NON-FUNGIBLE TOKEN BUBBLE PREDICTION USING EXTENDED LOG- PERIODIC POWER LAW MODEL

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ABSTRACT

Non-fungible token (NFT) bubbles are a problematic issue, and this study aims to predict NFT bubbles using an extended log-periodic power law singularity (LPPLS) model. The classic LPPLS model targets the endogenous nature of bubbles caused by the mimetic behavior of investors without external influences; however, the extended model attempts to incorporate exogenous influences. First, we compare the performance of the two models for NFT price prediction. The exogeneous variable in the extended model is cryptocurrency volatility. Then, we calculate the bubble confidence using both models. The results show that the explanatory power and forecasting accuracy of the extended model are superior in all projects. We also find that the bubble confidence indicator reinforces the results of bubble prediction.

KEYWORDS

Bubble, Cryptocurrency, Log-periodic power law, NFT

1. INTRODUCTION

The prediction of non-fungible token (NFT) bubbles is an industry-wide problem. Despite the strong growth of the NFT market, bubbles—faster-than-exponential price increases—have been causing crashes(see Section 3.1.3). All stakeholders (e.g., buyers, creators, and platformers) have been thrown into disarray. For buyers, prices are highly volatile, and the risk of substantial loss in a short time is high. As noted by Kong and Lin [1], the geometric mean of monthly returns for NFTs is 13.92% so far, whereas for stocks, bonds, and gold, they are 1.01, −0.61, and 0.63%, respectively. Meanwhile, the standard deviation of returns for NFTs is the highest (65.57%): approximately 14 times greater than that of stock returns. For NFT creators, bubbles are a disincentive to improving NFT quality. Hence, the mass production of low-quality NFTs for speculative purposes dominates creator motivations. On OpenSea, the world's largest NFT marketplace, more than 80% of new NFTs generated using the free mint function were found to be fraudulent (i.e., plagiarized, spam, or fake), including many copyright violations [2]. Platformers also experience significant negative impacts from price fluctuations. According to DappRadar [3], OpenSea saw a 99% decline in trading volume in just four months in 2022. Obviously, there is a domain-wide demand for bubble prediction. This study responds to this societal demand.

Notably, there are no current studies on NFT bubble prediction that consider both endogeneity and exogeneity. In the stock market, one line of research focuses on the endogenous and

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exogenous nature of bubbles [4], [5], [6], endogeneity refers to a dramatic change in prices in one direction caused by mimetic investor behaviors without external shocks [7]. On the other hand, exogeneity refers to price changes caused by external shocks, such as the Nazi invasion of Western Europe in 1940 and the COVID-19 pandemic [7]. For NFTs, a few studies [8], [9] have focused on endogenous bubbles. However, none have considered both endogeneity and exogeneity [5], [10]. To answer this academic requirement is the aim of this study and its novelty.

To answer the call for a combined model, this study answers two hypotheses:

(1) Extending an existing model that focuses on endogenous bubbles to incorporate external variables improves the model's explanatory power and forecasting accuracy.
(2) The extended model reinforces the results of bubble prediction.

To support these hypotheses, we use an extended log-periodic power law singularity (LPPLS) model that assumes rational expectations and investors who behave in irrational and mimetic manners. The mechanism by which the price of an asset rises rapidly and crashes due to the collective mimetic behaviors of investors is thus examined. Although the original LPPLS model targets the endogenous nature of bubbles, the extended model includes exogenous influences. The extended approach consists of three steps:

(1) Using the concept of drawdown [10], we select NFT projects for LPPLS analysis.
(2) We compare the performance of traditional vs. extended LPPLS models on NFT bubble prediction. The exogeneous variable is *cryptocurrency volatility*.
(3) We calculate the LPPLS bubble confidence indicator [11] to appraise the potential for nearterm price decreases or increases based on price data from which external influences are eliminated. We then compare the indicators of both models.

The results show that the explanatory power and forecasting accuracy of the extended model increase when external variables are considered. However, the explanatory power of the original model varies from project to project. We also find that a type-2 error in predicting the bubble risk may have occurred. That is, the exogenous factors may have offset the endogenous changes, causing the fluctuations to lose their bubble characteristics.
The rest of the paper is organized as follows. Section 2 discusses previous studies on the LPPLS model, Section 3 explains the drawdown construct, the LPPLS model, its extended version, and bubble confidence indicators, and Section 4 describes the data used for the analysis. Section 5 then discusses the implications of the results, and Section 6 concludes this study.

2. RELATED WORKS

Sornette [12] was the first to apply LPPLS to financial markets, and its success has been tracked by many scholars in different markets. Gonçalves *et al*. [13] used LPPLS to analyze the Portuguese stock market. Johansen [14] did so for the US stock markets, and Indiran *et al.* [15] did so for Malaysia. Separately, Takagi [16] detected numerous bubbles forming in memetic stocks in the US, but he had difficulty predicting social media-induced exogenous rallies, as LPPLS is meant to detect only endogenous bubbles.

In recent years, LPPLS has been applied to cryptocurrency and NFT markets. For cryptocurrency, Shu and Zhu [17] proposed an adaptive multilevel time-series LPPLS detection method to analyze a finer-than-daily timescale for Bitcoin (BTC) price data. Geuder *et al*. [18] revealed the existence of some frequent bubble periods in BTC prices. Regarding NFTs, Ito *et al.* [8] applied LPPLS to the time-series price data of major NFT projects and detected bubbles with

a discernable bubble confidence indicator. Wang *et al*. [9] used LPPLS for robustness testing in the detection of NFT bubbles using supremum augmented Dickey–Fuller (SADF) and generalized SADF tests.
As can be inferred from the extension of the model in conventional markets, both endogeneity and exogeneity should be grasped in NFT markets. However, no studies have thus far made this attempt.

3. METHODOLOGY

In this section, we describe the drawdown concept[10], which is used to select NFT projects to which we apply the LPPLS model. Next, we explain the LPPLS model and how it is extended to incorporate external variables. Finally, we explain how to calculate the bubble confidence indicator.

3.1. Drawdown for Capturing Downward Price Trends

In this study, we apply the drawdown concept when selecting the NFT projects to be analyzed with LPPLS. Drawdown is needed because the target data for the LPPLS model must be carefully selected to ensure that the time range does not include a single crash. Otherwise, the LPPLS model will not fit the data after the crash. Furthermore, a key LPPLS parameter is the *date of highest crash probability*. Hence, if there are multiple crashes in the range, the data will fail to set a unique parameter. Therefore, it is necessary to remove data containing extant crashes.

In this section, we describe the original and coarse-grained types of drawdowns. We then explain how to identify crashes from ordinary drawdowns. The variables in this section are listed in Table 1.

Table 1. Drawdown variables.

Variable	Explanation
$P_{\min}$	Local minimum price
$P_{\max}$	Local maximum price
ε	Threshold of drawdown being smaller than coarse-grained
σ_d	Volatility used to set ε
N_d	Number of data
r	Log price return
$E[*]$	Expected value of *
$p(t)$	Price at a given period t
$R(\text{Drawdown})$	rank number of Drawdown

3.1.1. Original Drawdown for the Detail Trend

Drawdown and drawup concepts were introduced by Johansen and Sornette [10] to capture trends in price fluctuations, as shown in Figure 1. Drawdown is the percentage of change from one local maximum to the next local minimum price. Thus, as long as prices continue to fall, it is considered a drawdown. A drawup, in contrast, is the percentage change in price from one local minimum to the next local maximum. Both are given, respectively, as follows:

$$\text{Drawdown} = \frac{P_{\min} - P_{\max}}{P_{max}}, \tag{1}$$

$$\text{Drawup} = \frac{P_{\max} - P_{\min}}{P_{\min}}. \tag{2}$$

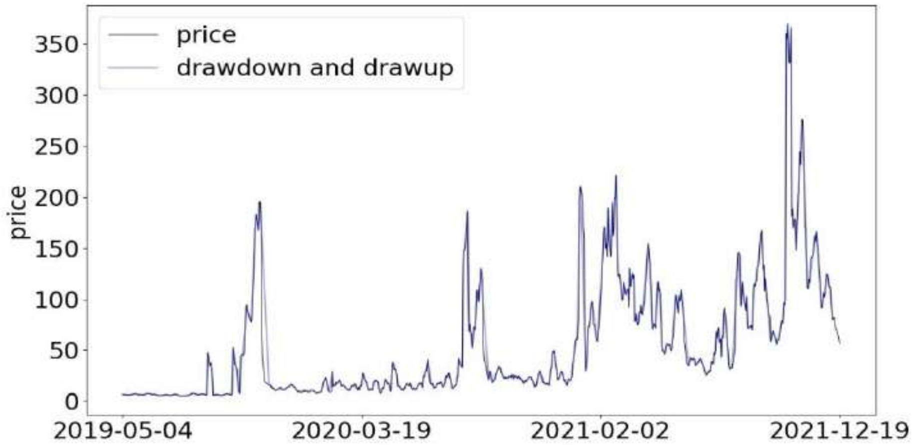


Figure 1. Drawdown using Ethereum Name Service data series

3.1.2. Coarse-Grained Drawdown for Macroscopic Trends

A coarse-grained drawdown is used to capture downward price trends based on the "big picture." A sequence of price declines for which a drawdown is calculated can be terminated by any small price increase. Generally, price fluctuations contain a great deal of noise. Therefore, the original drawdown concept may not adequately capture downward price trends.

Therefore, we introduce a threshold epsilon such that the coarse-grained drawdown reflects the rate of change from the local maximum to the local minimum, where the minimum is the point at which the first drawup larger than ε occurs after the local maximum. The local maximum is the point at which the first drawdown smaller than -ε occurs after the local minimum. ε is obtained by using the following σ_d:

$$\sigma_d = \sqrt{\frac{\sum_{i=1}^{N_d}(r_{i+1} - E[r])^2}{N_d}}, \tag{3}$$

where

$$r_{i+1} = \log p(t_{i+1}) - \log p(t_i). \tag{4}$$

In this study, we set $\sigma_d/4$, $\sigma_d/2$, and σ_d as the values for ε based on previous studies [19].

3.1.3.Crash as an Outlier

Drawdowns large enough to be called "crashes" must be distinguished statistically from other relatively small or ordinary drawdowns. Johansen and Sornette [10] analyzed past markets and showed that nearly all drawdowns were well-fitted based on a stretched exponential function. However, some drawdowns that are not well-fitted are considered outliers. The stretched exponential function is given as

$$\mathrm{R(Drawdown)} = \mathrm{A_d exp}(-\mathrm{b}|\mathrm{Drawdown}|^{\mathrm{z}}), \quad (5)$$

where A_d is a constant and $b = \lambda^{-\mathrm{z}}$.

Per Johansen and Sornette [10], by taking the logarithm of (5) for convenient and efficient fitting, the following equation is obtained:

$$\log \mathrm{R(Drawdown)} = \log \mathrm{A_z} - \mathrm{b}|\mathrm{Drawdown}|^{\mathrm{z}}. \quad (6)$$

This model is calibrated for ordinary least squares (OLS) applications using the dogbox algorithm [20]. In this study, an outlier is a drawdown value that deviates from the fit of (6) in the 99.5% tail of the distribution.

3.2. LPPLS Model

The LPPLS model [12], [20], [21] is used to detect and predict bubbles (i.e., faster-than exponential increases in asset prices). We use the LPPLS extension for bubble prediction, which targets the average NFT prices of all projects. Although each NFT has a unique price, we treat them as homogeneous. The variables used for this work are shown in Table 2.

Table 2. LPPLS model variables.

Variable	Explanation
$p(t)$	Price at a given period, t
A	Price at t_c
B	Power law acceleration amplitude
C	Log-periodic oscillation amplitude
D'	Parameter of $\int_{t_0}^{t} r(\tau) + \sigma(\tau)\varphi(\tau)d\tau$
t_c	Most probable time at which the bubble ends
m	Superexponential acceleration degree
ϕ	Oscillation period
$r(t)$	Interest rate at a given period, t
$\sigma(t,n)$	Historical volatility at t for n days
n	Number of volatile days
$\varphi(t)$	Market price of the stochastic discount factor risk at t
ω	Oscillation frequency
$C1$	$C \cos \phi$
$C2$	$C \sin \phi$
D	Parameter of $\upsilon(t)$
p_i	Price on the day i of a given period
$p(t)$	Average price over n days

3.2.1. Original LPPLS Model

The LPPLS model [12], [20], [21] is written in a suitable form for fitting time-series data:

$$\ln[p(t)] \approx A + B|t_c - t|^m + C\,|t_c - t|^m \cos(\omega \ln|t_c - t| - \phi) + D' \int_{t_0}^{t} r(\tau) + \sigma(\tau,n)\varphi(\tau)d\tau\,, \quad (7)$$

where $A = \ln[p(t_c)]$, t_c denotes the critical time at which the bubble is likely to end, m is the degree of super exponential acceleration, ω is the frequency of the oscillation, and ϕ is its period.

Assuming that $r(\tau) = \varphi(\tau) = 0$, the original LPPLS model is obtained as

$$\ln[p(t)] \approx A + Bf(t) + Cg(t), \quad (8)$$

where

$$f(t) = |t_c - t|^m, \quad (9)$$
$$g(t) = |t_c - t|^m \cos(\omega \ln|t_c - t| - \phi)\,. \quad (10)$$

Using the method of Filimonov and Sornette [21], the nonlinear parameter, ϕ, can be eliminated to obtain the following equation:

$$\ln[p(t)] \approx A + Bf(t) + C_1 g_1(t) + C_2 g_2(t), \quad (11)$$

Where

$$g_1(t) = |t_c - t|^m \cos(\omega \ln|t_c - t|)\,, \quad (12)$$
$$g_2(t) = |t_c - t|^m \sin(\omega \ln|t_c - t|)\,. \quad (13)$$

3.2.2. Extended LPPLS Model

To extend the original model so that external effects can be considered, we loosen the assumption that $r(\tau)$ and $\varphi(\tau)$ are equal to zero. Although both $r(\tau)$ and $\varphi(\tau)$ equal zero in the original model, they are not zero in reality. As performed by Zhou and Sornette [5], we assume that $\varphi(\tau)$ is a constant φ, and we employ the historical volatility of a specified asset as a proxy for the volatility factor, $\sigma(\tau)$. Note that we still assume that the interest rate is zero for simplification. However, please note that Hu and Li [6] used real values (i.e., the risk-free interest rate and deposit reserve rate in China).
Therefore, we obtain

$$\ln[p(t)] \approx A + Bf(t) + C_1 g_1(t) + C_2 g_2(t) + Dv(t), \quad (14)$$

where

$$D = D'\varphi, \quad (15)$$
$$v(t) = \int_{t_0}^{t} \sigma(\tau,n)d\tau\,, \quad (16)$$
$$\sigma(t,n) = \sqrt{\frac{\sum_{t=1}^{n}(p_t - \bar{p})}{n-1}}. \quad (17)$$

Following Zhou and Soenette [5], we use the trapezoid scheme to integrate $\sigma(\tau, n)$ as follows:

$$\upsilon(t) = \sum_{\tau=t_0+1}^{t} \frac{[\sigma(\tau-1, \mathrm{n}) + \sigma(\tau, n)]}{2}. \tag{18}$$

As mentioned, the original LPPLS model cannot account for external impacts. Hence, we propose this extended model based on the cryptocurrency's volatility.

Although studies [22] and [23] showed that volatility transmission effects between cryptocurrencies and NFTs are limited, Ante [24] revealed that a BTC price shock caused an increase in NFT sales. Furthermore, between BTC and Ethereum (ETH), a bidirectional relationship between returns and long-term spillovers was found.

Therefore, it is advisable to use cryptocurrency volatilities as the proxy for the volatility factor, $\sigma(\tau,\ n)$. Specifically, we use the historical volatilities for 7, 30, and 90 days of each BTC and ETH blockchain and compare the models while incorporating each external variable (see Section 3.2.7 for model evaluation details).

3.2.3.Calibration

Both LPPLS models can be calibrated using OLS to minimize the sum of squared residuals using a modified Python module [25].

3.2.4.Evaluation

To compare the original and extended LPPLS models using each explanatory variable, we apply the Akaike information criterion (AIC)[26]and adopt the model with the minimum AIC value. As performed by Zhou and Sornette [5], AIC is calculated by fitting models (11) and (14) with each external factor (see Section 4). We also calculate the adjusted R^2 from (11) and (14) to compare the models' explanatory power. Finally, we test the significance of each external explanatory factor by fitting each model (14) and calculating the p-values of the external factor coefficients.

Although these tests require that residual errors be i.i.d. with a Gaussian distribution and that the errors remain somehow dependent, the tests are still helpful in comparing relative model performance [5], [6].

3.2.5. Bubble Confidence Indicator

We use the bubble confidence indicator [11] to detect and predict bubbles. This index shows how well the targeted data fit the price movements from empirical bubble evidence from previous studies [11], [27], [28]. The larger the bubble confidence indicator, the more reliable the pattern (i.e., a crash is more likely).

A bubble confidence indicator for a given t_2 is calculated using the following steps:

(1) Iterate calibration for each time window where the start time, t_1, moves toward the end time, t_2, with a specific step, dt. In this study, we set the initial time range as 120 days and dt as 5 days, following Ito *et al*. [8]. Thus, each t_2 has 24 time windows.

(2) Count the number of cases in which $B < 0$ for each 24-calibration outcome, denoted as $[B< 0]_{\text{count}}$
(3) Count the cases in which the parameters satisfy the conditions listed in Table 4 and name them as $[B< 0]^{*}_{\text{count}}$, as derived from previous studies [11], [27], [28]. (4)Obtain the bubble confidence indicator as

$$\text{bubbleindicator} = \frac{[0 < 0]^{*}_{\text{Count}}}{[0 < 0]_{\text{Count}}}, \tag{19}$$

A higher bubble confidence indicator means that the price is likely to experience faster-thanexponential growth [29].

The bubble confidence indicator assumes that if a bubble is endogenous, the parameters must satisfy certain conditions, and the conditions are obtained inductively from empirical evidence [11], [27], [28]. Therefore, there are two drawbacks. Even if no apparent endogenous bubble trend is detected, it may only be offset by exogenous influences, and the hidden endogenous bubble may continue to grow. Furthermore, an apparent endogenous fluctuation may actually be a false endogenous detection caused by exogenous influences.

To test for these errors, we first calculate the bubble confidence indicator by fitting the original LPPLS model to the price data for each NFT project. We then calculate the indicators by fitting the original LPPLS model to the processed price data from which external effects are eliminated using the D and $\upsilon(t)$ from (12). The model from which we adopt the D and $\upsilon(t)$ depends on the comparison. If both calculate higher values, then there is a strong possibility that an endogenous bubble is occurring.

Table 3. Filtering conditions for each item in calculating the bubble confidence indicator.

Item	Condition
$\frac{\omega}{2\pi} ln \frac{t_c - t_1}{t_c - t_2}$	$[2.5, +\infty)$
$\frac{m\lvert B\rvert}{\omega\lvert C\rvert}$	$[0.5, +\infty)$

4. DATA

To compare the results with those of previous studies [8], the data used for our analysis included the same projects and periods as [8]. Namely, we used data from four major NFT projects: Decentraland (from March 19, 2018, to December 20, 2021), CryptoPunks (from May 17, 2018, to December 12, 2021), Ethereum Name Service (from May 4, 2019 to December 20, 2021), and ArtBlocks (from November 27, 2020 to December 20, 2021). These price data are available at https://nonfungible.com/ [30].

We used the same processed data as Ito *et al.*, provided by the Non-Fungible Corporation [30]. These data include weekly moving averages and average daily values of all NFT projects [8]. Notably, NFTs are not necessarily traded frequently, and huge price differences can be found within the same project.

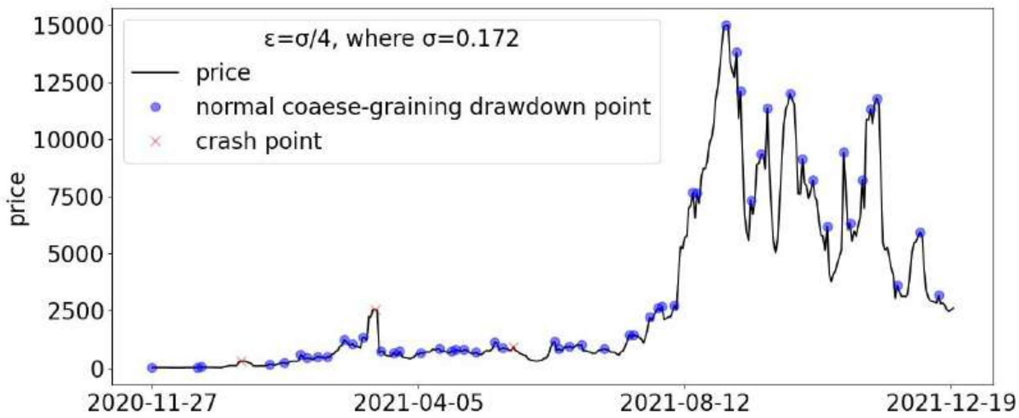
Regarding the volatility of cryptocurrency data, we applied the historical volatilities of 7, 30, and 90 days per BTC and ETH. These data are available at https://finance.yahoo.com/ [31].

5. RESULTS

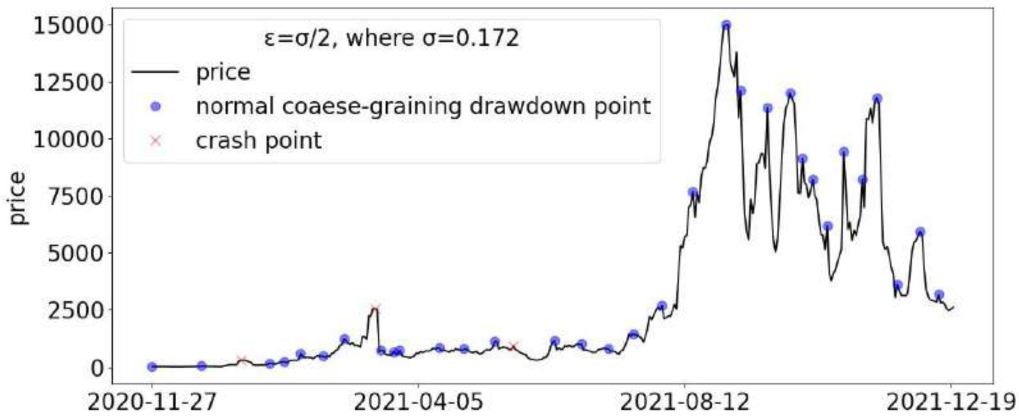
5.1. Selection of NFT Projects for the LPPLS Model

First, we obtained coarse-grained draw downs for the four NFT projects. Then, we extracted crash points by fitting a stretched exponential function with a 99.5% distribution point to indicate a crash. Figure 2 illustrates the coarse-grained drawdown points for ArtBlocks, some of which were considered crashes. Decentraland, CryptoPunks, and Ethereum Name Service did not indicate crashes, regardless of the epsilon value set.

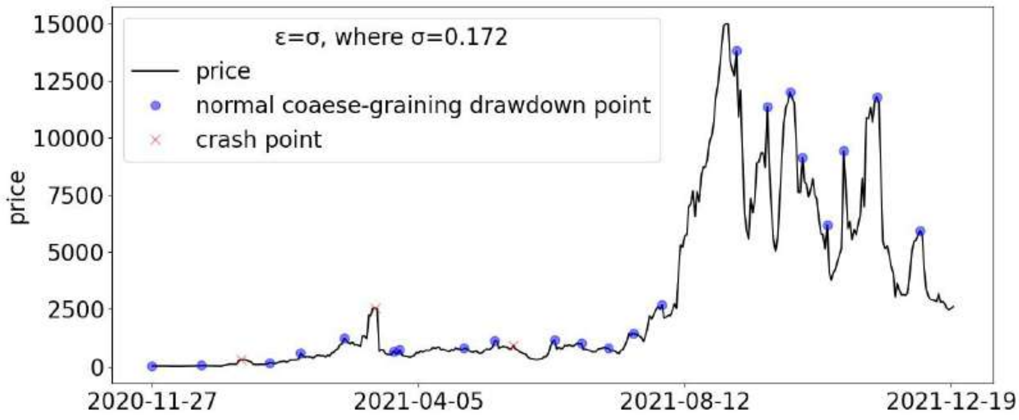
The following analytical results were obtained for the three crypto services. Although proper data extraction would improve LPPLS model fitting, even for these projects, extending the data extraction and their pretreatments was outside of the scope of this study. This may provide a future research opportunity in the near future.



(a)



(b)



(c)

Figure 2. Coarse-grained drawdowns in ArtBlocks: (a) $\varepsilon = \sigma/4$, (b) $\varepsilon = \sigma/2$, and (c) $\varepsilon = \sigma$.

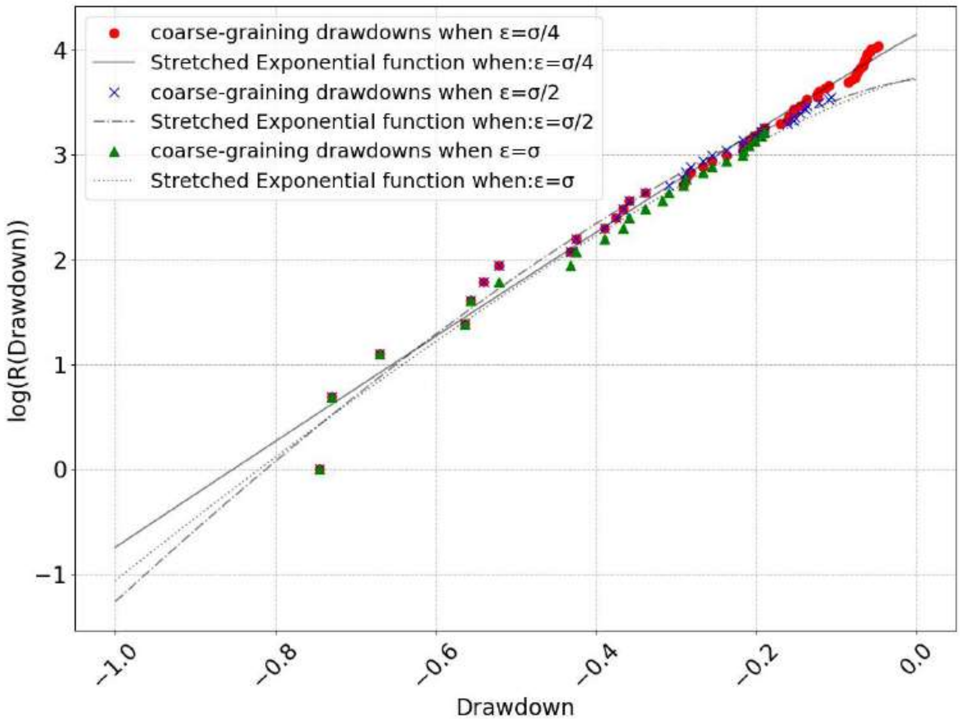


Figure 3. Stretched exponential function for drawdowns in ArtBlocks

Table 4. Logarithmic parameters of the stretched exponential function for ArtBlocks.

ε	**A**	**b**	**z**
σ/4	63.336	4.894	1.040
σ/2	41.271	4.986	1.403
σ	41.855	4.780	1.263

Table 5. Largest ArtBlockscrash.

Rank	**Size**	**Start Date**
1	−0.745	3/15/2021
2	−0.730	1/9/2021
3	−0.670	5/21/2021

5.2. Comparison of Original and Extended Models

When comparing the original LPPLS and extended models based on each explanatory variable, we calculated the AIC[26]and adjusted the R^2 values by fitting models (11) and (14) to each external factor. We also tested the levels of significance for each external explanatory factor by fitting model (14) and calculating the p-value of each external factor's coefficient.
The results revealed the following:

(1) As listed in Table 13, each adjusted R^2 in the extended model was higher than that of the original, and the AIC shrank in the extended models. This means that the explanatory power and forecasting accuracy, respectively, increased when the external variables were considered.
(2) For each project, the extended model was superior for detecting 90-day BTC volatility in Decentraland, 30-day ETH volatility in CryptoPunks, and seven-day ETH volatility in Ethereum Name Service.
(3) The explanatory power of the original model was much higher for CryptoPunks (Table 6-b) than for Decentraland (Table 6-a) and Ethereum Name Service (Table 6-c).
(4) In each best case, the external factor had a positive effect in Decentraland (Table 6-a), while it was negative in CryptoPunks (Table 6-b) and Ethereum Name Service (Table 6-c).
The implications inferred from these results are as follows:
(1) Assets are more sensitive to the cryptocurrency with which NFTs are traded.
(2) Owing to the fact that some projects had a good fit in the original model, the LPPLS model's assumption of mimetic irrational investors may be valid.
(3) Considering that there are collection-oriented NFTs and others for practical applications, a cross-sectional study based on these characteristics may be needed in the future (see Section 6).

Table 6. Adjusted R^2, AICs,and significance test results of the original and extended models.
(a) Decentraland

Model	Original	Extended					
External variable	-	7-day BTC volatility	30-day BTC volatility	90-day BTC volatility	7-day ETH volatility	30-day ETH volatility	90-day ETH volatility
Coefficient t-value	-	64.017 ***	−3.120 ***	64.453 ***	−17.506 ***	−18.617 ***	−19.517 ***
Adjusted R^2	0.579	0.763	0.584	0.770	0.656	0.665	0.669
AIC	3,403	2,612	3,3878	2,575	3,124	3,089	3,071
N	1,373	1,373	1,373	1,373	1,373	1,373	1,373

(b) CryptoPunks

Model	Original	Extended					
External variable	-	7-day BTC volatility	30-day BTC volatility	90-day BTC volatility	7-day ETH volatility	30-day ETH volatility	90-day ETH volatility
Coefficient t-value	-	−3.266 ***	−1.946	−7.784 ***	−17.852 ***	−19.529 ***	−19.363 ***
Adjusted R^2	0.969	0.954	0.954	0.970	0.963	0.974	0.973
AIC	2,169	2,688	2,694	2,138	2,412	1,932	1,987
N	1,314	1,314	1,314	1,314	1,314	1,314	1,314

(c) Ethereum Name Service

Model	Original	Extended					
External variable	-	7-day BTC volatility	30-day BTC volatility	90-day BTC volatility	7-day ETH volatility	30-day ETH volatility	90-day ETH volatility
Coefficient t-value	-	−8.931 ***	−8.690 ***	−8.341 ***	−9.788 ***	−11.369 ***	−11.361 ***
Adjusted R^2	0.622	0.664	0.664	0.663	0.707	0.669	0.667
AIC	1,953	1,839	1,840	1,841	1,707	1,825	1,830
N	962	962	962	962	962	962	962

Coefficient t-values are of the external variables. *** shows the significance level at the 1% confidence level.

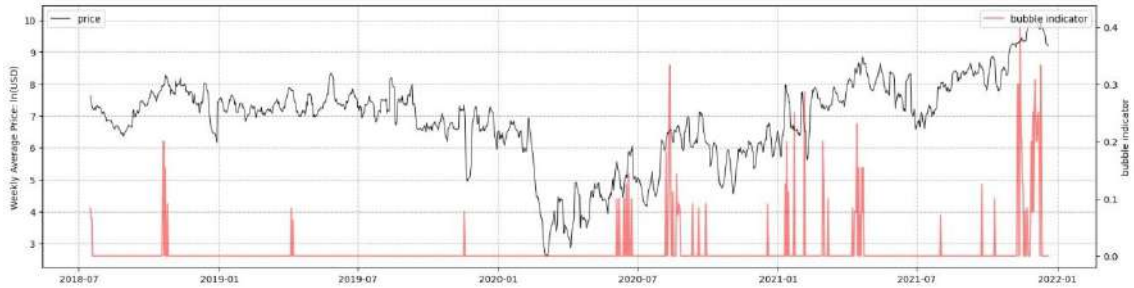
5.3. Comparison of Bubble Confidence Indicators

We calculated the bubble confidence indicators for the original data using the logarithm of the weekly moving average price of each NFT project (Fig. 4-a, b, c) and the processed price data from which external effects were eliminated using the D and $\upsilon(t)$ in (12) and shown in (Fig.

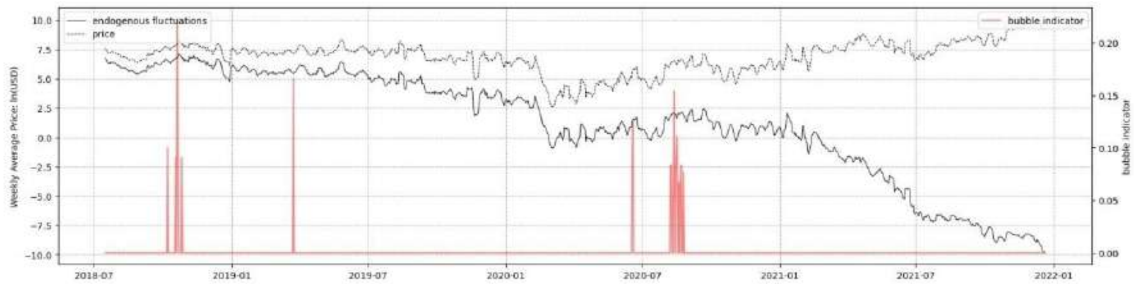
4a',b', and c'). The model D and $\upsilon(t)$ parameters adopted were determined by comparing the model evaluation results.

Overall, the original indicator correctly detected endogenous bubbles because the trends revealed by the indicators were roughly aligned. However, as in July 2022 with CryptoPunks, the original bubble confidence indicator did not detect a high endogenous risk; however, the indicator from the processed data did.

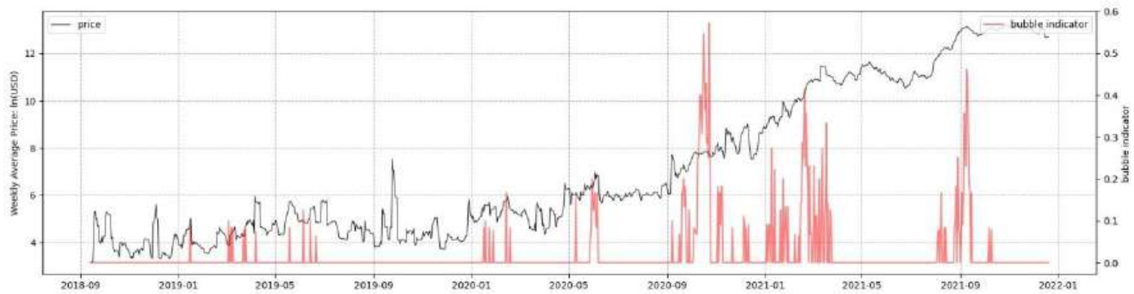
Therefore, a type-2 error may have occurred when predicting the bubble risk based on the confidence indicators calculated from the original data. However, a more realistic bubble confidence indicator might not be determined, as we did not examine the bubbles using multiple approaches (see Section 6).



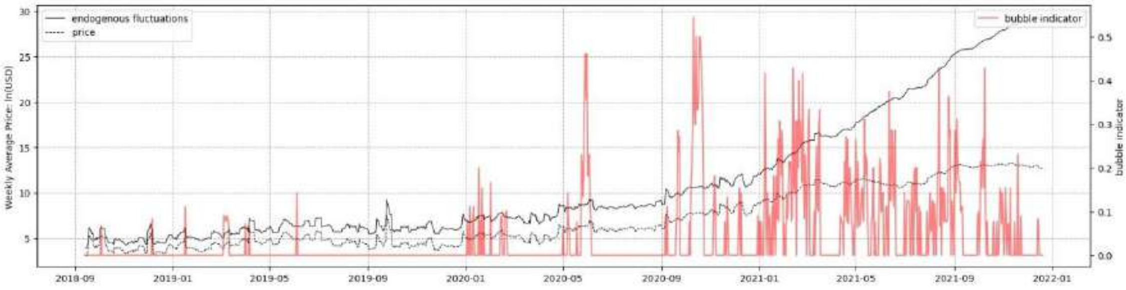
(a) *Decentraland* (the bubble confidence indicator calculated from the original price data)



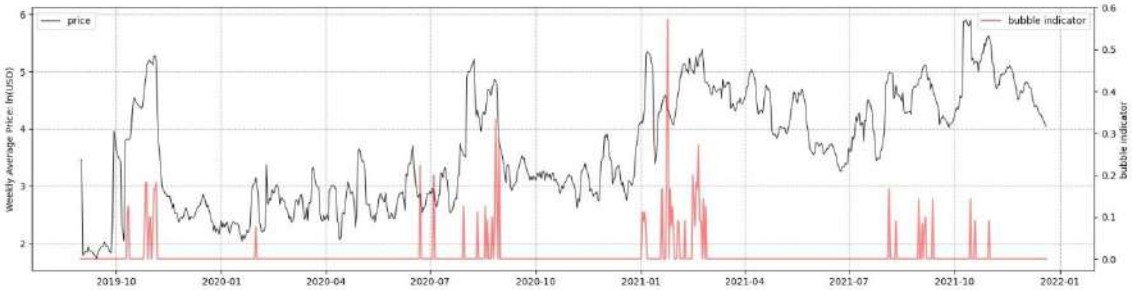
(a') *Decentraland* (the bubble confidence indicator calculated from the processed price data)



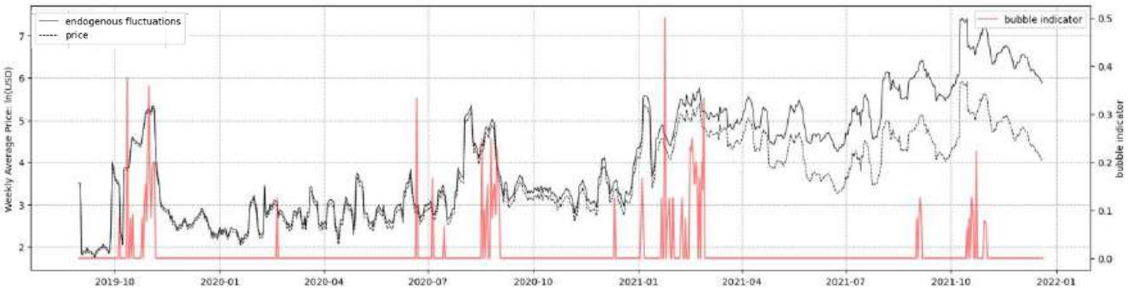
(b) *CryptoPunks* (the bubble confidence indicator calculated from the original price data)



(b') *CryptoPunks* (the bubble confidence indicator calculated from the processed price data)



(c) *Ethereum Name Service* (the bubble confidence indicator calculated from the original price data)



(c') *Ethereum Name Service*(the bubble confidence indicator calculated from the processed price data)

Figure 4. Results of the bubble confidence indicator of the original and processed data

6. CONCLUSION

Our conclusions are summarized as follows:

(1) Using the concept of drawdown, we successfully selected NFT projects to fit LPPLS analysis appropriately.

(2) We extended the original LPPLS model by incorporating external variables to improve its explanatory power and forecasting accuracy.

(3) By comparing the two kinds of confidence indicators, the results of endogenous bubble detection were confirmed.

Notably, the methods used to select appropriate NFT projects can be improved. For example, a crash might also be defined by considering its course-grained time horizon. It may also be possible to use the LPPLS model to analyze projects by cropping the period to be analyzed.

Moreover, there is room to consider additional explanatory variables. Cryptocurrency volatilities were employed in this study as explanatory variables, but as noted, their relationship with NFT prices remains unclear. Notably, an attention index that reflects public interest in a given project could be leveraged as an alternative variable.

We showed that bubble confidence indicators can appear differently depending on whether external variables are considered. However, this did not necessarily improve the predictive power of the model. We must continue to examine the bubbles using different approaches (e.g., Metcalfe's Law) for triangulation purposes.

Finally, because only a few projects were covered, it was difficult to make cross-sectional comparisons that considered the characteristics of each project. By broadening the scope in the future, it may be possible to identify project characteristics that influence their susceptibility to internal or external influences.

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