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A HYBRID OPTIMIZATION ALGORITHMS FOR SOLVING METRIC DIMENSION PROBLEM

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ABSTRACT

In this paper, we consider the NP-hard problem of finding the minimum resolving set of graphs. A vertex set B of a connected graph G resolves G if every vertex of G is uniquely identified by its vector of distances to the vertices in B. The cardinality of the minimal resolving set is the metric dimension of G. The metric dimension appears in various fields such as network discovery and verification, robot navigation, combinatorial optimization and pharmaceutical chemistry, etc.

In this study, we introduce a hybrid approach (WCA_WOA) for computing the metric dimension of graphs t hat combines the water cycle algorithm and a whale optimisation algorithm.

The WOA algorithm hybridises the WCA in order to obtain the optimal result and manage the optimization process.

The results of the experiments show that the WCA_WOA hybrid algorithm outperforms the WCA, WOA, an d particle swarm optimization methods.

KEYWORDS

Metric dimension; Water cycle algorithm; Whale optimization algorithm; Optimization

1. INTRODUCTION

Let $G=(V, E)$ be a connected graph and $d(u,v)$ be the shortest path between two vertices $u,v \in V(G)$. An ordered vertex set $B=\{x_1,x_2,...,x_k\}\subseteq V(G)$ is a metric basis of G if the following two conditions are satisfied:

i) $\forall v\in V(G)$, the representation $r(v| B) = (d(v, x_1), d(v, x_2),....,d(v, x_k))$ is unique .

ii) B with minimum cardinality.

Slater[1,2] introduced the notion of metric basis as a locating set of G and used the cardinality of B as a locating number to uniquely identify the location of an intruder in a network. Harary and Melter in [3] introduced independently the notion of metric basis as a resolving set of G and the cardinality of B as a metric dimension, denoted dim (G), which has been used in several applications such as robot navigation in networks [4,5], pharmaceutical chemistry Chartrand et al. [6], pattern recognition Melter et al. [7] and wireless sensor network localization [8].

The metric dimension of several graphs is computed theoretically in the literature [9-15]. Along them are gear graphs[9], wheel related graphs [10], jahangir graphs [11], generalized Petersen graphs [12], flower graphs and convex polytopes [13], power of total graphs [14], mobius ladder [15],etc.

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Integer linear programming is an efficient process for determining the metric dimension of graphs with large orders. The metric dimension problem was first proposed as an integer programming problem by Chartrand et al.[6]. In the computation of the metric dimension of a graph, the size of its solution space grows exponentially with the problem dimension. To deal with this problem, approximation and heuristic approaches can be applied. Although heuristic methods do not offer a guarantee of reaching the optimum solution, they give good results in a reasonable amount of time. The use of a recently found class of heuristic algorithms, called metaheuristic algorithms, related to metric dimension problems has been studied by Kratica et al.[16]. In [16], Kratica et al. proposed a genetic algorithm (*GA*) that uses binary encoding and the standard genetic operators adapted to the metric dimension problem. In [17], Murdiansyah et al. provided a particle swarm optimization (PSO) algorithm that is obtained by adapting a standard discrete PSO to solve the metric dimension problem. In [18], Mladenovic presented an efficient variable neighborhood search approach for solving the metric dimension problem and the problem of determining minimal doubly resolving sets. For some networks, including the trapezoid network, Z-(P_n) network, open ladder network, and tortoise network, Mohamed et al. [25] computed the exact value of the secure resolving set. They also calculated the domination number for other networks, including the twig network, double fan network, bistar network, and linear kc_4-snake network. The first attempt to heuristically calculate the minimally connected dominating resolving set of graphs using a binary version of the equilibrium optimization technique was made by Mohamed et al. [26].The first work to heuristically calculate the lowest connected resolving set of graphs using a binary implementation of the Enhanced Harris Hawks Optimization was performed by Mohamed et al. [27].According to Mohamed [28], when a robot is travelling a network represented by the (2,1)C_4-snake graph, 22-snake graph, and 3C_4-snake graph, the metric dimension is investigated in terms of contraction and bijection.

For example, consider the graph *G* of Fig. 1. The set $B= \{v_1;v_2\}$ is not a resolving set for *G* since $r(v_4 \mid B) = (3; 2) = r(v_5 \mid B)$. On the other hand, $B_1= \{v_1;v_4\}$ is a resolving set for *G* since the representations for the vertices of *G* with respect to B_1arer$(v_1,B_1) = (0, 3)$, $r(v_2,B_1) = (1, 2)$, $r(v_3,B_1) = (2, 1)$, $r(v_4,B_1) = (3, 0)$,$r(v_5,B_1) = (3, 1)$, $r(v_6,B_1) = (2, 2)$, $r(v_7,B_1) = (1, 3)$.

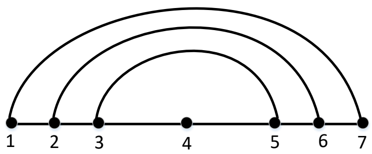

Figure 1: Graph *G*

In this work, we introduce a hybrid algorithm using the water cycle algorithm (WCA) and the whale optimization algorithm (WOA). The results and comparison with pure WCA,WOA and PSO algorithms confirm the effectiveness of the proposed algorithm WCA-WOA for solving metric dimension problem that are applied to graphs such as ladder graphs, middle graphs and square graphs.

The remaining of this paper is organized as follows: Details of the Water Cycle Algorithm details and its procedure are described in Section 2. In Section 3, detailed descriptions of the Whale Optimization Algorithm and its concepts are introduced. The proposed algorithm is discussed in Section 4.Section 5 reports computational results.

2. WATER CYCLE ALGORITHM

The WCA was developed by monitoring the water cycle process and simulating the movement of rivers and streams towards the sea. Assume there is some sort of precipitation or rain phenomenon. After the sampling process, a random population of design variables is formed. The best individual, classified in terms of having the lowest cost function (for minimization problems), is chosen as the sea [19].

The remainder of the streams flow into rivers and the sea, while a few of the better streams are designated as rivers. The production of an initial population represents a matrix of streams with a size of $N_{pop} \times D$, where D is the dimension and (N_{pop}) is the population size. As a result, this randomly generated matrix is provided as follows:

$$\text{Totalpopulation} = \begin{bmatrix} Sea \\ River_1 \\ River_2 \\ \vdots \\ Stream_{Nsr+1} \\ Stream_{Nsr+2} \\ Stream_{Nsr+3} \\ \vdots \\ Stream_{Npop} \end{bmatrix} = \begin{bmatrix} x_1^1 & x_2^1 & x_3^1 & \dots & x_D^1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_1^{N_{pop}} & x_2^{N_{pop}} & x_3^{N_{pop}} & \dots & X_D^{N_{pop}} \end{bmatrix} \quad (1)$$

In the first step, $Npop$ streams are created. Then, a number of the best individual *Nsr* (minimum values) are chosen as the sea and rivers. The stream that is least valuable among the others is referred to as the sea. Actually, *Nsr* is the total number of rivers (which is defined by the user) and a single sea. The remaining population (*Nstream*) is considered as stream flowing into the rivers or may alternatively flow directly into the sea [20]. Each river absorbs water from streams depending on the size of the flow.This means that from stream to stream, different amounts of water enter a river and/or the sea. Additionally, rivers flow to the sea, which is the most downward. Using the following formulas [21], the designated streams for each river and the sea are determined:

$$NS_n = round\left\{\left|\frac{Cost_n - Cost_{Nsr+1}}{\sum_{n=1}^{Nsr} C_n}\right| \times Nstreams\right\}, n = 1,2,3, \dots \dots .., Nsr \quad (2)$$

where NS_n is the total number of streams feeding into a given river and the sea. New locations for streams and rivers have been proposed for the WCA's exploitation phase [19] as follows:

$$X_{stream}(t+1) = X_{stream}(t) + rand \times C \times \left(X_{sea}(t) - X_{stream}(t)\right) \quad (3)$$
$$X_{stream}(t+1) = X_{stream}(t) + rand \times C \times \left(X_{river}(t) - X_{stream}(t)\right) \quad (4)$$
$$X_{river}(t+1) = X_{river}(t) + rand \times C \times \left(X_{sea}(t) - X_{river}(t)\right) \quad (5)$$

where t is an iteration index, $1 < C < 2$, the best value for C may be selected as 2 and rand is a uniformly distributed random number between [0,1]. In (4) and (5), the streams that drain into the sea and the related rivers are discussed. If the solution given by a stream is more optimal than that of its connecting river, the positions of the river and stream are changed. A similar exchange can be performed between a river and the sea. Furthermore, the evaporation process operator is also incorporated to prevent premature convergence to local optima (exploitation phase) [19]. In principle, as rivers and streams enter the sea, evaporation causes sea water to evaporate. This causes new precipitation to occur. Therefore, it is important to determine whether the river or stream is close enough to the sea for evaporation to actually occur. The evaporation condition between a river and the sea is measured using the criteria listed below [21]:

$$\left\|X_{sea}^{t} - X_{riverj}^{t}\right\| < d_{\max} \text{or rand} < 0.1 \quad j = 1,2,3,\dots\dots, N_{sr} - 1 \qquad (6)$$

where $d_{\max}$ is a small value close to zero. Following evaporation, the raining process is used, and new streams are created at various locations. Indeed, the evaporation operator is responsible for the exploration phase in the WCA. The new locations of the newly formed streams are determined via a uniform random search. Additional searches are discouraged by large values for $d_{\max}$, whereas searches near the sea are encouraged by low values. Thus, $d_{\max}$ regulates the level of search activity close to the sea. The following is an adaptive decrease in the $d_{\max}$ value [22]:

$$d_{max}(t+1) = d_{max}(t) - \frac{d_{max}(t)}{\text{max iteration}} t = 1,2,\dots\dots\dots, \text{max iteration} (7)$$

We see [23,24] for more information on the metaheuristic approach.

3. WHALE OPTIMIZATION ALGORITHM

The WOA may be mathematically represented in three phases: encircling prey, bubble-net attacking, and prey search. Exploitation includes encircling prey and attacking with a bubble net, while exploration includes random searching.

3.1. Encircling prey

Humpback whales have the ability to locate prey and encircle it. This represents the solution space in WOA. Another factor that encourages other search agents to move towards the best candidate solution is the assumption that the present best candidate solution is the target prey or very near to it. Equations [8,9] simulate this behaviour as follows:

$$\vec{D} = \left|\vec{C}.\overrightarrow{X^{*}}(t) - \vec{X}\,(t)\right|, (8)$$
$$\vec{X}(t+1) = \overrightarrow{X^{*}}(t) - \vec{A}.\vec{D}, \; (9)$$

where $\overrightarrow{X^{*}}(t)$ indicates the whale's earlier best location at iteration t. $\vec{X}(t+1)$ is the whale's current position, and $\vec{D}$ is the distance vector between whale and prey. The $\vec{A}$ and $\vec{C}$ are coefficient vectors calculated as follows:

$$\vec{A} = 2\vec{a}.\vec{r} - \vec{a}, \; \vec{C} = 2.\vec{r} (10)$$

where components of $\vec{a}$ are decreased linearly from 2 to 0 during iterations, and r_1,r_2 are random vectors in [0,1].

3.2. Bubble-Net Attacking Method (Exploitation Phase)

The shrinking encircling mechanism and spiral updating position were introduced in order to mathematically simulate the bubble-net attacking behaviour of humpback whales. The mechanism is realized by setting random values for A in [-1, 1], and the spiral updating position strategy is explained, as shown by Equations [11,12]:

(11) $\vec{D'} = \left|\vec{X^*}(t) - \vec{X}(t)\right|$

$\vec{X}(t+1) = \vec{D'}.e^{bl}\cos(2\pi l) + \vec{X^*}(t)$ (12)

where $\vec{D'}$ denotes the distance of the whale to the prey, while b is a constant used to define the random number l ∈ [−1, 1], spiral shapes that are logarithmic. On the basis of the 50% probability of switching between either the encircling prey mode or the spiral model, which may be stated mathematically as follows, to update the position of whales:

$$\vec{X}(t+1) = \begin{cases} \vec{X^*}(t) - \vec{A}.\vec{D} & p < 0.5 \\ \vec{D'}.e^{bl}\cos(2\pi t) + \vec{X^*}(t) & p \geq 0.5 \end{cases} \quad (13)$$

Where p is a random number in [0,1].

3.3. Search for Prey (Exploration Phase)

If A> 1, the search agent is updated as indicated by a randomly chosen search agent in place of the best search agent to have global optimizers.

$\vec{D} = \left|\vec{C}.\vec{X}_{rand} - \vec{X}(t)\right|$ (14)

$\vec{X}(t+1) = \vec{X}_{rand} - \vec{A}.\vec{D}$ (15)

where $\vec{X}_{rand}$ is a random position vector (a random whale).

4. The Proposed WCA-WOA Algorithm

In this section, we propose a new hybrid algorithm, WCA-WOA is a collaborative combination of WCA and WOA techniques. In this hybrid, first, WCA investigates the search space to isolate the most promising region of the search space. Secondly, in order to improve global search and prevent traps in local optima, it is introduced to whale optimization algorithm to expand search space (starting with WCA 's solution) and locate new populations closer to the optimal solution. The main structure of the hybrid WCA-WOA algorithm is presented in Algorithm 1.

Algorithm 1: Hybrid WCA-WOA Algorithm

Begin

Input the parameters of the algorithm and initial data Generate M initial possible solutions randomly Evaluate fitness values for all solutions

Classified solutions into streams, rivers and the sea and assigned each stream to a river or the sea

it=1

While (it<Maxiter/2)

For j = 1 to S (total number of streams)

Flow stream j toward the corresponding river or the sea

If the new stream is better than the river or the sea

Change the flow direction

End if

Next j

For *j* = 1 to *R* (total number of rivers) flow river*j* toward the sea

Ifthenewgeneratedriverisbetterthanthesea ,reverse the flowdirection

End if *Next* j

Iftheevaporation condition is satisfied, start the raining process

End if

Reduce the value of δ

End while

Obtain the most recent population

Initialize a, *A, C, l* and *p*

Calculate the fitness of each search agent

*X**= the best search agent

it=1+Maxiter/2

while (it <Maxiter)

For each search agent

if (*p*< 0.5)

if (|*A*| < 1)

Update the position of the current search agent

else if (|*A*| ≥ 1)

Choose a search agent atrandom (*X*_rand)

Update the position of the current search agent

end

else if (p ≥ 0.5)

Update the position of the current search

end

end

Calculate the fitness of each search agent

Update *X** if there is a better solution

it=it+1

Update *a, A, C, l* and *p*

end while

Using equations.11,12 ,13,14 and 15

return*X**

5. RESULTS AND DISCUSSIONS

5.1. Experimental Results

In this section, these experiments have been run using MatLab2018a installed on Windows 10 Pro which runs on a Core i5 and 4 GB RAM. WCA-WOA has been compared with WOA, WCA and PSO Algorithms for the metric dimension of graphs are given in Tables 2-4. From these table, we can conclude that WCA-WOA achieve the best results.

Table 1: Parameter Setting.

Parameter	Value
Population size	**30**
Number of iterations	**500**
Number of dimensions	**33**

-The WOA, WCA, WCA-WOA and particle swarm optimization PSO have been run 20 times for each graph and the results are summarized in Tables 2–4. The tables are organized as follows:

-The columns contain the test the number of nodes *N*, edges *M*, exact metric dimension *Md*, the CPU time (*t*) used to indicate the exact metric dimension and iteration: the average number of iterations for finishing the algorithms to achieve best solution respectively.

Table 2: Comparison of WOA, WCA, WCA-WOA and PSO algorithms for the metric dimension of ladder, middle and square graphs. (*N* is the number of vertices).

Table 2: Results on ladder graph L_n

Ladder graph														
N	***M***	**WCA-WOA**			**WCA**			**WOA**			**PSO**			
		Md	*t* (sec)	iteration (generation)	*Md*	*t* (sec)	iteration	*Md*	*t* (sec)	iteration	*Md*	*t* (sec)	iteration
4	4	2	**0.072**	1	2	0.48	1	2	2.49	1	2	0.32	1
6	7	2	**1.63**	1	2	3.71	2	2	7.12	3	2	4.09	5
8	10	2	**3.88**	2	2	6.25	2	2	15.37	4	2	11.72	7
10	13	2	**6.92**	4	2	9.18	3	2	21.58	6	2	32.37	10
12	16	2	13.63	2	2	**12.39**	4	2	47.94	17	2	45.24	18
14	19	2	**17.37**	7	2	89.64	11	2	71.36	32	2	69.71	26
16	22	2	**25.11**	5	2	114.17	8	2	98.61	68	2	95.07	53
18	25	2	**33.94**	4	2	148.21	19	2	157.44	91	2	203.19	104
20	28	2	**38.39**	9	2	195.67	33	2	331.72	127	2	289.43	81
22	31	2	**53.91**	10	2	282.75	55	2	498.15	141	2	346.04	53
24	34	2	**75.32**	11	2	343.11	63	2	664.23	84	2	459.58	99
26	37	2	**94.89**	13	2	497.32	75	2	721.69	102	2	511.26	134
28	40	2	**123.87**	18	2	578.56	191	2	893.51	78	2	725.47	61
30	43	2	**159.98**	24	2	692.16	85	2	1064.18	139	2	802.13	98
32	46	2	**191.22**	32	2	753.81	56	2	1179.27	71	2	941.38	64

Table 3: Results on middle graph

Middle graph														
N	*M*	WCA-WOA			WCA			WOA			PSO			
		Md	*t* (sec)	iteration (generation)	*Md*	*t* (sec)	iteration	*Md*	*t* (sec)	iteration	*Md*	*t* (sec)	iteration
5	5	2	**0.13**	1	2	0.19	1	2	0.26	1	2	1.52	1
7	8	2	3.19	1	2	3.24	5	2	**2.18**	5	2	3.01	5
9	11	2	**5.38**	1	2	7.03	9	2	9.41	18	2	10.66	12
11	14	2	**11.91**	2	2	18.36	3	2	15.09	22	2	24.01	9
13	17	2	27.74	3	2	**23.81**	7	2	69.32	15	2	52.13	14
15	20	2	**35.43**	10	2	82.97	32	2	173.01	38	2	79.61	25
17	23	2	**57.06**	16	2	139.14	18	2	311.19	21	2	123.49	31
19	26	2	**81.75**	31	2	211.18	54	2	425.07	82	2	299.76	43
21	29	2	**104.32**	28	2	377.01	67	2	508.52	99	2	471.11	137
23	32	2	**142.18**	19	2	421.33	29	2	647.84	11	2	565.13	59
25	35	2	**179.32**	85	2	507.42	106	2	754.16	103	2	648.03	115
27	38	2	**265.12**	24	2	603.16	144	2	886.09	187	2	761.14	42
29	41	2	**327.15**	38	2	699.25	11	2	971.45	152	2	893.02	91
31	44	2	**199.01**	32	2	816.39	84	2	1048.78	39	2	982.31	67
33	47	2	**278.13**	17	2	982.15	69	2	1193.89	86	2	1049.03	36

Table 4: Results onsquare graph

Square graph

N	*M*	WCA-WOA			WCA			WOA			PSO		
		Md	*t* (sec)	iteration (generation)	*Md*	*t* (sec)	iteration	*Md*	*t* (sec)	iteration	*Md*	*t* (sec)	iteration
3	3	2	0.06	1	2	0.24	1	2	1.54	1	2	1.19	1
4	6	2	2.18	1	2	5.95	3	2	6.12	6	2	2.47	1
5	7	2	5.32	5	2	10.39	8	2	18.89	14	2	**4.18**	3
6	9	2	**7.98**	2	2	24.31	15	2	21.77	9	2	41.92	6
7	11	2	**19.76**	9	2	39.81	26	2	29.47	17	2	78.23	13
8	13	2	**25.43**	17	2	86.97	22	2	47.11	28	2	109.05	21
9	15	2	**39.15**	12	2	159.23	38	2	115.35	59	2	128.12	17
10	17	2	**61.78**	31	2	272.65	54	2	131.19	42	2	176.01	89
11	19	2	84.32	45	2	387.19	118	2	168.14	73	2	**82.07**	57
12	21	2	**115.24**	12	2	411.33	77	2	246.84	101	2	251.26	83
13	23	2	**172.42**	75	2	507.48	106	2	299.58	89	2	365.88	65
14	25	2	**185.61**	82	2	595.12	42	2	317.29	148	2	621.58	134
15	27	2	**226.35**	25	2	671.28	89	2	435.52	71	2	729.01	53
16	29	2	**139.19**	31	2	726.22	106	2	602.13	94	2	851.13	73
17	31	2	**208.14**	18	2	817.75	74	2	889.38	52	2	944.93	61

5.2. Comparison

To further demonstrate the excellence of WCA-WOA, we chose WCA, WOA and PSO algorithms to conduct experiments under the same conditions and compared the results.

The results on graphs are shown in Tables 2,3 and 4, which indicate that WCA-WOA algorithm, outperforms other algorithms on graphs, reaching 191.22sec inWCA-WOA,753.81 sec in WCA , 1179.27 sec in WOA and 941.38 sec in PSO for ladder graph, and 278.13 sec in WCA-WOA, 982.15 sec in WCA, 1193.89sec in WOA and 1049.03 sec in PSO for middle graph and 208.14 sec in WCA-WOA, 817.75 sec in WCA, 889.38 sec in WOA and 944.93 sec in PSO for square graph. It proves the correctness and superiority of WCA-WOA.

6. CONCLUSION

In this work, we introduced a hybrid algorithm for solving metric dimension problem that is based on the Water Cycle Algorithm. We run the proposed algorithm on three graphs including ladder graph, middle graph and square graph. The experimental results analysis proved that the proposed hybrid algorithm WCA-WOA overcomes the WCA,WOA and PSO algorithms.

Conflicts of Interest

Authors must declare all and any conflicts of interest. If there are noconflicts of interest, it should also be declared as such: “The authors declare that they have no conflicts of interest to report regarding the present study.”

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