ON DIRECT PROOFS OF FERMAT'S LAST THEOREM: ABEL CONJECTURE, THE EVEN AND NON-PRIME EXPONENTS, AND THE FIRST CASE.

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ABSTRACT

In this paper, we study Fermat's equation,

$$x^n + y^n = z^n \qquad (1)$$

with $n > 2, x, y, z$ positive integers such that $xy \not\equiv 0 \; mod(p)$. Consider the set F_n of hypothetical solutions of equation (1) and $(a, b, c) \in F_n$. Let $p > 2$ be a prime, we establish the following results:

- $a^p + b^p \neq (b+1)^p$. This completes the direct proof of Abel's conjecture.
- $F_{2p} = \emptyset$. This completes the direct proof of the second case of even exponent FLT.
- $F_n = \emptyset$ if n is a non-prime odd integer.
- If $ab \not\equiv 0 \; [p]$ then $a^p + b^p \neq c^p$. This provides simultaneous Diophantine evidence for the first case of FLT and the second case $c \equiv 0 \; mod(p)$.

We analyse each of the evidence from the previous results and propose a ranking in order of increasing difficulty to establish them.

KEYWORDS

Fermat Last Theorem, Fermat equation, First case, Secund case, Abel Conjecture, Kimou main divisors Theorem. The even exponent, The odd non-prime exponent, a prime number.

1. INTRODUCTION AND MAIN RESULTS

In 1670 Fermat wrote that "It is impossible for a cube to be written as the sum of two cubes or for a fourth power to be written as the sum of two fourth powers or, in general, for any number equal to a power greater than two to be written as the sum of two powers" [1] p.1-2. Fermat claimed to have "woven" a wonderful proof of his problem. He gave the principle, the infinite descent, and illustrated it by proving the exponent 4 of his problem. For a little more than three centuries, Fermat's proposition, hitherto called Fermat's conjecture, had not yet been demonstrated in generality, even for the first case. However, non-obvious elementary proofs based on the principle of Fermat's infinite descent or not have been obtained for the small exponents of 3, 5, ...,100 (first case) and 3, …, 14 (general case) [1] p. 64. Using computer tools, these limits had been pushed to $57*10^9$ (Morishima and Gunderson, 1948) for the first case and to 125 000

(Wagstaff,1976) for the general case [1] p.19. Apart from these results concerning precise values of the exponents or its programming, there are other partial results involving families of prime exponents and based on relatively elementary theories [2] p. 109-122,203-211,360-361:

- In 1823, Sophie Germain established the first case of FLT for exponents n less than 100. It also states that if $n = p$ is prime such that $2p + 1$ is still prime, then the first case of FLT for exponent p is true.
- In 1846, Kummer used the theory of cyclotomic fields to obtain some very remarkable results: The impossibility of Fermat's equation for prime and regular n and deduce that the first case of FLT fails for all prime exponents less than 100 except 37, 59 and 67.
- In 1977, Terjanian proved the first case of even exponent of FLT. He considered $x^{2p} + y^{2p} = z^{2p}$ with p a prime and he used the law of reciprocity to prove an important lemma involving quotients $\frac{z^p - y^p}{z - y}, \frac{z^q - y^q}{z - y}$ and Jacobi's symbols.

Despite these results, general proof was still slow to be found. It was in 1985 that Andrews Wiles provided the first recognized proof by the scientific community of Fermat's conjecture, which would become the Fermat-Wiles theorem [2] [4]. In 2023, Kimou K. P. took the decisive step by introducing Kimou 's divisors for a hypothetical solution of $x^n + y^n = z^n$ with $n = 4, p, 2p$ and proposing new proofs of FLT for, for the first case of the Abel conjecture, and proved some properties related to Fermat problem [4]-[9]. Then, he proved new fundamental and decisive results for this problem: A crucial relationship and a fundamental theorem that will allow him to reach the "Heart" of the problem [10]-[11]. The objective of this paper is to give a Diophantine proof for following main results.

Theorem 1.1. Let $p > 2$ be a prime. Consider $F_p(\mathbb{N}^*) = \{(x, y, z) \in \mathbb{N}^{*3}, x^p + y^p = z^p\}$ and $(a, b, c) \in F_p(\mathbb{N}^*)$. Then

$$c - b = 1 \Longrightarrow a^p + b^p \neq c^p.$$

Theorem 1.2. Let p be a prime. Then,

$$p > 2 \Longrightarrow F_{2p}(\mathbb{N}^*) = \emptyset.$$

Theorem 1.3. Let n an odd non-prime integer. Then,

$$n > 2 \Longrightarrow F_n(\mathbb{N}^*) = \emptyset.$$

Theorem 1.5. Let $n > 2$ a non-prime integer. Consider

$F_p(\mathbb{N}^*) = \{(x, y, z) \in \mathbb{N}^{*3}, x^p + y^p = z^p\}$ and $(a, b, c) \in F_p(\mathbb{N}^*)$. Then

$$ab \not\equiv 0 \ mod(p) \Longrightarrow a^p + b^p \neq c^p.$$

2. PRELIMINARIES

In this section we define commonly used terms, state and prove theorems and lemmas necessary for the proofs of our main results.

Classically, we say that the first case of FLT for the odd prime exponent is true when:

$$xyz \not\equiv 0 \ mod(p) \Longrightarrow x^p + y^p \neq z^p$$

and the secund case of FLT for the same exponent is true if:

$$xyz \equiv 0 \; mod(p) \Longrightarrow x^p + y^p \neq z^p$$

Definitions 1.1

1. A solution (x, y, z) of equation (1) is said to be the primitive or primitive (or eigen) solution of (1), if $x > 0, y > 0, z > 0$, and $\gcd(x, y, z) = 1$.
2. A solution of equation (1) is said to be trivial if $xyz = 0$.
3. Let $n > 2$ be a integer. Here we give an explicit definition of the set F_n introduced above:
$F_n = \{(x, y, z) \in \mathbb{N}^{*3} \setminus x^n + y^n = z^n, \gcd(x, y, z) = 1\}$.

We'll need the following lemmas.

Theorem 2.1. Let $n > 2$ be an integer and let $p > 2$ be a prime. Then

$$F_p = \emptyset \Longrightarrow F_n = \emptyset.$$

Proof. Proving lemma 2.1 is equivalent to proving that if $F_n \neq \emptyset$ then $F_p \neq \emptyset$. We proceed by contraposed reasoning. Let us consider that $F_n \neq \emptyset$. We distinguish two cases.
On the one hand, if $n = 2^l$ avec $l \geq 2$. Consider the case $l = 2$. In that case $n = 4$ and equation (1) becomes

$$x^4 + y^4 = z^4.$$

This is Fermat's bisquare equation and we all know that it does not admit non-trivial solutions. Consider the case where $l > 2$ then $n = 2^l \equiv 0 \; (mod\; 4)$. Therefore, there exists a natural number k such that $n = 4k$. Equation (1) becomes $x^{4k} + y^{4k} = z^{4k}$. As a result

$$x^{4k} + y^{4k} = z^{4k} \Longrightarrow (x^k)^4 + (y^k)^4 = (z^k)^4 \\ \Longrightarrow \square.$$

Hence $n \neq 2^l$, with $l > 2$. In short $n \neq 2^l$ with $l \geq 2$.

On the other hand, if $n \neq 2^l$ then n admits a prime factor $q > 2$. There are $k \geq 2$ such as $n = kq$. Then

$$\begin{aligned} F_n \neq \emptyset &\Longrightarrow \exists (a, b, c) \in F_n, abc \neq 1 \\ &\Longrightarrow (a, b, c) \in F_{kq} \\ &\Longrightarrow a^{kq} + b^{kq} = c^{kq} \\ &\Longrightarrow (a^k)^q + (b^k)^q = (c^k)^q \; with \; q > 2 \; a \; prime \\ &\Longrightarrow (a^k, b^k, c^k) \in F_q, a^k b^k c^k \neq 0 \\ &\Longrightarrow F_q \neq \emptyset. \end{aligned}$$

Hence if $F_p = \emptyset$ then $F_n = \emptyset$.

Lemma 2.2. Let $p > 2$ be a prime and let $(a, b, c) \in F_p$. Then,

$$b = a + 1 \Longrightarrow q_1 = q_2 = 1.$$

Proof. See [8].

Lemma 2.3. Let $p > 2$ be a prime and let $(a, b, c) \in F_p$. Consider $c = aq_2 + r_2$ *with* $r_2 < a$ and $e = \gcd(b, c - a)$. Then,

$$q_2 = 1 \implies \begin{cases} r_2 = \dfrac{e^p}{p} \text{ if } b \equiv 0 \; mod(p) \\ r_2 = e^p \text{ otherwise} \end{cases}.$$

Proof. See [8].

Theorem 2.2. (Kimou-Fermat). Let $p > 2$ be a prime and let (a, b, c) be a triple of primitive positive integers solution of equation (1) . Then,

$$(a, b, c) \in F_p \implies \begin{cases} b - a = 1 \text{ if } c \equiv 1 \; mod(2) \\ b - a = 2 \text{ otherwise} \end{cases}.$$

Proof. See [10].

Lemma 2.4. Let $p > 2$ be a prime and let $(a, b, c) \in F_p$. Then

$$b - a = 1 \iff c \equiv 1 \; mod(2).$$

Proof.

On the one hand, let us show that if $b - a = 1$ then $c \equiv 1 \; mod(2)$. We have:

$$\begin{aligned} b - a = 1 &\implies b = a + 1 \\ &\implies a, b \text{ are the same parity} \\ &\implies c \text{ is odd} \end{aligned}$$

Reciprocally, let us show that if $c \equiv 1 \; mod(2)$ then $b - a = 1$. We have:

$$\begin{aligned} c \equiv 1 \; mod(2), b - a = 2 &\implies b, a \text{ have the same parity} \\ &\implies b, a \text{ are odd because } \gcd(a, b) = 1. \\ &\implies c \text{ is even} \\ &\implies \square. \end{aligned}$$

Hence $b - a = 1$.

Lemma 2.5. Let $p > 2$ be a prime and let $(a, b, c) \in F_p$ be a triple of primitive positive integers solution of equation (1). Then

$$b - a = 2 \iff c \equiv 0 \; mod(2).$$

Proof. Can be deduced by contraposition of the previous lemma.

3. PROOFS OF THE MAIN RESULTS

In this section we define commonly used terms, state and prove theorems and lemmas necessary for the proofs of our main results.

3.1. Proof of Abel Conjecture

Conjecture (Abel). Let $p > 2$ be a prime and let (a, b, c) a triple of primitive positive integers. If $(a, b, c) \in F_p$ then none of the a, b or c is the power of a prime number.

When b or c is a prime power, the proof is easy [11]. When a is a prime power, the problem becomes tricky. The first case of this conjecture was proved by Abel himself. The second case has yet to receive direct proof.

Theorem 3.1. Let $p > 2$ be a prime and let $(a, b, c) \in F_p$ be a triple of primitive positive integers. If $\forall x \in \{a, b, c\} \exists (\pi, m) \in \mathbb{N}^{*2}$ with π is a prime such that $x = \pi^m$ then $x \not\equiv 0\ (mod\ p)$.

Proof.

$$\begin{aligned} b = \pi^m, b \equiv 0\ (mod\ p) &\Rightarrow \pi^m \equiv 0\ (mod\ p) \\ &\Rightarrow \pi = p \\ &\Rightarrow b = p^m \\ &\Rightarrow e\beta = p^m \\ &\Rightarrow \square \text{ because } \gcd(e, \beta) = 1 \end{aligned}$$

Hence $b \not\equiv 0\ [p]$.

Lemma. Let $p > 2$ be a prime and let $(a, b, c) \in F_p$. Then

$$b = \pi^m \Rightarrow e = 1.$$

Proof. We proceed by reasoning from the absurd. Let's assume that $(a, b, c) \in F_p$ *and* $b = \pi^m$. We have:

$$\begin{aligned} (a, b, c) \in F_p, b = \pi^m &\Rightarrow e\beta = \pi^m \textit{ with } \gcd(e, \beta) = 1, \beta > e \\ &\Rightarrow e = 1; \end{aligned}$$

Theorem 3.1. Let $p > 2$ be a prime and let $(a, b, c) \in F_p$. Then

$$(a, b, c) \in F_p \Rightarrow \nexists (\pi, m) \in \mathbb{N}^{*2}, b = \pi^m \textit{ with } \pi \textit{ is a prime}$$

Proof.

$$\begin{aligned} b = \pi^m &\Rightarrow e = 1, a \equiv 0\ [p] \text{ and } b > a \\ &\Rightarrow \beta > d\alpha \\ &\Rightarrow d < \frac{\beta}{\alpha} < \sqrt{p} \Rightarrow d < 2 \\ &\Rightarrow d = 1 \\ &\Rightarrow \square \text{ because } d \equiv 0\ [p] \end{aligned}$$

Lemma 3.2. Let $p > 2$ be a prime. Then

$$(a, b, c) \in F_p \Rightarrow \nexists (\pi, m) \in \mathbb{N}^{*2}, c = \pi^m \text{ with } \pi \text{ is a prime}$$

Proof. We reason from the absurd by supposing that $(a, b, c) \in F_p$ *and* $c = \pi^m$. We have:
On the one hand

$(a,b,c) \in F_p$ *and* $c = \pi^m \Rightarrow f\gamma = \pi^m$ with $\gcd(e,\beta) = 1, \beta > e$.
$\Rightarrow f = 1;$

Moreover,

$$(a,b,c) \in F_p, c = \pi^m \Rightarrow a + b = f^p = 1$$
$$\Rightarrow ab = 0 \Rightarrow \square.$$

Hence $\nexists(\pi, m) \in \mathbb{N}^{*2}, c = \pi^m$ with π is a prime.

Lemma 3.3. Let $\pi > 2$ be a prime and let $(a,b,c) \in F_p$ be a triple of primitive positive integers solution of equation (1). Then,

$$a = \pi^m \Rightarrow c - b = 1.$$

Proof.

$$a = \pi^m \text{ and } d > 1 \Rightarrow d\alpha = \pi^m, pgcd(d,\alpha) = 1$$
$$\Rightarrow \exists(r,s) \in \mathbb{N}^{*2}, d = \pi^r, \alpha = \pi^s, r + s = m, s > r$$
$$\Rightarrow \exists(r,s) \in \mathbb{N}^{*2}, 1 = pgcd(\pi^r, \pi^s) \geq \pi$$
$$\Rightarrow \square.$$

Hence $d = 1$ and as a result $c - b = d^p = 1$.

Lemme 3.4. Let $p > 2$ be a prime and let $(a,b,c) \in F_p$. Then

$$c - b = 1 \Rightarrow a \equiv 1\ [2].$$

Proof.

$$c - b = 1 \Rightarrow c = b + 1$$
$$\Rightarrow c \text{ or } b \text{ is even}$$
$$\Rightarrow a \text{ is odd.}$$

Lemme 3.5. Let $p > 2$ be a prime and let $(a,b,c) \in F_p$ be a triple of primitive positive integers solution of equation (1). Then

$$c - b = 1 \Rightarrow \begin{cases} c - a = 2 \text{ if } c \equiv 1\ [2] \\ c - a = 3 \text{ otherwise} \end{cases}$$

Proof.

$$c - b = 1 \Rightarrow b = c - 1$$
$$\Rightarrow \begin{cases} b - a = 1 \text{ if } c \equiv 1\ (mod\ 2) \\ b - a = 2 \text{ otherwise} \end{cases} \quad [Theorem\ 2.2.]$$
$$\Rightarrow \begin{cases} c - 1 - a = 1 \text{ if } c \equiv 1\ (mod\ 2) \\ c - 1 - a = 2 \text{ otherwise} \end{cases}$$
$$\Rightarrow \begin{cases} c - a = 2 \text{ if } c \equiv 1\ (mod\ 2) \\ c - a = 3 \text{ otherwise} \end{cases}$$

Theorem. Let $p > 2$ be a prime and let $(a,b,c) \in F_p$. Then

$$c - b = 1 \Rightarrow a^p + b^p \neq c^p$$

Proof.

On the one hand, if $b \not\equiv 0\ [p]$,, then

$$c - b = 1 \Rightarrow \begin{cases} c - a = 2 \text{ if } c \equiv 1\ (mod\ 2) \\ c - a = 3 \text{ otherwise} \end{cases} \quad [Lemma\ 3.5]$$
$$\Rightarrow \begin{cases} e^p = 2\ if\ c \equiv 1\ mod(2) \\ e^p = 3 \text{ otherwise} \end{cases}$$
$$\Rightarrow \begin{cases} k^p 2^{vp-1} = 1 \text{ if } c \equiv 1\ mod(2) \\ k^p 3^{vp-1} = 1 \text{ otherwise} \end{cases}$$
$$\Rightarrow \begin{cases} 2^{vp-1} = 1 \text{ if } c \equiv 1\ mod(2) \\ 3^{vp-1} = 1 \text{ otherwise} \end{cases}$$
$$\Rightarrow \begin{cases} p = \frac{1}{v_1} \text{ if } c \equiv 1\ mod(2) \\ p = \frac{1}{v_2} \text{ otherwise} \end{cases} \Rightarrow \begin{cases} p = 1 \text{ if } c \equiv 1\ mod(2) \\ p = 1 \text{ otherwise} \end{cases}$$
$$\Rightarrow p = 1$$
$$\Rightarrow \square\ because\ p > 2.$$

Hence $c - b > 1$

On the other hand, if $b \equiv 0\ [p]$, then

$$c - b = 1 \Rightarrow \begin{cases} c - a = 2 \text{ if } c \equiv 1\ mod(2) \\ c - a = 3 \text{ otherwise} \end{cases}$$

$$\Rightarrow \begin{cases} \frac{e^p}{p} = 2 \text{ if } c \equiv 1\ mod(2) \\ \frac{e^p}{p} = 3 \text{ otherwise} \end{cases}$$
$$\Rightarrow \begin{cases} k^p 2^{p-1} = p \text{ if } c \equiv 1\ mod(2) \\ k^p 3^{p-1} = p \text{ otherwise} \end{cases}$$
$$\Rightarrow \begin{cases} k^p 2^{p-1} p^{p-1} = 1 \text{ if } c \equiv 1\ mod(2) \\ k^p 3^{p-1} p^{p-1} = 1 \text{ otherwise} \end{cases}$$
$$\Rightarrow \begin{cases} (2p)^{p-1} = 1 \text{ if } c \equiv 1\ mod(2) \\ (3p)^{p-1} = 1 \text{ otherwise} \end{cases}$$
$$\Rightarrow p = 1$$
$$\Rightarrow \square.$$

Hence $c - b > 1$.

3.2. Proof of FLT for Even Exponent

Proof. Let $p > 2$ be a prime and let (a, b, c) be a triple of primitive positive integers. Then According to the Kimou-Fermat theorem, we have:

$$(a, b, c) \in F_{2p} \Rightarrow (a^2, b^2, c^2) \in F_p, c \equiv 1\ mod(2)$$
$$\Rightarrow b^2 - a^2 = 1 \text{ according to the crucial relation}$$
$$\Rightarrow (b - a)(a + b) = 1$$

$$\Rightarrow a + b = 1 \; et \; b - a = 1$$
$$\Rightarrow b = 1$$
$$\Rightarrow \Box \text{ because } b > 1.$$

Hence $(a, b, c) \notin F_{2p}$ and consequently $F_{2p} = \emptyset$.

Remark 3.1. Because of this theorem, Fermat's theorem is true for all even exponents.

3.3. Proof of FLT for the No-Prime Exponent

3.3.1. Proof of FLT for the odd No-Exponent

Theorem 3.3. Let $m > 2$ be an odd integer. Then.

$$m \; is \text{ no} - \text{prime} \Rightarrow F_m = \emptyset.$$

Proof. Let $m > 2$ be an odd no-prime integer. $(a, b, c) \in F_m$.

Then, on the one hand,

$$\begin{aligned} c \equiv 1 \; (mod\; 2) &\Rightarrow \exists (r, s), r > 2, s > 2 \text{ are odd prime}, a^{krs} + b^{krs} = c^{krs} \\ &\Rightarrow \left(a^{ks}\right)^r + \left(b^{ks}\right)^r = \left(c^{ks}\right)^r \\ &\Rightarrow b^{ks} - a^{ks} = 1 \qquad [Theorem\; 2.2] \\ &\Rightarrow \left(b^k - a^k\right) T_s\left(a^k, b^k\right) = 1 \text{ because s is odd} \\ &\Rightarrow \left(b^k - a^k\right) T_s\left(a^k, b^k\right) = 1, b^k - a^k \geq 1, T_s\left(a^k, b^k\right) \geq 1 \\ &\Rightarrow 1 > s\left(b^k - a^k\right) a^{k(s-1)} \\ &\Rightarrow 1 > s > 2 \\ &\Rightarrow \Box; \end{aligned}$$

On the other hand,

$$\begin{aligned} c \equiv 0 \; (mod\; 2) &\Rightarrow \exists (r, s), r > 2, s > 2 \text{ are odd prime}, a^{krs} + b^{krs} = c^{krs} \\ &\Rightarrow \left(a^{ks}\right)^r + \left(b^{ks}\right)^r = \left(c^{ks}\right)^r \\ &\Rightarrow b^{ks} - a^{ks} = 2 \qquad [Theorem\; 2.2] \\ &\Rightarrow \left(b^k - a^k\right) T_s\left(a^k, b^k\right) = 2 \text{ because s is odd} \\ &\Rightarrow 2 > s\left(b^k - a^k\right) a^{k(s-1)} \\ &\Rightarrow 2 > s > 2 \\ &\Rightarrow \Box; \end{aligned}$$

Hence m cannot be an odd no-prime integer and consequently $F_m = \emptyset$.

3.3.2. Proof of FLT for the No-Prime Exponent

This is a consequence of Theorem 1.2. and Theorem 1.3. Indeed, let $(a, b, c) \in F_n$ with n nonprime, then.

$$\begin{cases} a^n + b^n \neq c^n & \text{if } n \text{ is even} \\ a^n + b^n \neq c^n & \text{if } n \text{ is odd no} - \text{prime} \end{cases}$$

As a result,

$$a^n + b^n \neq c^n, \forall n, \qquad \text{n is not a prime}$$

Proof. Let $p > 2$ be a prime and let (a, b, c) be a triple of primitive positive integers. Then According to the Kimou-Fermat theorem, we have:

3.4. Proof of FLT for the First Case and Secund Case C=0 (Mod P)

We distinguish two cases:

Lemma 3.8. Let $p > 2$ be a prime and let $(a, b, c) \in F_p$ such that $ab \not\equiv 0 \ (mod\ p)$. Then

$$c \equiv 1\ mod\ (2) \Longrightarrow F_p = \emptyset.$$

Proof. Let $(a, b, c) \in F_p$ with $p > 2$. We have:

$$\begin{aligned} ab \not\equiv 0\ mod(p), \qquad c \equiv 1\ mod(2) &\Longrightarrow b - a = 1 \\ &\Longrightarrow e^p - d^p = 1 \\ &\Longrightarrow e^p = 2\ or\ 3. \\ &\Longrightarrow 1 > p(e-d)\ d^{p-1} \\ &\Longrightarrow e = d \\ &\Longrightarrow \square; \end{aligned}$$

Hence $ab \equiv 0\ (mod\ p)$.

Lemma 3.9. Let $p > 2$ be a prime and let $(a, b, c) \in F_p$ such that $ab \not\equiv 0\ (mod\ p)$. Then

$$c \equiv 0\ mod(2) \Longrightarrow F_p = \emptyset.$$

Proof. Let $p > 2$ be a prime and let $(a, b, c) \in F_p$ such that $ab \not\equiv 0\ (mod\ p)$. Then:

$$\begin{aligned} c \equiv 0\ mod(2) &\Longrightarrow b - a = 2 \\ &\Longrightarrow e^p - d^p = 2 \\ &\Longrightarrow 2 > p(e-d)d^{p-1} \\ &\Longrightarrow e = d \\ &\Longrightarrow \square; \end{aligned}$$

Theorem 1 is thus established because of lemmas 3.8 and 3.9.

4. Conclusions

In this paper we establish Diophantine proofs of Abel's conjecture, FLT for the exponents even, odd non-prime, and integer non-prime. The analysis of these proofs leads us to classify these problems in order of increasing difficulty: In the first position we find Abel's conjecture, then comes the case of the exponent 2p which is at the same level of difficulty as the first case of the first exponent and its second case $z \equiv 0\ mod(p)$, then there is the case of the non-prime odd exponent and finally the case of the non-prime integer exponent. Moreover, rise the second cases $x \equiv 0\ mod(p)$ and $y \equiv 0\ mod(p)$ of FLT. In perspective, we intend to establish a Diophantine proof of the second remaining cases, i.e. to prove that if $xy \equiv 0\ mod(p)$ then $x^p + y^p \neq z^p$.

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