UNSUPERVISED LEARNING OF SHAPE SEGMENT POINT DISTRIBUTION MODELS WITH THE EM ALGORITHM

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ABSTRACT

This paper demonstrates how 2D handwritten shapes can be classified by analyzing shape structure. The underlying framework is a one-layer architecture where the shapes are segmented to a series of connected segments. Each segment is represented by a set of uniformly distributed landmarks along the skeleton of the character. We follow by representing each segment using the Point Distribution Model (SPDM). We then capture shape variations by learning Gaussian mixture of segment point distribution models in a two-step Expectation Maximization algorithm. The approach is tested on a set of handwritten Arabic characters.

KEYWORDS

Handwritten Arabic characters, Shape analysis, Point distribution models, Machine learning, Expectation Maximization Algorithm.

1. INTRODUCTION

Pattern Recognition is a subfield of computer vision that attracted many scholars to find solutions describing pattern analysis. The task arises when the shape under study preserves a high degree of shape complexity. The complexity of the shape identifies the existence of many variations of such shape, for example, in the case of handwritten letters. Researchers addressed the problem of complexity of shape by approaching the divide and conquer methodology. The divide and conquer approach provide a simple and fast solution to address the complexity of the shape. The approach revolves around 3 step mechanism. The first of these is breaking down the shape into multiple minor pieces. The second is to analyze each piece individually and provide a solution. The third is to combine the multiple minor solutions yielding one final full solution to the original [1].

Generative models are examples in machine learning discipline are probabilistic models of the data itself, that determine how likely the example object belongs to a correct class [2]. Generative models predict objects similar in shape to those in the analyzed data set. Point Distribution models are examples of generative models [3].

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Point distribution Models (PDM) have been used widely by researchers in the field of pattern recognition to register two-dimension (2D) shapes. Atul Kanaujia [4] have used point distribution model to track facial features across training data, generating a large set of synthetic shape examples by randomly sampling a given statistical shape model [5], correcting facial appearances by warping the sample patterns to the training set mean shape [6].

There are several attempts to extend the point distribution models to add one mode dimension to the utility. Onyango M. [7] extended point distribution models to cover three dimensional to Model grey level surfaces for shapes, Estimating dimensions of free-swimming fish [7], building dense surface model of the human face using 3D point distribution models [9], reconstruction of a patient-specific 3D bone surface model [10], producing 3D shape from 2D shape by augmenting the generator using projection module [11], Creating large-scale city models from 3D-point clouds [12].

Expectation Maximization Algorithm (EM) has been used widely in machine learning as an optimization algorithm to reveal the local maximum likelihood estimates (MLE) for obtainable variables in statistical models [13]. Joko Purwadi [14] used EM algorithm to learn facial appearances for face tablet face recognition. Mohammed et al. have used the EM algorithm is estimate global parameters of Gaussian mixtures for the purpose of cluster segmentation [15]. Ajinkya N. Jadhav et al. Have used EM algorithm to establish voice recognition for on-line speaker [16]. Zhang, T et al. [17] used EM algorithm to cluster key-points using Fuzzy Gaussian mixture models (FGMMs) for the purpose of gesture recognition. Ross Greer et al. [18] utilized the EM algorithm for estimation of corner points and linear crossing segments for both marked and unmarked pedestrian crosswalks using the detections of pedestrians from processed point clouds or camera images.

2. Shape Representation

We are confronted with recognizing a 2D shape represented by a set of uniformly distributed landmarks along the skeleton of the shape in

$$X_i = ((x_{i1}, y_{i1}), (x_{i2}, y_{i2}), (x_{i3}, y_{i3}), \ldots, (x_{iL-1}, y_{iL-1}), (x_{iL}, y_{iL})) \qquad (1)$$

Each shape pattern X_i is segmented into a set of K non-overlapping consecutive segments in

$$X_i = (S_{i_1}, S_{i_2},, S_{i_3}, \ldots, S_{i_{k-1}},, S_{i_k}) \qquad (2)$$

For each segment κ is represented by a set of ordered landmarks in

$$S_{i_k} = \left(\left(x_{i_{k_1}}, y_{i_{k_1}}\right), \left(x_{i_{k_2}}, y_{i_{k_2}}\right) \ldots \left(x_{i_{k_{n-1}}}, y_{i_{k_{n-1}}}\right), \left(x_{i_{k_n}}, y_{i_{k_n}}\right)\right) \qquad (3)$$

Since we are concerned with recognizing $2\mathcal{D}$ shape segmented into set of segments to overcome the shape complexity. Here, we approach the class-shape model using the apparatus Cootes and Taylor Point Distribution models (PDM) []. The Cootes method commences by extracting the mean shape from a set of training patterns, hence, a mixture of shape patterns to be constructed. Assume that training pattern X_i^t belonging to the training set t; therefore, rewriting equation 2 to include full segment arrangements in more concrete parametric form.

$$\mathrm{X}_{i}^{t} = (S_{i_1}^{t}, S_{i_2}^{t}, S_{i_3}^{t} \dots, S_{i_{k-1}}^{t}, S_{i_k}^{t}) \tag{4}$$

With each segment $S_{i_k}^{t}$ belonging to the training pattern t that belong to the training-set ω is represented by the vector of coordinate landmarks distributed along the segment skeleton in

$$S_{i_k}^{t} = \left(\left({x^t}_{i_{k_1}}, {y^t}_{i_{k_1}}\right), \left({x^t}_{i_{k_2}}, {y^t}_{i_{k_2}}\right), \dots, \left({x^t}_{i_{k_n}}, {y^t}_{i_{k_n}}\right)\right) \tag{5}$$

The segment mean-shape is computed by averaging the segment over the total number of patterns in set t in

$$\bar{S}_{ik}^{t} = \frac{1}{t}\sum_{i=1}^{t}\sum_{k=1}^{n} S_{i_k}^{t} \tag{6}$$

Hence, the full-shape point distribution model mean-shape $\bar{\mathrm{X}}_{i}^{t}$ has the set of mean-segment shape denoted by

$$\bar{\mathrm{X}}_{i}^{t} = (\bar{S}_{i_1}^{t}, \bar{S}_{i_2}^{t}, \bar{S}_{i_3}^{t} \dots, \bar{S}_{i_{k-1}}^{t}, \bar{S}_{i_k}^{t}) \tag{7}$$

The covariance projection is a matrix used to describe the variance values between two shapes (mean-shape and a pattern=shape coordinates) stored in a random vector. It is also known as the variance-covariance matrix because the variance of each element is represented along the matrix's major diagonal and the covariance is represented among the non-diagonal elements. In Peter and Taylor [19] method the variance-covariance matrix is for each segment $\Sigma_{i_k}^{t}$ is computed by averaging the translation-correlation error between the segment mean shape $S_{i_k}^{t}$ and the segment shape $\bar{S}_{ik}^{t}$ pattern in

$$\Sigma_{i_k}^{t} = \frac{1}{t}\sum_{i=1}^{t}\sum_{k=1}^{n} (S_{i_k}^{t} - \bar{S}_{ik}^{t})(S_{i_k}^{t} - \bar{S}_{ik}^{t})^{T} \tag{8}$$

The set of full-shape variance matrix $\bar{\Sigma}_{i}^{t}$ consisting of the set of segments correlation is denoted by.

$$\bar{\Sigma}_{i}^{t} = (\Sigma_{i_1}^{t}, \Sigma_{i_2}^{t}, \Sigma_{i_3}^{t}, \dots, \Sigma_{i_k}^{t}) \tag{9}$$

Assume that the variance-covariance matrix is a square matrix, hence, the eigenmodes of the landmark's covariance matrix are used to construct the segment point distribution models PDM. To do that, the eigenvalues $\lambda_{i_n}^{t}$ are collected by solving linear equation in

$$\left| \ \bar{\Sigma}_{i}^{t} - \lambda_{i_n}^{t} I \ \right| = 0 \tag{10}$$

where I is the segment identity matrix of size $2n$ X $2n$. The eigenvalue $\lambda_{i_k}^t$ corresponding eigenvector $\phi_{i_k}^t$ is found by solving the eigenvector linear equation.

$$\bar{\Sigma}_i^t \phi_{i_k}^t = \lambda_{i_k}^t \phi_{i_k}^t \qquad (11)$$

According to Daniel and Taylor [20], the segment landmark points are allowed to undergo displacements relative to the segment mean-shape in directions defined by the eigenvectors of the covariance matrix. To compute the set of possible displacement directions, the n most significant eigenvectors are ordered according to the magnitudes of their corresponding eigenvalues to form the matrix of column-vectors

$$\phi_{i_k}^t = (\phi_{i_1}^t, \phi_{i_2}^t, \phi_{i_3}^t, \ldots, \phi_{i_n}^t) \qquad (12)$$

Similarly, the corresponding eigenvalues are ordered sorted ascendingly by

$$\lambda_{i_k}^t = (\lambda_{i_1}^t, \lambda_{i_2}^t, \lambda_{i_3}^t, \ldots, \lambda_{i_n}^t) \qquad (13)$$

Consistently, a new similar segment-shape landmarks $\widehat{S_{i_k}^t}$ could be generated by using linear projection of a training pattern

$$\widehat{S_{i_k}^t} = \bar{S}_{ik}^t + \phi_{i_k}^t \gamma_{i_k}^t \qquad (14)$$

Where $\gamma_{i_k}^t$ is the segment modal co-efficient that represent the free-parameters of global segment-model deformation estimated heuristically. The model deformation vector $\gamma_{i_k}^t$ for a single segment pattern $S_{i_k}^t$ can be computed using initially linear equation.

$$\gamma_{i_k}^t = {\phi_{i_k}^t}^{-1} (\bar{S}_{ik}^t - S_{i_k}^t) \qquad (15)$$

3. Learning mixture of Segment Point Distribution Model

Previous methods of training mixture of point distribution model involve extracting single variance-covariance matrix for a single training set; however, this strategy produce only variations of local segment landmark displacement variances; that is, does not gather knowledge about for the remaining training sets landmarks displacement of different classes. The need to approach mixture learning of segment patterns; is to produce more complex and optimal description of segment shape classes.

The literature demonstrates two main training mixture strategies, i.e., supervised and unsupervised learning. Supervised learning involves using labelled data set patterns to training algorithms to classify patterns under study. It's main use in classification and regression algorithms; however, in our case of segment mixture learning, this type of learning is limited to capturing the optimal modes of different segment pattern variations. While unsupervised learning involves learning algorithms to analyze and cluster unlabeled data sets. These algorithms discover hidden patterns in data without the need for human intervention. Therefore, unsupervised learning is an effective strategy to produce the knowledge of more complex segment patterns variations.

In this paper, we adopt an approach based on fitting mixtures of segment point distribution models (SPDM). The reason for such adoption is that we would like to model more complex segment distribution variations and deformations. The second reason for such training is to capture the variations and deformation knowledge of other segment mixture training sets. The method is based on fitting Gaussian mixtures segment model to a set of training segment training patterns. We further assume that segment training pattern $S_{i_k}^{t^{\omega}}$ belonging to the full-shape pattern $\mathrm{X}_{i}^{t^{\omega}}$ are individually independent and can be presented by a set of segment-class ω. Each segment-class ω has its own set of segment-mean shape $\bar{S}_i^{\omega}$ and a covariance matrix $\bar{\Sigma}_{i}^{\omega}$. With these ingredients, we can establish the distribution likelihood function for a set of segment training patterns in

$$p\left(S_{i_k}^{t^{\omega}}, t \in \omega, k \in i\right) = \prod_{i=1}^{t} \sum_{t \in \omega} \prod_{k=1}^{n} p(S_{i_k}^{t^{\omega}} | \bar{S}_i^{\omega}, \bar{\Sigma}_{i}^{\omega}) \tag{16}$$

Where $p\left(S_{i_k}^{t^{\omega}}, t \in \omega, k \in i\right)$ is the probability density function for addressing the segment training pattern $S_{i_k}^{t^{\omega}}$ to the segment-shape class ω belonging the full-shape class denoted by Ω. According to the Expectation Maximization algorithm [13], we would like to maximize the likelihood function provided above by adopting a two-step iteration process. The process involves around the expected log-likelihood function

$$Q_L\left(C^{(j+1)} \middle| C^{(j)}\right) = \sum_{i=1}^{t} \sum_{\omega \in \Omega} p\left(t \in \omega | \mathrm{X}_{i}^{t^{\omega}}, \bar{\mathrm{X}}_{i}^{\omega(j)}, \bar{\Sigma}_{i}^{\omega(j)}\right)$$
$$\mathrm{X} \prod_{k=1}^{n} \ln p\,(t \in \omega, k \in i, S_{i_k}^{t^{\omega}} | \bar{S}_i^{\omega(j+1)}, \bar{\Sigma}_{i}^{\omega(j+1)}) \tag{17}$$

Where $\left(\bar{\mathrm{X}}_{i}^{\omega(j)}, \bar{\Sigma}_{i}^{\omega(j)}\right)$ are the estimation of the full-shape set of segment-mean shape and related full-shape set of segment-covariance matrix at iteration j of the iterative algorithm. The magnitude $p\left(t \in \omega | \mathrm{X}_{i}^{t^{\omega}}, \bar{\mathrm{X}}_{i}^{\omega(j)}, \bar{\Sigma}_{i}^{\omega(j)}\right)$ is the $a\ posteriori$ probability that the full-shape training pattern which encompasses a set of segment-mean shape belongs to the class Ω at iteration j of the algorithm. The probability density for the set allowing to form the full-shape components of segment pattern vectors linked to the segment-shape class ω specified by the estimation of the product of the set of both segment-mean shape and related segment-covariance matrix at iteration $j+1$ is denoted by $\prod_{k=1}^{n} \ln p\,(S_{i_k}^{t^{\omega}} | \bar{S}_i^{\omega(j+1)}, \bar{\Sigma}_{i}^{t^{\omega(j+1)}})$. Regarding the Expectation Maximization algorithm[13], the algorithm is a two-step iterative process; The M, or maximization step of the algorithm involves in revising the estimation of both segment-mean shape $\bar{S}_{ik}^{\omega(j+1)}$ and segment-covariance matrix $\bar{\Sigma}_{i}^{\omega(j+1)}$ at iteration $j+$

1 which maximizes the expected loglikelihood function. While the E, or expectation step involves is updating the $a\,posteriori$ segment-class probabilities. This can be done by applying Bayes rule od factorization to the full-shape class conditional probability at iteration $j+1$ of the algorithm, hence the new revised estimate of the full-class is given by

$$p\left(t \in \omega | \mathrm{X}_{i}^{t\omega}, \bar{\mathrm{X}}_{i}^{\omega(j)}, \bar{\Sigma}_{i}^{\omega(j)}\right) = \frac{\sum_{k=1}^{n} p(S_{i_k}^{t\omega} | \bar{S}_{i}^{\omega(j)}, \bar{\Sigma}_{i}^{\omega(j)})\ \xi_{i}^{\omega(j)}}{\sum_{\omega \in \Omega} \sum_{k=1}^{n} p\left(S_{i_k}^{t\omega} \middle| \bar{S}_{i}^{\omega(j)}, \bar{\Sigma}_{i}^{\omega(j)}\right)\ \xi_{i}^{\omega(j)}} \tag{18}$$

Where the quantity $\xi_{\omega}^{(j)}$ is the mixing proportion of the full-class probability density function is given by

$$\xi_{i}^{\omega(j+1)} = \frac{1}{t} \sum_{i=1}^{t} \sum_{k=1}^{n} \xi_{i_k}^{\omega(j)} \tag{19}$$

While the segment-mean shape mixing probabilities quantity $\xi_{i_k}^{t\omega}$ is given by

$$\xi_{i_k}^{\omega} = \frac{1}{n} \sum_{i=1}^{t} \sum_{k=1}^{n} p(S_{i_k}^{t\omega} | \bar{S}_{i}^{\omega\,(j)}, \bar{\Sigma}_{i}^{\omega\,(j)}) \tag{20}$$

Taking into consideration, the case where the conditional density for the segment-training patterns is Gaussian distribution model. The segment-patterns are distributed according to Gaussian classification together utilizing the M, or maximizing step of the log-likelihood function criteria is computed by

$$\begin{aligned} p\left(S_{i_k}^{t\omega} \middle| \bar{S}_{i}^{\omega\,(j)}, \bar{\Sigma}_{i}^{\omega\,(j)}\right) &= \frac{1}{2\sigma^2 \sqrt{|\bar{\Sigma}_{i}^{t\omega(j)}|}} e^{[-\frac{1}{2}(S_{i_k}^{t\omega} - (\bar{S}_{i}^{\omega\,(j)} + \phi_{i_k}^{\omega(j)} \gamma_{i_k}^{\omega(j)}))\, x\, (S_{i_k}^{t\omega}} \\ &- (\bar{S}_{i}^{t\omega\,(j)} + \phi_{i_k}^{\omega(j)} \gamma_{i_k}^{\omega(j)}))^T \end{aligned} \tag{21}$$

At iteration $(j+1)$ of the Expectation Maximization algorithm, the revised estimate of the segment-mean shape $\bar{S}_{i}^{\omega\,(j)}$ for the training segment-patterns t belonging to the full-shape ω is given by

$$\bar{S}_{i}^{\omega\,(j+1)} = \sum_{i=1}^{t} \sum_{k=1}^{n} p(t \in \omega, k \in i | S_{i_k}^{t\omega}, \bar{S}_{i}^{\omega\,(j)}, \bar{\Sigma}_{i}^{\omega\,(j)})\ S_{i_k}^{t\omega} \tag{22}$$

Then full-shape mean PDM is a composition of a set of segment mean-shape SPDM

$$\bar{\mathrm{X}}_{i}^{\omega} = (\bar{S}_{i_1}^{\omega}, \bar{S}_{i_2}^{\omega}, \bar{S}_{i_3}^{\omega} \dots, \bar{S}_{i_{k-1}}^{\omega}, \bar{S}_{i_k}^{\omega}) \tag{23}$$

While the revised estimate of the segment-pattern covariance matrix $\bar{\Sigma}_i^{\omega\,(j)}$ is

$$\bar{\Sigma}_i^{\omega\,(j)} = \sum_{i=1}^{t} \sum_{k=1}^{n} p\left(t \in \omega, k \in i \middle| S_{i_k}^{t\omega}, \bar{S}_i^{\omega\,(j)}, \bar{\Sigma}_i^{\omega\,(j)}\right) (S_{i_k}^{t\omega} - \bar{S}_i^{\omega\,(j)}) (S_{i_k}^{t\omega} - \bar{S}_i^{\omega\,(j)})^T \quad (24)$$

Likewise, the full-shape covariance matrix is a composition of a set of related segment covariance matrices in

$$\bar{\Sigma}_i^{\omega} = (\Sigma_{i_1}, \Sigma_{i_2}^{\omega}, \Sigma_{i_3}^{\omega}, \ldots, \Sigma_{i_k}^{\omega}) \quad (25)$$

At algorithm convergence, the segment point distribution model for each segment-pattern is constructed and hence, the full-shape PDM resulted as a construction of a set of segment point distribution models.

4. Experiments

We have evaluated our learning approach on sets of handwritten Arabic characters. Here, we have used 14 sets of Arabic shapes with 2000 characters for each set. Figure 1 shows sample training sets used for our learning algorithm. Figure 2 illustrates handwritten characters and landmarks representation of the shape. Here we have used 50 $2D$ landmarks uniformly distributed along the skeleton of each shape.

Figure 1 : Sample training pattern sets used for learning

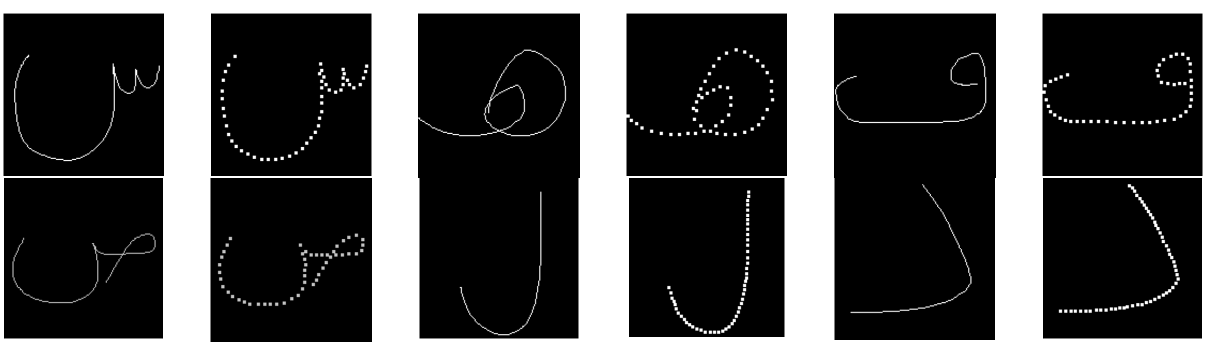

Figure 2: Sample characters and landmarks representation

In Figure 3, first column to the left shows shape-class training set used for the purpose of learning, while column 2 demonstrates the single sample shape, column 3 shows the shape landmarks representation, column 4 shows the Expectation Maximization learning algorithm shape-segment initialization, and column 5 shows the result obtained by the learning algorithm.

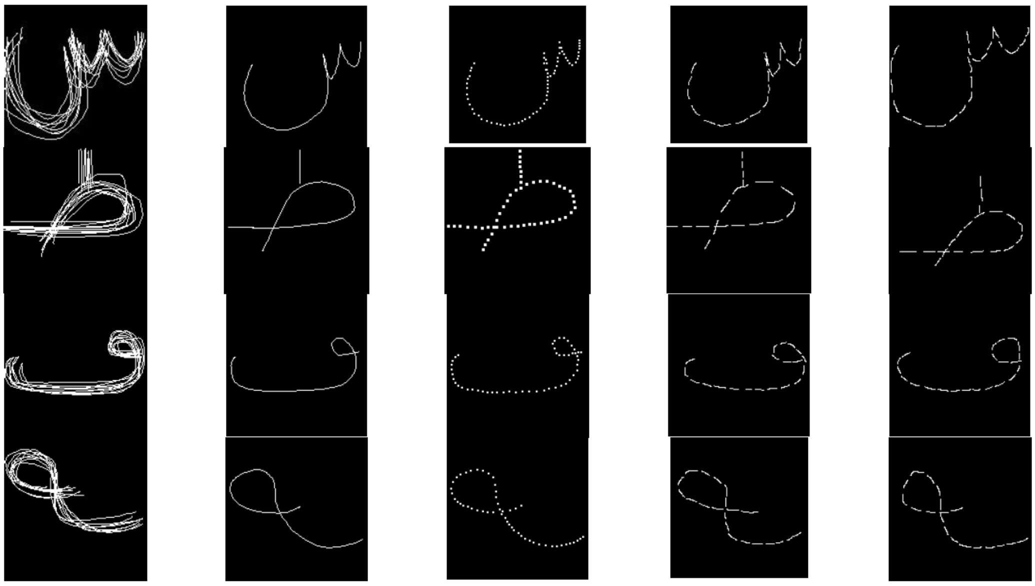

Figure 3: Sample EM initilization and result obtained

Figure 4 shows shape-segment-class $aposteriori$probabilities as a function per iteration number. It is obvious from the graph that class-segment converges within 5 iterations and faster than its counterpart full shape training stage.

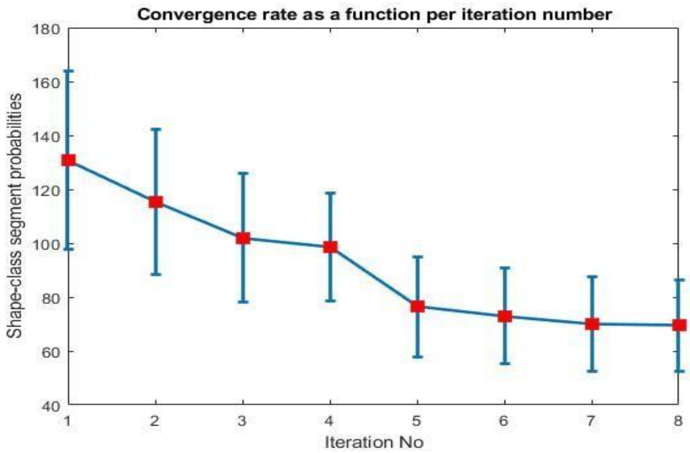

Figure 5: shape class segment convergence rate

We have used a test pattern of 28,000 handwritten characters and reached a correct pattern grouping of 27,524 shapes with a classification rate of 98.3%. The remaining wrong pattern grouping is a result from shape segmentation error and further investigation will be conducted in the near future.

5. CONCLUSION

In this paper, we have demonstrated that shapes of handwritten characters can be classified using shape analysis. Each shape-segment is represented by segment-point-distribution-model (SPDM). These models are then injected into a mixture of shape patterns to be trained and capture the shape modes of variations. The approach is then tested on a set of handwritten Arabic characters and results in a high classification rate. In the future we will use these models to recognize shapes using shape alignment.

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