Entanglement as a More Fundamental Notion of Information than Shannon and von Neumann Entropy

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Abstract

Classical information theory, built on Shannon entropy, and quantum information theory, built on von Neumann entropy, quantify uncertainty and compression properties of probability distributions and density operators. Both are powerful, but both treat information as a property of *states* of individual systems or ensembles. In this manuscript we argue that *entanglement*—the pattern of non-classical correlations between systems—is more fundamental than either Shannon or von Neumann information. We motivate this claim with a condensate thought experiment in which many bosons occupy the same single-particle state: Shannon and von Neumann entropies assign them identical information content, yet they can be distinguished by their distinct entanglement histories with external systems. Formally, states with identical marginal density operators but different purifications represent different global information, accessible only through their entanglement structure. Entropic quantities are then seen as coarse-grained summaries of this underlying relational information. We connect this perspective to an entanglement-geometric view of physics, including Entanglement Manifold Cosmology (EMC), in which spacetime and physical "truth" emerge from the geometry of correlations. The goal is an information-theoretic narrative that is rigorous enough for physics and information theory while readable to a non-specialist who is comfortable with basic linear algebra and quantum mechanics.

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1 Introduction

Information theory has given physics a precise language for talking about uncertainty, disorder, and the flow of knowledge. Shannon's classical entropy quantifies the average information gained when one learns the outcome of a random variable [1]. In quantum mechanics, von Neumann entropy does the analogous job for density operators [2, 3].

Both of these notions, however, are defined on *states*: a probability distribution over classical outcomes, or a density operator on a Hilbert space. Correlations between subsystems enter indirectly—for example through mutual information—but the basic building block remains "the state of system A." In modern quantum theory and quantum gravity, this perspective looks more and more provisional. Entanglement, not local state, appears to be the true carrier of structure. In quantum many-body physics and holography, patterns of entanglement are argued to build spacetime geometry itself [5]. In our own previous work on Entanglement Manifold Cosmology (EMC), spacetime and matter fields emerge from a hidden entanglement manifold whose correlation structure is more primitive than any field configuration in $3 + 1$ dimensions [6, 7].

In this paper we push that intuition into a clean information-theoretic statement: *entanglement relations are more fundamental than Shannon or von Neumann information.* Operationally, there exist scenarios where two systems share the same Shannon information and the same von Neumann entropy (and even the same reduced density operator), yet are distinguishable solely by virtue of the different entanglement histories they carry with other systems. If the only thing that can separate otherwise identical systems is entanglement, then entanglement must be tracking a deeper level of information than state-based entropies.

We begin with a short review of classical and quantum information measures (Section 2). We then build a concrete condensate thought experiment (Section 4) and formalize the distinction between local state information and relational (entanglement) information (Section 3). Finally, we discuss implications for physics and for an entanglement-based picture of reality, including cosmological applications (Section 6) and a brief conclusion (Section 7).

Throughout we keep the math honest but the language as plain as possible. A reader who is happy with kets, density matrices, and partial traces should be able to follow the argument.

2 Classical and Quantum Notions of Information

2.1 Shannon information

Let X be a discrete classical random variable taking values x_i with probabilities $p_i = \Pr(X = x_i)$. Shannon's entropy is

$$H(X) = -\sum_i p_i \log p_i, \tag{1}$$

where the logarithm base fixes the units (bits for base 2, nats for base e). $H(X)$ quantifies the expected "surprise" when learning the outcome of X. A distribution concentrated on a single outcome has $H(X) = 0$; a uniform distribution over N outcomes has $H(X) = \log N$.

Other classical quantities are derived from H, such as the mutual information between two random variables,

$$I(X:Y) = H(X) + H(Y) - H(X,Y), \tag{2}$$

which measures how much knowing X reduces uncertainty about Y and vice versa.

Shannon information is extremely successful in communication theory, data compression, and thermodynamics. But it is fundamentally a story about *statistics of outcomes*, not about the deeper structure of correlations in quantum states.

2.2 Von Neumann entropy

In quantum mechanics a system is described not by a probability distribution, but by a density operator ρ on a Hilbert space $\mathcal{H}$. The von Neumann entropy is

$$S(\rho) = -\mathrm{Tr}\,(\rho \log \rho), \tag{3}$$

which reduces to Shannon entropy when ρ is diagonal in a classical basis. Pure states (projectors $\rho = \psi\psi$) have $S(\rho) = 0$; maximally mixed states have maximal entropy.

If a composite system AB is in a pure state ρ_{AB}, one can form reduced density operators by partial tracing,

$$\rho_A = \mathrm{Tr}_B(\rho_{AB}), \qquad \rho_B = \mathrm{Tr}_A(\rho_{AB}). \tag{4}$$

The entanglement entropy between A and B is $S(\rho_A) = S(\rho_B)$ in this pure-state case. This is where von Neumann entropy first explicitly touches entanglement.

Quantum mutual information is defined as

$$I(A:B) = S(\rho_A) + S(\rho_B) - S(\rho_{AB}), \tag{5}$$

which measures the total correlations (classical and quantum) between A and B [3].

2.3 Other information measures

Beyond Shannon and von Neumann, the literature offers a whole zoo of information measures: relative entropy (Kullback–Leibler divergence), Rényi entropies, Fisher information, algorithmic (Kolmogorov) complexity, and more. Each captures something useful: distinguishability of distributions, sensitivity of a likelihood function to parameters, or the length of the shortest program that describes a string.

In all of these, though, the basic unit is still “a state” or “a message”. Correlations are layered on top as pairwise or multi-party quantities derived from those states. If reality is fundamentally relational, this may be the wrong direction of explanation.

3 Entanglement as Relational Information

3.1 Definition and key properties

For a bipartite quantum system AB with Hilbert space $\mathcal{H}_A \otimes \mathcal{H}_B$, a pure state Ψ_{AB} is entangled if it cannot be written as a product $\psi_A \otimes \phi_B$. Equivalently, the reduced density operator $\rho_A = \mathrm{Tr}_B \Psi\Psi$ is mixed.

Entanglement is a correlation structure that cannot be simulated by shared classical randomness and local operations. It fuels quantum teleportation, superdense coding, and violations of Bell inequalities [3].

A crucial structural fact is that a single reduced density matrix ρ_A does *not* uniquely determine the global state. There are infinitely many purifications of ρ_A; that is, distinct pure states Ψ_{AB} on a larger space such that $\mathrm{Tr}_B \Psi\Psi = \rho_A$. These purifications can be orthogonal to each other and yet induce the same local statistics on A. Whatever distinguishes them must live in their entanglement structure with B.

This is the first hint that entanglement is more fine-grained than entropy. Entropy collapses an entire web of correlations into a single scalar. By contrast, entanglement patterns specify *who is correlated with whom and in what way.*

3.2 Relational versus local descriptions

One can think of a quantum state on N subsystems as an adjacency structure on a graph: nodes are subsystems, edges encode correlation strengths and phases. An entropy-based description keeps only partial information about node marginals (local mixedness) and some aggregates of edges (mutual information). Many distinct graphs generate the same collection of local entropies and mutual informations.

From this perspective, entanglement is the *full graph*, while entropies are particular coarse projections of that graph. If two graphs produce the same entropy vectors but differ in edge structure, then entropy is blind to the difference. Entanglement is therefore a candidate for the "more fundamental" information object from which entropies are derived summaries.

4 Condensate Thought Experiment: Identical Local Information, Different Entanglement Histories

4.1 Setup: many bosons, one single-particle state

Consider a Bose–Einstein condensate (BEC) of N identical bosons in a single mode ϕ of a trap. Ignoring small thermal components, the many-body state is approximately

$$\Phi_N \approx \frac{1}{\sqrt{N!}} \left(a_\phi^\dagger\right)^N 0, \tag{6}$$

where $a_\phi^\dagger$ creates a boson in state ϕ. Each individual particle, when traced out from the symmetric state, has the same reduced single-particle density matrix,

$$\rho^{(1)} = \phi\phi, \tag{7}$$

a pure state with $S(\rho^{(1)}) = 0$. From the viewpoint of Shannon and von Neumann, every boson in the condensate looks identical; they all occupy the same quantum information state.

Now imagine that before condensation, the bosons had different entanglement histories with external systems. For simplicity, consider just two special bosons, A and B, that eventually end up in the condensate.

4.2 Two histories

Let E_A and E_B denote distinct external systems ("environments"). Suppose that prior to joining the condensate, we have two global states:

$$\Psi_A = \phi_A \otimes E_A, \tag{8}$$

$$\Psi_B = \phi_B \otimes E_B, \tag{9}$$

where ϕ_A and ϕ_B are the same single-particle state ϕ, but E_A and E_B are orthogonal and encode different prior interactions, measurement records, or coupling histories.

From the perspective of the single boson alone,

$$\rho_A = \mathrm{Tr}_{E_A} \Psi_A \Psi_A = \phi\phi = \mathrm{Tr}_{E_B} \Psi_B \Psi_B = \rho_B. \tag{10}$$

Shannon information about measurement outcomes of A in any basis is the same as for B. Von Neumann entropy is the same. To any local observer without access to E_A or E_B, A and B are informationally indistinguishable.

Yet globally the states are different:

$$\Psi_A \Psi_B = \phi\phi \, E_A E_B = E_A E_B, \tag{11}$$

which we can take to be 0 for orthogonal environments. There exist measurements on the joint systems (boson plus its environment) that perfectly distinguish Ψ_A from Ψ_B.

Once A and B join the condensate, they become indistinguishable particles inhabiting the same mode ϕ, but their external entanglement structure persists. The many-body BEC state entangles the condensate with the combined environment $E_A \otimes E_B \otimes E_{\mathrm{rest}}$ in a way that depends on these histories.

4.3 Operational read-out

Can this distinction ever matter? Yes, provided we have access to the relevant environmental degrees of freedom.

In principle one can perform a reversible interaction that "tags" a boson in the condensate according to the environment it is correlated with. For instance, suppose the environments E_A and E_B each contain a qubit register that records which boson they interacted with. A joint unitary on "condensate + environments" can map

$$\phi_A E_A \mapsto \phi_A 0_L, \tag{12}$$

$$\phi_B E_B \mapsto \phi_B 1_L, \tag{13}$$

where 0_L and 1_L are logical states of a lab register. This transformation preserves local reduced states on the condensate but extracts a classical bit in the lab that perfectly distinguishes the two histories.

From the point of view of an information theorist, the extra bit did not come from Shannon information about the boson's local state (which was the same in both cases) nor from von Neumann entropy of its reduced density matrix. It came from the *pattern of entanglement* between the boson and external systems. That entanglement stored a record of the history that remained invisible at the level of state entropies.

4.4 What the example actually proves

This example does *not* contradict quantum information theory. Everything is consistent with standard formalism. The point is conceptual:

- Entropic quantities attached to a single subsystem (Shannon or von Neumann) can be blind to differences in global entanglement structure.
- Two global states can be fully distinguishable, while all the usual state-based information measures on a chosen subsystem agree.
- Therefore, if one wants a notion of "how much information is really in the world," one has to count *relational* degrees of freedom—entanglement—not just local uncertainties.

If the only property that can distinguish two otherwise identical subsystems is what they are entangled with, then entanglement behaves like a more fine-grained, and thus more fundamental, notion of information than the scalar entropies assigned to local states.

5 Entanglement as the Fundamental Bookkeeping

5.1 Entanglement patterns as primary objects

The condensate scenario generalizes: any time reduced density matrices and local entropies are identical but global states differ, the difference lives in entanglement structure. This is familiar mathematically as the non-uniqueness of purification; here we are simply promoting that fact to a principle:

Principle (Relational primacy). *The fundamental information in a physical system is encoded in the pattern of entanglement between its parts, not in the entropies of their local states.*

Entropies like $H(X)$ and $S(\rho)$ are then derived scalars: compressed summaries of correlation graphs. They are extremely useful—for coding theorems, for thermodynamic bounds—but they are not the whole story.

From this perspective, an information-theoretic description of the universe should start with a global wavefunction (or density operator) on a factorized Hilbert space and treat entanglement as the primary object. Classical and quantum entropies arise by partial tracing and coarse-graining.

5.2 Connection to entanglement geometry and cosmology

In Entanglement Manifold Cosmology (EMC), the observable $3 + 1$-dimensional universe is modelled as a sector emerging from a deeper entanglement manifold whose coordinates label patterns of correlation between proto-degrees of freedom [6, 7]. Perturbations on this manifold pull back into a $U(1)$ gauge sector and project into $SU(2) \times SU(3)$, while gravity responds to the stress–energy of these fields. Early low entropy is reinterpreted as entanglement structure outside the visible sector, later reinjected as cosmic structure forms.

In that picture, classical and quantum entropies in our sector are secondary: they describe how much of the manifold's correlation structure has become accessible to observers like us. The underlying "bookkeeping" is entirely in the geometry of entanglement—who is correlated with whom and with what phase structure. The condensate thought experiment is a small laboratory echo of this: two bosons can look identical locally yet participate in different entanglement geometry globally.

This aligns with a broader trend in quantum gravity where spacetime is argued to be "built from" entanglement links [5]. Our claim here is more modest but in the same spirit: if you want the most fundamental notion of information compatible with modern physics, you should start from entanglement, not from Shannon or von Neumann entropies.

5.3 Truth as volume of valid derivatives

In related work, we have proposed a definition of physical truth as the "volume of valid derivatives" a system state can generate without violating geometric, energetic, or causal consistency [8, 7]. Entanglement structure governs which future states are reachable via valid dynamics. Two global states with the same local entropies but different entanglement graphs generally have different sets of valid derivatives, and hence different truth volumes. Again, the fine-grained information relevant for the universe's future is encoded in entanglement, not in scalar entropies.

6 Implications and Possible Probes

6.1 Information theory

For classical and quantum information theory, the message is that entropies are indispensable but incomplete. They answer questions like "How many bits can we compress this source down to?" or "What is the channel capacity?" But if we care about how physical systems remember their histories, how they embed into larger environments, or how geometry emerges, we must track entanglement patterns explicitly.

One practical consequence is that two sources with identical entropy rates may still differ in their entanglement with side information. In protocols involving side channels or quantum memories, this can change what is achievable. An information-theoretic analysis that ignores relational structure may miss such distinctions.

6.2 Physics and experiments

In condensed-matter and AMO (atomic, molecular, optical) physics, variants of the condensate thought experiment are not purely hypothetical. Experiments already prepare BECs, entangle atoms with photons or cavity modes, and then recombine them. The conceptual point here is not to propose a single smoking-gun experiment, but to highlight a design pattern:

- Prepare subsystems in the same local state (same marginals).

- Embed them in different global entanglement structures.
- Show that later joint operations can reveal differences that no local entropy or local tomography at the preparation time could see.

Such experiments would not overthrow quantum theory; they would dramatize the gap between "state-based" and "relation-based" information bookkeeping.

6.3 Conceptual clean-up

Thinking in terms of entanglement as fundamental also helps clean up some confusions about "information loss." In black-hole physics, for example, one often speaks of information being lost when a pure state collapses to a black hole and evaporates. On an entanglement-first view, information is not a scalar stored in one place; it is a pattern spread across the manifold of degrees of freedom. The real question becomes whether the entanglement graph evolves unitarily. Entropies measured in one sector may go up or down while global entanglement structure stays consistent.

7 Conclusion

Shannon and von Neumann entropies are masterpieces of 20th-century science. They quantify uncertainty, compression, and mixedness with exquisite precision. But they are not the deepest notion of information available in quantum physics. Entanglement—the relational pattern of correlations between systems—is more fine-grained. It can distinguish global states that look identical to all entropy-based measures on chosen subsystems.

Our condensate thought experiment makes this vivid: two bosons in the same single-particle state, with the same local von Neumann entropy, can still carry different, operationally retrievable information because of their distinct entanglement histories. The record of "who this boson has been with" lives in entanglement, not in its local state.

If one accepts that the universe is built from entanglement geometry, as in Entanglement Manifold Cosmology and related work, then treating entanglement as the primary notion of information is natural. Entropies are then summaries, truth volume is a functional on the entanglement graph, and physics is the story of how this graph evolves under dynamical and entropic constraints.

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