and every region of an image by particularly dealt with each region, as they extracted and detected objects/stuff in an image, and their attributes such as adjectives which provide extra useful information about that particular region, also details about the spatial connections between those regions, and based on these Conditional Random Field (CRF) is constructed and labels of graphs are predicted, and finally based on these labelling sentence is generated. This model contains few limitations such as it didn't provide semantically related texts on input images, so the accuracy of the model is compromised in this way.

Most of the work in this field is related to providing a visual interpretation of the image and relation to that to given captions in the dataset. In [7], Desmond et al. introduced representation to contain connections between different objects/stuff of an image, it worked based on similarities between the objects or image regions and based how these regions relate to each other. They used image parser to get information for each region of an image. But this model have certain limitations as the output of an object detector is not used to obtain a fully automated model, and there are various improvements that can be done to the image parser to enhance the efficiency of this model.

3. OVERVIEW OF METHOD

Image captioning involves machine learning algorithms and as well many mathematical simulations so that an accurate annotation can be provided.

3.1. Motivation

As human can perceive an image just by looking at it, we must have a robust and accurate model that can cope up with a human in case of captioning an image and express what the objects are doing in that image. Our model should have all important characteristics like accuracy, rich in sentence generation, consistent as could not be a biased model and concise as it much includes only relevant regions of the image. And there are many real life applications of image captioning like image search, tell stories from the pictures uploaded on the web and helpful for visually impaired people so that they can be aware of relevant information from the web.

3.2. Problem Statement

Image content description by using the neural networks and concepts of deep learning.

3.3. Input

A query image.

3.4. Dataset

MSCOCO, Flickr30k, and Flickr8k.

3.5. Output

Captioning of that query image, according to the learning of the model.

3.6. Feature Extraction

For a query image, firstly we extract its features based on its colour, texture, boundaries, objects, stuff, and position of things in it. This process of feature extraction is very crucial in our model, as it provides all basic required information about the image. This process is done with help of CNN models. As CNN contains certain specified number of layers and those layers are responsible for storing the features which are extracted from the images, then these features are passed forward to LSTM model so make a common representation for sentence generation. As shown in figure 2, an image is given to the model, and then model extracts all relevant features of that image such as:

- Low-level features, it involves around details of pixels level such as the colour of a pixel and edges and corners in images.
- Mid-level features, it involves between low-level and high-level features, it discuss any curves in the image of an object present in it.
- High-level features denote detection of the object in the image. It is hard to predict the exact object and scene of that object in that image, so image captioning is all about minimizing the gap between this low-level and high-level features methods.

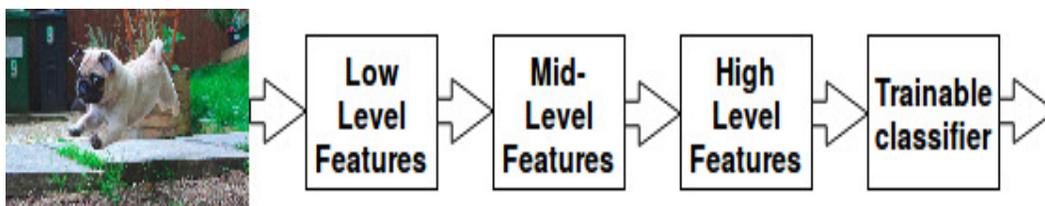


Figure 2. An image and extraction of its features by CNN

After obtaining all relevant features, CNN gives it to a trainable classifier. And from that to the common representation model. It is a neural network which is fully trainable with the help of mathematical simulations like stochastic gradient descent.

The model takes an input image and provides the possible caption C from the available dictionary of 1-to- T words, such as:

$$C = C_1, C_2, \dots, C_N, \text{ where } C_i \in R^T \quad (1)$$

, where T equals to the available number of keywords of the vocabulary and N is the possible length of the sentence.

We use CNN to extract the image features as a set of feature vectors. Then it produces M vectors, which belongs to a part of the image and having an L -dimensional form, such as:

$$B = B_1, \dots, B_M, B_i \in R^L \quad (2)$$

As shown in figure 2, CNN extracts features from lower levels, so that each relevant region is completed for sentence generation. So that decoder, LSTM model could focus on only relevant portions of the image by using feature vectors, $r(I)$ for an image I having a specified dimension.

$$CNN_MODEL(I) = W^i r(I) + b \quad (3)$$

As per the measurement of accuracy of the features extracted by our model, we trained our model based on visualization parameters, which helps in examining of the different feature activations and their relation to features embedding. Also, our model worked on both image classification and the localization tasks. It is analysed that as the network grows, there is rise in the number of filters used. So, the accuracy is optimised by incorporating more number of filters.

3.7. Language Modeling

In our model, we have used n-gram model for language modelling, it means a statistical probability function based on conditional factors such as for N words =

$$P(a_i | a_{i-N+1}, \dots, a_{i-1}) \quad (4)$$

, it means the possibility of the next word of the sequence is based on the previously occurred words in the sequence.

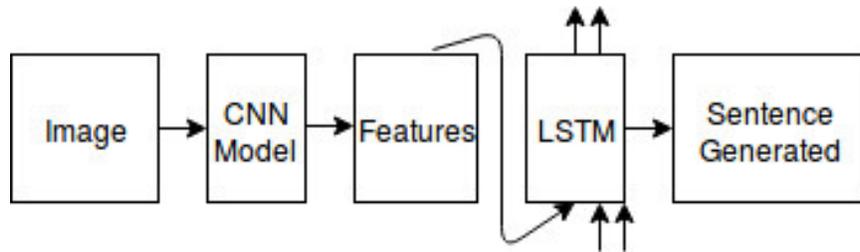


Figure 3. Basic Framework of our model.

3.8. LSTM

Long Short-Term Memory (LSTM) [15,16] acts as the decoder in our model when features are transferred to it, it uses the common representation of all gathered information and the based on it provide sentence. As shown in figure 3, the basic framework of our model is depicted, as shown an image is provided as an input to our model, then CNN used for feature extraction and then extracted features are given as input to the LSTM unit, which finally generates sentence, as shown in figure 3, which provides the basic framework of our model.

LSTM model generally consists of three important gates such as input gate, forget gate and output gate. And the main part of an LSTM model is its memory cell c , which keeps the whole information about the image features, previously generated words and track functionality of all three gates.

3.9. Words Representation

It is based on the size of the vocabulary of our model, like we taken image dimension as $I_D = 4096$, so word form will become of order:

$$T \times I_D \quad (5)$$

4. DESIGN OF PROPOSED METHOD

In our model, image features are extracted by CNN, then LSTM model acts like a decoder of that features. As CNN_MODEL(I) is passed to LSTM model, as an input. It takes it, and further evaluate values of gates defined in its inner working system. And whole working information of the system is stored in memory cells, c . These gates units are trained to learn when to open and close permit of access to information to memory cells. Three gates used as whether the current cell value is to forget(forget gate f), input gate (i) is to read input and output gate (o) to whether output the new cell value.

LSTM model computes on the basis of the memory cell information and previously calculated words of the sequence, such as:

$$P(S_t|I, S_0, \dots, S_{t-1}) \quad (6)$$

, where I is an image and S is a possible sentence which depends on previously generated words.

These are the main equations which explains our model:

$$a_{t-1} = CNN_MODEL(I) \quad (7)$$

$$a_t = W_e S_t \quad (8)$$

$$p_{t-1} = LSTM_MODEL(a_t) \quad (9)$$

, where a_t means input to LSTM model, as CNN_MODEL(I) is initial input to LSTM model, and then after it works recursively and obtain one word of sentence at each time.

Updating of gates and cell values in a LSTM model as such:

$$i_t = \sigma(W_{ia} a_t + W_{ik} k_{t-1}) \quad (10)$$

$$f = \sigma(W_{fa} a_t + W_{fk} k_{t-1}) \quad (11)$$

$$o = \sigma(W_{oa} a_t + W_{ok} k_{t-1}) \quad (12)$$

$$c = f_i \odot c_{t-1} + i_t \odot h(W_{ca} a_t + W_{ck} k_{t-1}) \quad (13)$$

$$k_t = o_t \odot c_t \quad (14)$$

$$p_{t+1} = \text{Softmax}(k_t) \quad (15)$$

where, \odot means multiplication of a gate value, $\text{Softmax}()$ is used as for higher dimension balancing between the various stages of values. And the pair (k_t, c_t) is passed as the present form of hidden state to the upcoming hidden state. And k_t is given to Softmax , which provide a probability distribution.

As shown in figure 4, LSTM models work recursively after a word is found, and use that information to predict the next word of the sentence.

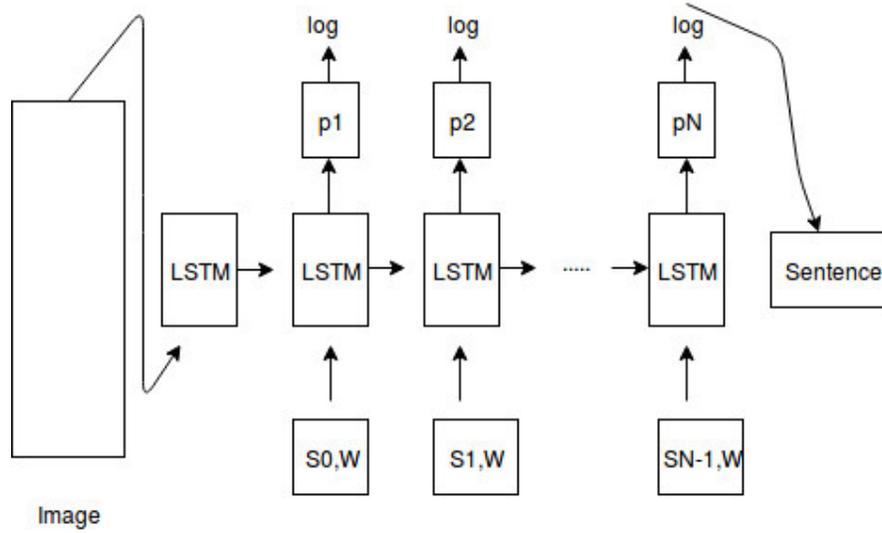


Figure 4. Working on LSTM model for sentence generation.

4.1. Sentence Generation

In our model, LSTM is used for sentence generation, the process of sentence generation involves certain basic steps, such as it starts from “##START##” or any other sentence generation reference words, which conveys that next word that will be generated will be the first word of our desired sentence. Our method calculates the probability distribution for the upcoming word, $P(S_i|I, S_0, \dots, S_{i-1})$. After that we use this distribution method and previously calculated words for the calculating probability of the next word. And cycle goes on until we encounter the last word of sequence and then after model produce output as end sign “##END##”. We use our model to calculate the probability of generating a sentence given an image. The sentence generation task is incorporated by using the perplexity of a sentence conditioned on the averaged image feature across the training set as the reference perplexity to normalize the original perplexity as discussed in [8].

For an example, in figure 5, an input image is given, our model starts predicting a word at a time and by using the previously calculated word, with the further calculation of probability distribution, it predicts the next word. Like first word predicted based on the image features is “man”, then by using it and model parameters, it predicts the next word as “bench”, and process

goes on until the model encounters the stop word. In this way, LSTM generates the most optimal sentence for a new input image.

5. IMPLEMENTATION AND RESULTS

Our model includes the novel combination of CNN and LSTM techniques with using deep learning approaches. We test our method on benchmark datasets like MSCOCO [12], Flickr30k [13] and Flickr8k [14]. The dataset MSCOCO contains around 80k images, and each image has at least 5 different captions of different lengths related to it. This contains images of almost everything such as sports, landscapes, portraits, persons, groups etc. Out of 80k images, we have taken 5k images for testing phase and check the implementation of our model on that testing dataset.

We used deep learning approach for implementation of our model, and it is implemented in python by using Keras [17] library, a high-level neural network for fast and accurate implementation which is run on Theano as backend.

5.1. Input

Human caption: A man lying on the bench and a dog sitting on the ground.



Figure 5. An input image

5.2. Output

Caption generated by our model: A woman sitting on a bench with a dog.

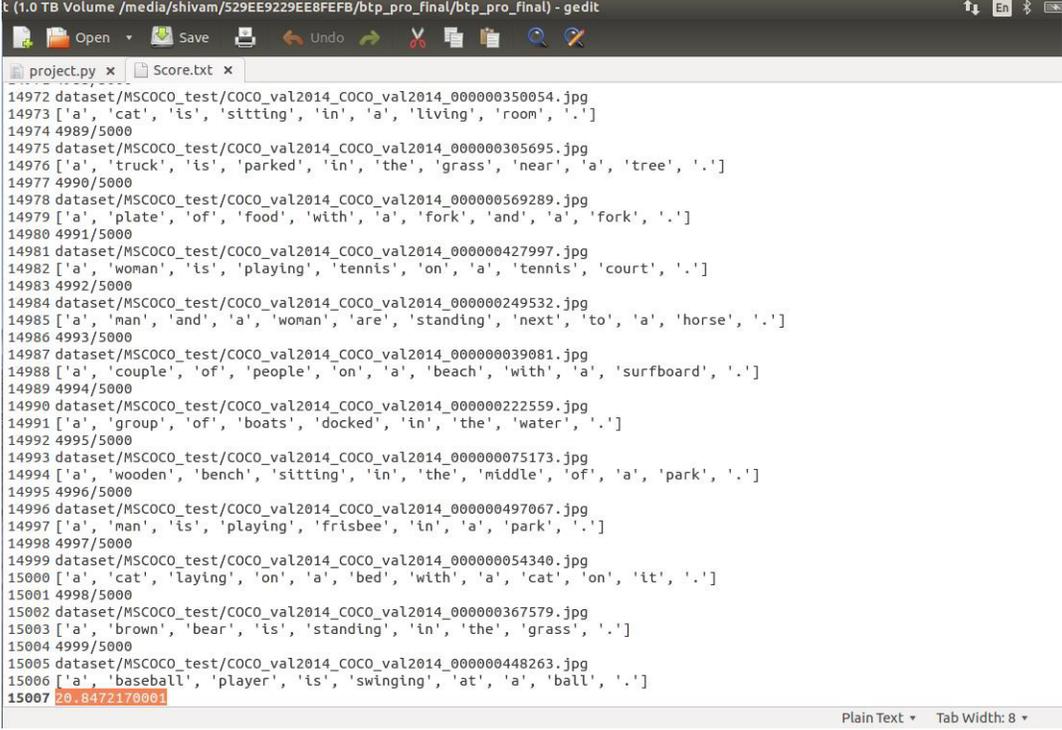
```
shivan@shivan-Inspiron-3543:~/Music/btp_pro_flnal$ python project.py
Using Theano backend.
Using model:1
Loading VGG model
Loading model
models/MSCOCO/image_captioning_LSTMmodel_1_output_rnn_512.json
/home/shivan/Documents/BTP_Work/projectBTP/4.jpg
['a', 'woman', 'sitting', 'on', 'a', 'bench', 'with', 'a', 'dog', '.']
A woman sitting on a bench with a dog.
shivan@shivan-Inspiron-3543:~/Music/btp_pro_flnal$
```

Figure 6. Output: A woman sitting on a bench with a dog.

5.3. Metric

We have used BLEU-4 metric instead of BLEU-1 metric, as it is a far better metric for measuring the efficiency of our model.

BLEU-4 metric score of our model: 20.84



```

t (1.0 TB Volume /media/shivam/529EE9229EE8FEFB/btp_pro_final/btp_pro_final) - gedit
project.py x Score.txt x
14972 dataset/MSCOCO_test/COCO_val2014_COCO_val2014_000000350054.jpg
14973 ['a', 'cat', 'is', 'sitting', 'in', 'a', 'living', 'room', '.']
14974 4989/5000
14975 dataset/MSCOCO_test/COCO_val2014_COCO_val2014_000000305695.jpg
14976 ['a', 'truck', 'is', 'parked', 'in', 'the', 'grass', 'near', 'a', 'tree', '.']
14977 4990/5000
14978 dataset/MSCOCO_test/COCO_val2014_COCO_val2014_000000569289.jpg
14979 ['a', 'plate', 'of', 'food', 'with', 'a', 'fork', 'and', 'a', 'fork', '.']
14980 4991/5000
14981 dataset/MSCOCO_test/COCO_val2014_COCO_val2014_000000427997.jpg
14982 ['a', 'woman', 'is', 'playing', 'tennis', 'on', 'a', 'tennis', 'court', '.']
14983 4992/5000
14984 dataset/MSCOCO_test/COCO_val2014_COCO_val2014_000000249532.jpg
14985 ['a', 'man', 'and', 'a', 'woman', 'are', 'standing', 'next', 'to', 'a', 'horse', '.']
14986 4993/5000
14987 dataset/MSCOCO_test/COCO_val2014_COCO_val2014_00000039081.jpg
14988 ['a', 'couple', 'of', 'people', 'on', 'a', 'beach', 'with', 'a', 'surfboard', '.']
14989 4994/5000
14990 dataset/MSCOCO_test/COCO_val2014_COCO_val2014_000000222559.jpg
14991 ['a', 'group', 'of', 'boats', 'docked', 'in', 'the', 'water', '.']
14992 4995/5000
14993 dataset/MSCOCO_test/COCO_val2014_COCO_val2014_000000075173.jpg
14994 ['a', 'wooden', 'bench', 'sitting', 'in', 'the', 'middle', 'of', 'a', 'park', '.']
14995 4996/5000
14996 dataset/MSCOCO_test/COCO_val2014_COCO_val2014_000000497067.jpg
14997 ['a', 'man', 'is', 'playing', 'frisbee', 'in', 'a', 'park', '.']
14998 4997/5000
14999 dataset/MSCOCO_test/COCO_val2014_COCO_val2014_000000054340.jpg
15000 ['a', 'cat', 'laying', 'on', 'a', 'bed', 'with', 'a', 'cat', 'on', 'it', '.']
15001 4998/5000
15002 dataset/MSCOCO_test/COCO_val2014_COCO_val2014_000000367579.jpg
15003 ['a', 'brown', 'bear', 'is', 'standing', 'in', 'the', 'grass', '.']
15004 4999/5000
15005 dataset/MSCOCO_test/COCO_val2014_COCO_val2014_000000448263.jpg
15006 ['a', 'baseball', 'player', 'is', 'swinging', 'at', 'a', 'ball', '.']
15007 20.842170001
  
```

Figure 7. BLUE-4 Score: 20.84.

5.4. Parameters for Implementation

Our LSTM model is implemented by using 2 layers, as we have checked it with 1 layer too. But former method produces more optimal captions. We have observed that on increasing further number of layers, generated captions efficiency degraded. So, we concluded that a number of layers are 2, is the most optimal method for implementation. And weights are initialized uniformly from [-0.06, 0.06].

We have taken maximum caption length = 16.

Batch size = 200

Dimension of LSTM output = 512

Image dimension parameter = 4096

Word vector dimension = 300

5.5. Comparison

We have compared our model with state-of-art techniques [1, 2, 3, 4], and based on BLEU-4 metric.

Table 1. Comparison of models

Dataset	Model	BLEU-4 Score
MSCOCO	Random	4.6
MSCOCO	Nearest Neighbour[2]	9.9
MSCOCO	CNN & RNN[8]	19.5
MSCOCO	Karpathy[3]	20.4
MSCOCO	Human	20.51
MSCOCO	Our model	20.84

From Table 1, we can see that our model is far better and efficient than previous works done in the field of automatic image captioning.

6. CONCLUSIONS

Our novel method showed that it is an efficient and a robust system, and can produce the description of any unseen image, which is more specific or related to the content of that image. And, it also is shown that our model is much better than state-of-art models and others previous automated works. As the measure of the efficiency of our model, we calculated BLEU-4 metric which is around 20.84 for our model. Several experiments are performed on different datasets, which depicts the robustness of our method.

In future works, we can make the current model more fast and efficient by applying fast machine learning algorithms. Also, we can fine-tune features extracted by CNN to improve correctness of our model. Also, we can test our model on more number of testing the dataset for better results.

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COMPARISON OF VOLUME AND DISTANCE CONSTRAINT ON HYPERSPECTRAL UNMIXING

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ABSTRACT

Algorithms based on minimum volume constraint or sum of squared distances constraint is widely used in Hyperspectral image unmixing. However, there are few works about performing comparison between these two algorithms. In this paper, comparison analysis between two algorithms is presented to evaluate the performance of two constraints under different situations. Comparison is implemented from the following three aspects: flatness of simplex, initialization effects and robustness to noise. The analysis can provide a guideline on which constraint should be adopted under certain specific tasks.

KEYWORDS

Hyperspectral unmixing, volume constraint, distance constraint, relative capacity

1. INTRODUCTION

Characterized as extremely high spectral resolution and numerous narrow continuous bands, hyperspectral remote sensing has raised extensive concerns [1] [18]. Hyperspectral images usually consist of mixed pixels due to limited spatial resolution of sensors. Thus, hyperspectral unmixing whose purpose is to decompose the mixed pixels into material signature (endmembers) and the corresponding abundance fractions, has become a challenging task.

Hyperspectral data unmixing is commonly based on linear mixed model (LMM) [2]. LMM hypothesizes that each pixel vector can be represented as the product of the endmember matrix and abundance vector. The abundance vector need to satisfy nonnegative and sum-to-one constraints. Based on LMM, there are mainly two groups of methods solving hyperspectral unmixing problem. The algorithms in the first group require existing pure signatures in hyperspectral image, such as Pixel Purity Index (PPI) [3], N-FINDR [4], Vertex Component Analysis (VCA) [5], iterative error analysis (IEA) [6]. The algorithms in the second group can process image without requirement of pure signatures, such as single individual evolutionary strategy (SIE) [7], nonnegative matrix factorization (NMF) [8], minimum volume constraint NMF

(MVC-NMF) [10], iterative constrained endmember method (ICE) [11], robust nonnegative matrix factorization [19], minimum volume simplex analysis(MVSA) [20].

2. RELATED WORK

Since there is no requirement on pure signatures for the algorithms in the second group, they have been widely used in hyperspectral unmixing [9] [10] [11]. Among them, NMF with minimum volume constraint (VC) [10] considers simplex volume composed of the unknown endmembers during endmember extraction. VC adopts the commonly used formula to measure the ‘volume’ of simplex enclosed by endmembers. Though VC can efficiently restrain simplex volume, it often involves massive computation. On the other hand, ICE [11] imposes sum of squared distances constraint (SSD) on the original objective function, which can generally achieve satisfied result.

Though algorithms based on VC and SSD have been proposed for several years, there hasn’t been any further analysis and comparison of these two methods. Thus, this paper mainly gives a detailed comparison of these two methods under various situations. The comparison analysis aims to give an instruction on how to choose the constraint under certain situation. To achieve this, the comparison includes flatness of simplex analysis, initialization analysis and robustness to noise analysis.

3. HYPERSPECTRAL UNMIXING ALGORITHM

In this section, we’ll briefly introduce linear mixing model and unmixing algorithms based on VC and SSD.

3.1. Linear Mixture Model (LMM)

LMM [2], [12] assumes that the hyperspectral data is a linear combination of endmember spectra, with the weights being proportions. Mathematically, the model is given as:

$$\begin{aligned} \mathbf{x} &= \mathbf{A}\mathbf{s} + \boldsymbol{\varepsilon}, \\ \text{subjected to: } s_k &\geq 0, k = 1, \dots, M, \sum_{k=1}^M s_k = 1, \end{aligned} \quad (1)$$

where, \mathbf{x} is L -dimensional vector (L is the number of bands) which is one of the pixel in image, \mathbf{s} denotes corresponding abundance, $\boldsymbol{\varepsilon}$ represents possible errors. In real data processing, abundance should satisfy two constraints called as *nonnegative* constraint and *sum-to-one* constraint, as shown in (1).

The matrix involving all pixels in image is shown as Equation(2)

$$\mathbf{X} = \mathbf{A}\mathbf{S} + \boldsymbol{\Theta}, \quad (2)$$

where $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N]$ represents hyperspectral data which is assumed to be composed with material signatures $\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_M]$ and abundance fractions $\mathbf{S} = [s_1, s_2, \dots, s_N]$. N is the number of pixel. M is the number of endmembers. $\boldsymbol{\Theta}$ is the error matrix.

3.2. Constraints on Endmembers

For real data, there are substantial local minimum problem due to non-convexity of unconstrained objection function. As shown in

Figure 1, the red polyline indicates real simplex for hyperspectral data. Meanwhile, blue and black polyline is the solution obtained with different initial value. Since there must be corresponding abundance if endmembers enclose all scatters, it is necessary to put certain constraint on endmembers, as shown in (3):

$$f(\mathbf{A}, \mathbf{S}) = \frac{1}{2} \|\mathbf{X} - \mathbf{A}\mathbf{S}\|_F^2 + \lambda J(\mathbf{A}) \quad (3)$$

where, $J(\mathbf{A})$ is the constraint added to endmember, λ is regularization factor to tradeoff the reconstruction error and constraint.

Before minimizing (3), several preprocessing steps will be taken to remove noises and reduce the dimension of original data, which aims to reduce computation complexity. Then, appropriate optimal strategy is used to minimize (3) and update \mathbf{A} and \mathbf{S} iteratively: first, given endmember matrix \mathbf{A} , calculate the abundance matrix \mathbf{S} by the optimal strategy. Then update \mathbf{A} by fixed \mathbf{S} in the same way. After several iterations, (3) will approach its minimum value with \mathbf{A} and \mathbf{S} well-decided.

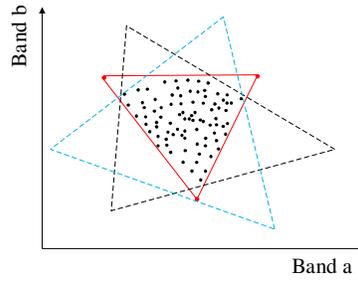


Figure 1. Endmember extraction by unconstrained algorithms. There exist many local minimum solutions due to the non-convex property. Although many solutions have relatively minor linear square error, the obtained points are still far from scattering, which cannot be regarded as appropriate endmembers. The volume and distance constraints are briefed as follows:

1. Minimum Volume Constraint

VC minimize volume [15] of simplex in its model. In VC, the expression of $J_V(\mathbf{A})$ is as for those.

$$J_V(\mathbf{A}) = \frac{1}{2(M-1)!} \det^2 \left(\begin{bmatrix} \mathbf{1}_M^T \\ \mathbf{A} \end{bmatrix} \right) \quad (4)$$

After adding volume constraint, volume of simplex will be compressed as small as possible. Meanwhile, the hyperspectral data reconstructed by the extracted endmembers and corresponding abundance matrix can also close to real data. Therefore, we can get relatively accurate solution for endmember and abundance.

2. Sum of Squared Distance Constraint

In ICE [11], the constraint which minimizes SSD among several endmembers on hyperplane is adopted and given as equation (5). Like VC, SSD can also efficiently control the shape of simplex during the iteration by minimizing the distance between any two endmembers.

$$J_D(\mathbf{A}) = \sum_{k=1}^{M-1} \sum_{l=k+1}^M (\mathbf{a}_k - \mathbf{a}_l)^T (\mathbf{a}_k - \mathbf{a}_l) , \quad (5)$$

Where, M is the number of endmembers, \mathbf{a}_k and \mathbf{a}_l are any two endmember vectors.

In equation (3), the first term intends to decrease reconstruction error, and the second term is used to limit the overall ‘volume’ of simplex contrasted by endmembers. During the optimization process, we can control the tradeoff between spectral reconstruction accuracy and the distance/volume constraint $J(\mathbf{A})$ via λ .

Furthermore, since abundance must satisfy sum-to-one constraints, we can adjust equation (5) by adding $\mathbf{1}_M$ and $\mathbf{1}_N$ to endmember matrix \mathbf{A} and original hyperspectral matrix \mathbf{X} respectively. To control the influence of sum-to-one constraints, we introduce a regulation factor α to $\mathbf{1}_M$ and $\mathbf{1}_N$, as shown in Equation (6):

$$\mathbf{A} \leftarrow \begin{bmatrix} \alpha \mathbf{1}_M^T \\ \mathbf{A} \end{bmatrix} \quad \mathbf{X} \leftarrow \begin{bmatrix} \alpha \mathbf{1}_N^T \\ \mathbf{X} \end{bmatrix} \quad (6)$$

4. SIMPLEX PATTERN AND PARAMETER SELECTION

Since algorithm’s performance fluctuated significantly with the variation of simplex pattern, analysis of simplex pattern should be significant. Two constraints mentioned above differ in most situations. It’s necessary to give a comparative analysis to decide which constraint is more operative given certain simplex pattern.

4.1. Analysis of Simplex Pattern

As both VC and SSD can restrict simplex of endmember closing to original hyperspectral scattering, it is necessary to analyze the equivalence for these two constraints. We mainly do analysis in following two situations: First, for regular or quasi-regular simplex, there is a one-to-one correspondence between volume and distance for simplex, as show in Figure 2(a). In this situation, the VC and SSD have a similar performance.

However, when the simplex is not regular, namely non-regular simplex, the relationship between volume and distance of simplex is indefinite, as shown in Figure 2(b). In this case, it is necessary to analysis which one is better. Since in real hyperspectral data set, simplex for endmembers are not always regular, analysis for non-regular case is very important. For convenience, we define η as shown in (7) to measure degree of flatness in simplex.

$$\eta = \frac{\theta}{\pi} \quad (7)$$

where, θ represents the maximum generalized angle of simplex.

Because of the inequivalence in non-regular simplex, flatness analysis is given to identify the performance for each constraint. Additionally, to further differentiate VC and SSD, random initialization and anti-noise analysis are implemented to compare algorithm performance.

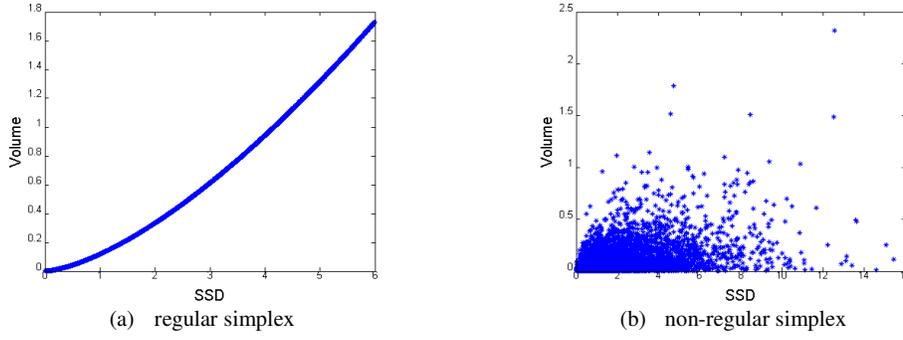


Figure 2. Relationship between volume and distance for regular/non-regular simplex. In the case of regular simplex, the relationship between two constraints is definite, while in the case of non-regular, the relationship is indefinite.

4.2. Parameter Selection

To present a fair comparison, we need to guarantee all variables except for the constraint item to be the same during unmixing process. For further details, (a) the update rule involved in these two constraints is fixed to quadratic programming method and steepest descent method respectively. (b) The regulation factor λ is carefully chosen to ensure similar ratio between constraint value and reconstruction error in each case.

To decide λ_d and λ_v for each constraint, we need to find out the relationship between volume V and sum of squared distance SSD for regular simplex. The relationship satisfies following equation:

$$\frac{V}{SSD} = c(M)d^{M-3} \quad (8)$$

where, M is the number of endmember, c is a variable only related to M , d is the distance between any two endmembers.

Thus, in order to ensure equivalence of these two constraints, we need to make:

$$\frac{\lambda_d}{\lambda_v} = \frac{1}{V / SSD} = \frac{1}{c(M)d^{M-3}} \quad (9)$$

5. EXPERIMENT

In this part, we analyse application range of these two algorithms. Then we apply these two algorithms to real hyperspectral data unmixing and compare the performance. As simplex pattern, initial value and SNR are the most important factors in hyperspectral unmixing, we mainly conduct the comparative analysis in these three aspects.

5.1. Comparison Criterion

In the process of comparing VC and SSD, a suitable comparison metric is needed to measure the unmixing performance. Since, endmember data and abundance map can be transferred to corresponding vector, we adopt angle distance(AD) which measures angular difference between two vectors as criterion, as shown in equation (10). For spectral endmember and abundance map, we refer to angle distance as spectral angle distance(SAD) and abundance angle distance(AAD) separately.

$$SAD = \cos^{-1} \left(\frac{\mathbf{x}_1^T \mathbf{x}_2}{\|\mathbf{x}_1\| \|\mathbf{x}_2\|} \right) \quad (10)$$

where, \mathbf{x}_1 and \mathbf{x}_2 are two transformed vectors. AAD is calculated in the similar way.

5.2. Analysis on Synthetic Data

For synthetic data, we pick several spectra from spectral library as endmembers. Then we create hyperspectral data by multiplying normalized endmember data with abundance map generated according to dirichlet distribution. Additionally, Gaussian noise with certain level is also added to data.

1. Flatness Analysis

In this experiment, we reconstruct hyperspectral data with two bands and three endmembers. To demonstrate unmixing capability, we increase the degree of flatness by certain value at each experiment.

Then, we implement unmixing algorithm based on VC and SSD on hyperspectral data. The initial values for both algorithms are randomly set. We compare experimental result of endmember extraction with chosen spectra from library and compute SAD. The result is shown in figure3.

According to Figure 3, we can see that SAD of VC based algorithm is becoming increasingly bigger compared to SSD with increase of degree of flatness. Whereas the resulting endmember based on VC and SSD are similar when the simplex approaches to the regular form. Therefore, we can draw the conclusion that SSD is better than VC when scattering is *flat*. In real data processing, scattering on hyperplane is usually non-regular, so algorithm based on SSD can handle most of these cases according to the result above.

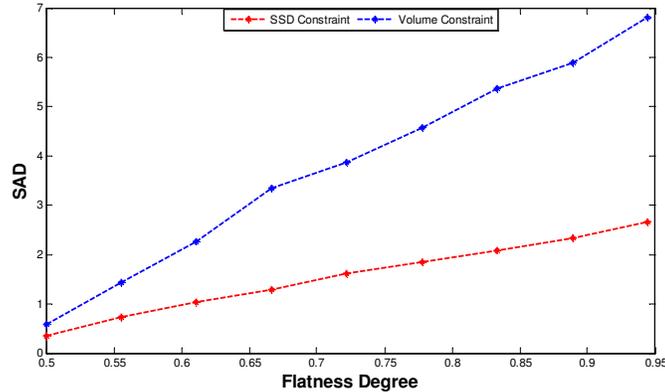


Figure 3 SAD of VC and SSD based algorithm.

2. Random Initialization Analysis

Normally, it is necessary to give an initial value. In this study, we used PPI or N-FINDR to obtain a relatively suitable initial value for following iterations. For abundance matrix, it is often initialized as random value. However, due to the complexity of real data set, these methods do not always perform well on finding well-conditioned initial values. Consequently, the result may be trapped into some local minima. Thus, random initial value analysis can give us a view that which one is more susceptible to ill initial conditions. We carry out the same random initialization on both VC and SSD algorithms, then we compare the result with original true endmember value.

We use random values following Gaussian normal distribution as initial value of endmember and abundance matrices. We conduct 50 comparison experiments with different initial values. Then we compute SAD of resulting endmember data and endmember data in spectral library, as shown in Figure 4.

We can see that endmember extraction result based on SSD is obviously closer to original endmember data. However, some of the result based on VC absolutely deviate from original endmember data. In several cases, though VC-based algorithm also can fit original hyperspectral data well, the evaluated endmember data completely differs from original endmember and consequently abundance matrix obtained by volume constraint is completely wrong.

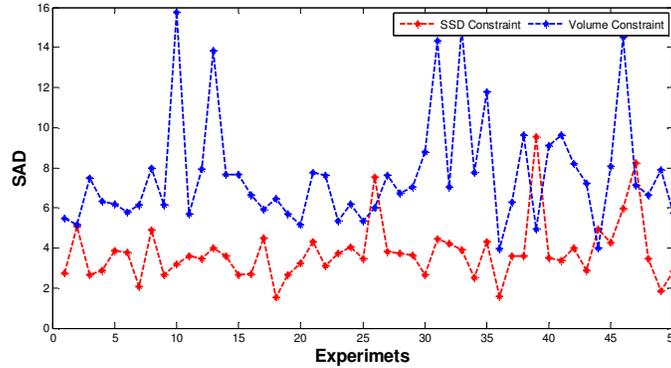
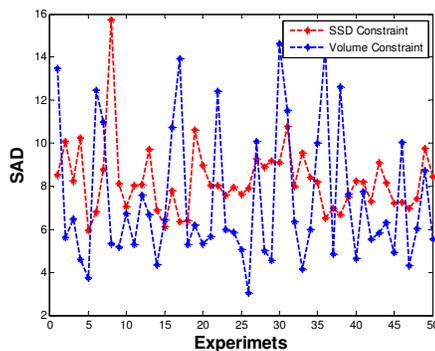


Figure 4 Solution between two constrained algorithms with random initial values. Simulated hyperspectral data consists of 5 bands and 6 endmembers.

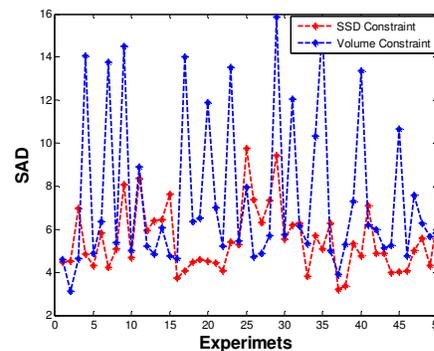
3. Robustness Analysis for Noise

Since real data consists of much noise, original unmixing algorithm is sensitive to noise for the sake of fitting every individual data sample. However, unmixing algorithms based on VC and SSD can be applied to unmixing hyperspectral image by minimizing ‘volume’ of simplex. Thus, reconstructed data will not inevitably approximate all data samples. As a result, these algorithms show strong anti-noise capacity. However, as these two constraints are considered to be inequivalent in many cases, noise-sensitivity may be different with each other. Furthermore, we need to evaluate the suitable degree of SNR for VC and SSD.

We create 50×50 hyperspectral data including 5 bands and 6 endmembers. Then, we add white noise with different levels to synthetic data. The SNR is 10db, 15db, 20db, 30db. During the process of unmixing based on two constraints, we use identical iteration method with same upper limit construction error. We implement two constrained algorithms under each SNR ratio with the same initial value following Gaussian distribution during each experiment.



(a) SAD with 10db SNR



(b) SAD with 15db SNR

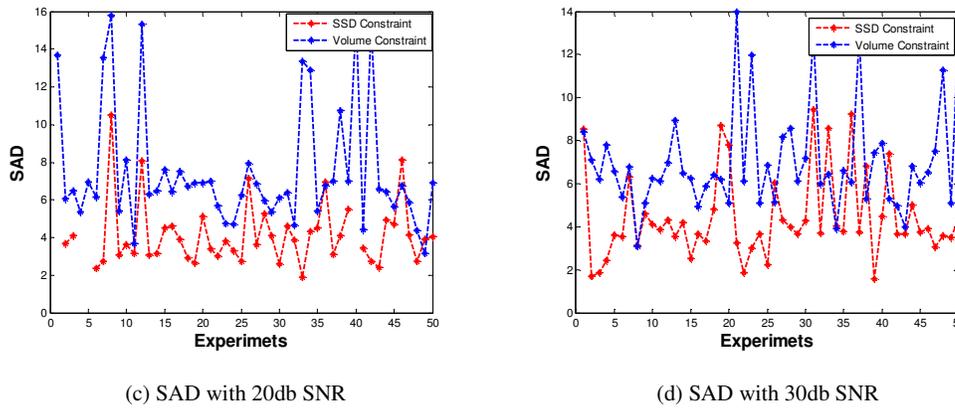


Figure 5 SAD for two constraints with different SNR

We can see from

Figure 5, SAD for VC between extracted and real endmember value change little with the decline of SNR. On the contrary, SAD for SSD fluctuates with the SNR significantly. Thus, algorithm based on VC is more robust than SSD in the sense of noise robustness.

5.3. Analysis on Real Data

After finishing the analysis above, we can conclude that SSD is better than VC when hyper scattering with high degree of flatness or under ill-conditioned initial values. While VC is better in the sense of robustness to noise. However, above-mentioned experiments are based on synthetic data. In this experiment, we will utilize real data(AVIRIS data) to identify these two algorithms.

The used AVIRIS data over Cuprite, Nevada totally contains 400*350 pixels and 50 bands. We do some pre-processes to raw data to reduce computation complexity before iteration. Firstly, we utilize principal component analysis (PCA) [16] to reduce data dimension and select principal band numbers. Secondly, we need to find good initial endmember and abundance value to ensure algorithms can extract real ground objects efficiently. We utilize endmember data extracted by N-FINDR as initial endmember matrix. Since hyperspectral data can be regarded as the product of endmember matrix and abundance matrix, we use unconstrained NMF algorithm to calculate corresponding abundance matrix \mathbf{S} as initial value by fixing endmember abundance \mathbf{A} . In addition, experiment shows that we can achieve much better results by assigning regulation factor λ_v as 0.15 for VC and λ_d as 0.01 for SSD.

Table 1 SAD among different algorithm

	N-FINDR	Volume Constraint	SSD Constraint
Alunite	4.43	5.45	4.00
Kaolinite	3.28	5.29	5.35
Andradite	4.41	5.03	4.35
Nontronite	4.14	7.58	4.19
Muscovite	6.16	2.34	4.71
Chalcedony	3.75	6.86	3.57
Average	4.36	5.43	4.46

Then, we begin to do iteration for two constraint algorithms until it satisfies terminating condition. we can find out best matching mineral obtained by two algorithms via comparing with each mineral reflectance in spectral library [17].

As shown in Figure 6, it's the endmember extraction result based on SSD. Solid line and dashed line represent extraction result obtained by SSD and best matched data in spectral library respectively. The reflectance is normalized to [0, 1]. From several subgraph results, endmembers extracted by SSD are very close to the spectral of real data, like Alunite and Muscovite. The closer they are, the smaller SAD is. Table 1 represents SAD for different algorithms. For some minerals, SSD gives a better solution. For other minerals, VC performs better.

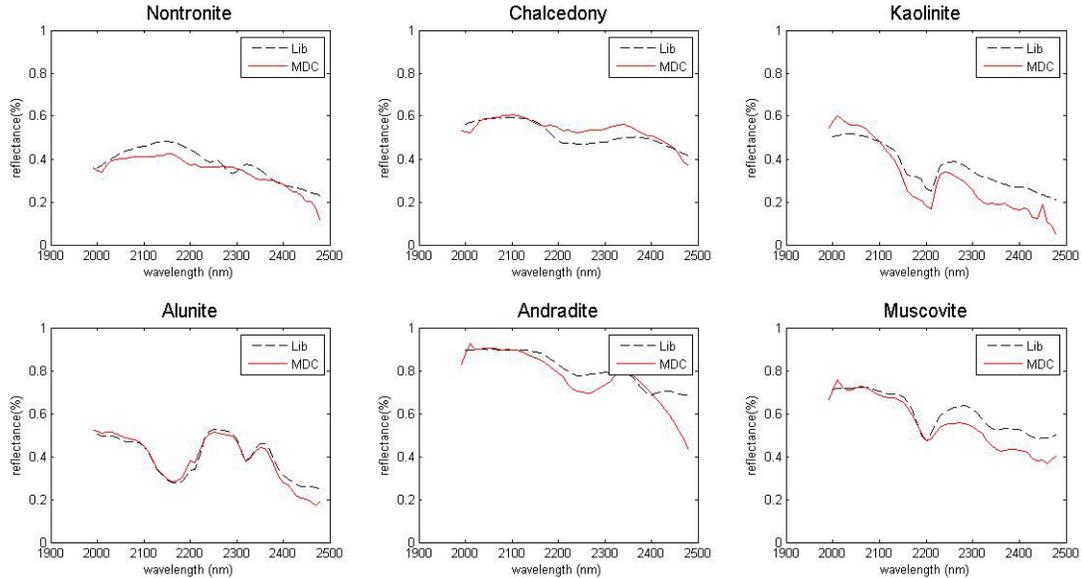


Figure 6 Extracted endmember by SSD.

6. CONCLUSION

In this paper, we analyse VC and SSD algorithm from flatness of simplex, anti-noise and initialization to discriminate these two algorithms. We aim to provide a guidance on which constraint is more suitable under some special conditions.

First, we do analysis for flatness and conduct three comparative experiments using synthetic data. For the pattern of scattering, SSD is better than VC when the scattering is flat. Whereas these two algorithms' performance resemble each other while the simplex is regular. As for initialization, endmember extracted under SSD is closer to original data in random initialization. For anti-noise performance, VC is more robust in different level of noise.

Eventually, on real data, similar solutions can be achieved for these two constrains with well-conditioned initial value. Quantitively, for some minerals, SAD of SSD is smaller, like Chalcedony in Table 1. Yet, for other minerals, like Muscovite, VC works better. Thus, VC and SSD both work similarly in hyperspectral unmixing task.

According to what mentioned above, relatively practical instruction on how to choose constraints can be attained.

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ADVANCEMENT IN THE MOBILE APP REVIEW SYSTEM TO ENHANCE QUALITY OF MOBILE APPLICATIONS

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ABSTRACT

The market for mobile applications has enormously increased in the past five years. According to the App Annie reports, the gross annual revenue is projected to exceed \$189 billion by 2020. Developing and enhancing mobile applications will vastly facilitate in increasing the market for mobile applications. User reviews of mobile apps play a vital role in enhancing the quality of currently existing mobile applications, and, in the development of new high quality mobile applications. Typically, user reviews are either in the form of several lines of text or numerical rating or both. Both forms of rating are meant to aid the prospective users in app installation/purchasing. User reviews also assist the developers in identifying technical glitches (if any) and in providing further advanced updates to the mobile app. The objective of this paper is twofold. Firstly, this paper identifies the currently existing app review systems and analyzes each of these app review systems in detail. Secondly, based on the analysis, we have introduced a four-step app review system that will be instrumental in enhancing the quality of mobile applications. In addition, we have presented the results of a survey (sample size = 100) that was administered to comprehend the effectiveness of the proposed app review system.

KEYWORDS

Mobile App, App Reviews, Mining Reviews, User Rating, User Feedback, User Satisfaction, App Distribution Platforms, IOS & Android OS

1. INTRODUCTION

Mobile applications have significantly increased in number in the past five years and will continue to grow in the coming years [1]. Mobile applications are presently available through various application distribution platforms, such as Apple store, Google Play store, Windows Phone store and many others. These app stores enable users to search, buy and install various software applications. The popularity of mobile apps is growing very rapidly. According to a survey [2], As of March 2017, over 2,800,000 apps are available in the Google play store. The app downloads are anonymous, with around one billion app downloads per month.

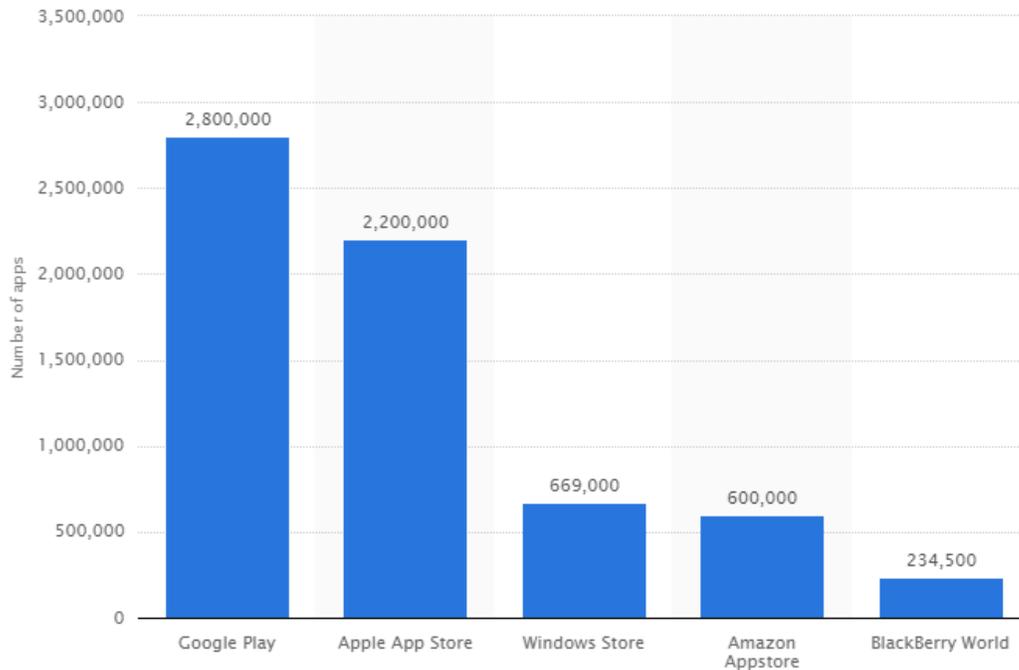


Figure 1: Apps available in various Mobile App Distribution Platforms

Mobile applications have now become a fundamental marketing tool for various businesses. Mobile apps are a smart means to connect to the target audience [3]. Mobile apps aid in: increasing customer engagement, promoting new products and services, online ecommerce transactions, boosting traffic to the products/services and in enhancing end user experience. With the rising popularity of mobile applications, app developers, are, constantly investing a significant amount of time, in gathering and analyzing the user reviews, to, further advance the quality of mobile applications[14]. Several times, the developers experience the absence of effective user reviews. Usually, the user reviews and feedbacks for mobile apps, are, provided by the application distribution platforms, through which the app is distributed. Users who purchase or download an app from a particular platform, can rate and write a review for that app. Ratings and reviews are publicly visible to all the users and developers of that app. Applications with higher ratings will rank higher in the apps list of a distribution platform, which, will increase the visibility of the app to a larger audience. The increased visibility of the app will escalate the number of downloads of the app thereby increasing the revenue. Mobile app reviews primarily consist of the following valuable information: bug reports, issue reports, feature requests among several others [4]. The information gathered from the user reviews is valuable to three stakeholders: App Developers, App Users and App Distribution Platforms.

2. MOBILE APP USER REVIEWS

Mobile app – User reviews play a crucial role in the purchasing/installation decisions of users. Positive reviews and high star ratings by the users will empower an app to be ranked in the list of top 100 apps in the distribution platforms. Ranking success will enable an app to be easily discoverable in the app store; thereby increasing the number of downloads of the app and making

the app successful [13]. The mobile app reviews are useful for three stakeholders: Users, Developers and App Distribution Platforms. After deliberate analysis of the user reviews and its applications, the authors have developed a block diagram depicting the prominence of user reviews for Users, Developers and App Distribution Platforms.

By adapting the approach of the block diagram developed by (Venkata N Inukollu et al, 2017) [5], that depicts the different views of a system, the authors have developed a block diagram that depicts three different views of the mobile app review system: User view, System view and Developer view.

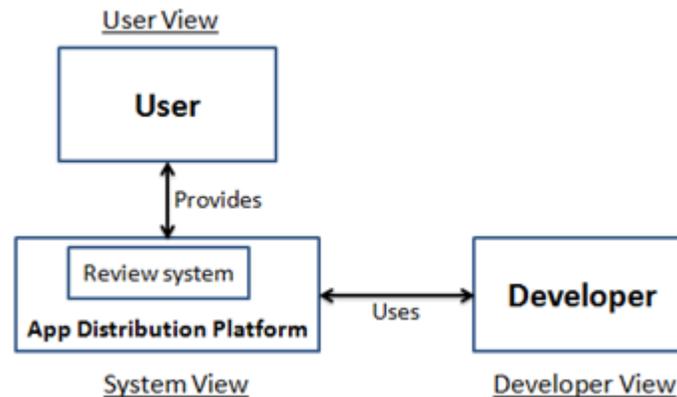


Figure 2: Block Diagram that depicts different views (User, System and Developer view)

- **User View:** This view describes how effectively and efficiently the users are able to provide the reviews to mobile applications (from this section, we will refer mobile application reviews as app reviews). From the user's point of view, a proficient review system is required by the users to effectively express their views and experiences (positive, negative or neutral) with respect to the mobile application.
- **System View:** This view describes about the role of the app distribution platforms such as Apple store, Google Play store, Windows Phone store and many others in providing the collection of app reviews to both the users and the developers. From the system point of view, the system should provide an interface that will be helpful: to the users to express their views and to the developers to understand the views expressed by the users with ease. The review system should be robust and truly simple with the end goal that the App Distribution platforms can gather and present the collected reviews to the users and developers with ease.
- **Developer view:** The developers require an adept platform that will greatly aid in analyzing the requirements and reviews of the users. Understanding the user requirements and reviews will enable the developers in creating innovative applications, and in providing impactful improvements and updates to the existing applications.

Based on the analysis of the roles and requirements of the three stakeholders (classified above), the authors felt the need for an improved review system that will be greatly beneficial both to the users and the developers.

3. RELATED WORK

Understanding user requirements is fundamental for developing superior quality mobile apps and enhancing the quality of currently existing mobile apps. User requirements for mobile applications can be collected in the form of user reviews for the apps. An app is downloaded by several users for versatile purposes. Every user is unique and has a unique perspective, which is applicable even to reviews about mobile applications. Different segments of users, view/download an app for different purposes. For example, students use the apps for education and entertainment purposes, whereas businesses use the apps for promoting/marketing their products and services. Thus, the user segments have been classified as follows: Businesses, Employees/Working Individuals, Students and Researchers. The classified segments can provide feedback related to technical features of an app and also about non-technical features. Technical features include Security, Compatibility, Stability, Accuracy and Bugs among several others whereas; non-technical aspects include Performance, Attractiveness, Costs and features. Currently, the user feedback with respect to different features is analyzed by using the following review systems:

a) Star Rating

Star rating is one of the most commonly used review system. The star rating generally ranges from 1 to 5. Users' rate the app based on their experience with the app. High star rating indicates positive reviews. Based on the inputs from the users, the average rating is calculated and is displayed as a single number (4.2/5) as shown in figure 3. Along with the star rating, users will also be provided with a text area to express their comments/reviews in the text form.



Figure 3: Five Star Rating

b) Polling

The Polling system is one of the most simple review systems. Here the users provide the reviews in a few clicks (depending on the number of questions). The users are prompted for their responses in the form of Yes/no (or) True/false answer format. Based on the responses provided by the users, the results/numbers are calculated and the resultant output is demonstrated as Pie charts, Bar graphs, Percentages and other pictorial forms. User's opinions cannot be justified with the resultant numbers as it is just a Polling system and users do not explain their choices of options or their opinions in any textual form.

Do you believe in horoscopes?

Yes

No

Maybe

Vote

Figure 1: Polling System

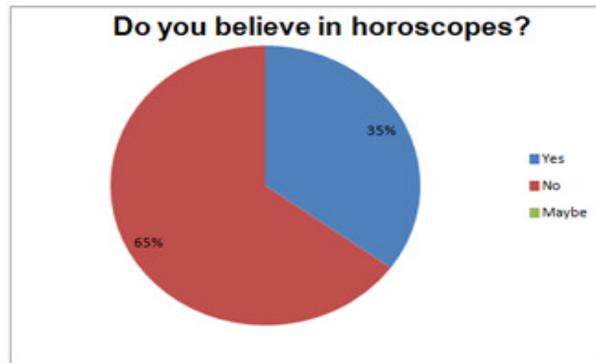
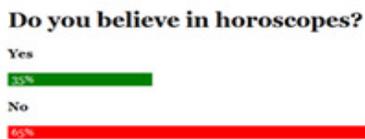


Figure 5: Polling Results

c) Voting

Voting system is similar to Polling system with few differences. The major difference between Polling and Voting is: Unlike the Polling system where the user has only 2 options (T/F or Y/N) to choose from, the voting system provides the users with more than two options to choose from. Also, the count for each vote will be displayed in this case.

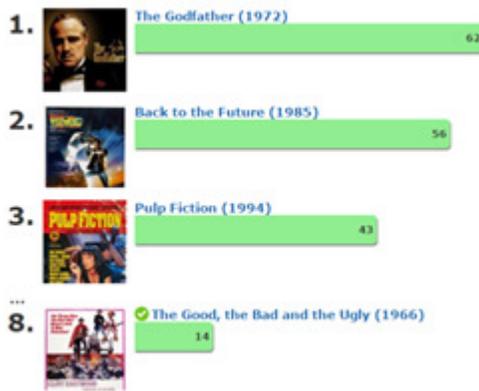


Figure 6 Voting Results

d) Comment

Users will be given an option to comment along with their rating. Using this option, users can support/justify their rating by describing few key points as shown below. The comments assist: the developers in providing necessary updates to the apps and the users in installation/ purchasing decisions i.e. whether an app is of their expected standards and is worth downloading/purchasing.

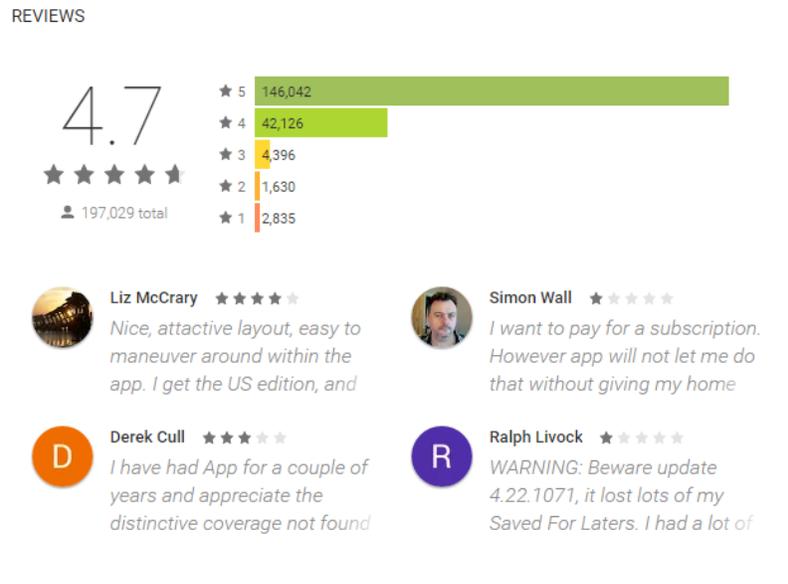


Figure 7: Comment along with 5-Star Rating system

Most of the App Distribution platforms commonly use the 5-star rating system along with the comment system. However, this review system is not comprehensive and robust, and thus, does not provide effective output to the developers and the users. According to the study[6], they analyzed 10k+ mobile apps in Google play store and found an issue related to the current 5-star rating system. They found that the app stores are very resilient — once the app reaches substantial number of ratings, then the Google play store rating system is resilient to fluctuations because the current rating system provides the average of the ratings of the entire life cycle of that app. This provides the inappropriate details to both users and developers. As the current app rating system is five star rating system, it does not depicts the actual satisfaction of user. The current rating system does not provide any facility for the technical people in providing their feedback. These are some of the drawbacks that the current app rating system has. On the other hand, we have external tools, which analyze the data [7] and provide the results to the developers. Also, there are empirical studies on mining and summarizing the user reviews based on various perspectives [8][9]. Phong Minh Vu et al. performed the mining of user reviews by a keyword-based approach and Minqing Hu et al. worked on the mining and summarizing of customer reviews. The above-mentioned research methods follow an arduous approach and require installation of additional external tools for the purpose of analysis of user reviews. By considering all the limitations, we have proposed a new approach, which helps users in providing as well as developers in getting the appropriate reviews for the mobile app.

4. PROPOSED SYSTEM

Considering the limitations of the existing systems in collecting the user reviews, the authors have devised a new and enhanced review system, which will be useful to all the three stakeholders (Users, Developers and Application distribution platforms).

DEVELOPER-CUSTOMER-REVIEW (DCR) CYCLE:

To achieve an enhanced and superior quality app, it is greatly required to collect the reviews from the users in an effective and proficient format. To accomplish the above-mentioned objective, the authors have introduced the Developer-Customer-Review (DCR) cycle, shown in figure 7.

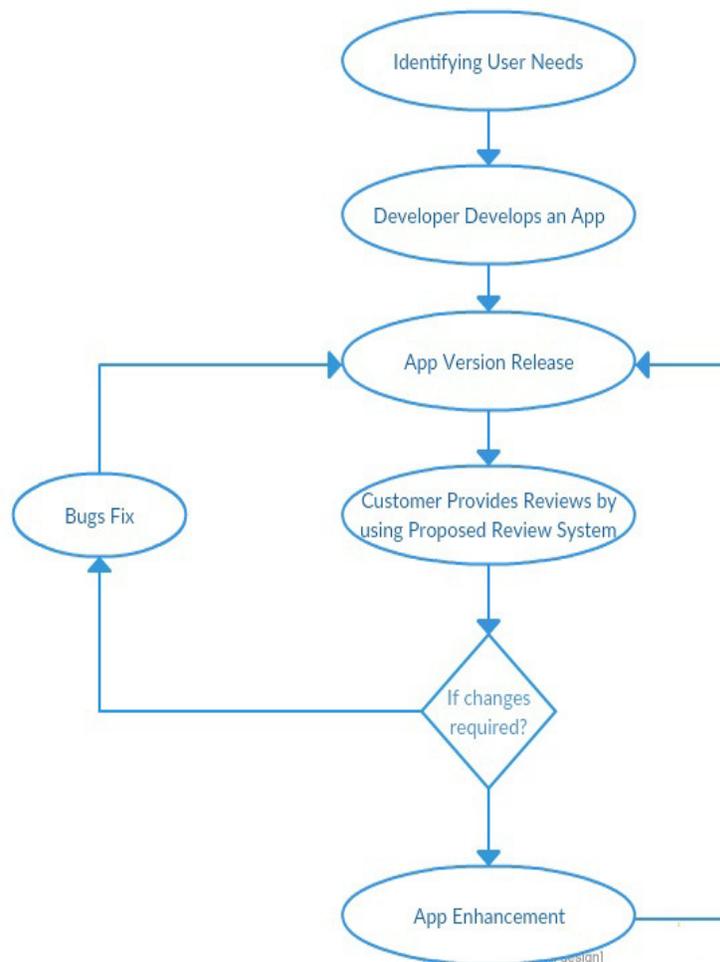


Figure 8: Developer-Customer Cycle (DCR)

In the first step, before the commencement of the actual development, user needs will be identified and gathered. Based on the inputs from the users, the actual development will begin. Once the app is developed, the app will be released as versions. The actual review process starts here. After the collection of various reviews, the developers will be provided with the gathered

information in different formats. Post analysis of reviews, if any changes are required, the appropriate process will be initiated (Bug fixes or App enhancement).

The proposed review system meets the expectations of both the technical and non-technical user segments, as the system includes both the generalized review and the technical review. Technical savvy users can provide the reviews by choosing the technical terms, and the non-technical users can skip those steps. The process is explained in detail in the column: Steps involved in proposed review system.

The proposed review system is beneficial to both the users and the app developers. Developers can merely look at the reported technical defective factors, and, can rapidly estimate the glitches in the app, and, can very quickly decide on the next move to remove the bugs and to enhance the performance of the app.

In the proposed review system, the user has to complete only four steps to finish the review process, and, thus, is fairly simple and easy to use. A non-technical user does not require reviewing the technical factors and is provided with an optional comment box which can be used to provide user reviews not related to technical factors. A technical user can provide a technical review by choosing out of the 8 relevant technical defective factors including: crashing, compatibility, interface, security, hidden cost, performance, error reports, and resource heavy. The list of the technical defective factors is the resultant of the research conducted by Thung et al. [10] Tian et al. [11], who performed mining of information and categorized the issues into various complaint types. The related information from various research efforts [10][11][12] has greatly helped us in creating the list of different technical defective factors.

STEPS INVOLVED IN PROPOSED REVIEW SYSTEM:

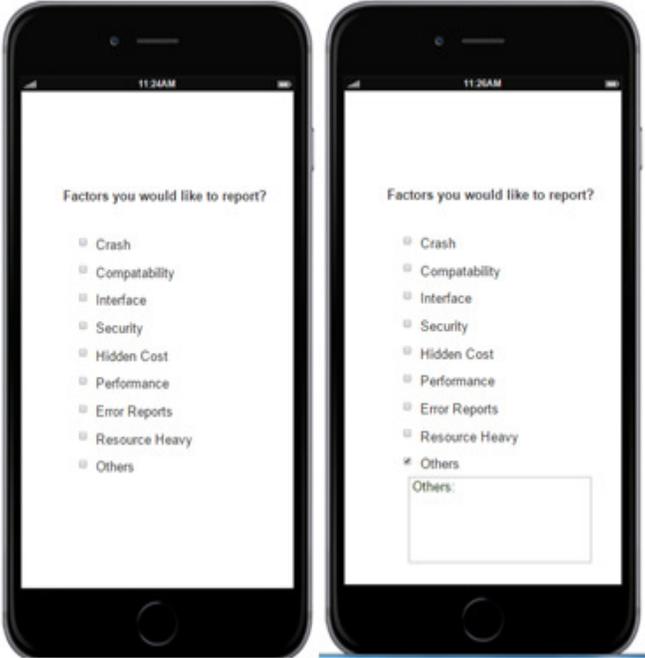
Step 1:In this step, the system will ask whether the user is willing to provide a review. If the user is interested in providing a review, the user can select the accept button, else, the user can opt for the reject button.

Step 2:The user can rate his level of satisfaction with regards to using the app and the level varies from user to user. In this step, a user can give his satisfaction level on a scale of 1 to 5. According to [13]: Companies deploying emotional-connection-based strategies, and, metrics to design, prioritize, and measure the customer experience will definitely have increasing number of customers. This particular step of collecting the satisfaction levels, captures, both the positive and negative emotions of the users towards the app. This step will greatly help in recording the feelings and emotions of the users. Gathering and measuring the satisfaction levels of users is only half of the story, and the next half of the story would include: gathering of information regarding the list of technical factors that are not up to the expected standards and thereby have lowered the satisfaction levels of the users.

Step3:In step 2, both the technical and non-technical users provide their reviews based on their overall level of satisfaction with the app. In Step 3, as shown below, the users can select from the list of technical factors that they would like to report. The technical issues most commonly faced by the users are short-listed in this step. Here, non-technical users who are unfamiliar with the technical terms can choose “Others” as an option and can type their comments in the provided comments box (as shown in the below figure).



Step 1 & Step 2

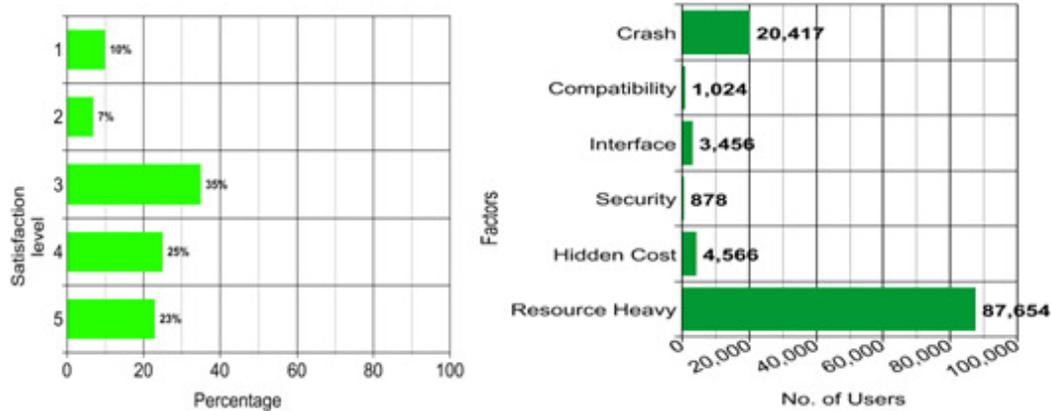


Step 3

Final Output:

Based on the reviews provided by the users, the output is generated in the form of two graphs that can be comprehended by both technical savvy users and non-technical users. Below shown is a sample of output graphs based on the user reviews.

For non-technical users, the Satisfaction-level graph will be more comprehensible and helpful in assisting decisions related to downloading or updating the desired app.



For technical savvy users, the second graph along with the first (Satisfaction level) graph will aid in the comprehension process of the nature and performance of the app. The two graphs combined together will aid the users in arriving at the best decision regarding the installation / updating of an app.

5. SURVEY RESULTS

We have conducted a Literature Survey to determine the issues related to the current app review system and to comprehend the effectiveness of the proposed review system in terms of the ease of the users with the proposed review system. We then developed a draft of the survey. A panel of experts assembled and reviewed the draft survey (Panel includes four professors from the University, Software Engineering Department). According to the feedback from the experts, we have modified the survey and finalized. This finalized survey consists of three sections: First section discusses about the Methodology we have chosen for the review system; second section discusses about the User Interface of our proposed review system and final section will includes the questions that will ask the feedback for further improvements of our proposed system. These questions are categorized into three types Agree/Disagree, Yes/No and Feedback.

The survey was conducted with a sample size of 100 students (3 sections, each section includes 35 students (approx) and each section at a time) in our university. Before providing the survey handouts, we have presented our topic to the students for better understanding of our concept. Then we distributed the handouts to all the students in the section. After survey was conducted successfully in all the three sections, we have collected the papers and performed analysis on the survey feedback. After the analysis, we got to know that 78% of the people liked our proposed review system compared with the current app review system. The output was generated in the form of a pie chart, is as shown below. In the below pie chart, the green color demonstrates a positive response and the red color demonstrates a negative response.

Survey Summary Stats

Total Hits

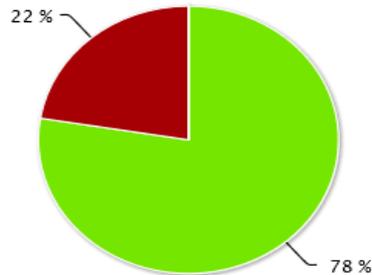


Figure 9. Summary of Survey Results

6. CONCLUSION & FUTURE WORK

In this research effort, we have proposed a theoretical approach that will significantly improve the mobile app review system. All the users (that we have mentioned in section 2) have many advantages by adopting this approach. We have also provided the usability test results by conducting survey. By analyzing the results, it was clearly mentioned that most of the people liked the proposed review system. By considering these results, In future we are going to develop a system that depicts the proposed review system and will conduct usability analysis on the system.

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ON ANNIHILATOR OF INTUITIONISTIC FUZZY SUBSETS OF MODULES

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ABSTRACT

In the theory of rings and modules there is a correspondence between certain ideals of a ring R and submodules of an R -module that arise from annihilation. The submodules obtained using annihilation, which correspond to prime ideals play an important role in decomposition theory. In this paper, we attempt to intuitionistic fuzzify the concept of annihilators of subsets of modules. We investigate certain characterization of intuitionistic fuzzy annihilators of subsets of modules. Using the concept of intuitionistic fuzzy annihilators, intuitionistic fuzzy prime submodules and intuitionistic fuzzy annihilator ideals are defined and various related properties are established.

KEYWORDS

Intuitionistic fuzzy submodule, intuitionistic fuzzy prime submodule, intuitionistic fuzzy ideal, intuitionistic fuzzy annihilator, semiprime ring.

1. INTRODUCTION

The concept of intuitionistic fuzzy sets was introduced by Atanassov [1], [2] as a generalization to the notion of fuzzy sets given by Zedah [16]. Biswas was the first to introduce the intuitionistic fuzzification of the algebraic structure and developed the concept of intuitionistic fuzzy subgroup of a group in [5]. Hur and others in [8] defined and studied intuitionistic fuzzy subrings and ideals of a ring. In [7] Davvaz et al. introduced the notion of intuitionistic fuzzy submodules which was further studied by many mathematicians (see [4], [9], [12], [13], [14]).

The correspondence between certain ideals and submodules arising from annihilation plays a vital role in the decomposition theory and Goldie like structures (see [6]). A detailed study of the fuzzification of this and related concepts can be found in [10], [11] and [15]. Intuitionistic fuzzification of such crisp sets leads us to structures that can be termed as intuitionistic fuzzy prime submodules. In this paper, we attempt to define annihilator of an intuitionistic fuzzy subset of a module using the concept of residual quotients and investigate various characteristic of it. This concept will help us to explore and investigate various facts about the intuitionistic fuzzy aspects of associated primes, Godlie like structures and singular ideals.

2. PRELIMINARIES

Throughout this section, R is a commutative ring with unity 1 , $1 \neq 0$, M is a unitary R -module and θ is the zero element of M . The class of intuitionistic fuzzy subsets of X is denoted by $\text{IFS}(X)$.

Definition (2.1)[4] Let R be a ring. Then $A \in \text{IFS}(R)$ is called an intuitionistic fuzzy ideal of R if for all $x, y \in R$ it satisfies

- (i) $\mu_A(x-y) \geq \mu_A(x) \wedge \mu_A(y)$, $\nu_A(x-y) \leq \nu_A(x) \vee \nu_A(y)$;
- (ii) $\mu_A(xy) \geq \mu_A(x) \vee \mu_A(y)$, $\nu_A(xy) \leq \nu_A(x) \wedge \nu_A(y)$.

The class of intuitionistic fuzzy ideals of R is denoted by $\text{IFI}(R)$.

Definition (2.2)[4] An intuitionistic fuzzy set $A = (\mu_A, \nu_A)$ of an R -module M is called an intuitionistic fuzzy submodule (IFSM) if for all $x, y \in M$ and $r \in R$, we have

- (i) $\mu_A(\theta) = 1$, $\nu_A(\theta) = 0$;
- (ii) $\mu_A(x+y) \geq \mu_A(x) \wedge \mu_A(y)$, $\nu_A(x+y) \leq \nu_A(x) \vee \nu_A(y)$;
- (iii) $\mu_A(rx) \geq \mu_A(x)$, $\nu_A(rx) \leq \nu_A(x)$.

The class of intuitionistic fuzzy submodules of M is denoted by $\text{IFM}(M)$.

Definition (2.3)[2, 12] Let $\alpha, \beta \in [0, 1]$ with $\alpha + \beta \leq 1$. An intuitionistic fuzzy point, written as $x_{(\alpha, \beta)}$, is defined to be an intuitionistic fuzzy subset of X , given by

$$x_{(\alpha, \beta)}(y) = \begin{cases} (\alpha, \beta) & ; \text{if } y = x \\ (0, 1) & ; \text{if } y \neq x \end{cases}. \text{ We write } x_{(\alpha, \beta)} \in A \text{ if and only if } x \in C_{(\alpha, \beta)}(A),$$

where $C_{(\alpha, \beta)}(A) = \{x \in X : \mu_A(x) \geq \alpha \text{ and } \nu_A(x) \leq \beta\}$ is the (α, β) -cut set (crisp set) of the intuitionistic fuzzy set A in X .

Definition (2.4)[9] Let M be an R -module and let $A, B \in \text{IFM}(M)$. Then the sum $A + B$ of A and B is defined as

$$\mu_{A+B}(x) = \begin{cases} \mu_A(y) \wedge \mu_B(z) & ; \text{if } x = y + z \\ 0 & ; \text{otherwise} \end{cases} \quad \text{and} \quad \nu_{A+B}(x) = \begin{cases} \nu_A(y) \vee \nu_B(z) & ; \text{if } x = y + z \\ 1 & ; \text{otherwise} \end{cases}$$

Then, $A + B \in \text{IFM}(M)$.

Definition (2.5) Let M be an R -module and let $A \in \text{IFS}(R)$ and $B \in \text{IFM}(M)$. Then the product AB of A and B is defined as

$$\mu_{AB}(x) = \begin{cases} \mu_A(r) \wedge \mu_B(m) & ; \text{if } x = rm \\ 0 & ; \text{otherwise} \end{cases} \quad \text{and} \quad \nu_{AB}(x) = \begin{cases} \nu_A(r) \vee \nu_B(m) & ; \text{if } x = rm \\ 1 & ; \text{otherwise} \end{cases}, r \in R, m \in M.$$

Clearly, $AB \in \text{IFM}(M)$.

Definition (2.6) [9, 12] Let M be an R -module and let $A, B \in \text{IFM}(M)$. Then the product AB of A and B is defined as

$$\mu_{AB}(x) = \begin{cases} \mu_A(y) \wedge \mu_B(z) & ; \text{if } x = yz \\ 0 & ; \text{otherwise} \end{cases} \quad \text{and} \quad \nu_{AB}(x) = \begin{cases} \nu_A(y) \vee \nu_B(z) & ; \text{if } x = yz \\ 1 & ; \text{otherwise} \end{cases}, \text{ where } y, z \in M.$$

Definition (2.7) [3] An $P \in IFI(R)$ is called an intuitionistic fuzzy prime ideal of R if for any $A, B \in IFI(R)$ the condition $AB \subseteq P$ implies that either $A \subseteq P$ or $B \subseteq P$.

Definition (2.8) Let X be a non empty set and $A \subset X$. Then an intuitionistic fuzzy set

$\chi_A = (\mu_{\chi_A}, \nu_{\chi_A})$ is called an intuitionistic fuzzy characteristic function and is defined as

$$\mu_{\chi_A}(x) = \begin{cases} 1 & ; \text{if } x \in A \\ 0 & ; \text{if } x \notin A \end{cases} \quad \text{and} \quad \nu_{\chi_A}(x) = \begin{cases} 0 & ; \text{if } x \in A \\ 1 & ; \text{if } x \notin A \end{cases}.$$

Definition (2.9) Let M be R -module and χ_θ is an IFS on M defined as $\chi_\theta(x) = (\mu_{\chi_\theta}(x), \nu_{\chi_\theta}(x))$,

$$\text{where } \mu_{\chi_\theta}(x) = \begin{cases} 1 & ; \text{if } x = \theta \\ 0 & ; \text{if } x \neq \theta \end{cases} \quad \text{and} \quad \nu_{\chi_\theta}(x) = \begin{cases} 0 & ; \text{if } x = \theta \\ 1 & ; \text{if } x \neq \theta \end{cases}$$

and χ_0 and χ_R are IFSs on R defined by

$$\chi_0(r) = (\mu_{\chi_0}(r), \nu_{\chi_0}(r)) \quad \text{and} \quad \chi_R(r) = (\mu_{\chi_R}(r), \nu_{\chi_R}(r)), \text{ where}$$

$$\mu_{\chi_0}(r) = \begin{cases} 1 & ; \text{if } r = 0 \\ 0 & ; \text{if } r \neq 0 \end{cases}; \quad \nu_{\chi_0}(r) = \begin{cases} 0 & ; \text{if } r = 0 \\ 1 & ; \text{if } r \neq 0 \end{cases} \quad \text{and} \quad \mu_{\chi_R}(r) = 1; \quad \nu_{\chi_R}(r) = 0, \quad \forall r \in R.$$

Theorem (2.10) Let $x \in R$ and $\alpha, \beta \in (0, 1]$ with $\alpha + \beta \leq 1$. Then $\langle x_{(\alpha, \beta)} \rangle = (\alpha, \beta)_{\langle x \rangle}$,

where $(\alpha, \beta)_{\langle x \rangle}(y) = \begin{cases} (\alpha, \beta) & ; \text{if } y \in \langle x \rangle \\ (0, 1) & ; \text{if } y \notin \langle x \rangle \end{cases}$ is called the (α, β) -level (or cut set) intuitionistic

fuzzy ideal corresponding to $\langle x \rangle$.

Proof. Case(i) When $y \in \langle x \rangle$ and let $y = x^n$, for some positive interget n , then

$$\mu_{(\alpha, \beta)_{\langle x \rangle}}(y) = \alpha = \mu_{x_{(\alpha, \beta)}}(x) \leq \mu_{x_{(\alpha, \beta)}}(x^n) = \mu_{x_{(\alpha, \beta)}}(y) \quad \text{and}$$

$$\nu_{(\alpha, \beta)_{\langle x \rangle}}(y) = \beta = \nu_{x_{(\alpha, \beta)}}(x) \geq \nu_{x_{(\alpha, \beta)}}(x^n) = \nu_{x_{(\alpha, \beta)}}(y)$$

Case(ii) When $y \notin \langle x \rangle$, then

$$\mu_{(\alpha, \beta)_{\langle x \rangle}}(y) = 0 = \mu_{x_{(\alpha, \beta)}}(y) \quad \text{and} \quad \nu_{(\alpha, \beta)_{\langle x \rangle}}(y) = 1 = \nu_{x_{(\alpha, \beta)}}(y).$$

Thus in both the cases we find that $(\alpha, \beta)_{\langle x \rangle} \subseteq x_{(\alpha, \beta)}$.

Now $\langle x_{(\alpha, \beta)} \rangle = \bigcap \{A : A \in IFI(R) \text{ such that } x_{(\alpha, \beta)} \subseteq A\}$ implies that $(\alpha, \beta)_{\langle x \rangle} = \langle x_{(\alpha, \beta)} \rangle$.

3. ANNIHILATOR OF INTUITIONISTIC FUZZY SUBSET OF R-MODULE

Throughout this section, R is a commutative ring with unity 1, $1 \neq 0$, M is a unitary R -module and θ is the zero element of M .

Definition (3.1) Let M be a R -module and $A \in IFS(M)$, then the annihilator of A is denoted by $\text{ann}(A)$ and is defined as: $\text{ann}(A) = \bigcup \{B : B \in IFS(R) \text{ such that } BA \subseteq \chi_\theta\}$.

Lemma (3.2) Let M be a R -module, then $\text{ann}(\chi_\theta) = \chi_R$.

Proof. Since $\chi_\theta \in IFS(M)$ and $\chi_R \in IFS(R)$, therefore, $\chi_R \chi_\theta \in IFS(M)$.

Also, $\chi_R \chi_\theta(x) = (\mu_{\chi_R \chi_\theta}(x), \nu_{\chi_R \chi_\theta}(x))$, where

$\mu_{\chi_R \chi_\theta}(x) = \vee \{\chi_R(r) \wedge \chi_\theta(m) : r \in R, m \in M, rm = x\}$ and

$\nu_{\chi_R \chi_\theta}(x) = \wedge \{\chi_R(r) \vee \chi_\theta(m) : r \in R, m \in M, rm = x\}$.

$$\begin{aligned} \text{Now, } \mu_{\chi_R \chi_\theta}(x) &= \vee \{\chi_R(r) \wedge \chi_\theta(m) : r \in R, m \in M, rm = x\} \\ &= \vee \{\chi_\theta(m) : r \in R, m \in M, rm = x\} \\ &= \begin{cases} 1 & ; \text{ if } x = \theta & [\because \text{ if } x = \theta \Rightarrow \text{one } m = \theta] \\ 0 & ; \text{ if } x \neq \theta & [\because \text{ if } x \neq \theta \Rightarrow m \neq \theta] \end{cases} \end{aligned}$$

$$\begin{aligned} \text{Also, } \nu_{\chi_R \chi_\theta}(x) &= \wedge \{\chi_R(r) \vee \chi_\theta(m) : r \in R, m \in M, rm = x\} \\ &= \wedge \{\chi_\theta(m) : r \in R, m \in M, rm = x\} \\ &= \begin{cases} 0 & ; \text{ if } x = \theta \\ 1 & ; \text{ if } x \neq \theta \end{cases} \end{aligned}$$

Thus, $\chi_R \chi_\theta(x) = \chi_\theta(x)$.

So, $\chi_R \subseteq \bigcup \{B : B \in IFS(R), B \chi_\theta \subseteq \chi_\theta\} = \text{ann}(\chi_\theta) \subseteq \chi_R$.

Hence $\text{ann}(\chi_\theta) = \chi_R$.

Lemma (3.3) Let M be a R -module and $A \in IFS(M)$, then $\chi_0 \subseteq \text{ann}(A)$.

Proof. Now, $\mu_{\chi_0 A}(x) = \vee \{\mu_{\chi_0}(r) \wedge \mu_A(m) : r \in R, m \in M, rm = x\}$

When $x \neq \theta \Rightarrow r \neq 0 ; \forall r \in R$, such that $rm = x$

$\Rightarrow \mu_{\chi_0}(r) = 0 \forall r \in R$, such that $rm = x$. So, $\mu_{\chi_0 A}(x) = 0 = \mu_{\chi_0}(x)$.

When $x = \theta \Rightarrow \mu_{\chi_0 A}(\theta) \leq 1 = \mu_{\chi_0}(\theta)$. Thus, $\mu_{\chi_0 A}(x) \leq \mu_{\chi_0}(x)$.

Similarly, we can show that $\nu_{\chi_0 A}(x) \geq \nu_{\chi_0}(x)$. Therefore, $\chi_0 A \subseteq \chi_\theta$.

Hence $\chi_0 \subseteq \bigcup \{B : B \in IFS(R) \text{ such that } BA \subseteq \chi_\theta\} = \text{ann}(A)$.

Lemma (3.4) Let M be a R -module and $A, B \in IFS(M)$. If $A \subseteq B$, then $\text{ann}(B) \subseteq \text{ann}(A)$.

Proof. Let $A, B \in IFS(M)$, $C \in IFS(R)$. Then $CA(x) = (\mu_C(x), \nu_{CA}(x))$, where

$\mu_{CA}(x) = \vee \{\mu_C(r) \wedge \mu_A(m) : r \in R, m \in M, rm = x\}$ and

$\nu_{CA}(x) = \wedge \{\nu_C(r) \vee \nu_A(m) : r \in R, m \in M, rm = x\}$.

Now, $\mu_C(r) \wedge \mu_A(m) \leq \mu_C(r) \wedge \mu_B(m)$

$$\begin{aligned} \text{Therefore, } \mu_{CA}(x) &= \vee \{\mu_C(r) \wedge \mu_A(m) : r \in R, m \in M, rm = x\} \\ &\leq \vee \{\mu_C(r) \wedge \mu_B(m) : r \in R, m \in M, rm = x\} \\ &= \mu_{CB}(x). \end{aligned}$$

Similarly, we can show that $v_{CA}(x) \geq v_{CB}(x)$. Thus $CA \subseteq CB$.

So, $CB \subseteq \mathcal{X}_\theta \Rightarrow CA \subseteq \mathcal{X}_\theta$.

$$\begin{aligned} \therefore \bigcup \{C : C \in IFS(R) \text{ such that } CB \subseteq \mathcal{X}_\theta\} &\subseteq \bigcup \{C : C \in IFS(R) \text{ such that } CA \subseteq \mathcal{X}_\theta\} \\ \Rightarrow \text{ann}(B) &\subseteq \text{ann}(A). \end{aligned}$$

Theorem (3.5) Let M be a R -module and $A \in IFS(M)$. Then

$$\text{ann}(A) = \bigcup \{r_{(\alpha, \beta)} : r \in R, \alpha, \beta \in [0, 1] \text{ with } \alpha + \beta \leq 1 \text{ such that } r_{(\alpha, \beta)}A \subseteq \mathcal{X}_\theta\}$$

Proof. We know that

$$\{r_{(\alpha, \beta)} : r \in R, \alpha, \beta \in [0, 1] \text{ with } \alpha + \beta \leq 1 \text{ such that } r_{(\alpha, \beta)}A \subseteq \mathcal{X}_\theta\} \in IFS(R)$$

$$\begin{aligned} \therefore \{r_{(\alpha, \beta)} : r \in R, \alpha, \beta \in [0, 1] \text{ with } \alpha + \beta \leq 1 \text{ such that } r_{(\alpha, \beta)}A \subseteq \mathcal{X}_\theta\} \\ \subseteq \{B : B \in IFS(R) \text{ such that } BA \subseteq \mathcal{X}_\theta\} \end{aligned}$$

$$\begin{aligned} \Rightarrow \bigcup \{r_{(\alpha, \beta)} : r \in R, \alpha, \beta \in [0, 1] \text{ with } \alpha + \beta \leq 1 \text{ such that } r_{(\alpha, \beta)}A \subseteq \mathcal{X}_\theta\} \\ \subseteq \bigcup \{B : B \in IFS(R) \text{ such that } BA \subseteq \mathcal{X}_\theta\} = \text{ann}(A). \end{aligned}$$

Let $B \in IFS(R)$ such that $BA \subseteq \mathcal{X}_\theta$.

Let $r \in R$ and $B(r) = (\alpha, \beta)$, i.e., $\mu_B(r) = \alpha$ and $v_B(r) = \beta$.

Now, $(r_{(\alpha, \beta)}A)(x) = (\mu_{r_{(\alpha, \beta)}A}(x), v_{r_{(\alpha, \beta)}A}(x))$, where

$$\begin{aligned} \mu_{r_{(\alpha, \beta)}A}(x) &= \vee \{ \mu_{r_{(\alpha, \beta)}}(s) \wedge \mu_A(y) : s \in R, y \in M, sy = x \} \\ &\leq \vee \{ \mu_B(r) \wedge \mu_A(y) : y \in M, ry = x \} [\because \mu_{r_{(\alpha, \beta)}}(s) \leq \mu_B(r) = \alpha] \\ &= \vee \{ \mu_B(s) \wedge \mu_A(y) : s \in R, y \in M, sy = x \} \\ &= \mu_{BA}(x) \\ &\leq \mu_{\mathcal{X}_\theta}(x) \end{aligned}$$

i.e., $\mu_{r_{(\alpha, \beta)}A}(x) \leq \mu_{\mathcal{X}_\theta}(x)$, $\forall x \in M$.

Similarly, we can show that $v_{r_{(\alpha, \beta)}A}(x) \geq v_{\mathcal{X}_\theta}(x)$, $\forall x \in M$. Thus, $r_{(\alpha, \beta)}A \subseteq \mathcal{X}_\theta$.

So, $\text{ann}(A) \subseteq \bigcup \{r_{(\alpha, \beta)} : r \in R, \alpha, \beta \in [0, 1] \text{ with } \alpha + \beta \leq 1 \text{ such that } r_{(\alpha, \beta)}A \subseteq \mathcal{X}_\theta\}$

Hence $\text{ann}(A) = \bigcup \{r_{(\alpha, \beta)} : r \in R, \alpha, \beta \in [0, 1] \text{ with } \alpha + \beta \leq 1 \text{ such that } r_{(\alpha, \beta)}A \subseteq \mathcal{X}_\theta\}$.

Theorem (3.6) Let M be a R -module and $A \in IFS(M)$. Then $\text{ann}(A)A \subseteq \mathcal{X}_\theta$.

Proof. Now, $(\text{ann}(A)A)(x) = (\mu_{\text{ann}(A)A}(x), v_{\text{ann}(A)A}(x))$, where

$$\mu_{\text{ann}(A)A}(x) = \vee \{ \mu_{\text{ann}(A)}(r) \wedge \mu_A(y) : r \in R, y \in M, ry = x \} \text{ and}$$

$$v_{\text{ann}(A)A}(x) = \wedge \{ v_{\text{ann}(A)}(r) \vee v_A(y) : r \in R, y \in M, ry = x \}.$$

$$\begin{aligned}
\text{Therefore, } \mu_{ann(A)A}(x) &= \vee \{ \mu_{ann(A)}(r) \wedge \mu_A(y) : r \in R, y \in M, ry = x \} \\
&= \vee [\vee \{ \mu_B(r) : B \in IFS(R), BA \subseteq \chi_\theta \} \wedge \mu_A(y), r \in R, y \in M, ry = x] \\
&= \vee \{ \mu_B(r) \wedge \mu_A(y) : B \in IFS(R), BA \subseteq \chi_\theta, r \in R, y \in M, ry = x \} \\
&\leq \vee \{ \mu_{BA}(ry) : B \in IFS(R), BA \subseteq \chi_\theta, r \in R, y \in M, ry = x \} \\
&\leq \vee \{ \mu_{\chi_\theta}(x) : BA \subseteq \chi_\theta \} \\
&= \mu_{\chi_\theta}(x)
\end{aligned}$$

i.e., $\mu_{ann(A)A}(x) \leq \mu_{\chi_\theta}(x)$, $\forall x \in M$.

Similarly, we can show that $\nu_{ann(A)A}(x) \geq \nu_{\chi_\theta}(x)$, $\forall x \in M$.

Thus $(ann(A)A) \subseteq \chi_\theta$.

Corollary (3.7) If $A \in IFS(M)$ be such that $\mu_A(\theta) = 1$ and $\nu_A(\theta) = 0$, then $ann(A)A = \chi_\theta$.

Proof. By Lemma (3.3) we have $\chi_\theta \subseteq ann(A)$

$$\Rightarrow \mu_{\chi_\theta}(0) \leq \mu_{ann(A)}(0) \text{ and } \nu_{\chi_\theta}(0) \geq \nu_{ann(A)}(0)$$

$$\text{i.e., } 1 \leq \mu_{ann(A)}(0) \text{ and } 0 \geq \nu_{ann(A)}(0)$$

$$\Rightarrow \mu_{ann(A)}(0) = 1 \text{ and } \nu_{ann(A)}(0) = 0.$$

$$\begin{aligned}
\text{Now, } \mu_{ann(A)A}(\theta) &= \vee \{ \mu_{ann(A)}(r) \wedge \mu_A(m) : r \in R, m \in M, rm = \theta \} \\
&\geq \mu_{ann(A)}(0) \wedge \mu_A(\theta) \\
&= 1 \wedge 1 = 1
\end{aligned}$$

i.e., $\mu_{ann(A)A}(\theta) = 1$. Similarly, we can show that $\nu_{ann(A)A}(\theta) = 0$.

Therefore, $\chi_\theta \subseteq ann(A)A$. Hence by Theorem (3.6) we get

$$ann(A)A = \chi_\theta.$$

Note (3.8) If $A \in IFM(M)$, then $ann(A)A = \chi_\theta$.

Theorem (3.9) Let M is a R -module and $B \in IFS(R)$, $A \in IFS(M)$ such that $BA \subseteq \chi_\theta$ if and only if $B \subseteq ann(A)$.

Proof. By definition of annihilator $BA \subseteq \chi_\theta \Rightarrow B \subseteq ann(A)$.

Conversely, let $B \subseteq ann(A) \Rightarrow BA \subseteq ann(A)A \subseteq \chi_\theta$.

Corollary (3.10) If in the above theorem (3.8) $\mu_B(0) = 1$, $\nu_B(0) = 0$ and $\mu_A(\theta) = 1$, $\nu_A(\theta) = 0$, then $BA = \chi_\theta$ if and only if $B \subseteq ann(A)$.

Theorem (3.11) Let M is a R -module and $A, B \in IFS(M)$. Then the following conditions are equivalent:

- (i) $ann(B) = ann(A)$, for all $B \subseteq A$, $B \neq \chi_\theta$.
- (ii) $CB \subseteq \chi_\theta$ implies $CA \subseteq \chi_\theta$, for all $B \subseteq A$, $B \neq \chi_\theta$, $C \in IFS(R)$.

Proof. For (i) \Rightarrow (ii) Let $CB \subseteq \chi_\theta$. Then by theorem (3.9) we have

$C \subseteq \text{ann}(B) = \text{ann}(A)$ (by (i)). Again by the same theorem we have $CA \subseteq \chi_\theta$.

For (ii) \Rightarrow (i) By theorem (3.6) we have $\text{ann}(B)B \subseteq \chi_\theta$.

So (ii) implies $\text{ann}(B)A \subseteq \chi_\theta$ where $B \subseteq A$, $B \neq \chi_\theta$.

By theorem (3.8) $\text{ann}(B) \subseteq \text{ann}(A)$.

Also, $B \subseteq A \Rightarrow \text{ann}(A) \subseteq \text{ann}(B)$. Thus $\text{ann}(A) = \text{ann}(B)$.

Corollary (3.12) If in the above theorem $\mu_A(\theta) = 1$, $\nu_A(\theta) = 0$ and $\mu_B(\theta) = 1$, $\nu_B(\theta) = 0$. Then the above theorem can be stated as: The following conditions are equivalents:

(i) $\text{ann}(B) = \text{ann}(A)$, for all $B \subseteq A$, $B \neq \chi_\theta$.

(ii) $CB = \chi_\theta$ implies $CA = \chi_\theta$, for all $B \subseteq A$, $B \neq \chi_\theta$, $C \in \text{IFS}(R)$ with $\mu_C(0) = 1$, $\nu_C(0) = 0$.

Theorem (3.13) Let M is a R -module and $A \in \text{IFS}(M)$. Then

$\text{ann}(A) = \bigcup \{B : B \in \text{IFI}(R) \text{ such that } BA \subseteq \chi_\theta\}$, where $\text{IFI}(R)$ is the set of intuitionistic fuzzy ideals of R .

Proof. Clearly, $\bigcup \{B : B \in \text{IFI}(R) \text{ such that } BA \subseteq \chi_\theta\} \subseteq \bigcup \{B : B \in \text{IFS}(R) \text{ such that } BA \subseteq \chi_\theta\} = \text{ann}(A)$.

Let $r \in R$, $\alpha, \beta \in [0, 1]$ with $\alpha + \beta \leq 1$ such that $r_{(\alpha, \beta)}A \subseteq \chi_\theta$.

Let $B = \langle r_{(\alpha, \beta)} \rangle$. Then $\langle r_{(\alpha, \beta)} \rangle A = (\alpha, \beta)_{\langle r \rangle} A$.

$$\begin{aligned} \text{Again, } \mu_{(\alpha, \beta)_{\langle r \rangle} A}(x) &= \vee \{ \mu_{(\alpha, \beta)_{\langle r \rangle}}(s) \wedge \mu_A(y) \mid r \in R, y \in M, sy = x \} \\ &= \vee \{ \alpha \wedge \mu_A(y) \mid s \in \langle r \rangle, y \in M, sy = x \} \\ &\leq \vee \{ \mu_{r_{(\alpha, \beta)} A}(ry) \mid t \in R, y \in M, t(ry) = x \} \\ &\leq \vee \{ \mu_{\chi_\theta}(ry) \mid t \in R, y \in M, t(ry) = x \} \\ &\leq \vee \{ \mu_{\chi_\theta}(t(ry)) \mid t \in R, y \in M, t(ry) = x \} \\ &= \mu_{\chi_\theta}(x). \end{aligned}$$

Thus, $\mu_{(\alpha, \beta)_{\langle r \rangle} A}(x) \leq \mu_{\chi_\theta}(x)$. Similarly, we can show that $\nu_{(\alpha, \beta)_{\langle r \rangle} A}(x) \geq \nu_{\chi_\theta}(x)$, $\forall x \in M$.

Therefore, we have $(\alpha, \beta)_{\langle r \rangle} A \subseteq \chi_\theta$.

Hence $\bigcup \{B : B \in \text{IFI}(R) \text{ such that } BA \subseteq \chi_\theta\} \supseteq$

$\bigcup \{r_{(\alpha, \beta)} : r \in R, \alpha, \beta \in [0, 1] \text{ with } \alpha + \beta \leq 1 \text{ such that } r_{(\alpha, \beta)}A \subseteq \chi_\theta\} = \text{ann}(A)$.

Hence $\text{ann}(A) = \bigcup \{B : B \in \text{IFI}(R) \text{ such that } BA \subseteq \chi_\theta\}$.

Theorem (3.14) Let M is a R -module and $A \in \text{IFS}(M)$. Then $\text{ann}(A) \in \text{IFI}(R)$.

Proof. Since $\chi_0 A \subseteq \chi_\theta$, so $\chi_0 \subseteq \text{ann}(A)$. Let $r_1, r_2 \in R$ be any elements. Then,

$$\begin{aligned}
& \mu_{ann(A)}(r_1) \wedge \mu_{ann(A)}(r_2) \\
&= \left(\vee \left\{ \mu_{A_1}(r_1) : A_1 \in IFI(R), A_1 A \subseteq \mathcal{X}_\theta \right\} \right) \wedge \left(\vee \left\{ \mu_{A_2}(r_2) : A_2 \in IFI(R), A_2 A \subseteq \mathcal{X}_\theta \right\} \right) \\
&= \vee \left\{ \mu_{A_1}(r_1) \wedge \mu_{A_2}(r_2) : A_1, A_2 \in IFI(R), A_1 A \subseteq \mathcal{X}_\theta, A_2 A \subseteq \mathcal{X}_\theta \right\} \\
&\leq \vee \left\{ \mu_{A_1+A_2}(r_1) \wedge \mu_{A_1+A_2}(r_2) : A_1, A_2 \in IFI(R), A_1 A \subseteq \mathcal{X}_\theta, A_2 A \subseteq \mathcal{X}_\theta \right\} \\
&\leq \vee \left\{ \mu_{A_1+A_2}(r_1 - r_2) : A_1 + A_2 \in IFI(R), A_1 A + A_2 A \subseteq (A_1 + A_2) A \subseteq \mathcal{X}_\theta + \mathcal{X}_\theta = \mathcal{X}_\theta \right\} \\
&\leq \vee \left\{ \mu_B(r_1 - r_2) : B \in IFI(R), BA \subseteq \mathcal{X}_\theta \right\} \\
&= \mu_{ann(A)}(r_1 - r_2).
\end{aligned}$$

Thus, $\mu_{ann(A)}(r_1 - r_2) \geq \mu_{ann(A)}(r_1) \wedge \mu_{ann(A)}(r_2)$.

Similarly, we can show that $\nu_{ann(A)}(r_1 - r_2) \leq \nu_{ann(A)}(r_1) \wedge \nu_{ann(A)}(r_2)$.

Again, $\mu_{ann(A)}(sr) = \vee \left\{ \mu_B(sr) : B \in IFI(R), BA \subseteq \mathcal{X}_\theta \right\} \geq \vee \left\{ \mu_B(r) : B \in IFI(R), BA \subseteq \mathcal{X}_\theta \right\} = \mu_{ann(A)}(r)$.

Thus, $\mu_{ann(A)}(sr) \geq \mu_{ann(A)}(r)$. Similarly, we can show that $\nu_{ann(A)}(sr) \leq \nu_{ann(A)}(r)$, $\forall r, s \in R$.

Hence $ann(A) \in IFI(R)$.

Theorem (3.15) Let M is a R -module and $A_i \in IFS(M)$, $i \in \Lambda$. Then

$$ann\left(\bigcup_{i \in \Lambda} A_i\right) = \bigcap_{i \in \Lambda} ann(A_i).$$

$$\begin{aligned}
\text{Proof. } ann\left(\bigcup_{i \in \Lambda} A_i\right) &= \bigcup \left\{ B : B \in IFS(R) \text{ such that } B\left(\bigcup_{i \in \Lambda} A_i\right) \subseteq \mathcal{X}_\theta \right\} \\
&= \bigcup \left\{ B : B \in IFS(R) \text{ such that } \bigcup_{i \in \Lambda} BA_i \subseteq \mathcal{X}_\theta \right\} \\
&\subseteq \bigcup \left\{ B : B \in IFS(R) \text{ such that } BA_i \subseteq \mathcal{X}_\theta \right\} \\
&= ann(A_i), \quad \forall i \in \Lambda.
\end{aligned}$$

$$\text{Hence } ann\left(\bigcup_{i \in \Lambda} A_i\right) \subseteq \bigcap_{i \in \Lambda} ann(A_i).$$

By Theorem (3.6), we have

$$\left(\bigcap_{i \in \Lambda} ann(A_i)\right)\left(\bigcup_{j \in \Lambda} A_j\right) = \bigcup_{j \in \Lambda} \left(\bigcap_{i \in \Lambda} ann(A_i)A_j\right) \subseteq \bigcup_{j \in \Lambda} (ann(A_j)A_j) \subseteq \bigcup_{j \in \Lambda} \mathcal{X}_\theta = \mathcal{X}_\theta.$$

$$\text{Thus } \bigcap_{i \in \Lambda} ann(A_i) \subseteq ann\left(\bigcup_{i \in \Lambda} A_i\right).$$

$$\text{Hence } ann\left(\bigcup_{i \in \Lambda} A_i\right) = \bigcap_{i \in \Lambda} ann(A_i).$$

Theorem (3.16) Let M is a R -module and $A, B \in IFM(M)$, then

$$ann(A+B) = ann(A) \cap ann(B).$$

Proof. Since $A, B \in IFM(M) \Rightarrow A + B \in IFM(M)$, we have

$$\mu_{A+B}(x) = \bigvee_{x=y+z} \{ \mu_A(y) \wedge \mu_B(z) \} \geq \mu_A(x) \wedge \mu_B(\theta) = \mu_A(x) \text{ and}$$

$$\nu_{A+B}(x) = \bigwedge_{x=y+z} \{ \nu_A(y) \vee \nu_B(z) \} \leq \nu_A(x) \vee \nu_B(\theta) = \nu_A(x), \forall x \in M.$$

This implies that $A \subseteq A+B$ and $B \subseteq A+B$.

So, $\text{ann}(A+B) \subseteq \text{ann}(A)$ and $\text{ann}(A+B) \subseteq \text{ann}(B)$

$\Rightarrow \text{ann}(A+B) \subseteq \text{ann}(A) \cap \text{ann}(B)$.

Now, $\text{ann}(A) \cap \text{ann}(B)$

$$= (\cup \{A_1 \mid A_1 \in IFI(R), A_1 A \subseteq \chi_\theta\}) \cap (\cup \{B_1 \mid B_1 \in IFI(R), B_1 B \subseteq \chi_\theta\})$$

$$= \cup \{A_1 \cap B_1 \mid A_1, B_1 \in IFI(R), A_1 A \subseteq \chi_\theta, B_1 B \subseteq \chi_\theta\}$$

$$\subseteq \cup \{C \mid C = A_1 \cap B_1 \in IFI(R), CA \subseteq \chi_\theta, CB \subseteq \chi_\theta\}$$

$$\subseteq \cup \{C \mid C = A_1 \cap B_1 \in IFI(R), C(A+B) \subseteq CA + CB \subseteq \chi_\theta\}$$

$$= \cup \{C \mid C \in IFI(R), C(A+B) \subseteq \chi_\theta\}$$

$$= \text{ann}(A+B).$$

Therefore, $\text{ann}(A) \cap \text{ann}(B) \subseteq \text{ann}(A+B)$.

Hence $\text{ann}(A+B) = \text{ann}(A) \cap \text{ann}(B)$.

Definition (3.17) Let M be R -module. Then $A \in \text{IFS}(M)$ is said to be faithful if $\text{ann}(A) = \chi_0$.

Lemma (3.18) Let $A \in \text{IFS}(M)$ be faithful, where M is R -module. If R is non-zero then $A \neq \chi_\theta$.

Proof. Since A is faithful $\Rightarrow \text{ann}(A) = \chi_0$.

If $A = \chi_\theta$ then $\text{ann}(A) = \text{ann}(\chi_\theta) = \chi_R$. Thus we have $\chi_0 = \chi_R \Rightarrow R = \{0\}$, a contradiction.

Therefore, $A \neq \chi_\theta$.

Theorem (3.19) Let $A \in \text{IFS}(R)$ with $\mu_A(0) = 1, \nu_A(0) = 0$. Then $A \subseteq \text{ann}(\text{ann}(A))$ and $\text{ann}(\text{ann}(\text{ann}(A))) = \text{ann}(A)$.

Proof. Let A be an intuitionistic fuzzy subset of R -module R . Then by corollary (3.7), we have $\text{ann}(A)A = \chi_0$.

By theorem (3.9), we have $A \subseteq \text{ann}(\text{ann}(A))$ (1)

$\Rightarrow \text{ann}(\text{ann}(\text{ann}(A))) \subseteq \text{ann}(A)$ [using lemma (3.4)]

Again using (1) : $\text{ann}(A) \subseteq \text{ann}(\text{ann}(\text{ann}(A)))$.

So, $\text{ann}(\text{ann}(\text{ann}(A))) = \text{ann}(A)$.

Theorem(3.20) Let $A \in \text{IFS}(M)$. Then

$$C_{(\alpha,\beta)}(\text{ann}(A)) \subseteq \text{ann}(C_{(\alpha,\beta)}(A)), \forall \alpha, \beta \in (0,1] \text{ with } \alpha + \beta \leq 1.$$

Proof. Let $x \in C_{(\alpha, \beta)}(\text{ann}(A))$. Then $\mu_{\text{ann}(A)}(x) \geq \alpha > 0$ and $\nu_{\text{ann}(A)}(x) \leq \beta < 1$
 $\Rightarrow \vee \{ \mu_B(x) : B \in \text{IFI}(R), BA \subseteq \chi_\theta \} \geq \alpha$ and $\wedge \{ \nu_B(x) : B \in \text{IFI}(R), BA \subseteq \chi_\theta \} \leq \beta$
 $\Rightarrow \mu_B(x) \geq \alpha$ and $\nu_B(x) \leq \beta$ for some $B \in \text{IFI}(R)$ with $BA \subseteq \chi_\theta$.

If $x \notin \text{ann}(C_{(\alpha, \beta)}(A))$ then \exists 's some $y \in C_{(\alpha, \beta)}(A)$ such that $xy \neq \theta$.

Now, $\mu_{BA}(xy) \geq \mu_B(x) \wedge \mu_A(y) \geq \alpha > 0$ and $\nu_{BA}(xy) \leq \nu_B(x) \vee \nu_A(y) \leq \beta < 1$,
 which is a contradiction. Hence $C_{(\alpha, \beta)}(\text{ann}(A)) \subseteq \text{ann}(C_{(\alpha, \beta)}(A))$.

Definition (3.21) $A \in \text{IFI}(R)$ is said to be an intuitionistic fuzzy dense ideal if $\text{ann}(A) = \chi_0$.

Definition (3.22) $A \in \text{IFI}(R)$ is called intuitionistic fuzzy semiprime ideal of R if for any IFI B of R such that $B^2 \subseteq A$ implies that $B \subseteq A$.

Theorem (3.23) If A is an IFI of a semi prime ring R , then $A \cap \text{ann}(A) = \chi_0$ and $A + \text{ann}(A)$ is an intuitionistic fuzzy dense ideal of R .

Proof. Since $A \cap \text{ann}(A) \subseteq A$, $A \cap \text{ann}(A) \subseteq \text{ann}(A)$ so $(A \cap \text{ann}(A))^2 \subseteq A \text{ann}(A) \subseteq \chi_0$.

Now R is a semiprime ring and it implies that 0 is a semiprime ideal of R so χ_0 is an intuitionistic fuzzy semi prime ideal of R .

Also, $(A \cap \text{ann}(A))^2 \subseteq \chi_0 \Rightarrow A \cap \text{ann}(A) \subseteq \chi_0$ and hence $A \cap \text{ann}(A) = \chi_0$.

Hence $\text{ann}(A + \text{ann}(A)) = \text{ann}(A) \cap \text{ann}(\text{ann}(A)) = \chi_0$ proving thereby $A + \text{ann}(A)$ is an intuitionistic fuzzy dense ideal of R .

Theorem (3.24) Let A be a non-zero intuitionistic fuzzy ideal of a prime ring R with $\mu_A(0) = 1$, $\nu_A(0) = 0$. Then A is an intuitionistic fuzzy dense ideal of R .

Proof. Now, $A \text{ann}(A) = \chi_0 \Rightarrow \text{ann}(A) = \chi_0$ or $A = \chi_0$. But $A \neq \chi_0$ so $\text{ann}(A) = \chi_0$.

Hence A is an intuitionistic fuzzy dense ideal of R .

Definition (3.25) If $A \in \text{IFS}(R)$. Then the intuitionistic fuzzy ideal of the form $\text{ann}(A)$ is called an intuitionistic fuzzy ideal. Thus if A is an intuitionistic fuzzy annihilator ideal if and only if $A = \text{ann}(B)$ for some $B \in \text{IFS}(R)$ with $\mu_B(0) = 1$, $\nu_B(0) = 0$.

Remark (3.26) In view of theorem (3.19) it follows that A is an annihilator ideal of R implies $\text{ann}(\text{ann}(A)) = A$.

Theorem (3.27) The annihilator ideals in a semiprime ring form a complete Boolean algebra with intersection as infimum and ann as complementation.

Proof. Since $\bigcap_{i \in I} \text{ann}(A_i) = \text{ann}\left(\sum_{i \in I} A_i\right)$, so any intersection of annihilator ideals is an intuitionistic fuzzy annihilator ideal. Hence these ideals form a complete semi-lattice with intersection as infimum. To show that they form a Boolean algebra it remain to show that:

$A \cap \text{ann}(B) = \chi_0$ if and only if $A \subseteq B$, for any annihilator ideals A and B .

If $A \subseteq B$ then $A \cap \text{ann}(B) \subseteq B \cap \text{ann}(B) = \chi_0$.

Conversary, let $A \cap \text{ann}(B) = \chi_0$.

Now, $A \cap \text{ann}(B) \subseteq A \cap \text{ann}(B) = \chi_0 \Rightarrow A \subseteq \text{ann}(\text{ann}(B)) = B$.

Theorem (3.28) Let M be a non-zero R -module. Suppose that there exist no ideal A maximal among the annihilators of non-zero intuitionistic fuzzy submodules (IFSMs) of M . Then A is an intuitionistic fuzzy prime ideal of R .

Proof. Since A is maximal among the annihilators of non-zero intuitionistic fuzzy submodules (IFSMs) of M . Therefore there is an IFSM $B (\neq \chi_0)$ of M such that $A = \text{ann}(B)$.

Suppose $P, Q \in \text{IFI}(R)$ properly containing A (i.e., $A \subset P$ and $A \subset Q$) such that $PQ \subseteq A$.

If $QB = \chi_0$ then $Q \subseteq \text{ann}(B) = A$, which is a contradiction to our supposition so $QB \neq \chi_0$.

Now, $PQ \subseteq A \Rightarrow P(QB) \subseteq AB = \text{ann}(B)B = QB = \chi_0$. So $Q \subseteq \text{ann}(QB)$.

Hence $A \subseteq \text{ann}(QB)$. This is a contradiction of the maximality of A . So A is an intuitionistic fuzzy prime ideal of R .

Remark (3.29) If $A \in \text{IFM}(M)$, $A \neq \chi_0$ satisfying one (hence both) the condition of Theorem (3.11) then A is called an intuitionistic fuzzy prime submodule of M .

Theorem (3.30) If A is an intuitionistic fuzzy prime submodule of M then $\text{ann}(A)$ is an intuitionistic fuzzy prime ideal of R .

Proof. Let A be an intuitionistic fuzzy prime submodule of M and $PQ \subseteq \text{ann}(A)$, where Q is not contained in $\text{ann}(A)$. Then

$\chi_0 \neq QA \subseteq A$. Now $PQ \subseteq \text{ann}(A) \Rightarrow (PQ)A \subseteq \text{ann}(A)A = \chi_0$.

So, $P \subseteq \text{ann}(QA) = \text{ann}(A)$, as A is prime. Hence $\text{ann}(A)$ is prime ideal of R .

4. CONCLUSIONS

In this paper we have developed the notion of annihilator of an intuitionistic fuzzy subset of a R -module. Using this notion, we investigate some important characterization of intuitionistic fuzzy annihilator of subsets of modules. The annihilator of union (sum) of intuitionistic fuzzy submodules are obtained. Annihilator of intuitionistic fuzzy ideal of prime ring, semi prime ring are also obtained. Using the concept of intuitionistic fuzzy annihilators, intuitionistic fuzzy prime submodules and intuitionistic fuzzy ideals are defined and various related properties are established.

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INTENTIONAL BLANK

A SURVEY OF GRAMMAR CHECKERS FOR NATURAL LANGUAGES

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ABSTRACT

Natural Language processing is an interdisciplinary branch of linguistic and computer science studied under the Artificial Intelligence (AI) that gave birth to an allied area called 'Computational Linguistic' which focuses on processing of natural languages on computational devices. A natural language consists of a large number of sentences which are linguistic units involving one or more words linked together in accordance with a set of predefined rules called grammar. Grammar checking is the task of validating sentences syntactically and is a prominent tool within language engineering. Our review draws on the recent development of various grammar checkers to look at past, present and the future in a new light. Our review covers grammar checkers of many languages with the aim of seeking their approaches, methodologies for developing new tool and system as a whole. The survey concludes with the discussion of various features included in existing grammar checkers of foreign languages as well as a few Indian Languages.

KEYWORDS

Natural Language Processing; Computational Linguistic; Grammar Formalism; Grammar Checker

1. INTRODUCTION

Language is a means of communication, particularly in human beings, that has origin in natural thoughts regardless of any planning or motivation. Language is subjective, creative, and dynamic system which encompasses vocal symbols of human being that play a crucial role in social affairs. Human natural language can be broadly defined as an interchangeability process within participants. It is contrasted to any natural communication system example includes bees' waggle dance or artificial/constructed language such as computer programming languages.

In today's era of technology, language engineering focuses on modeling of human languages under Computational Linguistic (CL) research domain. Computational Linguistic is an interdisciplinary field of computer science and linguistics has collaboration with Artificial Intelligence area, and is concerned with computational aspects of human natural language.

Computational linguistics is categorized into applied and theoretical components. Theoretical linguistic deals with linguistic knowledge needed for generation and understanding of language. Applied computational linguistic has concern with the development of tools, technologies, and applications to model human language. Although existing technologies are far from attaining human abilities and have major feats in related application development, computers are not able to correspond to human thoughts. As people share knowledge, ideas, thoughts and information with each other using natural language, it is also possible to share the same with computer with the help of applied CL that too, in natural languages only. To complete communication and to make it meaningful, used language must follow set of rules involved in it.

Grammar is the study of significant elements in language and set of rules that make it coherent. Words are grammatical basic units that combine together to form a sentence and collection of sentences completes the language. It is a bit easier for human beings to follow rules of native natural language as they are aware of it since infant phase. But it is a new and exciting challenge for language technology & applied CL to validate grammatical correctness of any natural language for computers. To deal with grammatical mistakes by human is also one of the challenging tasks. Even though grammar checker tools have been developed so far for many worldwide languages, it is relatively new in Indian languages. So there is scope to develop grammar checker for Indian languages.

The remaining of the paper is organized as follows: Section 2 gives idea about basic building block of language i.e. sentence and its types. Section 3 outlines fundamental concepts of grammar checking system. Section 4 gives general working of grammar checker. Section 5 describes available grammar checking approaches. Section 6 discusses how couple of world-wide natural languages, including few Indian natural languages use grammar checking approach. After descriptive study, analysis is shown in section 7. The review is summary and observations are recorded in section 8.

2. A BIT ABOUT LANGUAGE

Language is a mean of communication. Human beings exchange information between two or more parties using natural languages only. The prime objective of communication is to share information and request/impart knowledge. The information can be specified in *written-form* or *vocal-form*(spoken). The most important thing in information content form is the validity of sentences in the given language. Morphemes, phonemes, words, phrases, clauses, sentences, vocabulary and grammar are the building blocks of any natural language. All valid sentences of a language must follow the rules of that language (grammar). Invalid sentences are not worth and won't be effective to share knowledge, hence outrightly rejected.

Any natural language consists of countably infinite sentences and these sentences follow basic structure. A sentence structure is perceived hierarchically at different levels of abstraction, i.e. surface level(at the word level), POS(part-of-speech) level to abstract level(phrases: subject, object, verb etc.).The sentence formation strictly depends on the syntactically permissible structures coded in the language grammar rules. The basic sentence structures broadly depend on the positions of Subject, Object, Verb i.e. their permutations, accordingly SVO, SOV, OSV, OVS, VSO, VOS are possible, but not all(OVS, OSV) are followed in the grammar of natural languages of the world. These are all referred as word order. Depending the internal phrasal

structure of phrases especially the verb phrases, certain clauses, sentences are broadly classified as Simple, Complex and Compound sentence.

2.1 Simple Sentence

A simple sentence is a collection of predicate and one or more arguments. It has only single main clause and single verb (mostly verb root). It expresses a meaning that can stand by on its own. It does not contain any negation, question words and passivation. It has simpler sentence structure. Most of these sentences are kernel sentences and can span other sentence types such as complex and compound sentences. It is identified using morphology of the verb phrase, number of clauses and other cue words are negation words, question words and passivation.

2.2 Complex Sentence

A complex sentence has at least two clauses, having interdependence between main and dependent or subordinate clause. Subordinate clause gives additional information about main clause. Number of clauses is the important cue for identifying the Complex Sentences.

2.3 Compound Sentence

A compound sentence is a collection of multiple clauses connected through conjunctives (And/Or etc.), here it is important to note that each clause is a functionally complete sentence (having complete meaning). As there is no interdependency between clauses, they are considered at the same (peer) level. The use of conjunctive words is important cue for identifying this type of sentences.

3. GRAMMAR CHECKER

Now a days, people need not only mechanical support, but also expect intellectual assistance from machines. What if, we could have conveyed everything in natural languages to machines? Answers to such questions open new doors of possibilities and opportunities for intelligent systems. For realizing this idea into reality, it is a mandatory criterion for the machines to get aware of natural languages. It motivates us to build intelligent computer applications involving scientific and linguistic knowledge of human communication. Applications of NLP include QA (question-answering) system, Machine Translation, NL (Natural language) interface to databases, grammar checker, spell checker, chatter box, etc. Grammar checking is the fundamental application amongst these as it checks correctness of input sentence, which has a strong effect on other NLP applications. Correctness and validity of sentences are checked with the help of an underlying grammar of the natural language. The grammar consists of a set of rules which govern the formation and amalgamation of sentence constituents i.e. clauses/phrases. A valid phrase is one in which all constituent words are compatible with each other so to say they satisfy the morphological/syntactic agreement features, i.e. they agree in their *gender, number, person, case* (GNPC) features. The phrase structure is defined with the help of POS (part-of-speech) sequence. The same is true for a sentence in which the main verb agrees with either *subject, object* or none. Grammar checking involves testing the agreements between constituents on the scale of GNPC syntactic features as well as ontological semantic features.

At practical level, it is highly desired that the grammar checker software should not only check the correctness and validity of the sentences, but also correct the grammatical errors (auto correction, correction suggestion). The word order(kernel sentence structure) affects the parsing of the given natural language so does the NLP tasks and tools used for the grammar checking process.

4. GENERAL WORKING OF GRAMMAR CHECKER

Grammar checker takes as input, a sentence from the given document of a language and checks its correctness and validity w.r.t that language [1]. The sentence has to undergo some kind of preprocessing stage where in lexicalization, word setgenation, POS tagging. The actual grammar checking involves syntactic parsing of pre-processed sentence using chosen approach (discussed below) . The output of the Grammar Checker application should summarize the grammatical errors in the input document sentences and optionally auto correct the errors or at least provide correction suggestions. These tasks can be described in algorithmic form(step wise) as follows:

1. **Sentence Tokenization:** This involves sentence tokenization and word segmentation. The sentence is tokenized into words followed by breaking down words into constituent morphs and populating lexical information about the word from the lexicon.
2. **Morphological Analysis:** Morphological analysis returns word stems and associated affixes.
3. **Part-of-Speech(POS) tagging:** Assigning the appropriate POS tag to each word (morpheme)
4. **Parsing Stage:** Checks the syntactic constraints (agreement constraints) between input words and formation of Hierarchical phrasal/dependency structure of the input sentence using chosen approach/methodology. In case of failure flag grammatical error also provide auto correction mechanism or present suggestion list to the user.

Many grammar checkers have been developed for different foreign and Indian languages using different approaches. A detailed description is given in section 5.

5. GRAMMAR CHECKING APPROACHES

Broadly, there are three grammar checking approaches that are used, namely statistical, rule based and hybrid grammar checker.

- A) Statistical Grammar Checker
- B) Rule based grammar Checker
- C) Hybrid Grammar Checker

5.1 Statistical Grammar Checker

In this approach, corpus is maintained from a number of journals, magazines or documents. It ensures correctness of sentence by checking the input text against the corpus. There are two ways to check input text. First, input text is directly checked with corpus and matched input text, is tagged as grammatically correct otherwise it is considered as incorrect. In the second way, initially rules are generated from maintained corpus and input text is checked by these rules. But rules need to be updated when the corpus is modified or new data is added to it. It is easy to implement, but has some disadvantages also. As there is no specific error message, it is difficult to recognize error given by a system [3].

5.2 Rule based Grammar Checking

It is the most common approach. Input text is checked by rules formed from corpus, but unlike the statistical approach, rules are generated manually [12]. However, rules are easy to configure and also easy to add, remove or update. The most significant advantages include, rules can be handled by one who do not have programming language like linguistics and it provides a detailed error message. The main feature of this method is that it provides all feature of a language and sentences also need not to be complete, it can handle input text while writing.

5.3 Hybrid Grammar Checker

It combines both statistical and rule based grammar checker. So it is more robust and achieves higher efficiency.

6. EXISTING GRAMMAR CHECKER

This section will provide a detailed study of the existing grammar checker for world-wide languages. For study purposes, languages are divided into categories; foreign and Indian.

6.1 Grammar Checker for Foreign Languages

6.1.1 Afan Oromo Grammar Checker

Afan Oromo is the language of Ethiopia. The Afan Oromo grammar checker[2] is provided with paragraphs as an input, which is then, tokenized into sentences, further into words. Using tagger based Hidden Markov Model, which uses manually tagged corpus, each word is assigned a part of speech. A stemming algorithm removes certain types of affix using substitution rules, which only apply when certain conditions hold. By removing affix, an agreement between a subject - verb, subject-adjective, main verb-subordinate verb in number, gender, and the tense is identified. From identifying agreements, the system can provide alternative correct sentences in case of disagreement. The grammar checker provides prominent results but fails to detect compound and complex grammatically incorrect sentences. Also, as system is provided with incorrectly tagged words, it leads the system to generate false flags [2].

6.1.2 Amharic Grammar Checker

Speakers of Amharic language are found in Ethiopia. The Amharic grammar checker system [3] adopts two approaches. The first approach is a rule based approach for simple sentences. Another approach is a statistical approach for both simple and complex sentences. The rules are created manually and checked against the pattern of a sentence to be checked. To check grammatical errors in an Amharic sentence, n-gram and probabilistic methods are used along with a statistical approach. The pattern and the corresponding occurrence probabilities are extracted automatically from training corpus and maintained in the repository. Using stored patterns and probabilities, sentence probability can be calculated. Some threshold and probability of the sentence are used to determine correctness. The system shows good results, but false alarm is due to incomplete grammatical rules and quality of the corpus [3].

6.1.3 Swedish Grammatifx Grammar Checker

The speakers of Swedish are found in Sweden and parts of Finland. Grammatifx is an exploratory project, which has been developed as a Swedish grammar checker [4]. The study starts with investigating existing grammar checker of different languages. Swedish materials are collected for gathering error types and for the discovery of new ones. Error type in this classification is evaluated and subset of these error types is chosen for actual project development. Unlike other Swedish grammar checker [8], Grammatifx checks noun phrase internal agreement and verb chain consistency. To detect various error types, different technologies are selected. For detection of syntactic errors, constraint grammar formalism is used. Regular expression based technique is used to detect punctuation and number formatting convention violations. Word-specific stylistic marking is covered by style-tagging individual lexeme entries in the underlying Swedish two-level lexicon. Along with error detection, Grammatifx also has an error treatment module. It does not detect compound words mistakenly written separately.

6.1.4 Icelandic Grammar Checker

Icelandic is a language of Iceland. Rule based approach is used to implement Icelandic grammar checker [5]. Initially, input text has to pass through parsing, for syntactic analysis and POS tagging. After initial analysis of input text, the system then goes through part of finding process relevant to each rule. The system finds compliance with the rule. If it finds that in some way, input text is not in accordance with relevant rules, an error is generated. This system does only general error detection; it does not provide any detailed error message or correction suggestions. It also does not detect stylistic errors.

6.1.5 Nepali Grammar Checker

Nepali language is an official language of Nepal. Bal Krishna Bal and Prajol Shrestha have presented works on a Nepali grammar checker using rule based approach [6]. Input text is tokenized into words, then the word is passed to the morphological module for initial POS tagging. The next module is a POS tagger module, which tags untagged and undetermined tokenized words. Chunker and parser module identifies the chunks or phrases from POS tagged words. Thus, this module requires production rules and POS tags of input texts and after matching with the rule, it will return chunks and phrases. Identified chunks, and phrases are

assigned to grammatical roles like SUBJECT, OBJECT and VERB based on subject-verb agreement and agreement between modifier and head in the noun phrase. The syntax is checked then. The grammar checker only deals with simple sentences.

6.1.6 Portuguese Grammar Checker

Portuguese is a Romance language. Portuguese grammar checker named COGrOO is based on CETENFOLHA, a Brazilian Portuguese morphosyntactic annotated Corpus [7]. The system consists of local and structural error rules. Local rules include short word sequence rules, whereas structural rules include nominal-verbal agreement, nominal-verbal government, and misuse of adverb-adjective type complex rules. Initially, inputs are broken down into sentences by boundary detector module. Then each sentence is tokenized into words, POS tags are assigned to each of them. Chunker will chunk a tagged sentence into small verbal and a noun phrase with their respective grammatical roles. Local errors are detected by the local error checker whereas structural errors are detected by structural error checker.

6.2 Grammar Checker for Indian Languages

6.2.1 Urdu Grammar Checker

Kabir proposed the Urdu grammar checker system. The proposed two pass parsing approach analyzes the input text [15]. This approach was introduced to reduce redundancy in phrase structure, grammar rules developed for sentence analysis. Phrase structure grammar rules are used to parse the sentence. In case of failure, Movement rules are used to reparse the tree. The system works well, except for the module of Morphological disambiguation and POS guesser. The grammar checker checks grammatical and structural mistakes in declarative sentences and provide error correction suggestions.

6.2.2 Bangla Grammar Checker

Md. Jahangir Alam, Naushad UzZaman and Mumit Khan describes n gram based analysis of words and POS tags to decide the correctness of a sentence [9]. The system assigns tags to each word of a sentence using a POS tagger. Using-gram analysis, the system determines probability of tag sequence. If the probability is greater than zero, then it considers the sequence as correct. The same system is tested for both English and Bangla language. As it completely depends on POS tagging, author checked system against manual tagged sentences and automated tagged and results are more promising for Bangla compared to English, as there are large compound sentences in Brown corpus. The system also does not work for compound sentences of Bangla language.

6.2.3 Punjabi Grammar Checker

Mandeep Gill, Gurprit Lehal and Shiv Sharma Joshi have implemented grammar checking software for detecting grammatical errors in Punjabi texts and have provided suggestions [10]. The system performs morphological analysis using a full form lexicon and POS tagging and phrase chunking using rule based system. For literacy style Punjabi texts, the system also supports a set of carefully devised error detection rules, which can detect alteration for various

grammatical errors generated from each of the agreements, order of words in phrases etc. The rules in the system can be turned off or on individually. The main attraction of this system is that it provides a detailed description of detected errors and provides suggestions on the same. It also deals with compound and complex sentences.

6.2.4 Hindi Grammar Checker

Lata Bopche, Gauri Dhopavkar and Manali Kshirsagar describes a method for Hindi grammar checker [17]. This system consists a full-form lexicon for morphological analysis and rule based system. The input text passed through all basic processes like tokenization, morphological analysis, POS tagging. A POS tagged sentences match against a set of rules. This system gives prominent results for only simple sentences. The system only checks those patterns which have the same number of words present in the input sentence. It does not provide any suggestion for the errors

6.3 Commercial Grammar Checkers

A lot of work has gone into developing commercially sophisticated systems for widespread use, such as automatic translator, spell checkers, grammar checkers and so on for natural languages. However, most of such programs are available strictly on commercial basis, therefore no official documentation regarding their approaches/algorithms is available. Such commercial programs are available on Proprietary as well as Open Source office suite software such MS Microsoft word, open office, Libre office suite and many more. Majority of such grammar checkers are available for English language.

7. ANALYSIS

After descriptive study of various grammar checks for world-wide languages, some findings are analyzed. Table 1 summarizes language and respective grammar checker approach with performing and lacking features in each of them.

Table 1. Summarization of grammar checker features

Language	Feature of language	Grammar checker approach	Performing Features	Lacking Features
Afan Oromo[2]	<ul style="list-style-type: none"> Rich in morphology Agglutinative language 	Rule based	Provides alternative sentence in case of disagreement	Fails to detect compound and complex grammatically incorrect sentence Poor POS tagger
Amhari [3]	<ul style="list-style-type: none"> Rich in morphology Subject-Object-Verb structure 	Hybrid	Results are prominent in both simple and complex sentences. Able to detect multiple errors in a sentence	Gives false alarm due to incomplete rules and quality of statistical data

Swedish [4]	<ul style="list-style-type: none"> High amount of word independence 	Hybrid	Error detection and error correction module available Detects syntactic, stylistic and punctuation error	Does not detect compound words mistakenly written separately.
Icelandic [5]	<ul style="list-style-type: none"> Heavily inflected 	Rule based	Able to detect general errors	Does not provide detailed messages of error and also not correct it.
Nepali[6]	<ul style="list-style-type: none"> Family of Hindi and Bangala language 	Modular	Only simple sentences are checked and error messages are provided	Does not deal with compound and complex sentence
Portuguese [7]	<ul style="list-style-type: none"> Infinitive language 	Rule based	Local and structural errors detected	Lacks detection of stylistic error
Urdu[15]	<ul style="list-style-type: none"> Subject-Object-Verb structure 	Rule based	Checks structural and grammatical mistakes in declarative sentences. Provides error correction.	Weak in performance by morphological disambiguation and POS tagger
Bangla[9]	<ul style="list-style-type: none"> Loaded with the agreement 	Statistical	The system gives better results for Bangla language compared with English	Does not work for compound sentence.
Punjabi[10]	<ul style="list-style-type: none"> Concern with word order, case making, verb conjugation 	Hybrid	Provides detailed message of the error and give error correction suggestion. Works for simple, compound and complex sentences.	Due to word shuffling for emphasis, false alarm occurs. Emphatic intonation causes changes in meaning or class of the word.
Hindi[17]	<ul style="list-style-type: none"> Word dependency Inflectional rich 	Rule based	Gives promising results for simple sentences	The system only checks those patterns which have the same number of words present in the input sentence.

Table 2 summarizes error detection and/or correction, working on simple and/or compound and/or complex sentences, syntactic and stylistic error.

Table 2. Summarization of Detected errors by grammar checker

Language	Error Detection	Error Correction	Detect Error for			Syntactic Error	Stylistic Error
			Simple Sentence	Compound Sentence	Complex Sentence		
Afan Oromo[2]	✓		✓	✓	✓		
Amharic [3]	✓		✓		✓	✓	
Swedish[4]	✓	✓	✓			✓	✓
Icelandic[5]	✓		✓			✓	
Nepali[6]	✓		✓			✓	
Portuguese[7]	✓		✓			✓	

Urdu[15]	✓	✓	✓			✓	
Bangla[9]	✓		✓			✓	
Punjabi[10]	✓	✓	✓	✓	✓	✓	
Hindi[17]	✓		✓			✓	

Table 3 provides performance evaluation of studied grammar checkers.

Table 3. Performance evaluation of grammar checker

Language	Grammar checker approach	Performance Evaluation
Afan Oromo[2]	Rule based	Precision: 88.89% Recall: 80.00%
Amharic [3]	Rule Based	Precision: 92.45 % Recall: 94.23%
	Statistical	Precision: 67.14% Recall: 90.38%
Swedish[4]	Hybrid	Not reported
Icelandic[5]	Rule based	Precision: 84.21% Recall: 71.64%
Nepali[6]	Rule Modular	Not reported
Portuguese[7]	Rule based	A corpus used contains 51 errors based on which following parameters calculated True Positive: 14 & False Positive: 10
Urdu[15]	Rule based	Not reported
Bangla[9]	Statistical	Calculate probability of tag sequence and if it is above the threshold value, it is concluded as probabilistically correct otherwise not. Few examples are provided.
Punjabi[10]	Hybrid	Not reported
Hindi[17]	Rule based	Not reported

8. CONCLUSION

In this survey, we have reviewed different grammar checking approaches, methodologies along with key concepts and grammar checker internals. The aim of the survey was to study various Grammar Checkers on the scale of their features such as types of grammar errors, weaknesses and evaluation. Survey concludes with study of various features of grammar checkers thus leading to future scope for developing grammar checkers for uncovered languages with feasible approach. It is observed that most of the professionally available grammar checkers are available for English language, while for most of other languages, the work is in progress (early to final stages). Number of Indian Initiatives were also witnessed during our survey for popular languages like Hindi, Bangla, Urdu etc. No grammar checker could have been cited for Marathi Language. Hence our future research work aims to develop grammar checker for Marathi Language.

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