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EVEN GRACEFUL LABELLING OF A CLASS OF TREES

Mathew Varkey T.K and Rajesh Kumar T.J

Department of Mathematics,T.K.M College of Engineering, Kollam, Kerala, India

ABSTRACT

A labelling or numbering of a graph G with q edges is an assignment of labels to the vertices of G that induces for each edge uv a labelling depending on the vertex labels f(u) and f(v). A labelling is called a graceful labelling if there exists an injective function f: V (G) → {0, 1,2,......q} such that for each edge xy, the labelling $|f(x)-f(y)|$ *is distinct. In this paper, we prove that a class of* T_n *trees are even graceful.*

Key Words: *labelling, even graceful graph, tree.*

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1 INTRODUCTION

In this paper, we consider the finite graphs which are simple and undirected. We follow Harary [1] for basic definitions and notations in graph theory and Gallian [3] for all terminology regarding graceful labelling. Now a days, graph theory based results are using in every field of engineering and science. Graph labelling is one of the important areas in graph theory. There exist so many practical problems based on graph labelling. Graph labelling methods are used for application problems in communication network addressing system, database management, circuit designs, coding theory, X-ray crystallography, the design of good radar type codes, synch set codes, missile guidance codes and radio astronomy problems etc. Bloom and Golomb [2] presents graph labelling problems in various applications.

Graph labelling can also use for issues in mobile Ad hoc networks. In Ad hoc networks issues such as connectivity, scalability, routing, modelling the network and simulation are to be considered. For analyzing the issues in the Ad hoc network, it can be modelled as a graph. The model can be used to analyze the issues such as node density, mobility among the nodes and link information between the nodes.

In this paper, we apply even graceful labelling on elementary parallel transformation. Using elementary parallel transformation we have to join a new path between any two disconnected nodes by transforming the edges under certain conditions and form a Hamiltonian path. For even graceful labelling, there is no change in the labelling on edges when we transform an edge in elementary parallel transformation. In [5] Mathew Varkey.T.K and Shajahan.A proved that parallel transformation of tree are magic and also find their magic strength and super magic. In [6] it is proved that the parallel transformation of tree generates a class of super mean graph and the T_n class of trees are mean graph and odd mean graph.

Definition 1.

A graph G = (V(G),E(G)) is said to admit even graceful labelling if f:V(G) → {0, 1, 2, 3, ..., 2q-1} injective and the induced function f^* : E(G) → {2, 4, 6, ..., 2q-2} defined as $f^*(uv) = |f(u) - f(v)|$ is bijective .A graph which admits even graceful labelling is called an even graceful graph.

2 ON T_n CLASS OF TREES

Definition 2.

([4]) Let T be a tree and x and y be two adjacent vertices in T. Let there be two end vertices (non adjacent vertices of degree one) x_1, y_1 in T such that the length of the path x-x_1 is equal to the length of the path y-y_1. If the edge xy is deleted from T and x_1, y_1 are joined by an edge x_1y_1, then such a transformations of the edge from xy to x_1y_1 is called an elementary parallel transformation (or an EPT of T) and the edge xy is called a transformable edge.

Definition 3.

([4]) If a sequence of EPT's, the tree, T can be reduced to a Hamiltonian path, then T is called a T_n tree(transformed tree) and such a Hamiltonian path is denoted as $P^H(T)$. Any such sequence regarded as a composition mapping EPT's denoted by P is called parallel transformation of T.

Consider the T_n tree as follows in figure 1.

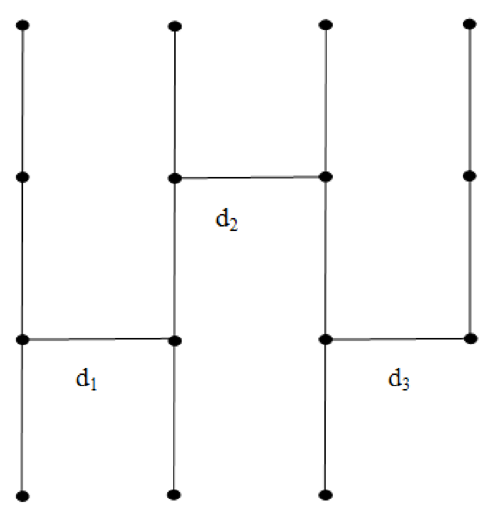

Figure- 1

Using a sequence of three EPT's the edges d_1, d_2, d_3 in figure 1 are replaced by edges e_1, e_2, e_3. The tree can be reduced to a Hamiltonian path as shown in the figure 2.

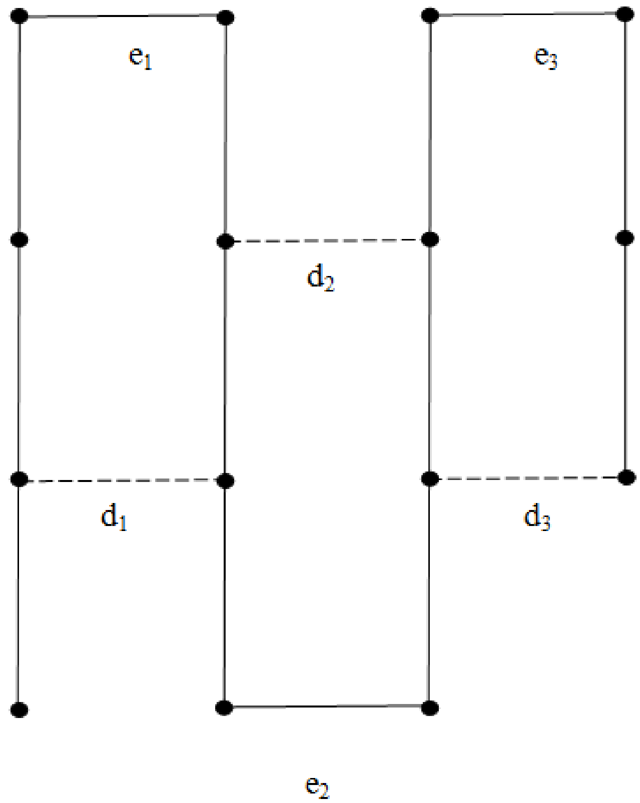

Figure- 2

Theorem 1.

Every T_n tree is even graceful.

Proof:

Let T be a T_n tree with n+1 vertices. By definition there exists a path $P^H(T)$ corresponding to T_n.

Using EPT's, for forming the Hamiltonian path $P^H(T)$ the set of edges $E_d = \{d_1, d_2, d_3... d_r\}$ can be deleted from tree T and E_p is the set of edges newly added through the sequence $\{e_1, e_2, e_3... ...,e_r\}$. Clearly E_p and E_d have the same number of edges. The number of vertices of T and the number of vertices of Hamiltonian path are same and edges of the Hamiltonian path is $\{E(T) - E_d\} \cup E_P$. Now denote the vertices of $P^H(T)$ successively as $v_1, v_2, v_3, ..., v_{n+1}$ starting from one pendant vertex of $P^H(T)$ right up to other . Consider the vertex numbering of $f: V(P^H(T)) \to \{0, 1, 2, 3, ..., 2q-1\}$ as follows

$$f(v_i) = \begin{cases} 2\left[\frac{i-1}{2}\right] \ if\ i\ is\ odd, 1 \le i \le n+1 \\ 2q - 2\left[\frac{i-2}{2}\right]\ \ i\ is\ even, 2 \le i \le n+1 \end{cases}$$

where [ ] denote the integral part. Clearly f is an injective function.

Let $g_f^*(uv) = |f(u) - f(v)|$ be the induced mapping defined from the edge set of $P^H(T)$ into the set {2, 4, 6, ..., 2q-2} whenever uv is an edge in $P^H(T)$.

Since $P^H(T)$ is a path, consider an arbitrary edge in $P^H(T)$ is of the form v_iv_{i+1}, i=1, 2,........., 2q.

Case (1): when i is even, then

$$\begin{aligned} g_f^*(v_i,v_{i+1}) &= |f(v_i) - f(v_{i+1})| \\ &= \left|2q - 2\left[\frac{i-2}{2}\right] - 2\left[\frac{i+1-1}{2}\right]\right| \\ &= \left|2q - 2\left[\frac{i-2}{2}\right] - 2\left[\frac{i}{2}\right]\right| \\ &= \left|2q - 2\left[\frac{i-2+i}{2}\right]\right| \\ &= \left|2q - 2\left[\frac{2i-2}{2}\right]\right| \\ &= \left|2q - 4\left[\frac{i-1}{2}\right]\right| \text{ .... (1)} \end{aligned}$$

Case (2): when n is odd, then

$$\begin{aligned} g_f^*(v_i,v_{i+1}) &= |f(v_i) - f(v_{i+1})| \\ &= \left|2\left[\frac{i-1}{2}\right] - (2q - 2\left[\frac{i+1-2}{2}\right])\right| \\ &= \left|2\left[\frac{i-1}{2}\right] - 2q + 2\left[\frac{i-1}{2}\right]\right| \\ &= \left|2q - 4\left[\frac{i-1}{2}\right]\right| \text{ .... (2)} \end{aligned}$$

From equation (1) and (2), we get for all i,

$$g_f^*(v_i,v_{i+1}) = \left|2q - 4\left[\frac{i-1}{2}\right]\right| \text{ ....(3)}$$

It is clear that g_f^* is injective and its range is {2, 4, 6,......, 2q}.

Then f is even graceful on $P^H(T)$.

In order to prove that f is also even graceful on T_n, it is enough to show that $g_f^*(d_s) = g_f^*(e_s)$.
Let $d_s = v_iv_j$, $1 \le i \le n + 1$; $1 \le j \le n + 1$ be an edge of T and d_s be deleted and e_s be the corresponding edge joined to obtain $P^H(T)$ at a distance k from u_i and u_j .Then $e_s = v_{i+k}v_{j-k}$. Since e_s is an edge in $P^H(T)$, it must be of the form $e_s = v_{i+k}v_{i+k+1}$.

We have $(v_{i+k}, v_{j-k}) = (v_{i+k}, v_{i+k+1})$, this implies j-k = i+k+1
j= i+2k+1

one of i,j is odd and other is even.

Case (1): when i is even, $2 \le i \le n$.

$$\begin{aligned} g_f^*(d_s) = g_f^*(v_i,v_j) &= g_f^*(v_i,v_{i+2k+1}) \\ &= |f(v_i) - f(v_{i+2k+1})| \end{aligned}$$

$= \left|2q - 2\left[\frac{i-2}{2}\right] - 2\left[\frac{i+2k+1-1}{2}\right]\right|$

$= \left|2q - 2\left[\frac{i-2}{2}\right] - 2\left[\frac{i+2k}{2}\right]\right|$

$= |2q - (2i + 2k - 2)|$

$= |2q - 2(i + k - 1)|$

$= |2(q - i - k + 1)|$ .... (4)

Case (2): when i is odd, $1 \le i \le n$

$g_f^*(d_s) = |f(v_i) - f(v_{i+2k+1})|$

$= \left|2\left[\frac{i-1}{2}\right] - (2q - 2\left[\frac{i+2k+1-2}{2}\right])\right|$

$= \left|2\left[\frac{i-1}{2}\right] + 2\left[\frac{i+2k-1}{2}\right] - 2q\right|$

$= |2(i + k - 1) - 2q|$

$= |2q - 2(i + k - 1)|$

$= |2(q - i - k + 1)|$ .... (5)

From (4) and (5) it follows that

$g_f^*(d_s) = |2(q - i - k + 1)|$,$1 \le i \le n$ ....(6)

Now,

$g_f^*(e_s) = g_f^*(v_{i+k}, v_{j-k}) = g_f^*(v_{i+k}, v_{i+k+1})$

$= |f(v_{i+k}) - f(v_{i+k+1})|$

$= \left|2q - 2\left[\frac{i+k-2}{2}\right] - 2\left[\frac{i+k+1-1}{2}\right]\right|$

$= |2q - 2(i + k - 1)|$

$= |2(q - i - k + 1)|$, $11 \le i \le n$ .... (7)

From (6) and (7)

$g_f^*(e_s) = g_f^*(d_s)$

Then f is even graceful on T_n also. Hence the graph T_n tree is even graceful.

Example: Even graceful labelling of the tree in figure 1 is shown below.

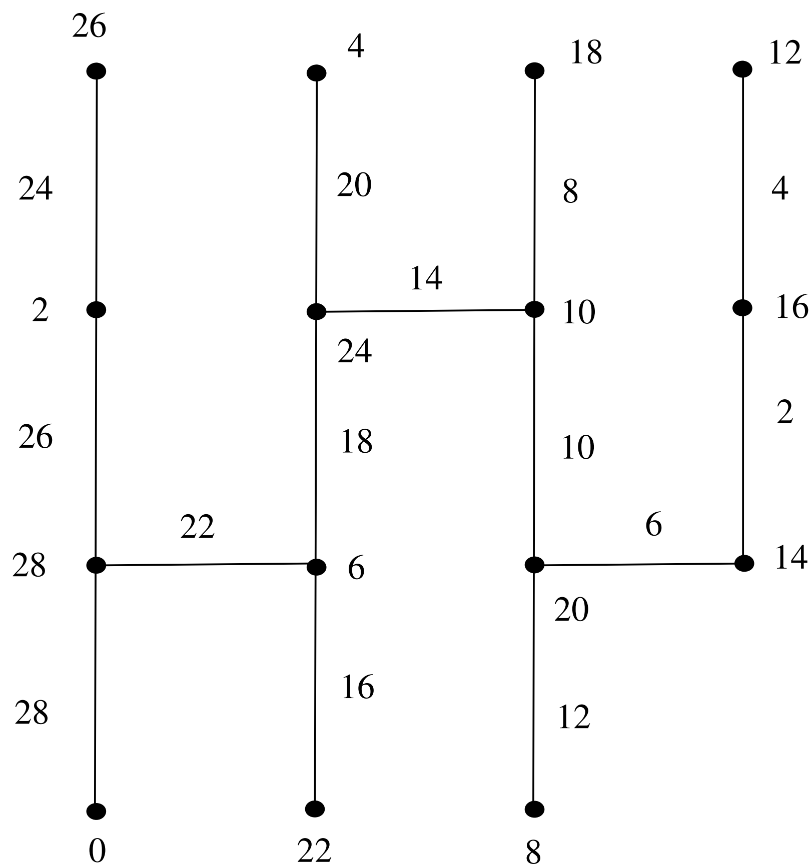

Figure- 3

Algorithm

The T_n tree has n+1 vertices and q edges. Let the class of tree T_n is demonstrated by listing the vertices and edges in the order $v_1e_1v_2e_2$.............$v_{q-1}e_{q-1}v_q$ e_qv_{q+1}.The algorithm starts at the vertex v_1 and label the vertex with the value f (v_1) = 0.Then traverses to the next vertex and label it with the value f (v_2) = 2q, traverses to the next vertex and label it with the value f (v_3) = 2, traverses to the next vertex and label it with the value f (v_4) = 2q−2 and so on. Let x_1 and y_1 be the two vertexes which have no path for connection. As in the definition of elementary parallel transformation joining x_1 and y_1 by an edge.

Then the tree can be reduced to a Hamiltonian path $v_1e_1v_2e_2$.................v_{x_1} , e_{x_1}, $v_{x_{1+1}}$........e_q v_{q+1}. Then label the vertices v_{x_1} and $v_{x_{1+1}}$ as in the same procedure given above. The edge labelling is the absolute value of the difference between the vertex labelling.

To label the T_n trees even graceful, perform the following operations starting with u_1
Number the vertex u_1 with the value f (u_1) = 0.
For (i = 3, i ≤ q + 1, i+ = 2)

$f(u_i) = f(u_{i-2}) + 2$

For $(i = 2, i \leq q, i+ = 2)$

$f(u_i) = 2q - i + 2$

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