

METHOD FOR THE DETECTION OF CARRIER-IN-CARRIER SIGNALS BASED ON FOURTH-ORDER CUMULANTS

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ABSTRACT

The method for the detection of Carrier-in-Carrier signals based on the calculation of fourth-order cumulants is proposed. In accordance with the methodology based on the “Area under the curve” (AUC) parameter, a threshold value for the decision rule is established. It was found that the proposed method provides the correct detection of the sum of QPSK signals for a wide range of signal-to-noise ratios. The obtained AUC value indicates the high efficiency of the proposed detection method. The advantage of the proposed detection method over the “radiuses” method is also shown.

KEYWORDS

Carrier-in-Carrier, Cumulants, QPSK.

1. INTRODUCTION

The development of digital communications creates the need to develop more efficient methods for using frequency resources. Although modern methods of error-correction coding allow working at rates close to the theoretical Shannon limit, channel capacity can be increased by using Carrier-in-Carrier (also known as PCMA, Paired-Carrier Multiple Access) technologies in which the transmitted signals occupy the same band (see, for example, [1]).

There are various methods of blind separation for the sum of several signals (see [2-3]), including PCI (Principal informative components), ICA (Independent Components Analysis), Particle Filtering and other approaches. In [4], a method of blind separation was also proposed, based on the iterative maximization of a posteriori probability density of the separated signals.

At the same time, the task of *detecting* the fact of transmitting a superposition of signals at a given frequency is practically not considered in the literature. This task has importance for the tasks of non-cooperative communication and electronic reconnaissance. Therefore, in this paper we propose a method for detecting such signals based on the use of the fourth-order cumulants. Cumulants, which are high-order mixed moments, are widely used in signal processing problems (see, for example, [5]). For example, there are many methods for applying them to the problem of detecting the modulation type of a transmitted signal (see [6]). It was shown that cumulants have high efficiency in detecting the type of signal modulation. This motivated our study of the application of cumulants to the problem of detecting Carrier-in-Carrier signals.

This paper provides an algorithm for the classification of Carrier-in-Carrier signals using a fourth-order cumulants. To select the optimal decision threshold, we used the Area Under the Curve

(AUC) criterion (see, for example, [7]). Numerical simulations confirm the effectiveness of the proposed method.

2. PRELIMINARIES

The mixture of two digitally modulated signals received by one antenna in a single channel can be expressed as:

$$x(t) = x_1(t) + x_2(t) + w(t),$$

where $x_u(t), u = 1, 2$ are the signals from two sources:

$$x_u(t) = a_u e^{j\phi_u} \sum_{n=-\infty}^{\infty} s_u(n) g(t - nT_s - \tau_u), u = 1, 2$$

and $s_u(n), u = 1, 2$ are original sequences to be estimated; T_s is a symbol period; a_u are the amplitudes; ϕ_u are the phases; τ_u are the time shifts; $g(t)$ is a total channel response (assumed to be raised square-root cosine with known roll-off); $w(t)$ is a white Gaussian noise. Here we assume that the signals have the same symbol period, although we will comment below that the proposed approach can be applied to signals with different symbol periods. The principle of formation of the received signal is shown in Figure 1.

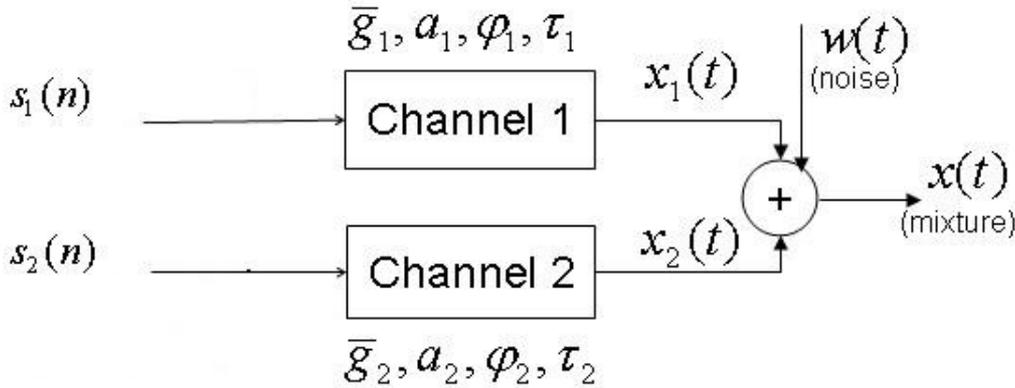


Figure 1: Statement of the problem.

The main criterion of the proposed method of detecting carrier-in-carrier in the channel is based on the demodulator's output constellation. Typical signal constellations of the QPSK (Quaternary Phase Shift Keying) and BPSK (Binary Phase Shift Keying) signals in the presence of noise with SNR (signal-to-noise ratio) equal to 10 dB are shown in Figure 2 (here $x_2(t) = 0$).

On the other hand, the carrier-in-carrier constellation has a distribution of points along circles around nominal points for simple QPSK (BPSK) modulation (Figure 3). This kind of constellation arises because of the sum of two M-PSK signals with a small difference of the carrier frequencies. The radius of the circle of distribution of points around the nominal point of the constellation depends on the ratio of the amplitudes of the signals, i.e. the greater the ratio of the amplitudes a_2 / a_1 , the smaller is the radius of the circle (see Figure 4).

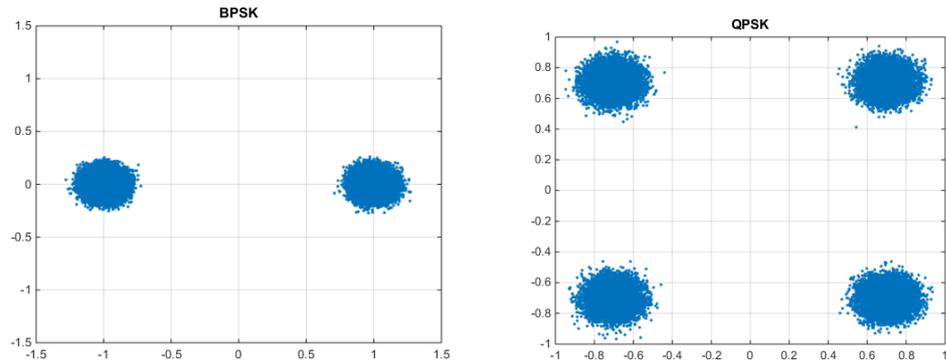


Figure 2: Output constellation of BPSK and QPSK signals

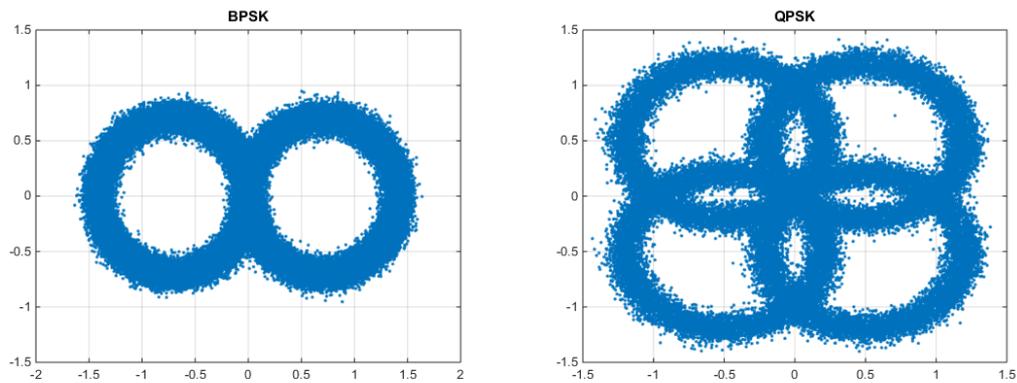


Figure 3: Output constellations for the sum of two BPSK signals and the sum of two QPSK signals with equal amplitudes

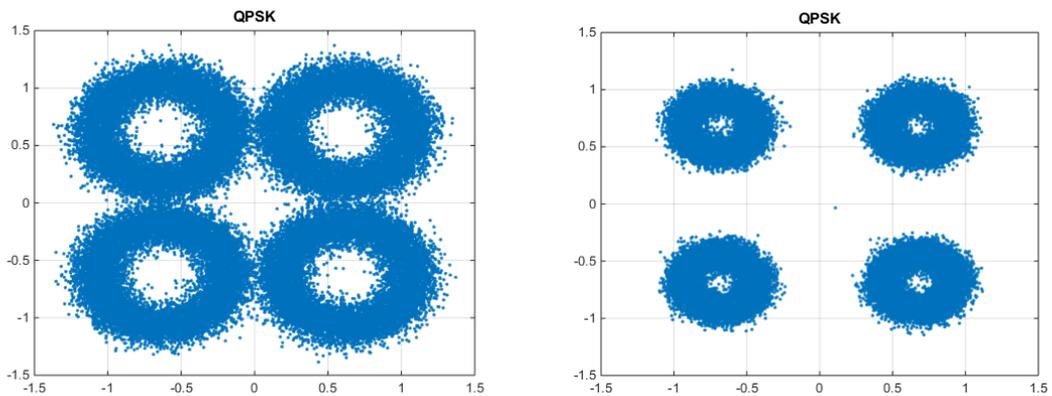


Figure 4: Output constellations for the sum of two QPSK signals with a ratio of amplitudes 2 and 4 respectively

Therefore, our proposed method for detecting Carrier-in-Carrier signals is based on detecting the transformation of the signal constellation. In accordance with this method, a fourth-order joint cumulant is calculated between the signal and its complex conjugate copy in one quadrant of the complex constellation.

The proposed detection algorithm is as follows. Signal samples are selected that correspond to one of the quadrants in the complex constellation (Figure 5):

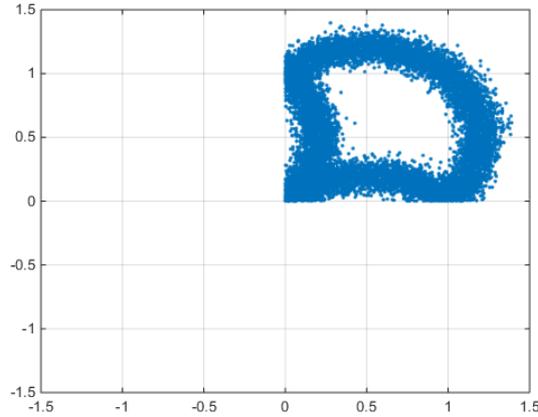


Figure 5: The QPSK samples corresponding to one of the quadrants of the complex constellation

Then the average value is subtracted from the received signal. Denote the received centered signal as $x(n)$. Then the value for the detection of Carrier-in-Carrier signals is fourth order cumulant:

$$C_{2,2} = cum[x, x, x^*, x^*], \quad (1)$$

where x^* is the conjugated signal.

The proposed criterion for detecting Carrier-in-Carrier signals consists of comparing the cumulant (1) with a threshold value. In that case, if the value defined by formula (1) does not exceed the threshold:

$$C_{2,2} < \Pi_C, \quad (2)$$

then a decision is made about the presence of a Carrier-in-Carrier signal. Otherwise, a decision is made about the absence of a Carrier-in-Carrier signal. The choice of the threshold will be discussed in the next section.

As is known, the quantity (1) can be expressed in terms of statistical moments:

$$C_{2,2} = E_{2,2} - |E_{2,0}|^2 - 2E_{1,1}^2,$$

where $E_{m,n}$ is the joint moment of the order (m,n) :

$$E_{m,n} = \frac{1}{N} \sum_{k=1}^N x^m(k) (x^*(k))^n.$$

From here follow simple formulas for calculating of $E_{1,1}$ and $E_{2,0}$:

$$E_{2,0} = \frac{1}{N} \sum_{k=1}^N x^2(k),$$

$$E_{1,1} = \frac{1}{N} \sum_{k=1}^N |x(k)|^2$$

In addition, in this paper, we compare the proposed detection method with the approach based on the standard deviation of the average radius of the signal constellation points in one quadrant of the complex constellation. In accordance with this criterion, the standard deviation for the calculated distances $|x(n)|$ is calculated normalized to the square of their average value:

$$\sigma_r = \frac{1}{e_x^2} \sqrt{\frac{1}{N} \sum_{k=1}^N (|x(n)| - e_x)^2} \quad (3)$$

Where

$$e_x = \frac{1}{N} \sum_{k=1}^N |x(n)|.$$

In accordance with this approach, in the event that the value σ_r defined by formula (3) does not exceed the threshold Π_r :

$$\sigma_r < \Pi_r, \quad (4)$$

then a decision is made about the presence of a Carrier-in-Carrier signal. Otherwise, a decision is made about the absence of a Carrier-in-Carrier signal.

3. EXPERIMENTAL RESULTS

In the following experiments, the cumulant values (1) and the normalized constellation radiuses (3) were calculated for signal-to-noise ratios (SNR) from 0 to 10 dB with an amplitude ratio of 0 (case of a single signal, $x_2(t) = 0$), 1, 2, 4 and also for the additive white Gaussian noise. The results of calculating the parameter (1) are shown in Figure 6.

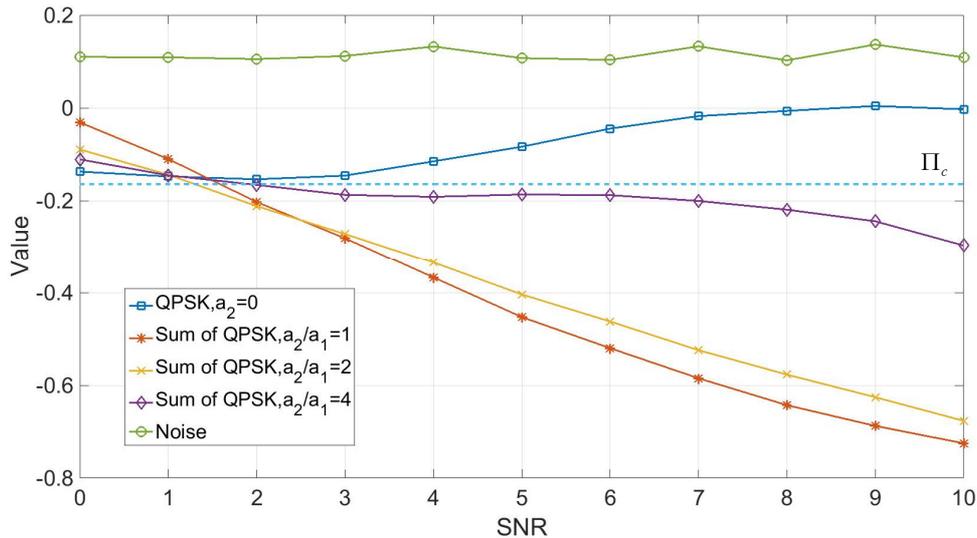


Figure 6: Dependence of cumulants on the SNR for various ratios of the amplitudes of the signals.

Figure 6 shows that the cumulant plots corresponding to Carrier-in-Carrier are separated from the plots corresponding to a single QPSK signal and background noise in the case when SNR is

greater than 1.5 dB (which is a common situation in practice). The “cumulants” method ensures the correct detection of a Carrier-in-Carrier (by comparing with a given threshold) for different ratios of signal amplitudes. It can also be seen that the cumulant graph (1), corresponding to white noise, is clearly separated from the cumulant graphs corresponding to the three considered cases of the sum of QPSK signals.

The results of calculating the parameter (3) for the radius method are shown in Figure 7. As can be seen in the figure, the detector based on the constellation radius does not provide a clear separation of the radius values necessary for the correct detection.

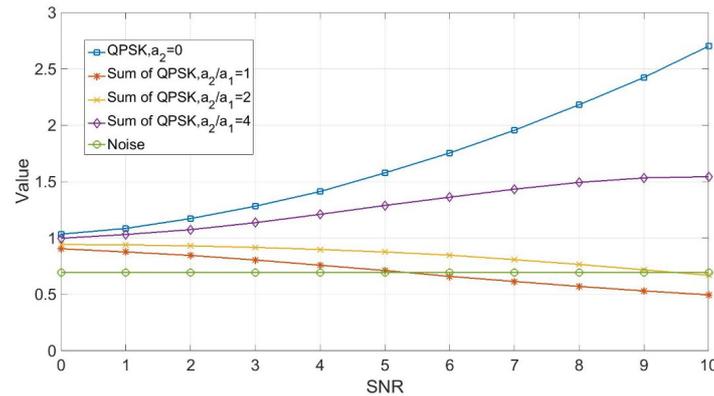


Figure 7: Dependence of normalized constellation radiuses on the SNR for various ratios of the amplitudes of the signals

In the next step, it is necessary to select the optimal thresholds for the rules (2) and (4). To select these thresholds and test the overall effectiveness of the proposed rule for detecting the Carrier-in-Carrier signals, we will use a technique based on the use of the ROC curve or, namely, Area under the Curve (AUC) parameter [6]. The calculation of this parameter requires the calculation of the statistical characteristics of a true positive rate (probability of correct detection) and a false positive rate (probability of false alarm) for all possible threshold values that lie within the range of values of the parameter selected as the detection criterion. The AUC plots for the cumulants method and the radius method are presented in Figures 8 and 9 respectively.

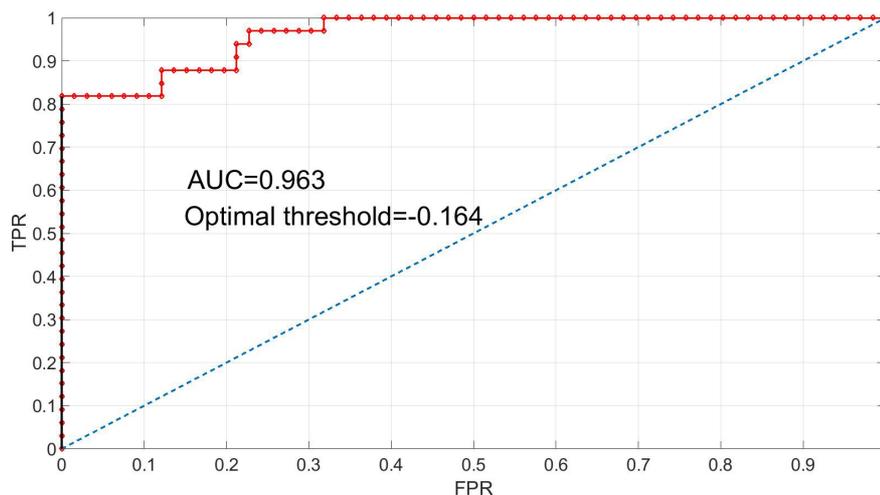


Figure 8: AUC for the cumulants method

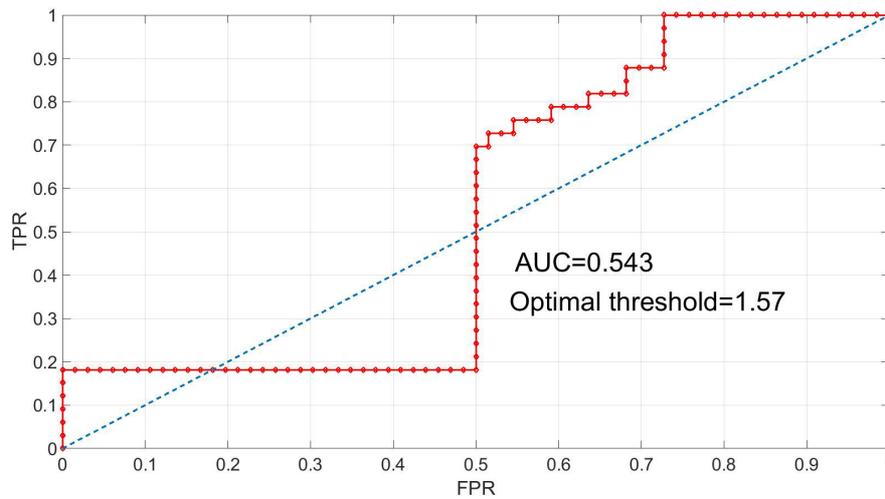


Figure 9: AUC for the radiuses method

From Figure 8 it follows that the AUC parameter for the cumulants method is 0.963, which indicates the high reliability of the proposed detection method. This is because the threshold $\Pi_c = -0.164$ chosen for the cumulants method is equally well suited for all SNRs and signal amplitude ratios. At the same time, the AUC value for the radius method (Figure 9) is significantly lower (0.543). The radius method is very sensitive to changes in the ratio of the amplitudes of the summable signals, as well as to variations of other parameters. Note also that the cumulants method showed high reliability when detecting the sum of signals with different symbol rates and other modulation types.

4. CONCLUSIONS

In this paper, a method for the detection of the Carrier-in-Carrier signals based on the calculation of fourth-order signal cumulants was proposed. In accordance with the methodology based on the “Area under the curve” (AUC) parameter, a threshold value for the decision rule was established. It was found that the proposed method provides the correct detection of the sum of QPSK signals with SNR greater than 1.5 dB, which corresponds to a wide class of practically encountered situations. The obtained AUC value for the cumulants method was 0.963, which indicates the high efficiency of the proposed detection method. The advantage of the proposed detection method over the “radiuses” method was also shown.

REFERENCES

- [1] Agne, Craig & Cornell, Billy & Dale, Mark & Keams, Ronald & Lee, Frank, (2010) “Shared-spectrum bandwidth efficient satellite communications”, *Proceedings of the IEEE Military Communications Conference (MILCOM' 10)*, pp341-346.
- [2] Gouldieff, Vincent & Palicot, Jacques, (2015) “MISO Estimation of Asynchronously Mixed BPSK Sources”, *Proc. IEEE Conf. EUSIPCO*, pp369-373.
- [3] Feng, Hao & Gao, Yong, (2016) “High-Speed Parallel Particle Filter for PCMA Signal Blind Separation”, *Radioelectronics and Communications Systems*, Vol.59, No.10, pp305-313.
- [4] Semenov, Vasyl, (2018) “Method of Iterative Single-Channel Blind Separation for QPSK Signals”, *Mathematical and computer modelling*, Vol. 17, No. 2, pp108-116.

- [5] Fernandes, Carlos Estevao R. & Comon, Pierre & Favier, Gerard, (2010) “Blind identification of MISO-FIR channels”, *Signal Processing*, Vol. 90, pp490–503.
- [6] Swami, Anantharam & Sadler, Brain M., (2000) “Hierarchical digital modulation classification using cumulants,” *IEEE Trans. Commun.*, Vol. 48, pp416-429.
- [7] Wunderlich, Adam & Goossens, Bart & Abbey, Craig K. “Optimal Joint Detection and Estimation That Maximizes ROC-Type Curves” (2016) *IEEE Transactions on Medical Imaging*, Vol. 35, No.9, pp2164– 2173.

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