

# AUGMENTED EFFICIENT ZERO-KNOWLEDGE CONTINGENT PAYMENTS IN CRYPTOCURRENCIES WITHOUT SCRIPTS

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## **ABSTRACT**

*Zero-Knowledge Contingent Payment presents how Bitcoin contracts can provide a solution for the so-called fair exchange problem. Banasik, W. et al. first presented an efficient Zero-Knowledge Contingent Payment protocol for a large class of NP-relations, which is a protocol for selling witness. It obtains fairness in the following sense: if the seller aborts the protocol without broadcasting the final message then the buyer finally gets his payment back. However, we find that the seller in the protocol could refuse to broadcast the final signature of the transaction without any compensation for the buyer. As a result, the buyer cannot get the witness from the final signature of the transaction and has the payment for the witness locked until finishing the large computation for a secret signing key.*

*In this paper, we fix this problem by augmenting the efficient Zero-Knowledge Contingent Payment protocol. We present a new protocol where the seller needs to provide the deposit before the zero-knowledge proof of knowledge of the witness being sold. And then the buyer could obtain the seller's witness if the seller broadcasts the final signature of the transaction and gets the payment and his deposit. Otherwise, the buyer could get back the payment and obtain the seller's deposit. This new augmented protocol is constructed without any new assumptions.*

## **KEYWORDS**

*fair exchange, Bitcoin, cryptocurrencies, zero-knowledge, without scripts.*

## **1. INTRODUCTION**

The concept of cryptocurrency emerged in the last few years and recently there has been a huge emphasis on constructing cryptocurrencies easy for circulation. The main valuable property of these cryptocurrencies is that their security does not need to rely on any single trusted third party. Bitcoin [1], the most prominent of the cryptocurrencies, is a decentralized payment system that is based on maintaining a public transaction ledger in a distributed manner. The list of transactions in this payment system is written on a public ledger, which is maintained jointly by the system users. With the public ledger, the system can implement an idea of the so-called smart-contracts. Consider the Zero-Knowledge Contingent Payment [2], which is a contract and shows that how Bitcoin contracts can provide a solution for the so-called fair exchange problem. With respect to [2], the Zero Knowledge Contingent Payment makes it possible to make payments using Bitcoin in a trustless manner where neither the payer or payee can cheat and that the payments are given

to the payee only in the case that some knowledge is disclosed by the payee. The execution of this contract is guaranteed by the rules of the underlying Bitcoin system.

To be more specific, it is executed between two parties that do not trust each other: the Seller and the Buyer. The Buyer is looking for some value  $x \in \{0,1\}^*$  and the valuable conditions of  $x$  for him can be described as a function  $f: \{0,1\}^* \rightarrow \{true, false\}$  (in a form of a polynomial-time computer program), such that finding  $x$  satisfying  $f(x) = true$  is much harder than verifying that  $f(x) = true$  holds. Hence then, the Buyer is willing to pay for  $x$  in the conditions that  $x$  is valuable for him. Imagine now that the Buyer is approached by a Seller through the internet, who is claiming that he knows  $x$  satisfying the valuable condition  $f(x) = true$  and willing to sell  $x$ . Then the parties face the following problem: they need to finish the transaction on the internet in a trustless manner where neither the Seller or the Buyer can cheat.

The Zero-Knowledge Contingent Payment protocol described in [2] is accomplished using the combination of a hash-locked transaction and some non-standard scripts so that the data revealed in the hashlock release is the data in need. Recently, It has been implemented in [3] for selling a proof of a sudoku solution.

Banasik, W. et al. first created non-trivial efficient smart contracts using only the standard transactions in the public ledger [4]. Under the assumption of semantically secure Paillier encryption and symmetric encryption, secure commitment and time-locked commitment schemes, the strongly unforgeable ECDSA scheme, and zero-knowledge proof of knowledge, they constructed efficient Zero-Knowledge Contingent Payment protocol for a large class of NP-relations, which is a protocol for selling the witness. In the protocol, the buyer first prepares a transaction  $T_1$  sending the funds from his public key to a public key shared by the buyer and the seller, and then they sign the transaction  $T_2$ , sending the funds from the shared public key to the seller's public key, using secret-shared signing keys in cooperation with the seller knowing the final signature of the transaction but the buyer not. After the seller giving a zero-knowledge proof of the witness being sold, the buyer broadcasts  $T_1$  and the seller broadcasts  $T_2$ . And then the buyer can reverse the witness from the final signature. However, we find that the seller in the protocol could refuse to broadcast the final signature of the transaction without any compensation for the buyer. As a result, the buyer cannot get the witness from the final signature of the transaction and has the payment for the witness locked until finishing the large computation for a secret signing key.

## 1.1. Our Results

In this paper, we fix this problem by augmenting the efficient Zero-Knowledge Contingent Payment protocol. We present a new protocol where the seller needs to provide the deposit before the zero-knowledge proof of knowledge of the witness being sold. And then the buyer could obtain the seller's witness if the seller broadcasts the final signature of the transaction and gets the payment and his deposit. Otherwise, the buyer could get back the payment and obtain the seller's deposit. This new augmented protocol is constructed without any new assumptions. A high-level overview is as the following.

The ECDSA signature scheme is denoted by  $(ECGen, ECSign, ECVer)$ . A user in Bitcoin system is identified by his public key  $pk$  in the ECDSA signature scheme and each key  $pk$  is called an address. A simple transaction denoted by  $[T]$  simply sends some Bitcoins  $x$  from address  $pk_0$  to  $pk_1$ .  $[T]$  contains the following tuple  $[T] := (TXid(T'), value: x, from: pk_0, to: pk_1)$ , where  $TXid(T')$  denotes the identifier of transaction  $[T']$  with value at least  $x$  that appeared earlier on the ledger and is redeemed by  $[T]$ .

A complete transaction denoted by  $T$  has a form  $([T], ECDSA_{sk_0}([T]))$ .  $TXid(T')$  is defined simply as a  $SHA256$  hash of  $([T], ECDSA_{sk_0}([T]))$ .

Under the assumption of semantically secure Paillier encryption and symmetric encryption, secure commitment and time-locked commitment schemes, the strongly unforgeable ECDSA scheme, and zero knowledge proof, our augmented efficient Zero-Knowledge Contingent Payment protocol consists of four stages.

In stage 1, the buyer and seller execute a key exchange protocol to generate two key pairs for the ECDSA signatures such that the secret keys are secret-shared between them. As a result, the buyer holds  $(PK, SK_0)$ ,  $(PK', SK'_0)$  and seller holds  $(PK, SK_1)$ ,  $(PK', SK'_1)$ .

In stage 2, the seller prepares transaction  $T_3$  sending the funds  $p$  from  $PK_S$  to  $PK'$  and  $T'_3$  sending the funds  $p$  from  $PK'$  to  $PK_S$ , and then sends the hash value of  $T_3$  to the buyer. The buyer prepares transaction  $T_1$  sending the funds  $v$  from  $PK_B$  to  $PK'$ ,  $T'_1$  sending the funds  $v + p$  from  $PK'$  to  $PK$ , and  $T_2$  sending the funds  $v + p$  from  $PK$  to the seller's public key  $PK_S$ . They execute the unique signature generation protocol to generate the signature of  $[T'_1]$  for the buyer and the signatures of  $T_2$  and  $[T'_3]$  for the seller. And then the seller broadcasts  $T_3$ . If  $T_3$  is not corresponding to the hash value of sent before, the buyer aborts. Otherwise, the parties execute the following stages.

In stage 3, the seller proves the knowledge of the witness being sold by executing a zero-knowledge proof of knowledge protocol with the buyer in the cut-and-choose technique. In the proof, the seller uses the signature of  $[T_2]$  to compute the secret key for encryption of a set of challenges and responses with the witness, and then it sends the commitments of the encryption to the buyer. After receiving the subset of the challenges from the buyer, the seller opens the commitments asked to open. The proof is valid if the output of the buyer's verification is true. Otherwise, the buyer aborts.

In stage 4, if the buyer broadcasts the valid  $T_1$  and  $T'_1$ , and the seller broadcasts the valid  $T_2$ , the buyer could reverse the witness from the signature of  $T_2$ . If the buyer refuses to broadcast  $T_1$  and  $T'_1$  or he broadcasts illegal  $T_1$  and  $T'_1$ , then the seller could broadcast  $T'_3$  to get the deposit back. If the buyer broadcasts the valid  $T_1$  and  $T'_1$ , but the seller refuses to broadcast  $T_2$ , the buyer could finally obtain his own funds together with the seller's deposit after finishing a large computation for a secret signing key.

Therefore, it holds that:

*assume the existence of semantically secure Paillier encryption and symmetric encryption, secure commitment and time-locked commitment schemes, the strongly unforgeable ECDSA scheme, and zero knowledge proof of knowledge. Then, there exists a secure efficient Zero-Knowledge Contingent Payment protocol in cryptocurrencies without scripts, which obtains fairness in the following sense: if the seller aborts protocol without broadcasting the final message then the buyer finally gets its payment back and gets an extra financial compensation from the seller.*

## 1.2. Related Works

Relevant to our work are the works on smart contracts that provide solutions for fair protocols in the cryptocurrency systems. Bentov, I. and Kumaresan, R. studied secure computations in the following model of fairness: a malicious user who aborts protocol after receiving the output is forced to pay a mutually predefined monetary penalty [5]. They then showed how to use Bitcoin

system to achieve secure computations with the above defined fairness in two-party setting as well as the multiparty setting (with a honest majority) by simulating with a new ideal functionalities as they proposed.

Andrychowicz, M. et al. showed how to obtain fair two-party secure computation protocol via the Bitcoin system with the fairness in the following sense: if one party aborts the protocol after learning the output but the other one not, then the other party gets a financial compensation from the aborted one [6]. They constructed the protocol with the two-party protocol of Goldreich and Vainish [7] additionally secured against an active adversary with zero-knowledge proofs. And they presented one possible application of the protocol to the fair contract signing: each party is forced to either complete the protocol or pay a fine to the other party.

### 1.3. Outline

In Section 2, we define the notations and definitions that are used through the paper. In Section 3, we describe the subprotocols used in the Banasik's efficient Zero-Knowledge Contingent Payment and give the new subprotocols used in our protocol. In Section 4, we present our augmented efficient Zero-Knowledge Contingent Payment protocol. And Section 5 is conclusion.

## 2. PRELIMINARIES

### 2.1. Notations

We use  $n$  to denote the security parameter. We use  $[k]$  for any  $k \in N$  to denote the set  $\{1, \dots, k\}$ . For any probabilistic algorithm  $A(\cdot)$ ,  $A(x)$  is the result of executing  $A$  with input  $x$  and uniformly chosen randomness. We use  $y = A(x)$  to denote that  $y$  is set to  $A(x)$ . For a set  $S$ , we use  $y \in S$  (or  $y \leftarrow S$ ) to denote that  $y$  is chosen from  $S$ . For any language  $L$  and instance  $x \in L$ , we use  $\{\mathcal{R}_L\}(x)$  to denote the set of witnesses for  $x \in L$ . A function  $\mu: N \rightarrow R$  is negligible (in  $x$ ) if for every positive integer  $c$  there exists an integer  $N_c$  such that for all  $x > N_c$  we have that  $|\mu(x)| < \frac{1}{x^c}$ .

### 2.2. Elliptic Curve Digital Signature Algorithm (ECDSA)

We recall the definition of digital signature algorithm [8, 4, 9].

**Definition 1** (Digital Signature Scheme): A digital signature scheme consists of a triple of probabilistic polynomial-time algorithms  $(Gen, Sign, Ver)$  satisfying the following conditions:

- Key-generator algorithm  $Gen$ : On input  $1^n$ ,  $Gen$  outputs a pair of keys  $(sk, vk)$ ;
- Signing algorithm  $Sign$ : On input secret key  $sk$  and a message  $\alpha$ ,  $Sign$  outputs signature  $\sigma$ ;
- (Deterministic) Verifying algorithm  $Ver$ : On input public key  $vk$ , message  $\alpha$ , and signature,  $\sigma$ ,  $Ver$  returns  $Output \in \{\text{accept}, \text{reject}\}$ ;
- For every pair  $(sk, vk)$  in the range of  $Gen(1^n)$  and for every  $\alpha \in \{0,1\}^*$ , the signing and verification algorithms  $Sign$  and  $Ver$  satisfy

$$\Pr[Ver(vk, \alpha, Sign(sk, \alpha)) = 1] = 1$$

where the probability is taken over the internal coin tosses of algorithms  $Sign$  and  $Ver$ .

**Definition 2** (Elliptic Curve Digital Signature Algorithm) The ECDSA signature scheme  $(ECGen, ECSign, ECVer)$  is a variant of digital signature scheme algorithm, in which the algorithms  $ECGen, ECSign, ECVer$  are defined as the followings.

- Key-generator algorithm  $ECGen$ : On input  $1^n$ ,  $ECGen$  chooses an elliptic curve group  $(G, O, g, +)$  over a prime field  $Z_p$ , where  $O$  is the neutral element,  $g$  is the generator of  $G$ , and the order of  $G$  is a prime number such that  $\lceil \log_2 |G| \rceil = n$ . And then  $ECGen$  samples a random  $d \in_R Z_{|G|}$ , and computes  $D := d \cdot g$ . The generated secret key is  $(d, (G, O, g, +))$ , and the public key is  $(D, (G, O, g, +))$ ;
- Signing algorithm  $ECSign$ : Let  $H$  be a hash function and  $f: G \rightarrow Z_{|G|}$  be a reduction function that we will define in a moment. On input secret key and message  $\alpha$ ,  $ECSign$  first chooses a random  $k \in_R Z_{|G|}$ , and then computes  $r = f(k \cdot g)$  and  $s = k^{-1}(H(\alpha) + d \cdot r) \bmod |G|$ . If  $r = 0$  or  $s = 0$  then the algorithm aborts. Otherwise, the  $ECSign$  outputs  $(r, s)$ ;
- (Deterministic) Verifying algorithm  $ECVer$ : On input public key, message  $\alpha$  and signature  $(r, s)$ ,  $ECVer$  first checks if  $r$  and  $s$  are non zero elements of  $Z_{|G|}$  and then verifies if  $r = f(H(\alpha) \cdot s^{-1} \cdot g + r \cdot s^{-1} \cdot D)$ . If this holds, then  $ECVer$  outputs  $ok$ , and otherwise it outputs  $\perp$ .

Recall that every element of  $G$  has the form  $(x, y) \in Z_p^2$ . The reduction function  $f: G \rightarrow Z_{|G|}$  is defined as: on input  $(x, y)$  ignores  $y$  and produces as output  $f(x, y) = x \bmod |G|$ .

Security of the ECDSA signature scheme. The ECDSA signature scheme satisfies existentially strong unforgeability under a chosen message attack. We consider the following game played by a polynomial time adversary  $A$ . First, a key pair is sampled as  $(vk, sk) := ECGen(1^n)$  and the adversary  $A$  is given the public verification key  $vk$ . Then,  $A$  chooses a sequence of messages  $\alpha_1, \dots, \alpha_k$  and learns each corresponding signature  $\sigma_i := ECSign_{sk}(\alpha_i)$ . He does it in an adaptive way that he chooses each  $\alpha_i$  after learning  $\sigma_1, \dots, \sigma_{i-1}$ . Finally,  $A$  outputs a pair  $(\alpha_{k+1}, \sigma_{k+1})$ . We say that  $A$  mauls a signature if the output  $(\alpha_{k+1}, \sigma_{k+1})$  satisfies that  $ECVer(\alpha_{k+1}, \sigma_{k+1}) = ok$  and  $\sigma_{k+1}$  has not been sent to  $A$  as one of the signatures in  $\sigma_1, \dots, \sigma_k$ . The ECDSA signature scheme is existentially strongly unforgeable under a chosen message attack if every polynomial-time adversary can maul a signature with at most negligible probability.

### 2.3. Homomorphic Encryption Schemes

We recall the definitions of encryption schemes in [4, 10, 11] and present the following definition.

**Definition 3** (Public Encryption Scheme) A public encryption scheme consists of a triple of probabilistic polynomial-time algorithms  $(EncGen, Enc, Dec)$ :

- Key-generator algorithm  $EncGen$ : On input  $1^n$ ,  $EncGen$  outputs a pair of keys  $(pk, sk)$ ;
- Encryption algorithm  $Enc$ : On input public key  $pk$  and message  $\alpha \in \{0,1\}^*$ ,  $Enc$  outputs ciphertext  $c = Enc_{pk}(\alpha) \in \{0,1\}^*$ ;
- Decryption algorithm  $Dec$ : On input private key  $sk$ , ciphertext  $c$ ,  $Dec$  outputs  $\alpha' = Dec_{sk}(c)$ ;
- For every pair  $(pk, sk)$  in the range of  $EncGen(1^n)$  and for every  $\alpha \in \{0,1\}^*$ , the encryption and decryption algorithms  $Enc$  and  $Dec$  satisfy

$$\Pr \left[ Dec_{sk} \left( Enc_{pk}(\alpha) \right) = \alpha \right] = 1$$

where the probability is taken over the internal coin tosses of algorithms  $Enc$  and  $Dec$ .

**Security.** To define security of an encryption scheme  $(EncGen, Enc, Dec)$  consider a polynomial time adversary  $A$  produces a pair of messages  $(m_0, m_1)$  with access to oracle  $Enc_{pk}(\cdot)$  and  $Dec_{sk}(\cdot)$ , where  $(pk, sk) = EncGen(1^n)$ . He then receives a ciphertext  $c = Enc_{pk}(m_b)$  for a random  $b \in \{0,1\}$ , and produces  $b' \in \{0,1\}$  without asking the oracle  $Dec_{sk}(\cdot)$  to decrypt  $c$ . We say that  $A$  wins if  $b' = b$ . We say that the encryption scheme  $(EncGen, Enc, Dec)$  is semantically secure if for every polynomial time adversary  $A$  the probability that he wins is at most  $\frac{1}{2} + \mu(n)$ , where  $\mu$  is some negligible function (in other words:  $Enc_{pk}(m_0)$  and  $Enc_{pk}(m_1)$  are computationally indistinguishable).

The definition of a symmetric-key encryption scheme is the same as that of a public encryption scheme except that there is only one secret key  $k$ , which is usually sampled uniformly at random from some space  $\mathcal{K}$  (that depends on  $1^n$ ). The adversary does not learn the key, but can try to get some partial information about  $k$  by choosing messages  $m_1, \dots, m_n$  and learning the corresponding ciphertexts  $Enc_{pk}(m_1), \dots, Enc_{pk}(m_n)$  in an adaptive way.

A public-key encryption scheme  $(EncGen, Enc, Dec)$  is additively homomorphic if the set of valid messages for the public key  $pk$  is an additive group  $(H_{pk}, +)$ , where the key pair  $(pk, sk)$  is generated by  $EncGen$ . Moreover, the homomorphic algorithm we require is defined by an operation  $\otimes: \{0,1\}^* \times \{0,1\}^* \rightarrow \{0,1\}^* \{\perp\}$ , such that for every valid key pair  $(pk, sk)$  and every pair of messages  $(m_0, m_1) \in H_{pk}$  we have that  $Dec_{sk}(Enc_{pk}(m_0) \otimes Enc_{pk}(m_1)) = m_0 + m_1$ .

## 2.4. Time-lock Commitment Schemes

We recall the definitions of time-lock commitment schemes [4, 12, 13]. A commitment scheme  $(Commit, Open)$  executed between two parties (a committer and a receiver) consists of two phases, the commit phase and the open phase.

- Commit Phase. The committer takes as input a message  $\alpha \in \{0,1\}^*$  and chooses a random  $r \in \{0,1\}^*$ . The committer then computes  $c := Commit(\alpha)$  with randomness  $r$  and sends it to the receiver;
- Open Phase. The committer reveals the message  $\alpha$  committed and the randomness  $r$  used in the commit phase. And the receiver verifies  $Open(c, \alpha, r)$ .

**Security.** A commitment scheme is secure if it is binding and hiding. The hiding property means that for every  $\alpha_0, \alpha_1 \in \{0,1\}^*$ ,  $Commit(\alpha_0)$  and  $Commit(\alpha_1)$  are computationally indistinguishable. The binding property means that for any PPT committer, he can reveal another message  $\alpha' \neq \alpha$  such that  $Open(c, \alpha, r) = \alpha'$  with negligible probability.

A commitment scheme  $(Commit, Open)$  is a time-lock commitment scheme if the receiver can open the commitment by himself with a significant computational effort. Every time-lock commitment comes with two parameters,  $\tau_0$  and  $\tau_1$  (with  $\tau_0 \leq \tau_1$ ), and is called  $(\tau_0, \tau_1)$ -secure, where  $\tau_0$  denotes the time that everybody, including very powerful adversaries, needs to force open the commitment, and  $\tau_1$  denotes the time needed by the honest users to force open the commitment.

## 2.5. Zero-knowledge Proofs of Knowledge

We recall the definitions of zero-knowledge protocols [4, 14, 15, 16] and give a simple introduction.

**Definition 4** (Interactive Proof System) A pair of interactive Turing machines  $\langle P, V \rangle$  is called an interactive proof system for any language  $L$  if machine  $V$  is polynomial-time and the following two conditions hold:

- Completeness: There exists a negligible function  $c$  such that for every  $x \in L$ ,

$$\Pr[\langle P, V \rangle(x) = 1] > 1 - c(|x|)$$

- Soundness: There exists a negligible function  $s$  such that for every  $x \notin L$  and every interactive machine  $B$ , it holds that

$$\Pr[\langle P, V \rangle(x) = 1] < s(|x|)$$

$c(\cdot)$  is called the completeness error and  $s(\cdot)$  is the soundness error.

Zero-knowledge protocols are interactive proof systems with zero-knowledge property, which means that the prover knows a witness  $\omega$  for some instance  $x \in L$  and convinces the verifier that  $x \in L$  without providing the verifier with any additional information beyond the fact that  $x \in L$ .

A zero-knowledge protocol is called a zero-knowledge proof of knowledge if  $L \in NP$  and for every prover  $P^*$  there exists a polynomial-time knowledge extractor, that can output a witness  $\omega$  for  $x \in L$  by interacting with  $P^*$ . We follow the requirement in [4], suppose that the last two messages in the zero-knowledge proof of knowledge protocol are: a challenge  $c$  sent by the verifier to prover, and the prover's response  $r$  corresponding to the challenge  $c$ . The knowledge extractor extracts witness  $\omega$  after being given transcripts of two accepting executions that are identical except that the challenges are different (and the responses may also be different).

## 2.6. Bitcoin Transaction Syntax

A complete transaction denoted by  $T$  has a form  $([T], EC\text{Sign}_{sk_0}([T]))$ , where  $[T] := (TXid(T'), value: x, from: pk_0, to: pk_1)$ . Another standard type of the transaction is called multisig transaction [4], where  $[T]$  has a form  $(TXid(T'), value: x, from: pk_0, to: k - out - ofm: pk_1, \dots, pk_m)$  and it is signed by  $sk_0$ . It can be redeemed by a transaction  $T''$  with the form  $([T''], \sigma_{i_1}, \dots, \sigma_{i_k})$ ,  $1 \leq i_1 < \dots < i_k \leq m$  and for every  $1 \leq j \leq k$  it holds that  $ECVer_{pk_{i_j}}([T''], \sigma_{i_j}) = ok$ .

## 3. THE SUBPROTOCOLS IN THE ZERO-KNOWLEDGE CONTINGENT PAYMENT PROTOCOL

In this section, we describe the subprotocols used in Banasik's efficient Zero-Knowledge Contingent Payment system and give the new formalization used in our protocol.

### 3.1. The Two-party ECDSA Key Generation Protocol

The first ingredient is a protocol called *SharedKGen*, in which two parties, the Seller and Buyer, generate a key pair (public key, private key) for ECDSA, in such a way that the secret key is secret-shared between the Seller and Buyer. This protocol is still used in our final protocol.

To be more precise, fix an elliptic curve  $(G, Og, +)$  constructed over a prime field  $Z_p$  and let  $Com = (Commit, Open)$  be a secure (computational hiding) commitment scheme, both parties take as input a security parameter  $1^n$ , the overview of *SharedKGen* is as following:

- Seller: samples  $d_S \leftarrow Z_{|G|}^*$ , computes  $D_S := d_S \cdot g$  and  $c := Commit(D_S)$ , and sends  $c$  to the Buyer
- Buyer: samples  $d_B \leftarrow Z_{|G|}^*$ , computes  $D_B := d_B \cdot g$  and sends  $D_B$  to the Seller;
- Seller: sends  $Open(D_S)$  to the Buyer;
- The Seller and Buyer compute the ECDSA public key separately:  $pk := d_S \cdot D_B = d_B \cdot D_S$  and abort if  $pk = 0$ . And the secret key  $sk := d_S \cdot d_B$  is secret-shared between the two parties.

### 3.2. The Unique Signature Generation Protocol

We will present two formalizations of the unique signature generation protocol,  $USG_1$  and  $USG_2$ .  $USG_1$  is a little different from the  $USG$  protocol used in Banasik's efficient Zero-Knowledge Contingent Payment system [4].

The  $USG$  protocol uses *KSignGen* as a subroutine, in which the parties jointly sign a message  $z$  using the secret-shared signing key generated by *SharedKGen*. Recall that  $G$  is an elliptic curve group for ECDSA signature scheme  $ECGen, ECSign, ECVer$ ,  $TLCCom = (TLCommit, TLForceOpen)$  is a time-locked commitment scheme, and  $(AddHomGen, AddHomEnc, AddHomDec)$  is a Paillier encryption scheme which is additively homomorphic over  $Z_{n'}$ , where  $n' > 2 \cdot |G|^4$ . Let  $F$  be a one-way function. Protocol  $KSignGen$  is described as follows.

- Seller and Buyer: They jointly create signing randomness  $K$  using the same way in *SharedKGen*. The Seller holds  $K_S \in Z_{|G|}^*$  and  $K_S := k_s \cdot g$  and the Buyer holds  $K_B \in Z_{|G|}^*$  and  $K_B := k_B \cdot g$ . Finally the parties both know the signing randomness  $K := k_s \cdot k_B \cdot g$ , parse  $K$  as  $(x, y)$ , compute  $r := x \bmod |G|$ , and abort if  $r = 0$ ;
- Seller: He creates a new key pair  $(pk_{AH}, sk_{AH}) := AddHomGen(1^n)$  in the Paillier encryption scheme, computes the ciphertext  $c_S := AddHomEnc_{pk_{AH}}(d_S)$ , and sends the public key  $pk_{AH}$  and the ciphertext  $c_S$ ;
- Buyer: On receiving  $pk_{AH}$  and  $c_S$  from the Seller, he computes  $c_0 := k_B^{-1} \cdot H(z) \bmod |G|$ ,  $c_1 := AddHomEnc_{pk_{AH}}(c_0)$ ,  $t := k_B^{-1} \cdot r \cdot d_B \bmod |G|$ ,  $c_2 := c_1 \otimes (c_S)^t$ , samples  $u \leftarrow \{1, \dots, |G|^2\}$ , computes  $c_B := c_2 \otimes AddHomEnc_{pk_{AH}}(u \cdot |G|)$ , and sends the ciphertext  $c_B$ ;
- Seller: On receiving  $c_B$ , he computes  $S_0 := AddHomDec_{sk_{AH}}(c_B)$ ,  $s := k_S^{-1} S_0 \bmod |G|$  and aborts if  $s = 0$ . The final signature of  $z$  is  $\sigma := (r, s)$  if  $ECVer_{pk}(z, \sigma) = ok$  and otherwise the Seller aborts. At the end he commits to  $S = F(\sigma)$ , creates a time-lock commitment to  $d_S$ , and sends  $\Gamma_i := Commit(S)$  and  $\Phi_i := TLCCommit(d_S)$ ;

We stress that when the two parties execute *KSignGen*, the one who is supposed to obtain the final signature at the end of *KSignGen* plays the role of "Seller".

The *USG* protocol is executed after the parties generate  $a$  key pairs  $(sk^1, pk^1), \dots, (sk^a, pk^a)$  using the SharedKGen protocol.

- Buyer: He chooses a random subset  $J \subseteq \{1, \dots, a\}$ , such that  $|J| = a - b$ . Let  $j_1, \dots, j_b$  denote the set  $\{1, \dots, a\} \setminus J$ . And then he chooses message  $z$  to be signed and sends it to the Seller ;
- Seller and Buyer: For  $i = 1$ , the parties execute the  $KSignGen(1^n)$  protocol with  $pk_i := d_S^i \cdot d_B^i \cdot g$ . And at the end of each execution, the Seller sends the commitment  $\Gamma_i := \text{Commit}(S^i)$ , where  $S^i = F(\sigma_i)$ , and the time-lock commitment  $\Phi_i := \text{TLCCommit}(d_S^i)$  ;
- Buyer: He sends  $J$  to Seller ;
- Seller: For each  $j \in J$ , the Seller opens the commitments to  $S^j$  and  $d_S^j$ , and sends  $\sigma^j, k_S^j$  and  $sk_{AH}^j$  to Buyer ;
- Buyer: He aborts if any of the commitments did not open correctly. Otherwise, for each  $j \in J$ , he verifies if the following holds:
  - $\text{ECVer}_{pk^j}(z, \sigma^j) = ok$ ;
  - $F(\sigma^j) = S^j$ ;
  - $d_S^j \cdot d_B^j \cdot g = pk^j$ ;
  - $\text{AddHomDec}_{sk_{AH}^j}(c_S^j) = d_S^j$ .
 If the verification fails then the Buyer aborts. Otherwise, the unique signature is the set  $\{\sigma^{j_i}\}$ , where  $i \in \{1, \dots, b\}$ .

In this paper, the  $USG_1$  protocol is the same as the *USG* protocol except that  $USG_1$  outputs the set  $\{\sigma^i\}$ , where  $i \in \{1, \dots, a\}$ , as the unique signature. For the technique used in the final protocol, we assume that each  $S^i$  from the  $USG_1$  protocol is divided into  $2n$  parts  $S^{i,1}, \dots, S^{i,2n}$  each of size  $n$ . In addition, we assume that each part  $S^{i,j}$  is committed separately.

The  $USG_2$  protocol is the same as the  $KSignGen(1^n)$  protocol except that there is no further computation after obtaining the signature  $\sigma := (r, s)$ , of which the verification result is *ok*.

## 4. THE AUGMENTED ZERO-KNOWLEDGE CONTINGENT PAYMENT PROTOCOL

In this section we show how to use the subprotocols to construct the augmented two-party Zero-Knowledge Contingent Payment Protocol protocol with fairness in the following sense: if the malicious seller aborts protocol without broadcasting the final message then the buyer finally gets his payment back and an extra financial compensation from the seller.

### 4.1. Security Definition

In our protocol, the Seller sells to the Buyer a value  $x$  such that  $f(x) = true$  (for some public function  $f: \{0,1\}^* \rightarrow \{true, false\}$ ). We assume that the price of  $x$  is  $v$  Bitcoins and the deposit of the Seller for this transaction is  $p$  Bitcoins. Hence, before an execution of the protocol starts, there is some unspent transaction  $T_0$  on the blockchain that sends  $v$  Bitcoins to  $pk_B$  and some unspent transaction  $T'_0$  on the blockchain that sends  $p$  Bitcoins to  $pk_S$ . The parties initially share the following common input: security parameter  $1^n$ , price  $v$  for the secret  $x$ , deposit value  $p$  for the computation of a time-lock committed message, parameters  $a, b \in N$  such that  $a > b$ , an elliptic curve group  $G, O, g, +$  for an ECDSA signature scheme, such that  $\lceil \log_2 |G| \rceil =$

$n$ , and parameters  $(\tau_0, \tau_1)$  for the time-lock commitment. We say that the final protocol is  $\varepsilon$  – *secure* if, except with probability  $\varepsilon + \mu(n)$ , the following properties hold:

- if the honest Buyer loses his funds then he learns  $x'$  such that  $f(x') = \text{true}$ ;
- if the honest Seller does not get Buyer's funds then the Buyer learns no information about  $x$ ;
- if an honest Buyer is forced to open a time-lock commitment then he finally does not lose his funds and obtains a financial compensation from the Seller.

## 4.2. Instantiations and Assumptions

**Instantiations.**  $F$  is a one-way function and we use a standard symmetric encryption scheme  $\text{EncGen, Enc, Dec}$  and the additively-homomorphic public key encryption scheme  $(\text{AddHomGen, AddHomEnc, AddHomDec})$  introduced by Pascal Paillier [11]. The elements on which we perform the addition operations are the exponents in the elliptic curve group of the ECDSA scheme. Hence the Paillier encryption scheme is homomorphic over  $Z_{n'}$ , where  $n' > 2 \cdot |G|^4$ , and  $[\log_2 |G|] = n$ .

We use a standard commitment scheme  $\text{Com} = (\text{Commit, Open})$  that is based on hash function and secure in the random oracle model [4]. Let  $H$  be a hash function and a commitment to message  $x$  is defined as  $\text{Commit}(x) := H(x||r)$ , where  $r \in \{0,1\}^*$ . And  $\text{Open}$  is to reveal  $(x, r)$ . The binding property follows from the collision-resistance of  $H$  because that a commitment that can be open in two different ways would form a collision for  $H$ . And the hiding property follows from the fact that  $H(x||r)$  does not reveal any information about  $x$ . We use the classic  $(\tau_0, \tau_1)$ -secure timed commitments  $\text{TLCCom} = (\text{TLCCommit, TLForceOpen})$  in [16] and assume that  $\tau_1 = 10 \cdot \tau_0$  as in [4].

We consider the two-party protocol, executed between a Buyer and a Seller, in which the Seller sells to the Buyer  $x$  such that  $f(x) = \text{true}$ , in the active security settings. The parties are connected by a secure channel, which can be easily obtained using the standard cryptographic techniques. One user in Bitcoin is identified by his public key in the ECDSA signature scheme, which helps to establish the secure channel between each other. Without loss of generality, we set  $(PK_B, SK_B)$  and  $(PK_S, SK_S)$  to be the ECDSA key pairs of the Buyer and Seller respectively.

**Assumptions.** We assume that Paillier encryption and symmetric encryption are semantically secure,  $\text{Com}$  and  $\text{TLCCom}$  are secure commitment schemes, and the ECDSA scheme is strongly unforgeable. Hence, the subprotocols in Section 3 are secure.

We keep the form of the assumption of zero-knowledge proof of knowledge protocol in [4]. We also assume that the public function  $f$  has a zero-knowledge proof of knowledge protocol, denoted by  $\mathcal{F}$ , in which the Seller can prove the knowledge of  $x$  such that  $f(x) = \text{true}$ . Protocol  $\mathcal{F}$  consists of two phases: the  $\text{Setup}_{\mathcal{F}}$  phase and the  $\text{Challenge}_{\mathcal{F}}$  phase. After executing the  $\text{Setup}_{\mathcal{F}}$  phase, the views of the Seller and Buyer are denoted by  $S_{\mathcal{F}}$  and  $B_{\mathcal{F}}$  respectively.

In the  $\text{Challenge}_{\mathcal{F}}$  phase, the Buyer generates challenge message as  $c_{\mathcal{F}} = \text{GenChallenge}_{\mathcal{F}}(B_{\mathcal{F}})$  and sends  $c_{\mathcal{F}}$  to Seller. Then the Seller calculates the corresponding response  $r_{\mathcal{F}} = \text{GenResponse}_{\mathcal{F}}(x, S_{\mathcal{F}}, c_{\mathcal{F}})$  and sends  $r_{\mathcal{F}}$  to the Buyer. At the end, the Buyer decides to accept or reject the proof according to the output of function  $\text{VerifyResponse}_{\mathcal{F}}(B_{\mathcal{F}}, c_{\mathcal{F}}, r_{\mathcal{F}}) \in \{\text{true, false}\}$ .

Protocol  $\mathcal{F}$  is a proof of knowledge since we require that there is a knowledge extractor  $\text{Extract}_{\mathcal{F}}$  such that  $\text{Extract}_{\mathcal{F}}(B_{\mathcal{F}}, c_{\mathcal{F}}^1, r_{\mathcal{F}}^1, c_{\mathcal{F}}^2, r_{\mathcal{F}}^2)$  and  $f(x') = \text{true}$  if only  $\text{VerifyResponse}_{\mathcal{F}}(B_{\mathcal{F}}, c_{\mathcal{F}}^i, r_{\mathcal{F}}^i) = \text{true}$  for  $i = 1, 2$  and  $c_{\mathcal{F}}^1 \neq c_{\mathcal{F}}^2$ . It means that a witness for  $f$  can be extracted from the correct answers corresponding to two different challenges. We also assume that the challenge  $c_{\mathcal{F}}$  is sampled uniformly from the set  $X_{A_{\mathcal{F}}} = \{0, 1\}$  in Seller's view.

### 4.3. The Protocol

Our protocol consists of four stages and the final protocol called *SellWitness* is depicted on Fig.1.

- **Stage 1.** Using the two-party ECDSA key generation protocol *SharedKGen*, the Buyer and Seller jointly generate  $a + 1$  key pairs  $(sk^1, pk^1), \dots, (sk^{a+1}, pk^{a+1})$ , where  $pk^i := d_S^i \cdot d_B^i \cdot g$  and  $sk^i := d_S^i \cdot d_B^i$ . As a result, the Buyer holds  $(PK, SK_0) := ((pk^1, \dots, pk^a), (d_B^1, \dots, d_B^a))$ ,  $(PK', SK'_0) := (pk^{a+1}, d_B^{a+1})$  and the Seller holds  $(PK, SK_1) := ((pk^1, \dots, pk^a), (d_S^1, \dots, d_S^a))$ ,  $(PK', SK'_1) := (pk^{a+1}, d_S^{a+1})$ ;
- **Stage 2.** The parties respectively produce the messages to be signed. If one prepares a transaction  $T$ , then the message to be signed is  $[T]$ . The procedure is called *GenMsg<sub>T</sub>*. The Seller prepares transaction  $T_3$  sending the funds  $p$  from  $PK_S$  to  $PK'$  and  $T'_3$  sending the funds  $p$  from  $PK'$  to  $PK_S$ , and then sends the hash value of  $T_3$  to the Buyer. And then the Buyer prepares transaction  $T_1$  sending the funds  $v$  from  $PK_B$  to  $PK'$ ,  $T'_1$  sending the funds  $v + p$  from  $PK'$  to  $a$ -out-of- $a + b - 1PK, PK_B, \dots, PK_B$  (consists of  $b - 1PK_B$ ), and  $T_2$  sending the funds  $v + p$  from  $PK$  to  $PK_S$ . They execute the unique signature generation protocol *USG<sub>2</sub>* using  $(PK', SK'_0)$  and  $(PK', SK'_1)$  to generate the signature of  $[T'_1]$  for the Buyer and the signature of  $[T'_3]$  for the Seller. And then they execute the *USG<sub>1</sub>* using  $(PK, SK_0)$  and  $(PK, SK_1)$  to generate the signatures of  $[T_2]$  for the Seller, in which the Seller will commit to  $SK_1$  using *TLCom* and send to the Buyer. And then the Seller broadcasts  $T_3$ . If  $T_3$  is not corresponding to the hash value of sent before, then the Buyer aborts; otherwise, the parties execute the following stages;
- **Stage 3.** The Seller proves the knowledge of  $x$  such that  $f(x) = \text{true}$  by executing a zero-knowledge proof of knowledge protocol  $\mathcal{F}$  with the Buyer in the cut-and-choose technique;
- **Stage 4.** The Buyer either extracts  $x$  such that  $f(x) = \text{true}$  or takes his funds back and obtains the Seller's deposit.

**Theorem.** Suppose Paillier encryption and symmetric encryption are semantically secure, *Com* and *TLCom* are secure commitment schemes, and the ECDSA scheme used in the construction of the *USG<sub>1</sub>* and *USG<sub>2</sub>* is strongly unforgeability. Additionally, there is a zero knowledge proof  $\mathcal{F}$  of knowledge of  $x$  s.t.  $f(x) = \text{true}$  of the required form. Then the *SellWitness* (Fig.1) is  $\varepsilon$ -secure for  $\varepsilon = (b/a)^b$ .

*Proof.* Recall that *SellWitness* is  $\varepsilon$ -secure if, except with probability  $\varepsilon + \mu(n)$ , the following properties hold: (1) if an honest Buyer loses funds then he learns  $x'$  such that  $f(x') = \text{true}$ ; (2) if an honest Seller does not get Buyer's funds then the Buyer learns no information about  $x$ ; (3) if an honest Buyer is forced to open a time-lock commitment, then he finally does not lose his funds and obtains a financial compensation from the Seller. Hence then, we show the security analysis of *SellWitness* in two cases: (i) the Buyer is honest and the Seller is malicious, and (ii) the Seller is honest and, while the Buyer is malicious.

At the beginning of this proof, we stress that the security of the unique signature generation protocol *USG<sub>1</sub>* follows that of the *USG* protocol used in Banasik's efficient Zero-Knowledge

Contingent Payment system [4], and the security of the  $USG_2$  protocol follows that of the ECDSA.

For case (i), the honest Buyer loses his funds only if he broadcasts transactions  $T_1$  and  $T'_1$  is redeemed by the Seller or it just locks the Buyer's funds forever. Since with probability  $\geq 1 - (b/a)^b - \mu_0(n)$  (for a negligible  $\mu_0(n)$ ) at least one of the  $b$  chosen executions, consist of execution  $i$  for  $i \in \{1, \dots, a\} \setminus j$ , of the KSignGen procedure was completed honestly by the Seller (guaranteed by the security of  $USG_1$  protocol). While if the Seller does not redeem  $T'_1$ , the honest Buyer will be able to redeem transaction  $T'_1$  and get back his funds together with the Seller's deposit after that the Buyer force-opens one time-locked puzzle  $\Phi_i$  in a honestly completed execution  $i$  of KSignGen, where  $i \in \{1, \dots, a\} \setminus j$ . Hence then, the property (3) follows and the Buyer's funds cannot be locked forever except with probability  $(b/a)^b - \mu_0(n)$ .

Assume that the Seller redeems transaction  $T'_1$  and he can redeem  $T'_1$  only via transaction  $T_2$ . Then the Buyer can use signatures  $\hat{\sigma}_i$  to calculate secrets  $S^{i,j}$ . Then he decrypts all the values  $\sigma^{i,j}$  to get all the challenges and responses  $c_k^{i,j}, r_k^{i,j}$ . At the end using any pair of responses he can calculate  $x' = \text{Extract}_{\mathcal{F}}(B_{\mathcal{F}}^{i,j}, c_1^{i,j}, r_1^{i,j}, c_2^{i,j}, r_2^{i,j})$  s.t.  $f(x') = \text{true}$  with probability  $\geq 1 - \mu_1(n)$ , where  $\mu_1(n)$  is negligible (see the proof of Lemma 2 in [4]). Hence then, the property (1) follows.

For case (ii), if an honest Seller does not get the Buyer's funds, the property (2) is guaranteed by the zero-knowledge property of the zero-knowledge proof of knowledge protocol.

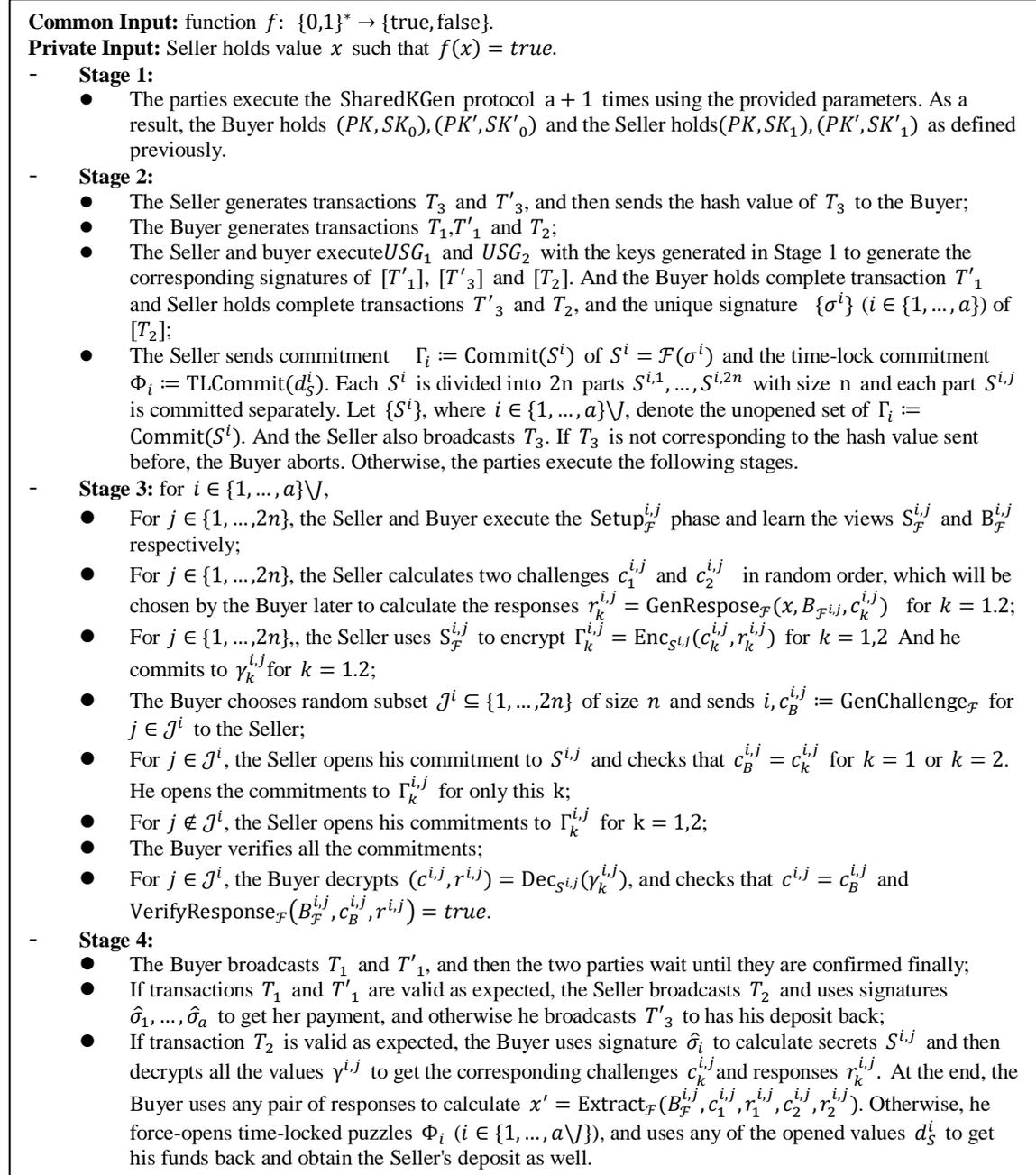


Figure 1. The SellWitnessProtocol.

## 5. CONCLUSIONS

In this paper, with respect to the unfairness between the buyer and seller in the Zero-Knowledge Contingent Payment protocol, we present protocol SellWitness to achieve that, if the seller is malicious, then the honest buyer will get back his payment and some penalty paid by the seller. In

the further work, we will optimize SellWitness, e.g., replacing the time-lock building block with some timeless primitives to allow the parties to have unlimited number of valid transactions.

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