INSTANTANEOUS FREQUENCY AND AOA ESTIMATION OF MULTICOMPONENT SIGNALS BASED ON BORN-JORDAN DISTRIBUTION

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ABSTRACT

Based on the spatial sample of the multi-component signals by the array antenna, using Born-Jordan distribution which is a kind of typical Cohen type time-frequency distribution, the detection and estimation approach of the spatial multi-component signals' angle-of-arrival and instantaneous frequency is proposed by comparison of the instantaneous frequency of the signals at the same time from the point of digital signal processing. The simulation results show that the proposed method not only has high noise performance, but also enhances the real-time performance of the spatial signal detection.

KEYWORDS

Angle-of-arrival Estimation, Time-frequency Distribution, BJD, Instantaneous Frequency.

1. INTRODUCTION

Based on the array signal processing, the time-frequency analysis has been introduced to estimate the angle of arrival and instantaneous frequency parameters of the spatial non-stationary signals in the recent years. It has become one of the hotspots of the signal processing research [1-4]. S. Ouelha et al [5,6] proposed a method to estimate the angle of arrival (AOA) of the non-stationary signals using quadratic spatial time-frequency distribution in the array signal processing. The paper [7] constructed the array signal model in the time-frequency domain through the time-frequency distribution with the symmetrical array, and estimated the AOA and instantaneous frequency parameters of the spatial signal by the eigenspace decomposition of the data correlation matrix of the adjacent time-frequency points. Wang Shu et al [8, 9] proposed a method to construct correlation matrix by combining the spatial sampling data with the temporal sampling data. This kind of research combines the concept of the spatial spectrum estimation with the method of the time-frequency analysis, and effectively solves the problem of the AOA and instantaneous frequency estimation of the spatial non-stationary signals. However, because the algorithm is based on the analysis of the signal spectrum, the eigenvalue decomposition of matrix and the search of one-dimensional or two-dimensional spectrum peak are needed, which leads to large amount of calculation and low real-time performance in the parameter estimation. For the multi-component signals, the estimation method based on the Wigner Ville distribution used in the reference [5] has some other problems such as the cross term interference etc.

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2. SPATIAL SAMPLING MODEL OF THE MULTIPLE SIGNALS

It is assumed that there is a symmetrical uniform linear array with elements $2L+1$, and the spacing between elements is d. We set the reference element as the center element and number the elements from left to right as $-L,-(L-1),\cdots,L$. Consider M signal sources are incident to the linear array at the same time from the far field, as shown in Figure 1. Then the output of the Nth element at the t_0 time can be expressed as:

$$x_n(t_0)=\sum_{i=1}^{M} s_i(t_0+n\tau_i)+v_n(t_0) \qquad n=-L,-(L-1),\cdots,L\ ;\ i=1,2,\cdots M \quad (1)$$

$$\tau_i=\frac{d}{c}\sin(\theta_i) \quad (2)$$

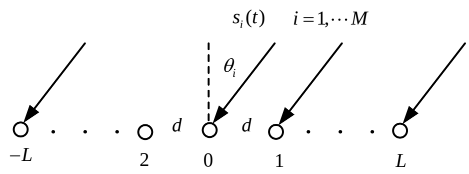

Figure 1. symmetrical linear array antenna

Where, $\{s_i(t)\}_{i=1}^{M}$ are the outputs of M unrelated signal sources in the reference element, and $\{\theta_i\}_{i=1}^{M}$ are the AOAs of the incident signals. τ_i is the delay of signal $s_i(t)$ between the adjacent elements, $\{v_n(t)\}_{n=-L}^{L}$ are the independent white Gaussian noise outputs of the $2L+1$ elements, which are independent of the input signals.

Let the output $\{x_n(t_0)\}_{n=-L}^{L}$ of $2L+1$ elements be the signal sequence obtained by spatial sampling of the linear array at time t_0. It can be seen from formula (1) that the spatial sampling signal $\{x_n(t_0)\}_{n=-L}^{L}$ at time t_0 can be regarded as the sum of M $2L+1$ point sequences and the noise signal, which are sampled by M signal sources $s_i(t)$ with $1/\tau_i$ as the sampling frequency in the short time window of $t\in[t_0-L\tau_i, t_0+L\tau_i]$. The sampling frequency of signal $s_i(t)$ is

$$f_s^{(i)}(\theta)=1/\tau_i=\frac{c}{d\sin(\theta_i)} \quad (3)$$

In order to eliminate the direction ambiguity, the array spacing should meet the half wavelength condition of $d\le\lambda_{\min}/2$, so the array output sequence naturally meets the sampling condition of sampling frequency $f_s^{(i)}\ge 2f_{\max}$. At the same time, it can be seen that the sampling frequency $f_s^{(i)}(\theta)$ of the signal $s_i(t)$ is directly related to the incident angle θ_i of the signal, which provides a basis for us to estimate the arrival angle θ_i of the signal source $s_i(t)$ using the method of time-frequency analysis.

3. INSTANTANEOUS FREQUENCY EXTRACTION OF THE SPATIAL SIGNAL

The instantaneous frequency (IF) is an important parameter to characterize the non-stationary signal, and the time-frequency distribution of the signal is an important means to obtain the instantaneous frequency. For multi-component signals, the cross terms of the time-frequency distribution must be suppressed in the process of the instantaneous frequency acquisition. Born Jordan distribution [10] (BJD), as a typical Cohen class time-frequency distribution, has strong cross term suppression ability and has been widely studied and applied [11]. BJD is defined as:

$$\mathrm{BJD}_s(t, f) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} s(u+\frac{\tau}{2})s^*(u-\frac{\tau}{2})\psi(t-u,\tau)e^{-j2\pi f\tau}dud\tau \tag{4}$$

Where $\psi(t,\tau)$ is the kernel function of BJD

$$\psi(t,\tau) = \begin{cases} 1/|\tau| & |\tau| \geq 2|t| \\ 0 & |\tau| < 2|t| \end{cases} \tag{5}$$

Sampling the signal $s(t)$ composed of M components and taking the time window with length of $2L+1$, the frequency distribution $\mathrm{C}(t_0, k)$ of the signal $s(t)$ at time t_0 can be obtained by using BJD:

$$\begin{aligned}\mathrm{C}(t_0,k) &= \mathrm{BJD}(t_0,k) \\ &= 2\sum_{m=-L}^{L}\sum_{n=-(L-|m|)}^{(L-|m|)} s(t_0+nT_s+mT_s)s^*(t_0+nT_s-mT_s)\psi_0(-n,2m)e^{-4\pi km/N}\end{aligned} \tag{6}$$

Where, T_s is the sampling period, N is the number of sampling points of the digital frequency in the frequency domain, and $\psi_0(n,m)$ is the discrete kernel function of BJD,

$$\psi_0(n,m) = \begin{cases} 1/|m| & |m| \geq 2|n| \\ 0 & |m| < 2|n| \end{cases} \tag{7}$$

By using the multi-peak detection method [10] for the frequency distribution $\mathrm{C}(t_0, k)$ of signal $s(t)$ at time t_0, the instantaneous frequency $k_1, k_2 \cdots, k_M$ of the M signal components at time t_0 can be obtained.

According to the above analysis, the Nth element output of the linear array at time t_0 : $x_n(t_0) = s(t_0 + nT_s)$, it is substituted into equation (6) to obtain:

$$\mathrm{C}(t_0,k) = 2\sum_{m=-L}^{L}\sum_{n=-(L-|m|)}^{L-|m|} x_{n+m}(t_0)x_{n-m}{}^*(t_0)\psi_0(-n,2m)e^{-4\pi km/N} \tag{8}$$

In the formula (8), if set

$$S(m) = 2\sum_{n=-(L-|m|)}^{L-|m|} x_{n+m}(t_0)x_{n-m}{}^*(t_0)\psi_0(-n,2m)$$

we can get:

$$\mathrm{C}(t_0,k/2) = \sum_{m=-L}^{L} S(m)e^{-2\pi km/N} \tag{9}$$

The formula (8) and (9) show that $C(t_0, k/2)$ can be obtained by the FFT transformation of N points by the vector $\{S(m)\}_{m=-L}^{L}$, and $C(t_0, k)$ is the frequency distribution curve of the multi-component signal $s(t)$ at time t_0. Using the multi peak estimation algorithm [12] for $C(t_0, k/2)$, we can get the twice digital instantaneous frequency k_i of the M component signals at time t_0, $i = 1, \cdots, M$. Since the sampling signal of $2L+1$ point is expanded to N point for FFT transformation, the corresponding unit quantity of k_i in frequency domain is $f_s^{(i)}(\theta_i)/N$, and the instantaneous frequency corresponding to k_i shall be $k_i f_s^{(i)}(\theta_i)/N$. The instantaneous frequency extraction process based on BJD can be composed of the steps shown in the Figure 2.

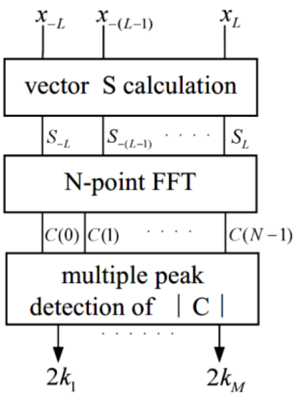

Figure 2. IF extraction based on BJD

The digital instantaneous frequencies $\{k_i\}_{i=1}^{M}$ of the M signal components obtained by the spatial sampling contain the arrival angle information of each component signal. So we can calculate and estimate the arrival angle θ_i of each signal source by comparing the instantaneous frequency at the same time.

4. AOA Detection based on the Instantaneous Frequency Comparison

In order to obtain the arrival angle θ_i of each spatial signal source, the actual instantaneous frequency $f_i(t)$ of each component signal $s_i(t)$ must be obtained. We can analyze the output signal of any array element in time domain. Set the output signal of the reference array element as $s(t)$. Using the signal processing process shown in Figure 3, the instantaneous frequency $f_i(cT_s)$ of each component signal at time cT_s can be obtained through the short-time signal sampled in the time domain.

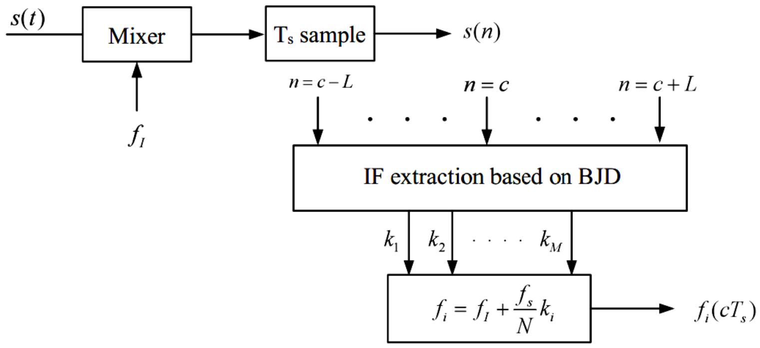

Figure 3. IF detection of $s(t)$

In Figure 3, T_s is the sampling period of the signal after mixing, and f_s is the sampling frequency. The number of the sampling signals selected is $2L+1$, which is equal to the number of the array elements. When the sampling time of the array signal is $t_0 = cT_s$, the digital instantaneous frequency k_i of each component signal at t_0 time can be obtained by using the short-time signal sampled in spatial domain. For the same signal source $s_i(t)$ at the same time cT_s , the instantaneous frequency obtained by spatial sampling signal and temporal sampling signal should be equal. Therefore, the following formula is established:

$$k_i \frac{f_s^{(i)}(\theta_i)}{N} = f_i(cT_s) \tag{10}$$

Therefore, from the formula (3), the arrival angle θ_i of the signal can be calculated according to $f_i(t_0)$,

$$\theta_i = \arcsin\left[(\frac{k_i c}{Nd}) / f_i(cT_S) \right] \tag{11}$$

The experimental results show that the error $\Delta\theta_i$ of the DOA estimation is mainly affected by the length of the array elements and the quantization error Δf_s of the digital frequency in the frequency domain. In addition, the incident angle θ_i of the signal also affects $\Delta\theta_i$ to some extent. For the spatial signal of the incident angle $0^\circ < \theta_i \le 90^\circ$, a large number of sampling points N in the frequency domain is used in the time-frequency transformation process of the instantaneous frequency extraction, and the number of linear array elements is increased appropriately, which can achieve satisfactory results.

5. EXPERIMENT AND ANALYSIS

Exp.1 It is assumed that there is a LFM signal s_1 and a single frequency signal s_2 whose incident angle changes at a constant speed with time in the airspace. They are incident at angles θ_1 and θ_2 on a symmetrical uniform linear array with $L+1$ elements. Where $s_1(t) = \exp(j2\pi(f_1 t + 0.5at^2))$, $s_2(t) = \exp(j2\pi f_0 t)$, $f_1 = 520\text{MHz}$, $a = 100\text{MHz/s}$, $f_0 = 650\text{MHz}$, $\theta_1 = 30^\circ$, $\theta_2(t) = \theta_0 - bt$, $\theta_0 = 40^\circ$,

$b = 5^{\circ}/s$, the element interval $d = 0.1m$, Intermediate frequency $f_I = 500MHz$, sampling frequency $f_s = 600MHz$, Sampling points in the frequency domain $N = 10000$. In the noise environment with SNR of 0dB, we use the proposed method to take different linear array element length L, and carry out the detection simulation experiment on the arrival angle and instantaneous frequency of s_1 and s_2. The results are shown in Figure 4 and Figure 5.

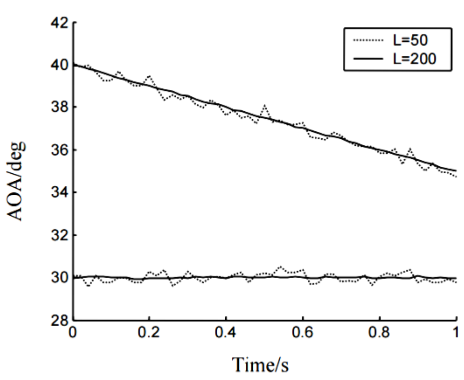

Figure 4. AOA estimation of s_1 and s_2

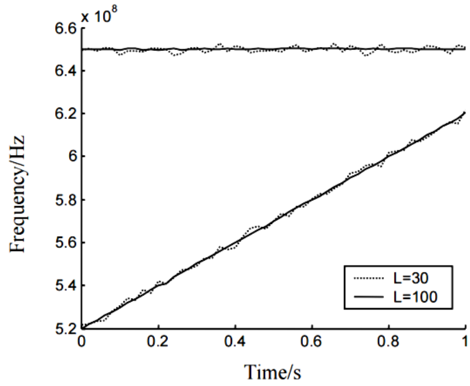

Figure 5. IF estimation of s_1 and s_2

It can be seen from Figure 4 and Figure 5 that in the noise environment with low SNR , the detection and comparison of instantaneous frequency of the spatial sampling signal and the time-domain sampling signal can analyze and judge the change of the instantaneous frequency and the arrival angle of each signal component in the spatial multi-source signal with the time. The estimation accuracy of the instantaneous frequency and the angle of arrival can be improved by increasing the array element number of the linear array, because with the increase of the sampling signal window length, the time-frequency aggregation performance of BJD distribution is also enhanced.

Exp.2 In order to study the influence of noise on the detection results, it is assumed that there are single frequency signal s_1 and LFM pulse signal s_2 in the airspace. s_1 and s_2 are simultaneously incident on the symmetrical uniform linear array with $L+1$ elements at angles $\theta_1 = 30^{\circ}$ and $\theta_2 = 40^{\circ}$ respectively. We have simulated the instantaneous frequency and angle of arrival of signal s_1 and s_2 at time $t = 0$. Where $s_1(t) = \exp(j2\pi f_1 t)$, $s_2(t) = \exp(j2\pi(f_2 t + 0.5bt^2))$, $f_1 = 600MHz$, $f_2 = 550MHz$, $b = 100MHz/\mu s$, $d = 0.1m$, $f_I = 500MHz$, $f_s = 600MHz$, $N = 10000$. Take the number of array elements as $L = 50$ and $L = 100$, and carry out 100-200 Monte Carlo experiments in each noise environment. Figure 6 and Figure 7 show the curve of RMS error of the arrival angle and the instantaneous frequency estimation varying with SNR.

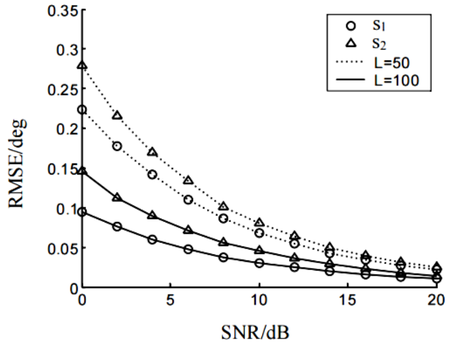

Figure 6. RMSE of AOA estimation

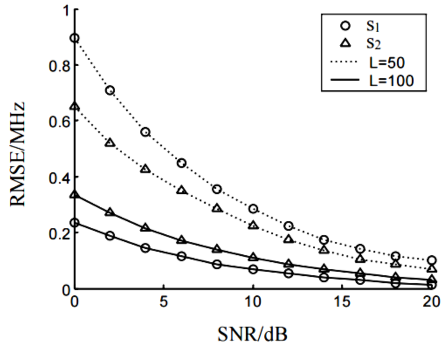

Figure 7. RMSE of IF estimation

It can be seen that the RMSE of the estimation method for the angle of arrival and the instantaneous frequency of the signal basically shows an exponential decay trend with the increase of the SNR. The increase of the number of array elements L can significantly reduce the error of the angle and frequency estimation when the SNR is low. It can also be seen from the figure that the estimation errors of the single frequency signal s_1 and the LFM signal s_2 do not differ greatly in the same case. This shows that the estimation method of the angle of arrival and the instantaneous frequency based on the BJD is not obviously affected by the time-frequency characteristics of the signal.

6. CONCLUSION

Based on the spatial sampling model of the linear array antenna for the incident signal, this paper proposes a method to detect the arrival angle and the instantaneous frequency parameters of the spatial multi-source signal by the digital signal process using the Born Jordan distribution which has good time-frequency aggregation characteristics and cross term suppression ability. The method realizes the estimation of the angle of arrival by means of the instantaneous frequency comparison, and reduces the computation of time-frequency spatial spectrum estimation. This method maintains the noise suppression ability of the second-order time-frequency distribution, and is not affected by the time-frequency characteristics of the incident signal. The simulation results show that the method has not only a good real-time performance, but also maintains a strong noise suppression ability.

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