DETERMINING THE PRIMALITY OF N-1 AND N+1 BY EXAMINING THE NATURAL NUMBER N

METHODS FOR DECIDING PRIMALITY OF A NUMBER WITHOUT USING FACTORIZATION OR SIEVING

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ABSTRACT

In Mathematics, for any Natural Number n,there is no general procedure to determine whether n-1 or n+1 is a prime or composite simply by examining n itself. Factorization of n fails to produce meaningful information regarding the primality of n-1 and n+1. The research being discussed in this paper shows how representing a number n as a distinct set of sequences, heuristically derived from a circle with n points, demonstrates the primality of not only n but of n-1 and n+1; i.e.,the Natural Number n "knows" whether its immediate neighbors n-1 and n+1 are either prime or composite. This method, although simple to comprehend, has significant implications for the Theory of Numbers.

KEYWORDS

Number Theory, Primality Checks, Prime Numbers, Composite Numbers

1. INTRODUCTION

This research took place from March 2022 through May 2024, being prompted simply from playing with numbered dots around a circle, counting increments, recording sequences, and noticing that prime numbers produce Latin Squares, where every sequence memberwhen placed into an array distinctly occurs in one row and one column. This curiosity led to examiningother patterns within these arrays of sequences to identify properties of Natural Numbers that are not apparent when considering n by itself. The objective of this paper has been to report on what happens to the sequences representing n when 1 is added to n to produce n+1 and its related sequences.

The approach only uses modular addition to discover the primality of numbers, avoiding the challenges of factorization, since multiplication and division are shown to be unnecessary to demonstrate that a number is prime. The research provides methods by which *any* number n-1 or n+1 can be determined to be a prime simply by examining the sequences denoting n.

Since every Natural Number n can be represented by a unique set of sequences, heuristically derived from a depiction of n as a circle with n points, it becomes easy to 1) Check the primality of the number n without using factorization or sieving, and, 2) Check the primality of n-1 and n+1 by only examining n.

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When put into an array, the sequences for composite numbers present a distinct pattern of 1s. Prime numbers, being Latin Squares, do not exhibit such a pattern. Since the "shadows" of the patterns for n-1 and n+1 can be found in the structure of the sequences derived for n, any Natural Number n "knows" whether its neighbors n-1 and n+1 are either prime or composite. The included github linksprovidecode for three C++ programs that substantiates the research presented, revealing the amazing engine underneath the Natural Number line.

1.1. Related Work

Initially, research has investigated the "Orbit" value of each row in the array. An Orbit value is the number of Zones, as defined below in Section 4.3, for any particular row. Intuitively, an Orbit value is the number of circuits required to hop around a circle with n points using a particular increment before landing on the starting point again. This simple concept opens up a beautiful investigation into the Theory of Numbers that has not been documented in mathematics. The results of that research are presented in the three-volume work, *Orbit Theory of Natural Numbers*, as listed in the References. However, there is a fundamental missing piece about how the sequences for n become the sequences for n+1. This paper presents the answer to this question.

2. PREMISE OF THE RESEARCH

The fundamental theorem of arithmetic states that any Natural Number n can be factored into a product of prime numbers. However, adding 1 to this product of primes, in other words, adding 1 to n, reveals little about n+1 without factoring n+1 itself. Take for example the prime factorization of 16, which is 2^4. If 1 is added to this factorization, $2^4 + 1$, the most that can be said is that $2^4 + 1$ is an odd number. Whether $2^4 + 1$ is prime or composite cannot be determined without calculating out $2^4 + 1 = 17$ and factoring 17 into 1 * 17, or applying some primality test or sieve to determine if 17 is prime or composite. Although this example is trivial, it demonstrates the problem with factorization, that it is not possible to determine if n+1 is prime or composite simply based upon the factorization of n.

Factorization is only one type of representation for a Natural Number. There is an "engine" beneath the Natural Number line that links every Natural Number together with its neighbors. The Natural Number line is simply the "hood" of the Natural Numbers.

The researchexplores another representation for a Natural Number n that reveals the underlying mechanisms beneath the number line and demonstrates how adding 1 to a number n can create a prime number. The domain of this analysis is only the Natural Number domain$\mathbb{N}$. No fractions, decimals, negative numbers, imaginary or transcendental numbers are used; not even 0 is defined.

3. THE ENGINE BENEATH $\mathbb{N}$

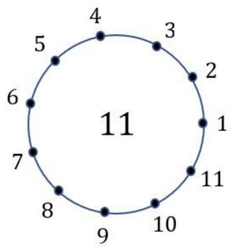

Figure 1 – The Natural Number 11 represented as a circle with 11 points.

This investigation of a number n involves viewing n as a unique entity by itself in representing n as a circle surrounded by n points numbered from 1 to n. From this simple geometric structure, a unique set of sequences can be derived representing n, by counting increments, less than or equal to n, around that circle. The collection of these sequences demonstrates when a number n itself is prime or composite and becomes a powerful tool. This tool is key to understanding what actually happens when 1 is added to n; i.e.,what is the effect to these sequences when 1 is added to n? How does this distinct set of sequences for n transform into the distinct set of sequences for n+1?

3.1. The Sequences for Primes Produce Latin Squares

Using various increments from 1 to n, count around the points of the circle starting at 1 and continuing until 1 is encountered again. List the numbers that are encountered. For example, if n=11, using the increment 1, move from point 1 to point 2 to point 3, etc. Do not list the initial point 1 because that is the starting point. Thus, the list for an increment d=1 is: {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 1}. Now do this for the next increment d=2 listing out the sequence: {3, 5, 7, 9, 11, 2, 4, 6, 8, 10, 1}. By continuing in this manner, the following sequences occur for n=11.

d=1 {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 1}
d=2 {3, 5, 7, 9, 11, 2, 4, 6, 8, 10, 1}
d=3 {4, 7, 10, 2, 5, 8, 11, 3, 6, 9, 1}
d=4 {5, 9, 2, 6, 10, 3, 7, 11, 4, 8, 1}
d=5 {6, 11, 5, 10, 4, 9, 3, 8, 2, 7, 1}
d=6 {7, 2, 8, 3, 9, 4, 10, 5, 11, 6, 1}
d=7 {8, 4, 11, 7, 3, 10, 6, 2, 9, 5, 1}
d=8 {9, 6, 3, 11, 8, 5, 2, 10, 7, 4, 1}
d=9 {10, 8, 6, 4, 2, 11, 9, 7, 5, 3, 1}
d=10 {11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1}
d=11 {1}

n=11

2	3	4	5	6	7	8	9	10	11
3	5	7	9	11	2	4	6	8	10
4	7	10	2	5	8	11	3	6	9
5	9	2	6	10	3	7	11	4	8
6	11	5	10	4	9	3	8	2	7
7	2	8	3	9	4	10	5	11	6
8	4	11	7	3	10	6	2	9	5
9	6	3	11	8	5	2	10	7	4
10	8	6	4	2	11	9	7	5	3
11	10	9	8	7	6	5	4	3	2

Figure 2 – Array of Square Sq(11).

Dropping the last member m of each sequence where m=1, and the last set entirely where d=11, consider the remaining numbers and put them into a n-1 x n-1 array as inFigure 2. This particular array for n = 11, exhibits the characteristics of a Latin Square, where each member appears only once in each column and row. This occurs when n is a prime number. All prime numbers, formed in this way, exhibit a Latin Square.

3.2. The Construction Rule

There is another way to determine the members of this array without creating a circle with n points and counting increments to produce sequences. It is known as the *Construction Rule*. The increments d are put into their own array from 1 to n-1 called the additive increment array.

1	2	3	4	5	6	7	8	9	10

The first row of the array for n = 11 is defined as {2, 3, …, 11}, or for an arbitrary n, {2, 3, …, n}.

2	3	4	5	6	7	8	9	10	11

All subsequent rows of the array are calculated, by using modular arithmetic for each pair of cells, to add the increment array to the previous row to get the next row,

2	3	4	5	6	7	8	9	10	11
3	5	7	9	11	2	4	6	8	10

and so forth. Arrays formed in this manner are denoted by Square Sq(n) for both prime and composite n and are distinct representations of any n. The Construction Rule can easily be automated using a spreadsheet or writing a program to produce the Square Sq(n) for any n.

3.3. Composite Numbers Produce Patterns of 1s

Consider a circle with n points where n is a composite number. Some of the recorded sequences, when counting increments around the circle,land on the initial point 1 without covering all the points. For example, let n=12, and count increments around the circle from 1 to 12.This results in the following sequences.

d=1 {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 1}
d=2 {3, 5, 7, 9, 11, 1}
d=3 {4, 7, 10, 1}
d=4 {5, 9, 1}
d=5 {6, 11, 4, 9, 2, 7, 12, 5, 10, 3, 8, 1}
d=6 {7, 1}
d=7 {8, 3, 10, 5, 12, 7, 2, 9, 4, 11, 6, 1}
d=8 {9, 5, 1}
d=9 {10, 7, 4, 1}
d=10 {11, 9, 7, 5, 3, 1}
d=11 {12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2,1}
d=12 {1}

Some of these are shorter sequences than those sets having all the numbers from 1 to 12 represented. When the increment d is relatively prime to n, counting the increment lands on every point until it hits the final 1. If the increment d is a factor of n, then the sequence skips various points.

n=12

2	3	4	5	6	7	8	9	10	11	12
3	5	7	9	11	**1**	3	5	7	9	11
4	7	10	**1**	4	7	10	**1**	4	7	10
5	9	**1**	5	9	**1**	5	9	**1**	5	9
6	11	4	9	2	7	12	5	10	3	8
7	**1**	7	**1**	7	**1**	7	**1**	7	**1**	7
8	3	10	5	12	7	2	9	4	11	6
9	5	**1**	9	5	**1**	9	5	**1**	9	5
10	7	4	**1**	10	7	4	**1**	10	7	4
11	9	7	5	3	**1**	11	9	7	5	3
12	11	10	9	8	7	6	5	4	3	2

Figure 3 – Square Sq(12) showing the distinct pattern of 1s.

When the Construction Rule is applied to form the Square Sq(12), these shorter sequences are filled in to the width of the array as in Figure 3. A distinct pattern of 1s appears. All composite numbers show a pattern of 1s unique to its Natural Number n. For prime numbers, the resulting Latin Square has no 1s within the array. If a 1 appears in the Square Sq(n), then n is a composite number.

3.4 SYMMETRY WITHIN SQUARE SQ(N)

Consider the Latin Square Sq(n) formed when n=13 as in Figure 4. Notice that the numbers within the cells are symmetric around the two diagonals drawn from corner to corner. The Square Sq(13) can be folded on the diagonals and the numbers match up.

Square Sq(13)

2	3	4	5	6	7	8	9	10	11	12	13
3	5	7	9	11	13	2	4	6	8	10	12
4	7	10	13	3	6	9	12	2	5	8	11
5	9	13	4	8	12	3	7	11	2	6	10
6	11	3	8	13	5	10	2	7	12	4	9
7	13	6	12	5	11	4	10	3	9	2	8
8	2	9	3	10	4	11	5	12	6	13	7
9	4	12	7	2	10	5	13	8	3	11	6
10	6	2	11	7	3	12	8	4	13	9	5
11	8	5	2	12	9	6	3	13	10	7	4
12	10	8	6	4	2	13	11	9	7	5	3
13	12	11	10	9	8	7	6	5	4	3	2

Figure 4 – Symmetry on either side of the diagonals for Square Sq(13).

Square Sq(15)

2	3	4	5	6	7	8	9	10	11	12	13	14	15
3	5	7	9	11	13	15	2	4	6	8	10	12	14
4	7	10	13	**1**	4	7	10	13	**1**	4	7	10	13
5	9	13	2	6	10	14	3	7	11	15	4	8	12
6	11	**1**	6	11	**1**	6	11	**1**	6	11	**1**	6	11
7	13	4	10	**1**	7	13	4	10	**1**	7	13	4	10
8	15	7	14	6	13	5	12	4	11	3	10	2	9
9	2	10	3	11	4	12	5	13	6	14	7	15	8
10	4	13	7	**1**	10	4	13	7	**1**	10	4	13	7
11	6	**1**	11	6	**1**	11	6	**1**	11	6	**1**	11	6
12	8	4	15	11	7	3	14	10	6	2	13	9	5
13	10	7	4	**1**	13	10	7	4	**1**	13	10	7	4
14	12	10	8	6	4	2	15	13	11	9	7	5	3
15	14	13	12	11	10	9	8	7	6	5	4	3	2

Figure 5 – Symmetry of the diagonals for Square Sq(15).

However, the Square Sq(13) is not symmetric along the two axes formed by the vertical and horizontal center lines. When folding it in half along the X and Y center lines, the numbers do not match. This is a characteristic of all Squares Sq(n) for both prime and composite numbers.

Consider the composite number n=15 in Figure 5. Square Sq(15) is symmetric around each diagonal. When folded, all the numbers matchup.When folded on the center lines of the X and Y axes, not all the numbers match up, but the pattern of 1s do! The 1s are symmetric around the center X and Y axes as well as both diagonals. The 1s form an invariant group within the Square Sq(n) for any composite number. The array can be turned 90^{o} in any direction, flipped on both diagonals as well as the X and Y axes, and the pattern of 1s do not change.

These Squares Sq(n) with their symmetries create a powerful tool for analyzing any Natural Number.

1) A number n is prime, if and only if the Square Sq(n) forms a Latin Square when created using the Construction Rule.
2) A number n is composite, if and only if the Square Sq(n) contains cells with a pattern of 1s.
3) When determining the primality of an arbitrarily large n, only a subset of the cells must be checked for the presence of 1s.

Square Sq(15)

2	3	4	5	6	7	8	9	10	11	12	13	14	15
3	5	7	9	11	13	15	2	4	6	8	10	12	14
4	7	10	13	**1**	4	7	10	13	**1**	4	7	10	13
5	9	13	2	6	10	14	3	7	11	15	4	8	12
6	11	**1**	6	11	**1**	6	11	**1**	6	11	**1**	6	11
7	13	4	10	**1**	7	13	4	10	**1**	7	13	4	10
8	15	7	14	6	13	5	12	4	11	3	10	2	9
9	2	10	3	11	4	12	5	13	6	14	7	15	8
10	4	13	7	**1**	10	4	13	7	**1**	10	4	13	7
11	6	**1**	11	6	**1**	11	6	**1**	11	6	**1**	11	6
12	8	4	15	11	7	3	14	10	6	2	13	9	5
13	10	7	4	**1**	13	10	7	4	**1**	13	10	7	4
14	12	10	8	6	4	2	15	13	11	9	7	5	3
15	14	13	12	11	10	9	8	7	6	5	4	3	2

Figure 6 – Only the bounded sector needs to be checked to determine the primality of n.

This third point suggests a way of determining the primality of any Natural Number n without using factorization or sieving. Only a sector, as in Figure 6,bounded by the diagonal and the X-axis, needs to be checked for the presence of 1s. If no cell with a 1 is found, then n is a prime number. If a single 1 is found, then n is a composite number.

4. PRIMALITY CHECKS ON SQUARE SQ(N)

The classical method of determining if a number n is a prime number is to divide it by all the prime numbers up through the square root of n. If all of these divisions result in a remainder, then n is prime. The research presented herein found several methods to determine primality of any Natural Number n that actually involve no division at all. Primality is determined by examining the cells within Square Sq(n). These methods include 1) Checking the entire Square Sq(n) to determine if it is a Latin Square, 2) Checking the sector using the Temple Search for 1s, and 3) The Staircase Search for 1s.Thefollowing sections review the last two of these.

Square Sq(15)

2	3	4	5	6	7	8
3	5	7	9	11	13	15
4	7	10	13	1	4	7
5	9	13	2	6	10	14
6	11	**1**	6	11	1	6
7	13	4	10	**1**	7	13
8	15	7	14	6	13	5

Figure 7 – The Temple Search for Primality.

4.1. The Temple Search

As pointed out in Section 3.4, only the sector bounded by the main diagonal and the X-axis, as identified in the previous Figure 6, needs to be checked for 1s to determine if n is either a prime or composite number, because of the symmetry of the 1s for any composite number. This sector can be further reduced as in Figure 7when it is observed that the first column and top row of Square Sq(n) never have a 1 based upon the definition of the Construction Rule.

This is called the *Temple Search* because it is similar to a Mayan temple. An algorithm can be constructed that evaluates each cell and tests it for the value of 1. If a 1 is found, then the search can end immediately since n is a composite number. For large values of n, this is still a great number of cells to check if n is indeed a prime number.

4.2. The Staircase Search

This search pattern is based upon the observation that for composite numbers, a 1 can be found on either the main diagonal or the diagonal of cells adjacent to the main diagonal as displayed in Figure 8. This is the *Staircase Conjecture* and has been verified for prime numbers less than one million.

Square Sq(21)

2	3	4	5	6	7	8	9	10	11
3	5	7	9	11	13	15	17	19	21
4	7	10	13	16	19	1	4	7	10
5	9	13	17	21	4	8	12	16	20
6	11	16	21	5	10	15	20	4	9
7	13	19	4	10	16	1	7	13	19
8	15	1	8	15	**1**	8	15	1	8
9	17	4	12	20	7	15	2	10	18
10	19	7	16	4	13	1	10	19	7
11	21	10	20	9	19	8	18	7	17

Figure 8 – The Staircase Search for Primality.

It is called the *Staircase Search* since the algorithm walks up the steps from the center to the upper left. If it finds a cell with a value of 1, then the search ends because a composite number has been recognized. If it completes the evaluation of all the cells in the staircase without finding a 1, then a prime has been found.

An algorithm for the Staircase Search can be derived where the value of a particular cell in the search area is:

$$\text{Value of Cell} = \text{mod}_{[n]}\left(\frac{n^2 - 2\,j\,n + j^2 + 3 + mod_2(j)}{4} \right).$$

The variable j is an index that marches up the stairs. The symbol $\text{mod}_{[n]}$ is a modified modular function since 0 does not exist as a Natural Number, and instead, defines $\text{mod}_{[n]}(0) = n$ and $\text{mod}_{[n]}(n)=n$. The symbol $\text{mod}_2(j)$ is the standard modular function that has values of 0 or 1 depending on j. If the value of any cell equals 1, then n is a composite number.

The following C++ implementation of this algorithm, for finding the primality of a number n based upon the Staircase Search, can be found in the github repository at this location:

https://github.com/alanverdegraal/OrbitTheory/tree/C++Programs/Staircase_Search_for_1.cpp

This algorithm only needs n-6 cell checks. Thus, it can evaluate a number n in linear time.

This C++ implementation of the Staircase Search can determine if a ten-digit number n is prime in about 60 seconds, and a composite number in a fraction of that time, limited by the precision and speed of the computing machine. It does not use factorization or sieving as a method for ruling out prime factors. Instead, it directly checks the cell values seeking 1s based upon the simple theory of Squares Sq(n) presented herein. Without understanding this underlying theory, the algorithm makes no sense in how it can identify prime and composite numbers.

4.3. Zone Analysis

Before demonstrating how the Natural Number n "knows" that both n-1 and n+1 are either prime or composite, without analyzing either n-1 or n+1, another important concept must be introduced regarding Squares Sq(n) created by the Construction Rule as shown in Figure 9.

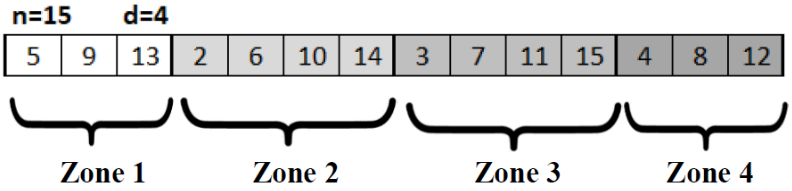

Figure 9 – Zones for row Sq(15,4).

Square Sq(15)

2	3	4	5	6	7	8	9	10	11	12	13	14	15
3	5	7	9	11	13	15	2	4	6	8	10	12	14
4	7	10	13	1	4	7	10	13	1	4	7	10	13
5	9	13	2	6	10	14	3	7	11	15	4	8	12
6	11	1	6	11	1	6	11	1	6	11	1	6	11
7	13	4	10	1	7	13	4	10	1	7	13	4	10
8	15	7	14	6	13	5	12	4	11	3	10	2	9
9	2	10	3	11	4	12	5	13	6	14	7	15	8
10	4	13	7	1	10	4	13	7	1	10	4	13	7
11	6	1	11	6	1	11	6	1	11	6	1	11	6
12	8	4	15	11	7	3	14	10	6	2	13	9	5
13	10	7	4	1	13	10	7	4	1	13	10	7	4
14	12	10	8	6	4	2	15	13	11	9	7	5	3
15	14	13	12	11	10	9	8	7	6	5	4	3	2

Figure 10 – Zones are Symmetric around the main diagonal but neither around the other diagonal nor the X-Y axes.

A *Zone* within a sequence is the subset of increasing values before a modular function resets the value to a lower number, so it stays within the set of modular sequence values. Denote the single row with increment d of a Square Sq(n) as Sq(n,d) to distinguish it from the other rows. In the above example where n=15 and the increment d=4, there are four zones for that row Sq(15, 4).
Recall the visual depiction of a number 15 as a circle with 15 points. When the increment d=4 moves around the circle, the resulting sequence are the numbers displayed in the cells above in Figure 9. Each time the movement passes 1, a new zone is established because the number has been reset using modular arithmetic to a lower value.

Each increment creates its own zones for its row. All of these can be put together, as in Figure 10, with each of the zones highlighted in shades of grey. Notice that the zones are symmetric around the main diagonal going from upper left to lower right. For any Square Sq(n), there are always n-1 zones.

5. TRANSFORMING SQUARES SQ(N) TO SQ(N+1) –THE FORWARD TRANSFORMATION RULE

What happens when 1 is added to n? It is easy to say that everyone knows that adding 1 to 11 gives 12. However, does that really give any information on how the “engine” is working beneath the hood? How do the sequences change from Square Sq(n) to become Square Sq(n+1)? How do the sequences of the Latin Square for Sq(11) become the pattern of 1s in Square Sq(12)? If this can be understood, then theory gets closer to knowing how a prime number transforms into a composite number, and how a composite number transforms into a prime number by adding 1.

5.1. Transforming a Prime Square Sq(n) into a Composite Square Sq(n+1)

Figure 12 illustrates the sequences of Square Sq(11) and those of Square Sq(12), to investigate how the cells for each row of Sq(11) convert to the cells in Sq(12), i.e., transforming the prime n = 11 into the composite number n = 12.

Square Sq(11)

2	3	4	5	6	7	8	9	10	11
3	5	7	9	11	2	4	6	8	10
4	7	10	2	5	8	11	3	6	9
5	9	2	6	10	3	7	11	4	8
6	11	5	10	4	9	3	8	2	7
7	2	8	3	9	4	10	5	11	6
8	4	11	7	3	10	6	2	9	5
9	6	3	11	8	5	2	10	7	4
10	8	6	4	2	11	9	7	5	3
11	10	9	8	7	6	5	4	3	2

Square Sq(12)

2	3	4	5	6	7	8	9	10	11	12
3	5	7	9	11	1	3	5	7	9	11
4	7	10	1	4	7	10	1	4	7	10
5	9	1	5	9	1	5	9	1	5	9
6	11	4	9	2	7	12	5	10	3	8
7	1	7	1	7	1	7	1	7	1	7
8	3	10	5	12	7	2	9	4	11	6
9	5	1	9	5	1	9	5	1	9	5
10	7	4	1	10	7	4	1	10	7	4
11	9	7	5	3	1	11	9	7	5	3
12	11	10	9	8	7	6	5	4	3	2

Figure 12 – How do the sequences for Sq(11) transform into the pattern of 1s in Sq(12)?

Examining the first row of Sq(11), it must be extended one cell to match the number of cells for the sequence of the first row of Sq(12). Remember from Section 3.1, that each of the sequences for Sq(11) had a 1 that was removed in creating the Latin Square for n=11.

Figure 13 shows both Sq(11,1) and Sq(12,1) with their differences noted between the two. Notice that nothing needs to be added or subtracted from Sq(11,1) in Zone 1 to generate the corresponding sequence members of Sq(12,1). For the last cell, subtracting 1 from 1 gives 0, yet the modular value of 0 with respect to the modulus 12 is defined to be 12. This fills out the row for Sq(12,1).

	Zone 1										Zone 2
Sq(11,1)	2	3	4	5	6	7	8	9	10	11	1
	0	0	0	0	0	0	0	0	0	0	1
Sq(12,1)	2	3	4	5	6	7	8	9	10	11	12

Figure 13 – Comparing Sq(11,1) against Sq(12,1).

Compare the next rows between Sq(11,2) and Sq(12,2). The difference in Zone 1 is 0 again. The difference in Zone 2 is 1 and that of Zone 3 is 2.

	Zone 1					Zone 2					Zone 3
Sq(11,2)	3	5	7	9	11	2	4	6	8	10	1
	0	0	0	0	0	1	1	1	1	1	2
Sq(12,2)	3	5	7	9	11	1	3	5	7	9	11

Figure 14 – Comparing Sq(11,2) and Sq(12,2).

In comparing the next rows between Sq(11,3) and Sq(12,3) in Figure 15,again, nothing needs to be subtracted from Zone 1 for Sq(11,3) to generate the corresponding members of Sq(12,3). For Zone 2, 1 needs to be subtracted. For Zone 3, 2 needs to be subtracted. For Zone 4, 3 is subtracted from 1 giving the value of 10 in modulus 12.

	Zone 1			Zone 2				Zone 3			Zone 4
Sq(11,3)	4	7	10	2	5	8	11	3	6	9	1
	0	0	0	1	1	1	1	2	2	2	3
Sq(12,3)	4	7	10	1	4	7	10	1	4	7	10

Figure 15 – Comparing Sq(11,3) and Sq(12,3).

With each row of Sq(11,d), the number of zones increases. Comparing Sq(11,4) and Sq(12,4), nothing needs to be subtracted from the members of Zone 1 to generate the corresponding members of Sq(12,4). From Zone 2, 1 is subtracted. From Zone 3, 2 is subtracted. From Zone 4, 3 is subtracted. Finally, for Zone 5, 4 is subtracted giving 9 in modulus 12. Notice how the 1s in Sq(12,4) appear when they were not apparent in Sq(11,4).

	Zone 1		Zone 2			Zone 3			Zone 4		Zone 5
Sq(11,4)	5	9	2	6	10	3	7	11	4	8	1
	0	0	1	1	1	2	2	2	3	3	4
Sq(12,4)	5	9	1	5	9	1	5	9	1	5	9

Figure 16 – Comparing Sq(11,4) and Sq(12,4).

Checking the next row comparing Sq(11,5) and Sq(12,5) in Figure 16, the same pattern is being followed. Subtracting 1 less than the zone number from the cell members of Sq(11,5) gives the cell members for Sq(12,5).

	Zone 1		Zone 2		Zone 3		Zone 4		Zone 5		Zone 6
Sq(11,5)	6	11	5	10	4	9	3	8	2	7	1
	0	0	1	1	2	2	3	3	4	4	5
Sq(12,5)	6	11	4	9	2	7	12	5	10	3	8

Figure 17 – Comparing Sq(11,5) and Sq(12,5).

Comparing one more set of rows in Figure 18, subtracting 1 less than the zone number generates the cell members for the row of the next n.

	Zone 1	Zone 2		Zone 3		Zone 4		Zone 5		6	7
Sq(11,6)	7	2	8	3	9	4	10	5	11	6	1
	0	1	1	2	2	3	3	4	4	5	6
Sq(12,6)	7	1	7	1	7	1	7	1	7	1	7

Figure 18 – Comparing Sq(11,6) and Sq(12,6).

Again, notice how the 1s on Sq(12,6) appear even though they were not present in Sq(11,6). Continuing in this fashion, the Latin SquareSq(11) for prime n=11 is converted into the patterns of 1s in the composite Square Sq(12) for n=12.

The row for Square Sq(11,11) that would match the row for Sq(12,11), does not actually existin Sq(11), but a row of 1s can be added so that subtracting 1 less than the zone numbers, gives the row of cells for Sq(12,11) as in the following Figure 19.

Zones	1	2	3	4	5	6	7	8	9	10	11
Sq(11,11)	1	1	1	1	1	1	1	1	1	1	1
	0	1	2	3	4	5	6	7	8	9	10
Sq(12,11)	12	11	10	9	8	7	6	5	4	3	2

Figure 19 – Comparing the added row of 1s for Sq(11,11) and the last row of Sq(12,11).

Thus, all the cell members for Square Sq(12) are generated from Square Sq(11) by subtracting 1 less than the zone numbers. If necessary, a cell value is adjusted for the modulus 12.

There is now a firm rule for transforming Square Sq(n) to Square Sq(n+1). This is called the *Forward Transformation Rule*. The example for Sq(11) being converted to Sq(12) is a case where a prime number n is transformed into a composite number having a pattern of 1s.

5.2. Transforming a Composite Square Sq(n) into a Prime Sq(n+1)

The same process can be applied to a composite number to transform the Square Sq(n) into a prime number with a Latin Square Sq(n+1). Consider applying the same rule from the previous section to Square Sq(12) in transforming it into the Latin Square Sq(13) as illustrated in Figure 20. This is where the pattern of 1s within Sq(12) vanish when it is transformed into Sq(13). Zones are still the key for this change. Subtracting 1 less than the zone number from each cell member eliminates the pattern of 1s when this composite is converted into a prime number.

Square Sq(12)

2	3	4	5	6	7	8	9	10	11	12
3	5	7	9	11	1	3	5	7	9	11
4	7	10	1	4	7	10	1	4	7	10
5	9	1	5	9	1	5	9	1	5	9
6	11	4	9	2	7	12	5	10	3	8
7	1	7	1	7	1	7	1	7	1	7
8	3	10	5	12	7	2	9	4	11	6
9	5	1	9	5	1	9	5	1	9	5
10	7	4	1	10	7	4	1	10	7	4
11	9	7	5	3	1	11	9	7	5	3
12	11	10	9	8	7	6	5	4	3	2

Square Sq(13)

2	3	4	5	6	7	8	9	10	11	12	13
3	5	7	9	11	13	2	4	6	8	10	12
4	7	10	13	3	6	9	12	2	5	8	11
5	9	13	4	8	12	3	7	11	2	6	10
6	11	3	8	13	5	10	2	7	12	4	9
7	13	6	12	5	11	4	10	3	9	2	8
8	2	9	3	10	4	11	5	12	6	13	7
9	4	12	7	2	10	5	13	8	3	11	6
10	6	2	11	7	3	12	8	4	13	9	5
11	8	5	2	12	9	6	3	13	10	7	4
12	10	8	6	4	2	13	11	9	7	5	3
13	12	11	10	9	8	7	6	5	4	3	2

Figure 20 – How does the pattern of 1s in Square Sq(12) transform into the Latin Square for Sq(13)?

Square Sq(13) is a Latin Square. Every number from 2 to 13 occurs distinctly on only one row and column. No 1s are present. If a pattern of 1s was there, then that number n would be a composite number.

	Zone 1			Zone 2				Zone 3				Zone 4
Sq(12,3)	4	7	10	1	4	7	10	1	4	7	10	1
	0	0	0	1	1	1	1	2	2	2	2	3
Sq(13,3)	4	7	10	13	3	6	9	12	2	5	8	11

Figure 22 – Comparing Sq(12,4) and Sq(13,4).

Rather than going through every row using the rule from Section 5.1, consider only some rows where 1s are present in Sq(12) to see how they disappear when being transformed by the rule into rows of distinct numbers in Sq(13). The following looks at rows where the increment d=3 and 4 and 6.

	Zone 1		Zone 2			Zone 3			Zone 4			Zone 5
Sq(12,4)	5	9	1	5	9	1	5	9	1	5	9	1
	0	0	1	1	1	2	2	2	3	3	3	4
Sq(13,4)	5	9	13	4	8	12	3	7	11	2	6	10

Figure 22 – Comparing Sq(12,4) and Sq(13,4).

	Zone 1	Zone 2		Zone 3		Zone 4		Zone 5		Zone 6		Zone 7
Sq(12,6)	7	1	7	1	7	1	7	1	7	1	7	1
	0	1	1	2	2	3	3	4	4	5	5	6
Sq(13,6)	7	13	6	12	5	11	4	10	3	9	2	8

Figure 23 – Comparing Sq(12,6) and Sq(13,6).

Notice the repetitive pattern formed by the 1s for Sq(12). The pattern for Sq(12,3) is {1, 4, 7, 10}, for Sq(12,4) is {1, 5, 9}, and for Sq(12,6) is {1, 7}. Each of these repetitive sequences is transformed into unique numbers in Sq(13). Once again, applying the zone rule of subtracting 1 less than the zone number, does give unique values in Sq(13) where no 1s are present, because a different number is being subtracted from each repetitive set, ensuring that no number is repeated. Applying the rule to every row of Sq(12) does transform Sq(13) into a Latin Square, demonstrating that n=13 is a prime number. This does not always occur since there are composite numbers that transform into other composite numbers such as 14, 15, and 16.

In summary, the Construction Rule creates the Square Sq(n) for any n. From that Square Sq(n), the Forward Transformation Rule can generate the Square Sq(n+1), which, when applied again, creates the Square Sq(n+2), and so forth. If the resulting Square Sq(n) is a Latin Square, then n is a prime number. If the Square Sq(n) exhibits a pattern of 1s, then that n is a composite number. All the numbers are interconnected through their Squares and the Forward Transformation Rule.
The Forward Transformation Rule shows how connected one number n is to the following number n+1. It demonstrates how the sequences of a composite number can be shifted into the sequences of a prime number, and how those of a prime number can be shifted into the sequences of a composite number.

Square Sq(12)

2	3	4	5	6	7	8	9	10	11	12
3	5	7	9	11	1	3	5	7	9	11
4	7	10	1	4	7	10	1	4	7	10
5	9	1	5	9	1	5	9	1	5	9
6	11	4	9	2	7	12	5	10	3	8
7	1	7	1	7	1	7	1	7	1	7
8	3	10	5	12	7	2	9	4	11	6
9	5	1	9	5	1	9	5	1	9	5
10	7	4	1	10	7	4	1	10	7	4
11	9	7	5	3	1	11	9	7	5	3
12	11	10	9	8	7	6	5	4	3	2

Square Sq(11)

2	3	4	5	6	7	8	9	10	11
3	5	7	9	11	2	4	6	8	10
4	7	10	2	5	8	11	3	6	9
5	9	2	6	10	3	7	11	4	8
6	11	5	10	4	9	3	8	2	7
7	2	8	3	9	4	10	5	11	6
8	4	11	7	3	10	6	2	9	5
9	6	3	11	8	5	2	10	7	4
10	8	6	4	2	11	9	7	5	3
11	10	9	8	7	6	5	4	3	2

Figure 24 – How does the pattern of 1s for Square Sq(12) transform into a Latin Square Sq(11)?

5.3. Transforming Squares Sq(n) to Sq(n-1) – The Reverse Transformation Rule

Similar to the Forward Transformation Rule that creates a Square Sq(n+1) from Sq(n), the process can go in reverse, creating a Square Sq(n-1) from Sq(n). This is known as the *Reverse Transformation Rule*. Consider how this works.

5.4. Transforming a Composite Square Sq(n) into a Prime Square Sq(n-1)

The first step is to drop or ignore the last column and last row so that the dimensions match between the two Squares. Then examine a particular row for the same increment d=3.

	Zone 1			Zone 2				Zone 3			
Sq(12,3)	4	7	10	1	4	7	10	1	4	7	10
	0	0	0	1	1	1	1	2	2	2	
Sq(11,3)	4	7	10	2	5	8	11	3	6	9	

Figure 25 – Comparing Sq(12,3) to Sq(11,3).

Instead of subtracting 1 less than the zone number, in the Reverse Transformation Rule, add 1 less than the zone number. For Zone 1, this is adding 0, which does not change any of the numbers. For Zone 2, 1 is added to obtain the value of the corresponding cell. For Zone 3, 2 is added. If we did add 2 to the last cell of Sq(12,3),which has a value of 10, the result would be 12; yet, because it is going into a Square with modulus 11, it converts through modular arithmetic to a 1. That 1 is dropped since it is at the end of the sequence.

Notice in the previousFigure 25how the two 1s on row Sq(12,3) convert to 2 and 3 respectively. No other 1s appear except for the last one that is being dropped because it is out of the bounds of the Square Sq(11).

Consider a row that has no 1s, Sq(12,5), as in Figure 26.

	Zone 1		Zone 2		Zone 3			Zone 4		Zone 5	
Sq(12,5)	6	11	4	9	2	7	12	5	10	3	8
	0	0	1	1	2	2	2	3	3	4	
Sq(11,5)	6	11	5	10	4	9	3	8	2	7	

Figure 26 – Comparing Sq(12,5) to Sq(11,5).

	Zone 1	Zone 2		Zone 3		Zone 4		Zone 5		Zone 6	
Sq(12,6)	7	1	7	1	7	1	7	1	7	1	7
	0	1	1	2	2	3	3	4	4	5	
Sq(11,6)	7	2	8	3	9	4	10	5	11	6	

Figure 27 – Comparing Sq(12,6) to Sq(11,6).

When adding 1 less than the zone number to each cell, the results have no 1s. Notice the modular addition that adds 2 to 12 giving 3 in Zone 3. This is obtained by 2 + 12 = 14, then subtracting the modulus 11 of the target row, which gives 3. This occurs also in Zone 4 where 10 + 3 = 13, and subtracting the modulus 11, gives 2.

For one last example, compare Sq(12,6) with Sq(11,6) as in Figure 27.

This is a case where the repetitive sequence {1,7} shows up. When 1 less than the zone number is added to each cell, the values end up being different since the zone numbers are different. Again, this produces a row in Sq(11) that has all distinct values.

Continuing in this manner for each of the rows results in a Square Sq(11) that has all distinct values, where a member occurs only once in a column and row. This meets the definition of a Latin Square that makes n=11 a prime number. The Reverse Transformation Rule has changed a composite number into a preceding prime number.

5.5. Transforming a Prime Square Sq(n) into a Composite Square Sq(n-1)

Consider the transformation of Sq(13) into Sq(12) illustrated in Figure 28. This uses the Reverse Transformation Rule to convert a prime number n into a composite numbern-1 with a distinctive pattern of 1s.

Square Sq(13)

2	3	4	5	6	7	8	9	10	11	12	13
3	5	7	9	11	13	2	4	6	8	10	12
4	7	10	13	3	6	9	12	2	5	8	11
5	9	13	4	8	12	3	7	11	2	6	10
6	11	3	8	13	5	10	2	7	12	4	9
7	13	6	12	5	11	4	10	3	9	2	8
8	2	9	3	10	4	11	5	12	6	13	7
9	4	12	7	2	10	5	13	8	3	11	6
10	6	2	11	7	3	12	8	4	13	9	5
11	8	5	2	12	9	6	3	13	10	7	4
12	10	8	6	4	2	13	11	9	7	5	3
13	12	11	10	9	8	7	6	5	4	3	2

Square Sq(12)

2	3	4	5	6	7	8	9	10	11	12
3	5	7	9	11	1	3	5	7	9	11
4	7	10	1	4	7	10	1	4	7	10
5	9	1	5	9	1	5	9	1	5	9
6	11	4	9	2	7	12	5	10	3	8
7	1	7	1	7	1	7	1	7	1	7
8	3	10	5	12	7	2	9	4	11	6
9	5	1	9	5	1	9	5	1	9	5
10	7	4	1	10	7	4	1	10	7	4
11	9	7	5	3	1	11	9	7	5	3
12	11	10	9	8	7	6	5	4	3	2

Figure 28 – How does the Latin Square Sq(13) transform into the Composite Square Sq(12) with its pattern of 1s?

Consider the Reverse Transformation Rule being applied to Sq(13,4) to create Sq(12,4).

	Zone 1			Zone 2			Zone 3			Zone 4		
Sq(13,4)	5	9	13	4	8	12	3	7	11	2	6	10
	0	0	0	1	1	1	2	2	2	3	3	
Sq(12,4)	5	9	1	5	9	1	5	9	1	5	9	

Figure 29 – Comparing Sq(13,4) to Sq(12,4).

When applied, the rule adds 1 less than the zone number to Sq(13,4) in deriving the cell value for the corresponding member in Sq(12,4). Notice in Zone 2, the modular addition of 1 + 12 gives 1 in modulo 12. The same occurs in Zone 3 where 11 + 2 gives 1 in modulo 12.
Notice that the pattern of three 1s appear in Sq(12,4) in the previous Figure 29after the transformation.

Consider one more example that compares Sq(13,6) and Sq(12,6) and produces the repeating pattern {1,7} using the Reverse Transformation Rule, as in Figure 30.

	Zone 1		Zone 2		Zone 3		Zone 4		Zone 5		Zone 6	
Sq(13,6)	7	13	6	12	5	11	4	10	3	9	2	8
	0	0	1	1	2	2	3	3	4	4	5	
Sq(12,6)	7	1	7	1	7	1	7	1	7	1	7	

Figure 30 – Comparing Sq(13,6) to Sq(12,6).

Notice the modular addition in Zone 1 that adds 0 to 13 but converts it to a 1 with the modulus 12. The same occurs in Zone 2 where 1 is added to 12 that converts it to a 1 in modulo 12. Again, in Zone 3, 2 + 11 converts to a 1 in modulo 12. In Zone 4, 3 + 10 gives 1 in modulo 12. Finally, 4 + 9 gives 1 in modulo 12, accounting for the pattern of 1s on row 6 where the increment d=6 in Sq(12,6).

Using the Reverse Transformation on all rows of Prime Square Sq(13) generates the Composite Square Sq(12) with its distinctive pattern of 1s.

Using the Construction Rule on any Natural Number n, the Reverse Transformation can generate all the Squares Sq(n-1) prior to Square Sq(n).

6. WHEN IS N+1 A PRIME?

The earlier Section 3.1illustrates how a prime number forms a Latin Square Sq(n). Section 3.3 shows how a composite number forms a distinctive pattern of 1s in the Square Sq(n). Sections 3.4and 3.5 demonstrate how the symmetric properties of these Squares Sq(n) can be used to obtain an optimal search looking for 1s in order to determine if n is prime or composite. Section 5 put forward the intimate connection between Squares of adjacent numbers, and how the Squares Sq(n-1) and Sq(n+1) can be derived from Sq(n). These next sectionsreveal a mystery about Natural Numbers where n "knows" that the number that follows it is either a prime or composite, and the number that precedes it is either a prime or composite, without examining either n-1 or n+1.

The connection between n and n+1 goes much deeper than simply the Forward Transformation of Square Sq(n) into Sq(n+1). The Natural Number n "knows" what is the pattern of 1s for n+1, or the lack of any pattern for n+1 that makes it a prime, even before the transformation takes place. Simply put, the "shadow" of any pattern for Square Sq(n+1) can be found in Square Sq(n).If this shadow does not exist, then n+1 is a prime number. The number n+1 does not need to be examined to determine if it is prime or composite.

Surely, the terms "knows" and "shadow" are not mathematical terms; however, through visual illustrations, their meanings become apparent, and rules for testing primality of n+1 by only examining n can be defined.

The Forward Transformation Rule states that subtracting 1 less than the zone number gives the value of the corresponding cell in Sq(n+1). In order to derive a cell value with a 1 in it, it requires that a cell in Sq(n) has a value that, when the subtraction takes place, generates the 1 with respect to the modulus n+1. There is a mapping, then, from cells in Square Sq(n) to the pattern of 1s in Square Sq(n+1), and a mapping back again.

Square Sq(11)

2	3	4	5	6	7	8	9	10	11
3	5	7	9	11	**2**	4	6	8	10
4	7	10	**2**	5	8	11	**3**	6	9
5	9	**2**	6	10	**3**	7	11	**4**	8
6	11	5	10	4	9	3	8	2	7
7	**2**	8	**3**	9	**4**	10	**5**	11	**6**
8	4	11	7	3	10	6	2	9	5
9	6	**3**	11	8	**5**	2	10	**7**	4
10	8	6	**4**	2	11	9	**7**	5	3
11	10	9	8	7	**6**	5	4	3	2

Square Sq(12)

2	3	4	5	6	7	8	9	10	11	12
3	5	7	9	11	**1**	3	5	7	9	11
4	7	10	**1**	4	7	10	**1**	4	7	10
5	9	**1**	5	9	**1**	5	9	**1**	5	9
6	11	4	9	2	7	12	5	10	3	8
7	**1**	7	**1**	7	**1**	7	**1**	7	**1**	7
8	3	10	5	12	7	2	9	4	11	6
9	5	**1**	9	5	**1**	9	5	**1**	9	5
10	7	4	**1**	10	7	4	**1**	10	7	4
11	9	7	5	3	**1**	11	9	7	5	3
12	11	10	9	8	7	6	5	4	3	2

Figure 32 – Mapping of the pattern of 1s in Square Sq(12) back to Sq(11).

Figure 32above shows the "shadow" pattern of the 1s of Sq(12) that can be found to the left within Square Sq(11). It is these cells that, when the Forward Transformation Rule is applied, result in the pattern of 1s in Square Sq(12) on the right. Unlike the pattern in Sq(12) that has all 1s, the values of the shadow pattern have various values. Since the number being subtracted increases as the zone number increases, the highlighted cells must have different values depending upon in which zone they are found.

Square Sq(11)

2	3	4	5	6	7	8	9	10	11	**Zone 1**
3	5	7	9	11	**2**	4	6	8	10	**Zone 2**
4	7	10	**2**	5	8	11	**3**	6	9	**Zone 3**
5	9	**2**	6	10	**3**	7	11	**4**	8	**Zone 4**
6	11	5	10	4	9	3	8	2	7	**Zone 5**
7	**2**	8	**3**	9	**4**	10	**5**	11	**6**	**Zone 6**
8	4	11	7	3	10	6	2	9	5	**Zone 7**
9	6	**3**	11	8	**5**	2	10	**7**	4	**Zone 8**
10	8	6	**4**	2	11	9	**7**	5	3	**Zone 9**
11	10	9	8	7	**6**	5	4	3	2	**Zone 10**

Figure 33 – Square Sq(11) with Zones and the "Shadow" values that produce the pattern of 1s in Sq(12).

In Figure 33 above, with the zones highlighted and the shadow pattern of boldfaced values that convert to 1s by the Forward Transformation Rule, notice an interesting property of the shadow numbers.

If there is a 2 in any cell in Zone 2, that value transforms into a 1 in Sq(12). If there is a 3 in any cell in Zone 3, that value transforms into a 1. If there is a 4 in any cell in Zone 4, that value transforms into a 1. If there is a 5 in Zone 5, that value transforms into a 1. If there is a 6 in Zone 6, that value transforms into a 1. If there is a 7 in the cells of Zone 7, that value transforms into a 1 in the Square Sq(12).

This produces a very easy rule for determining if n+1 is a composite number: If any zone has a cell with the value of the zone number, then n+1 is composite. This is what it means that the number n "knows" if n+1 is composite. Since the pattern of 1s within Sq(n+1) shows up as a shadow pattern in Sq(n), this new rule can be checked to see if n+1 is compositenumber without examining either n+1 or Sq(n+1).

Notice also that the numbers in bold in Figure 33 always occur on the left boundary of their zones. This is because these numbers are the lowest values in the sequences on each row for that zone.

Does the reverse of this rule hold? If all zones do not have any cells with the value of their zone numbers, then n+1 is a prime number.
Consider theSquare Sq(12) as in Figure 34. There should not be any shadow pattern since Sq(13) has no pattern of 1s.

Indeed, as seen in Figure 34, Zone 1 has no 1s; Zone 2 has no 2s; Zone 3 has no 3s; and, so forth with all the zones. This implies that Square Sq(13) has no pattern of 1s, and thus, Sq(13) is a Prime Latin Square.

Square Sq(12)

2	3	4	5	6	7	8	9	10	11	12	**Zone 1**
3	5	7	9	11	1	3	5	7	9	11	**Zone 2**
4	7	10	1	4	7	10	1	4	7	10	**Zone 3**
5	9	1	5	9	1	5	9	1	5	9	**Zone 4**
6	11	4	9	2	7	12	5	10	3	8	**Zone 5**
7	1	7	1	7	1	7	1	7	1	7	**Zone 6**
8	3	10	5	12	7	2	9	4	11	6	**Zone 7**
9	5	1	9	5	1	9	5	1	9	5	**Zone 8**
10	7	4	1	10	7	4	1	10	7	4	**Zone 9**
11	9	7	5	3	1	11	9	7	5	3	**Zone 10**
12	11	10	9	8	7	6	5	4	3	2	**Zone 11**

Figure 34 – Square Sq(12) with Zones. No zone has its own zone number in any cell. Thus, Sq(13) has no pattern of 1s, meaning 13 is a prime number.

Therefore, the number 12 with its Square Sq(12) knows that 13 is a prime number without needing to factor 13 or examine Sq(13).
With both of these rules, the number n knows that n+1 is either prime or composite without examining 13 itself.

6.1. Testing Primality of n+1 by Examining n

Section 6 demonstrates how the Forward Transformation Rule can be used to determine if n+1 is prime or composite even before applying the rule. If any zone has a number in its cells that is the same as the zone number, then n+1 is composite. Thus, all that needs to be done to determine if n+1 is prime or composite is to examine the first cells for each zone on each rowof Square Sq(n) to see if their values are equivalent to their zone numbers.

There is an even more powerful observation that could eliminate the need to examine all the zones.

> **The Zone Conjecture for Composite Numbers n+1.** All composite numbers n+1 have at least one 2 in Zone 2 of Square Sq(n).

> **The Zone Conjecture for Prime Numbers n+1**. All prime numbers n+1 do not have a 2 in Zone 2 of Square Sq(n).

The C++ code that implements these conjectures, checking for 2s in Zone 2 of Square Sq(n) to determine if n+1 is prime or composite, can be found in the github repository cited below. The procedure main() invokes a function called “check_4_2_in_zone_2(n)” that passes n and returns a Boolean variable called found_2. If found_2 is true, then n+1 is composite. If found_2 is false, then n+1 is prime. The code in main() can be easily changed to scan through a range of numbers reporting on the primality of each number. This function has been used to accurately verify all primes and composites using this method out to 10,000 and the count of primes in ranges out to 4,000,000.

https://github.com/alanverdegraal/OrbitTheory/tree/C++Programs/Zone_check_of_n_to_determine_primality_of_n+1.cpp

This demonstrates that factoring and sieving are not the only ways to determine if a number n+1 is prime or composite. The number n can be examined directly to determine the primality of n+1 without conducting a primality check of n+1.

7. When is n-1 Prime?

Sections 6 and 6.1 demonstrate how the Natural Number n “knows” that n+1 is either prime or composite without examining n+1. What about n-1 that precedes n? Does the number n know whether n-1 is prime or composite? Because of the connectivity between one Square Sq(n) and the next, the “shadow” of n-1 is carried over to its subsequent neighbor n.

Consider the Squares Sq(15) and Sq(14) illustrated in Figure 35. Sq(14) has a very distinctive cross pattern of 1s within its array. Can the shadow of this cross pattern be found within Square Sq(15)?

Square Sq(15)

2	3	4	5	6	7	8	9	10	11	12	13	14	15
3	5	7	9	11	13	15	2	4	6	8	10	12	14
4	7	10	13	1	4	7	10	13	1	4	7	10	13
5	9	13	2	6	10	14	3	7	11	15	4	8	12
6	11	1	6	11	1	6	11	1	6	11	1	6	11
7	13	4	10	1	7	13	4	10	1	7	13	4	10
8	15	7	14	6	13	5	12	4	11	3	10	2	9
9	2	10	3	11	4	12	5	13	6	14	7	15	8
10	4	13	7	1	10	4	13	7	1	10	4	13	7
11	6	1	11	6	1	11	6	1	11	6	1	11	6
12	8	4	15	11	7	3	14	10	6	2	13	9	5
13	10	7	4	1	13	10	7	4	1	13	10	7	4
14	12	10	8	6	4	2	15	13	11	9	7	5	3
15	14	13	12	11	10	9	8	7	6	5	4	3	2

Square Sq(14)

2	3	4	5	6	7	8	9	10	11	12	13	14
3	5	7	9	11	13	**1**	3	5	7	9	11	13
4	7	10	13	2	5	8	11	14	3	6	9	12
5	9	13	3	7	11	**1**	5	9	13	3	7	11
6	11	2	7	12	3	8	13	4	9	14	5	10
7	13	5	11	3	9	**1**	7	13	5	11	3	9
8	**1**	8	**1**	8	**1**	8	**1**	8	**1**	8	**1**	8
9	3	11	5	13	7	**1**	9	3	11	5	13	7
10	5	14	9	4	13	8	3	12	7	2	11	6
11	7	3	13	9	5	**1**	11	7	3	13	9	5
12	9	6	3	14	11	8	5	2	13	10	7	4
13	11	9	7	5	3	**1**	13	11	9	7	5	3
14	13	12	11	10	9	8	7	6	5	4	3	2

Figure 35 – Squares Sq(15) and Sq(14) with its distinctive cross pattern of 1s.

To help see how the Reverse Transformation Rule works to show the shadow of the pattern of 1s, examine a single row to see how the value of one cell in Sq(15,2) converts to Sq(14,2).

	Zone 1							Zone 2						
Sq(15,2)	3	5	7	9	11	13	15	2	4	6	8	10	12	14
	0	0	0	0	0	0	0	1	1	1	1	1	1	
Sq(14,2)	3	5	7	9	11	13	1	3	5	7	9	11	13	

Figure 36 – Comparing Sq(15,2) to Sq(14,2).

The Reverse Transformation Rule adds 1 less than the zone number to Sq(15,2) to derive the values of the cells for Sq(14,2). This shows that the 1 in Sq(14,2) maps to 15 in Sq(15,2). Using this cell as the top part of the cross pattern, the shadow shows up in Sq(15) as in Figure 37.

Square Sq(15)

2	3	4	5	6	7	8	9	10	11	12	13	14	15
3	5	7	9	11	13	15	2	4	6	8	10	12	14
4	7	10	13	1	4	7	10	13	1	4	7	10	13
5	9	13	2	6	10	14	3	7	11	15	4	8	12
6	11	1	6	11	1	6	11	1	6	11	1	6	11
7	13	4	10	1	7	13	4	10	1	7	13	4	10
8	15	7	14	6	13	5	12	4	11	3	10	2	9
9	2	10	3	11	4	12	5	13	6	14	7	15	8
10	4	13	7	1	10	4	13	7	1	10	4	13	7
11	6	1	11	6	1	11	6	1	11	6	1	11	6
12	8	4	15	11	7	3	14	10	6	2	13	9	5
13	10	7	4	1	13	10	7	4	1	13	10	7	4
14	12	10	8	6	4	2	15	13	11	9	7	5	3
15	14	13	12	11	10	9	8	7	6	5	4	3	2

Figure 37 – Square Sq(15) showing the shadow of the cross pattern of 1s found in Sq(14).

Looking at the zones for Sq(15), it is seen that the shadow of Sq(14) shows the boldface cells at the end of the sequences for the zones as in Figure 38.

Square Sq(15)

2	3	4	5	6	7	8	9	10	11	12	13	14	15	**Zone 1**
3	5	7	9	11	13	**15**	2	4	6	8	10	12	14	**Zone 2**
4	7	10	13	1	4	7	10	13	1	4	7	10	13	**Zone 3**
5	9	13	2	6	10	**14**	3	7	11	15	4	8	12	**Zone 4**
6	11	1	6	11	1	6	11	1	6	11	1	6	11	**Zone 5**
7	13	4	10	1	7	**13**	4	10	1	7	13	4	10	**Zone 6**
8	**15**	7	**14**	6	**13**	5	**12**	4	**11**	3	**10**	2	9	**Zone 7**
9	2	10	3	11	4	**12**	5	13	6	14	7	15	8	**Zone 8**
10	4	13	7	1	10	4	13	7	1	10	4	13	7	**Zone 9**
11	6	1	11	6	1	**11**	6	1	11	6	1	11	6	**Zone 10**
12	8	4	15	11	7	3	14	10	6	2	13	9	5	**Zone 11**
13	10	7	4	1	13	**10**	7	4	1	13	10	7	4	**Zone 12**
14	12	10	8	6	4	2	15	13	11	9	7	5	3	**Zone 13**
15	14	13	12	11	10	9	8	7	6	5	4	3	2	**Zone 14**

Figure 38 – Square Sq(15) with zones and the shadow pattern for Sq(14) in bold.

The shadow of Square Sq(14) appears in Sq(15) with the following formula: n – (zone-number – 1). The shadow cells are those with the following values.

Zone 1 = 15 – (1 – 1) = 15
Zone 2 = 15 – (2 – 1) = 14
Zone 3 = 15 – (3 – 1) = 13
Zone 4 = 15 – (4 – 1) = 12
Zone 5 = 15 – (5 – 1) = 11
Zone 6 = 15 – (6 – 1) = 10
Zone 7 = 15 – (7 – 1) = 9
Zone 8 = 15 – (8 – 1) = 8
Zone 9 = 15 – (9 – 1) = 7
Zone 10 = 15 – (10 – 1) = 6
Zone 11 = 15 – (11 – 1) = 5
Zone 12 = 15 – (12 – 1) = 4
Zone 13 = 15 – (13 – 1) = 3
Zone 14 = 15 – (14 – 1) = 2

If any of these numbers appear in the pertinent zones, then that number maps to a cell value of 1 in Sq(14). Remember, however, that the last column and row of Sq(15) are dropped and not included in any pattern for Sq(14). Thus, the shadow pattern representing 1s in Sq(14) can be found in Sq(15), indicating that Sq(14) is a composite number, without examining Square Sq(14).

If no zone in Square Sq(n) has cells with the values from the above formula, then n-1 is a prime number.

7.1. Testing Primality of n-1 by Examining n

The last section illustrates how the Reverse Transformation Rule can be used to determine if n-1 is prime or composite simply from examining Square Sq(n) to see if the pattern of 1s for n-1 is present. If any zone of Sq(n) has a value equivalent to n – (zone-number – 1), then n-1 is composite. If this is not the case, then n-1 is a prime number. This identifies the primality of n-1 without the use of sieving and factorization of n or n-1.

The Zone Conjecture for Composite Numbers n-1. If Zone 1 of Square Sq(n) has a cell with the value of n, then n-1 is composite.

The Zone Conjecture for Prime Numbers n-1. If Zone 1 of Square Sq(n) does not have a cell with the value of n, then n-1 is prime.

Similar to the conjectures in Section 6.1, only Zone 1 needs to be examined to see if the value n – (zone-number – 1) = n.

These conjectures have been demonstrated to be true for n > 3 through n = 1,000,000 finding all primes and composites.

Similar to the C++ code of the previous program, one can construct a C++ code that implements the primality test of n-1 by examining n. Create a procedure main() that invokes a function check_4_n_in_zone_1(n), which passes n and checks Square Sq(n) for the shadow pattern of 1s by identifying if there are any cells with the value of n in Zone 1. If n is found in Zone 1, the n-1 is a composite. If no n is found, then n-1 is prime. The C++ code for this can be found in the following github repository location.

https://github.com/alanverdegraal/OrbitTheory/tree/C++Programs/Check_Zone_1_for_n.cpp

8. THE PAST, PRESENT, AND FUTURE ARE FOUND IN N

The Square Sq(n) has the “present state” for n in it, whether n is prime or composite depending if it forms a Latin Square. If there is a pattern of 1s in Sq(n), then n is composite. If no pattern of 1s is found, indeed, if no 1 is found in any cell in Sq(n), then n is a prime number forming a Prime Latin Square Sq(n). The Staircase Search for a 1 in Square Sq(n) from Section 4.2 can verify that n is either prime or composite. Figures 39 – 41show how the past, present,and future states are found in Square Sq(17).

Square Sq(17) **The Past**

2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
3	5	7	9	11	13	15	**17**	2	4	6	8	10	12	14	16
4	7	10	13	16	2	5	8	11	14	17	3	6	9	12	15
5	9	13	**17**	4	8	12	**16**	3	7	11	**15**	2	6	10	14
6	11	16	4	9	14	2	7	12	17	5	10	15	3	8	13
7	13	2	8	14	3	9	**15**	4	10	16	5	11	17	6	12
8	15	5	12	2	9	16	6	13	3	10	17	7	14	4	11
9	**17**	8	**16**	7	**15**	6	**14**	5	**13**	4	**12**	3	**11**	2	10
10	2	11	3	12	4	13	5	14	6	15	7	16	8	17	9
11	4	14	7	17	10	3	**13**	6	16	9	2	12	5	15	8
12	6	17	11	5	16	10	4	15	9	3	14	8	2	13	7
13	8	3	**15**	10	5	17	**12**	7	2	14	**9**	4	16	11	6
14	10	6	2	15	11	7	3	16	12	8	4	17	13	9	5
15	12	9	6	3	17	14	**11**	8	5	2	16	13	10	7	4
16	14	12	10	8	6	4	2	17	15	13	11	9	7	5	3
17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2

Figure 39 – The Past Pattern from Square Sq(16) appears in Sq(17) after converting 1s in Sq(16). These are identified based upon any zone having numbers that satisfy the formula n – (zone-number – 1).

The previous Sections 6.1 and 7.1 have shown how the shadow of both n-1 and n+1 appear in the Square Sq(n). The shadow for n-1 found in Sq(n) shows from where n came, i.e., it illustrates the "past state" that immediately preceded Sq(n). Likewise, the shadow for n+1 found in Sq(n) shows where n is going, i.e., it demonstrates the "future state" that immediately follows Sq(n). These "states" within Sq(n) show whether n-1 and n+1 are prime or composite numbers.

Square Sq(17) **The Present**

2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
3	5	7	9	11	13	15	17	2	4	6	8	10	12	14	16
4	7	10	13	16	2	5	8	11	14	17	3	6	9	12	15
5	9	13	17	4	8	12	16	3	7	11	15	2	6	10	14
6	11	16	4	9	14	2	7	12	17	5	10	15	3	8	13
7	13	2	8	14	3	9	15	4	10	16	5	11	17	6	12
8	15	5	12	2	9	16	6	13	3	10	17	7	14	4	11
9	17	8	16	7	15	6	14	5	13	4	12	3	11	2	10
10	2	11	3	12	4	13	5	14	6	15	7	16	8	17	9
11	4	14	7	17	10	3	13	6	16	9	2	12	5	15	8
12	6	17	11	5	16	10	4	15	9	3	14	8	2	13	7
13	8	3	15	10	5	17	12	7	2	14	9	4	16	11	6
14	10	6	2	15	11	7	3	16	12	8	4	17	13	9	5
15	12	9	6	3	17	14	11	8	5	2	16	13	10	7	4
16	14	12	10	8	6	4	2	17	15	13	11	9	7	5	3
17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2

Figure 40 – The Present State for Square Sq(17). No 1s appear, so Sq(17) is a Latin Square indicating that n=17 is a prime number.

Thus, the Natural Number n has within its own structure of Square Sq(n), information about the past, present, and future primality states of itself and its neighbors within the Natural Number line.

These relationships, which intimately link n-1 and n+1 ton, establish the Natural Number line. Choosing any number n, no matter how large or small, the number line can be constructed at this sequence level in either direction, using the Forward Transformation Rule or the Reverse Transformation Rule.

Square Sq(17) **The Future**

2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
3	5	7	9	11	13	15	17	**2**	4	6	8	10	12	14	16
4	7	10	13	16	**2**	5	8	11	14	17	**3**	6	9	12	15
5	9	13	17	4	8	12	16	**3**	7	11	15	2	6	10	14
6	11	16	4	9	14	2	7	12	17	5	10	15	3	8	13
7	13	**2**	8	14	**3**	9	15	**4**	10	16	**5**	11	17	**6**	12
8	15	5	12	2	9	16	6	13	3	10	17	7	14	4	11
9	17	8	16	7	15	6	14	**5**	13	4	12	3	11	2	10
10	**2**	11	**3**	12	**4**	13	**5**	14	**6**	15	**7**	16	**8**	17	**9**
11	4	14	7	17	10	3	13	**6**	16	9	2	12	5	15	8
12	6	17	11	5	16	10	4	15	9	3	14	8	2	13	7
13	8	**3**	15	10	**5**	17	12	**7**	2	14	**9**	4	16	**11**	6
14	10	6	2	15	11	7	3	16	12	8	4	17	13	9	5
15	12	9	6	3	17	14	11	**8**	5	2	16	13	10	7	4
16	14	12	10	8	**6**	4	2	17	15	13	**11**	9	7	5	3
17	16	15	14	13	12	11	10	**9**	8	7	6	5	4	3	2

Figure 41 – The Future Pattern that will appear in Square Sq(18) based upon the Forward Transformation Rule where these numbers convert to 1s.

9. CONCLUSION

This paper presents a method for distinctly representing a Natural Number n as a set of sequences unique to n that can be placed into an array. If this array exhibits the properties of a Latin Square, where each member shows up only once in each row and column, then n is a prime number. If a pattern of 1s is found, then n is a composite number. The symmetry properties of this array enable the examination of a small subsection of the array cells to determine primality by searching for 1s. Algorithms can be formulated to search for 1s demonstrating whether n is prime or composite.

When adding 1 to n, the sequences for n must change into the sequences representing n+1. This paper shows the mechanism by which this occurs, demonstrating that there is a mapping between n and its neighbors of these patterns of 1s for a composite number, or lack thereof if it is prime. Thus, the primality of n-1 and n+1 can be determined simply from examining the “shadow” patterns of these mappings within the array of sequences for n.

With these insights, several links to C++ programs are provided, which determine if n is prime or composite, and if n-1 or n+1 are prime or composite simply from examining n, without using factorization or sieving.

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