A LOGIC OF SPATIAL QUALIFICATION USING QUALITATIVE REASONING APPROACH

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ABSTRACT

The qualification problem is well known within the field of artificial intelligence. This paper introduced a specific aspect of qualification problem that deals with knowing the possibility of an agent's presence at a specific location at a particular time as a qualification for carrying out an action or be participant in an event given its known location antecedents. A quantified modal logic was presented for reasoning with this problem. Logical axioms based on qualitative reasoning for inferring the possibility of an agent's presence at a certain location and time were presented. A formal semantics that clarified the fact that our first order modal logic is a fixed domain logic was also presented. The resulting spatial qualification model was compared with existing S4 and S5 modal systems. The logic was seen to have all the properties of the S4 system but failed to satisfy axiom B in S5 system.

KEYWORDS

Qualitative Reasoning, Quantified Modal Logic, Commonsense Reasoning, Spatial Qualification Problem, Possible World Semantics

1. INTRODUCTION

Qualification problem deals with the impossibility of knowing all the seemingly uncountable possible preconditions for an action to take place. This is a well known artificial intelligence (AI) problem [1]. This problem has been studied in the field of AI since 1977. A specific aspect of qualifications for an action is spatial qualification problem, which is concerned with knowing the possibility of an agent being present at a specific location at a certain time as a precondition for carrying out an action or participate in an event given its known antecedents. Existing formalisms attempting to address problems of this nature in the knowledge representation and reasoning literature have been using probabilistic and fuzzy approaches and not qualitative reasoning. Most knowledge representation formalisms avoid the use of modal logic and modalities. As it turns out, formalizing spatial qualification requires the use of a non-classical concept like "possible worlds" which require the use of modalities. Spatial qualification reasoning is applicable in several application domains such as:

- *Alibi Reasoning*: In a case where an accused person gives an alibi, to investigate the given alibi to be true that there is no possibility of the accused to be present at the scene of the incidence to be involved in the crime.

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- *Homeland Security*: In a case of an ATM Fraud, the model if built into the ATM machines can help to investigate the possibility of presence of an account holder at certain locations to carry out multiple transactions that are spatially questionable due the time difference between the repeated transactions.

- *Planning*: In planning, one needs to work out the feasibility of having an agent carry out an action at some future time, given its current location e.g. “I need to deliver a truck of oranges in Lagos in the next twenty minutes. I am now in Ibadan which is about 2 hours from Lagos.”

Qualitative reasoning allows us to abstract away from the quantities of physical domain and enable us build qualitative mechanisms without resorting to complex methods of calculus [2, 3]. Qualitative reasoning allows inferences to be made in the absence of complete knowledge without probabilistic or fuzzy techniques which may rely or arbitrarily assigned probability or membership values [4].

This work is aimed at creating and formalizing a logical theory that qualitatively investigates an agent’s spatial qualification in suitable application domains. The logical theory will answer the research question: ***Given a prior antecedent that an individual has been present at a certain location and therefore absent from the scene of incidence under investigation at a certain time, is it possible for the agent to have been at the scene of incidence at a certain later time?*** This paper is set to define and describe the axioms and derivation rules for our theory using an appropriate logical language; defining and describing the meaning of our logical model; and also relating our logical model with existing models and semantic structures.

This paper uses a purely logical approach to formalizing spatial qualification as opposed to other approaches in similar papers that use geometric and probabilistic techniques [5]. This paper demonstrates the fact that a purely logical approach is sufficient for solving certain spatial reasoning problems.

The rest of the paper is organized thus. Section 2 gives the theoretical background of spatial concepts featuring the methodologies used. The proposed spatial qualification logical system is formalized in section 3 with the axioms clearly stated, system’s semantics described and comparison of the system’s properties with that of the standard S4 and S5 systems of modal logics. Section 4 gives the conclusion of this paper.

2.THEORETICAL BACKGROUND OF SPATIAL CONCEPTS AND METHODS

It is well known that spatial knowledge is vague [6, 7] and cannot be completely represented. Several aspects of spatial knowledge are addressed using commonsense reasoning [4]. Attempts to categorize space using qualitative reasoning relate to concepts such as neighbourhood, region, district and location. [8]. Topological relations defined by [9] are widely used in this field. These relations are strictly qualitative. Although a qualitative approach alone cannot solve all spatial problems [10] without combining with some spatial quantities, it goes a long way to reduce the amount of data and some of complexities that the use of pure quantitative models such as the one used to model, adversarial abduction problems [5] would bring. Qualitative reasoning is fully explained not to mean the eschewal of quantitative information or mathematical approaches but warns that the mathematical method should not be judged better simply because it provides more information [11, 7]. Qualitative Spatial Reasoning has been done using Constraint Calculus [12, 13]. Due to the inability of most classical logics to handle uncertain knowledge, modalities have been introduced since the necessarily and possibly ◊ operators allow incomplete and uncertain

knowledge to be represented. For example, we can conveniently say that an agent is possibly present at a location l at time t, thus: ◊Present_at(x,l,t). Such representations are used in our definition of reachability which is used to draw inferences about an agent's presence at a spatial location at a certain time. This is seen in the use of Quantified Modal Logic with definite individuals [14]. Possible World Semantics offers the best semantic structure for interpreting modal logics [14]. The commonest application domain where spatiotemporal formalisms are involved in AI is planning [15, 16, 17]. But their formalisms did not consider the spatial qualification of the intelligent agent as one of the preconditions for an action to take place.
To represent the logic for reasoning about an agent's spatial qualification, we employ a qualitative reasoning approach and reuse the RCC-8 relations [9, 11]. Some of the RCC-8 employed in our logic are described in the table below.

Table 1. The RCC-8 Notations and Meanings

S/No.	Notation	Meaning
1.	$EQ(l_1,l_2)$	l_1 Equally connected with l_2
2.	$TPP(l_1,l_2)$	l_1 is a tangential proper part of l_2
3.	$TPP\ (l_2,l_1)$	l_2 is a tangential proper part of l_1
4.	$NTPP(l_1,l_2)$	l_1 is not a tangential proper part of l_2
5.	$NTPP(l_2,l_1)$	l_2 is not a tangential proper part of l_1
6.	$DC(l_1,l_2)$	l_1 has a disjoint connection with l_2
7.	$EC(l_1,l_2)$	l_2 is externally connected with l_1
8.	$PO(l_1,l_2)$	l_1 is partially overlapping with l_2

Our logic is built using a quantified modal logic (first-order modal logic), which combines the expressivity of first-order logic with the standard modalities (i.e. necessity, possibility) of modal logic [14]. We also employ the Possible World Semantics to explain our logic.

3.THE SPATIAL QUALIFICATION MODEL (SQM)

3.1Language of the Logic

The language of this logic is a many sorted first order modal logic. In the logic, constants are assumed to definitely refer to known individuals in the world, unlike in Fitting's quantified modal logic [14] where constant referents may not refer to a definite individual. As such, basic formulae in the logic take the form: $P(t_1, t_2, t_3 \ldots t_n)$ where P is an n-nary predicate symbol and $t_1, t_2, \ldots, t_n$ are terms. Each term can either be a constant symbol or variable symbol.

The rules for forming a formula are as follows: If φ and ψ are formulas, then so are $(\varphi\wedge\psi)$, $(\varphi\vee\psi)$, $\neg\varphi$, $\varphi\Rightarrow\psi$, $\varphi\Leftrightarrow\psi$, $\forall x.\varphi$, $\exists x.\ \psi$, $\Box\varphi$ and $\Diamond\varphi$, following the standard tradition of first order modal logics. The scope of variables in quantification is the formula following the dot after it. The meanings of the classical logic operators are as given in the model semantics for first order predicate logic. The modal operators have meaning attributed to them from the standard possible world semantics. The proposition $\Box\phi$ means ϕ is true in all possible worlds accessible from the current world, while $\Diamond\phi$ means ϕ is true in some world accessible from the current world.

There are three basic sorts of constant in the language. These are *Individuals*, *Location* and *Time points*. Locations in this logic denote the notion of regions in spatial logics. Apart from the predicates denoting the standard spatial relations from RCC, the major predicate is the *Present_at* predicate with the following signature.

$$Present\_at : Individuals \times Location \times Time\ point \rightarrow Boolean$$

Each proposition formed with *Present_at* is called a presence log. The fact that *x* is *Present_at* a location *l* at time *t* is defined by the fact that an individual occupies a region which is within the location *l*. That definition is presented thus:

$$\forall x, l, t.\ Present\_at(x, l, t) \Leftrightarrow \exists r.\ Occupy(x, r, t) \wedge (NTPP(r, l) \vee TPP(r, l) \vee r = l)$$

Occupy is a relation between individuals or objects and the exact 2-dimensional space they occupy at a certain time. If an object or individual occupies a space, that object does not occupy any larger region containing that region

$$\forall x, l, l_1, t.\ Occupy(x, l, t) \wedge l \neq l_1 \Rightarrow \neg Occupy(x, l_1, t)$$

In what follows, the major axioms of our logic of spatial qualification are presented. Subsequently the semantics is presented for it as well.

3.2The Logic of Spatial Qualification

Given that an agent was present at place *p* and at a time *t*. The question we want our representation to answer is: *Is it possible for the same agent to be present at a different place p_1 at a subsequent time t_1, given what known about its prior location*? This problem may be reduced in a sense to the problem of *determining whether or not the agent can travel between place p to place p_1 between time t and time t_1*. A human reasoning agent confronted with this problem would reason using the distance between place *p* and p_1, and the rate at which the agent could travel. Most human agents are able to estimate how long it takes to complete a journey on a certain highway (or path). As can be affirmed by most people, this kind of reasoning is commonsense reasoning because it can be answered experientially by anyone who has traversed the highway before or it can be estimated by anyone who knows the length of the highway. The person will use some prior knowledge of the distance and the speed limit allowed on the road. This knowledge can then be used to determine the time it will take simply by dividing the distance by the speed. It is obvious that the distance and the speed limit of the road to traverse have to be known in order to determine the minimum time it will take to traverse the road.

Our approach to solving this problem is based on qualitative modeling. Intelligent agents can use qualitative models to reason about quantities without having to resort to the nitty-gritty of mathematics and calculi. A particular approach that is powerful in this regard is that of discretization. In discretization, quantities are divided into chunks, and the solutions to our problems can be deduced from the solutions to the smaller versions of the problem. For example, if an agent being present at location l_1 at time t_1 implies he or she can be in location l_2 at a later time t_2, and an agent being at location l_2 at time t_2 implies he can be at location l_3 at a later time t_3 and l_3 is farther from l_1 than l_2, then *x* being present at l_1 at time t_1 implies *x* can be present at l_3 at time t_3. In other words the location, l_3 is *reachable* for the agent from l_1 within the time interval (t_1, t_3). The following basic definitions make up our qualitative logic for spatial qualification.

3.2.1Basic Definitions

Let *l* be a location (region) in space and l_1 a different location in space. Then following the definitions of the RCC-8 relations [18, 19, 20, 7], which is based on the region connection relation, for the definition of the eight disjoint pair of relations, we can go ahead to define the *Regionally_part_of* and the *Regionally_disjoint* relations as follows.

Def1: $\forall l,l_1$ $Regionally\_part\_of(l,l_1) \equiv EQ(l,l_1) \vee TPP(l,l_1) \vee TPP(l_1,l) \vee NTPP(l,l_1) \vee NTPP(l_1,l)$

Def2: $\forall l,l_1$ $Regionally\_disjoint(l,l_1) \equiv DC(l,l_1) \vee EC(l,l_1) \vee PO(l,l_1)$

The ability to reason with prior knowledge and tell of the possibility of an agent to be present at a location at a certain time is strongly dependent on the reachability of the two locations involved. This reachability axiom is built around regional connections of locations defined above.

3.2.2Persistence of Truth

Our logic treats any known fact as something that remains permanently true. As such if we know that an agent is present at a location *l* at time *t*, then that fact is always true.

For every agent x present at location *l* at time *t*, it implies that it is necessarily true that every agent *x* is present at location *l* at a certain time *t*.

$$T_{A1}: \quad \forall x.\ l.\ t. Present\_at(x,l,t) \Rightarrow \Box Present\_at(x,l,t)$$

3.2.3Possibility of Location Persistence

For every agent x present at location l at some time t, it implies that it is possible that the same agent is present at that location at a later time t_1.

$$T_{A2}: \quad \forall x.\ l.\ t.\ Present\_at(x,l,t) \Rightarrow (\exists t_1.\ t < t_1 \Rightarrow \Diamond Present\_at(x,l,t_1))$$

3.2.4Definition of *Reachability*

Now, defining what it means for an agent *x* to be able to reach location l_2 from l_1 in the interval (t_1, t_2) is given thus.

$$T_{A3}: \quad \forall x, l_1, l_2, t_1, t_2.\ Reachable(x, l_1, l_2, (t_1, t_2)) \Leftrightarrow (t_1 < t_2 \wedge (Present\_at(x, l_1, t_1) \Rightarrow \Diamond Present\_at(x, l_2, t_2)))$$

3.2.5Reachability is Reflexive

A location is reachable from itself for any agent within any interval of time no matter how small.

$$T_{A4}: \quad \forall x,l_1,l_2,t_1,t_2.\ l_1=l_2 \wedge t_1<t_2 \Rightarrow Reachable(x,l_1,l_2,(t_1,t_2))$$

3.2.6Reachability is Commutative

Generally, if one can reach l_2 from l_1 in a time interval, then it is possible to achieve a reverse of that feat within the same interval.

$$T_{A5}: \quad \forall x, l_1, l_2, t_1, t_2.\ Reachable(x, l_1, l_2, (t_1, t_2)) \Leftrightarrow Reachable(x, l_2, l_1, (t_1, t_2))$$

3.2.7Reachability depends on duration of time interval

Here is the definition of a property for the notion of being reachable. If it is possible for an agent to reach one location from another, it should still be possible for the same agent to perform the same feat within any interval of similar or longer duration.

$$T_{A6}: \quad \forall x, l_1, l_2, t_1, t_2. \quad Reachable(x, l_1, l_2, (t_1, t_2)) \wedge \forall t_3, t_4.\ t_3 < t_4 \wedge (t_4 - t_3) \geq (t_2 - t_1) \Rightarrow Reachable(x, l_1, l_2, (t_3, t_4)))$$

3.2.8Possibility of presence in regions at same time

The possibility of an agent to be present at two different locations at the same time can be determined by the topological relationship between the two locations. For every agent x said to be in location l at time t and also at location l_1 at the same time and the locations are regionally part of each other, it then implies that it is the case that the agent is present at both locations at the same time.

$$T_{A7}: \quad \forall x, l, l_1, t.\ (Present\_at(x, l, t) \wedge Regionally\_part\_of(l, l_1)) \Rightarrow (Present\_at(x, l_1, t)$$

3.2.9Persistence within regions

If an agent is at a certain location then for some time afterwards, the agent will be within some region surrounding the location.

$$T_{A8:} \quad \forall x, l, t.\ Present\_at(x, l, t) \Rightarrow \exists r, t_1.\ NTPP(l, r) \wedge Present\_at(x, r, t+t_1)$$

3.2.10Absence

For every agent x said to be in location l at time t and also at location l_1 at the same time and the locations are regionally disjoint, it then implies that it is not possible for the agent to be present at both locations at the same time.

$$T_{A9}: \quad \forall x, l, l_1, t.\ (Present\_at(x, l, t) \wedge Regionally\_disjoint(l, l_1)) \Rightarrow \neg\Diamond(Present\_at(x, l_1, t)$$

3.2.11 Reachability is transitive

For every agent x present at location l_1 at time t_1, it is possible for it to be at location l_2 at another time t_2. Also, being at location l_2 means it is possible for it to be at another location l_3 at time t_3' and the distance between l_1 and l_2 is smaller than the distance between l_1 and l_3 and t_1 is also less than t_2 then it implies that it is possibly true that the agent at location l_1 at time t_1 is at location l_3 at time t_3.

$$T_{A10}: \quad \forall x, l_1, l_2, l_3, t, t_2, t_3. \quad Reachable(x, l_1, l_2, (t, t_2)) \wedge \quad Reachable(x, l_2, l_3, (t_2, t_3)) \Rightarrow Reachable(x, l_1, l_3, (t, t_3)).$$

The axioms presented here are able to infer reachability when it is true. Otherwise they are not able to make the inference. In other words reachability is only semi-decidable. In order to make it decidable, we need a closure for the reachability concept.

Logic of spatial qualification must be able to reason about the presence of individuals at different locations. It is possible to view the problem of spatial qualification as the problem of reasoning about the accessibility of worlds. Each world contains a log of who is at what location.

3.3Formal Semantics of the Spatial Qualification Model

The SQM is built around a Kripke modal frame [21] which is the triple $<W, R, D>$ where W is a set of possible worlds, R is the accessible relation between pairs of worlds, and D is a definite domain from which individuals in the worlds are drawn. Our logic contrasts with Fitting's quantified modal logic [14], in which there is a domain function D associated with the modal frame such that the function D is defined for each world and returns a unique domain associated with that world. One may treat our modal frame as a special case of Fitting's modal frame, in which the domain function D is a constant function.

We assume the existence of an Interpretation function I which interprets constant and predicate symbols for each world. The function I maps each constant symbols to specific individuals in some specific world. The expression $I[c, w_1]$ denotes the application of the interpretation function I on the constant symbol c in the world w_1. All constant symbols are interpreted uniformly in all worlds. So that for any two worlds w_1 and w_2 from W: $I[c, w_1] = I[c, w_2]$. The function I also maps each n-ary predicate symbols to an appropriate n-ary relation in some appropriate world. For example the interpretation of *Present_at* $I[Present\_at, w_1]$ refers to the actual ternary relation that the predicate *Present_at* refers to in the world w_1. It is important to note that in any world $w \in W$:

$$I[Present\_at, w] \subseteq A \times L \times T$$

where A is the set of all agents, L is the set of all locations and T is time points.

Thus, we have a model M which is a 4-tuple $<W, R, D, I>$ and comprises the modal structure introduced earlier and the interpretation function, I. Let us denote the model by $M,w \models \varphi$, the fact that formula φ is true in a world w of the model M. Thus, the following statements hold for *Present_at* as well as for any other predicate.

$M, w \models Present\_at(Paul, Airport, Noon)$ *if and only if*
$\quad (I[Paul, w], I[Airport, w], I[Noon, w]) \in I[Present\_at, w]$
$M, w \models \Diamond Present\_at(Paul, Airport, Noon)$ *if and only if*
$\quad$ *For some* w_1 *such that* $(w, w_1) \in R$ *it is the case that:*
$\qquad (I[Paul, w_1], I[Airport, w_1], I[Noon, w_1]) \in I[Present\_at, w_1]$
$M, w \models \Box Present\_at(Paul, Airport, Noon)$ *if and only if*
$\quad$ *For every* w_1 *such that* $(w,w_1) \in R$ *it is the case that:*
$\qquad (I[Paul, w_1], I[Airport, w_1], I[Noon, w_1]) \in I[Present\_at, w_1]$
$M, w \models \neg Present\_at(Paul, Airport, Noon)$ *if and only if*
$\qquad (I[Paul, w], I[Airport, w], I[Noon, w]) \notin I[Present\_at, w]$
$M, w \models Present\_at(Paul, Airport, Noon) \wedge$
$\quad Present\_at(Paul, Swimming\text{-}pool, Noon)$ *if and only if*
$\quad (I[Paul, w], I[Airport, w], I[Noon, w]) \in I[Present\_at, w]$ *and*
$\quad (I[Paul, w], I[Swimming\text{-}pool, w], I[Noon, w]) \in I[Present\_at, w]$

$M, w \models Present\_at(Paul, Airport, Noon) \vee$
$Present\_at(Paul, Swimming\text{-}pool, Noon)$ *if and only if either*
$(I[Paul, w], I[Airport, w], I[Noon, w]) \in I[Present\_at, w]$ *or*
$(I[Paul, w], I[Swimming\text{-}pool, w], I[Noon, w]) \in I[Present\_at, w]$

In order to be able to interpret variables we need a valuation function such that *v* has the signature:

$$v: V \rightarrow D$$

where *V* is the set of all variables and *D* is our domain of individuals. It is important to note here that valuations do not depend on the world. Thus, in order to strengthen the interpretation function to deal with variables, we redefine the interpretation function as *Iv* so that for any item *t*:

$$Iv[t] = \begin{cases} \text{v(t)} & \text{if t is a variable} \\ \text{I(t)} & \text{otherwise} \end{cases}$$

Then, the model is now redefined as a *<W, R, D, Iv>* where *<W, R, D>* is our Kripke frame defined earlier. Thus, we can redefine what it means for propositions to be true in a world under our model for different terms x, l, l_1 and t:

$M, w \models Present\_at(x, l, t)$ *if and only if*
$(Iv[x, w], Iv[l, w], Iv[t, w]) \in Iv[Present\_at, w]$
$M, w \models \Diamond Present\_at(x, l, t)$ *if and only if*
For some w_1 *such that* $(w,w_1) \in R$ *it is the case that:*
$(Iv[x, w_1], Iv[l, w_1], Iv[t, w_1]) \in Iv[Present\_at, w_1]$
$M, w \models \Box Present\_at(x, l, t)$ *if and only if*
For every w_1 *such that* $(w,w_1) \in R$ *it is the case that:*
$(Iv[x, w_1], Iv[l, w_1], Iv[t, w_1]) \in Iv[Present\_at, w_1]$
$M, w \models \neg Present\_at(x, l, t)$ *if and only if*
$(Iv[x, w], Iv[l, w], Iv[t, w]) \notin Iv[Present\_at, w]$
$M, w \models Present\_at(x, l, t) \wedge Present\_at(x, l_1, t)$
if and only if $(Iv[x, w], Iv[l, w], Iv[t, w])$ *and*
$(Iv[x, w], Iv[l_1, w], Iv[t, w]) \in Iv[Present\_at, w]$
$M, w \models Present\_at(x, l, t) \vee Present\_at(x, l_1, t)$ *if and only if either*
$(Iv[x, w], Iv[l, w], Iv[t, w])$ *or*
$(Iv[x, w], Iv[l_1, w], Iv[t, w]) \in Iv[Present\_at, w]$

Finally, the interpretation of the quantifiers is presented. The universal quantifier is interpreted such that variables can take values from the worlds.

$M, w \models \forall x. P(x)$ *if and only if for every possible valuation that can be given to x in the world w through Iv, it is the case that* $(Iv[x, w]) \in Iv[P]$

Similarly, the existential quantifier is interpreted thus:

$M, w \models \exists x. P(x)$ *if and only if there is a possible valuation such that can be given to x in the world w through Iv, it is the case that* $(Iv[x, w]) \in Iv[P]$

It is important to emphasize that our Model is based on worlds in which the domains remain constant as opposed to worlds in which domains increase or decrease. As such, the following Barcan's axioms hold

$$\Box \forall x.\ P(x) \leftrightarrow \forall x.\ \Box P(x).$$

3.4Modal Properties of the Spatial Qualification Model

A logic of presence like ours exhibits the basic property of Kripke's minimal system, **K** along with every other property of the standard **S4** system: These properties are:

K: $\Box(\phi \Rightarrow \psi) \Rightarrow (\Box\phi \Rightarrow \Box\psi)$

T: $\Box\phi \Rightarrow \phi$

4: $\Box\psi \Rightarrow \Box\Box\psi$

However it falls short of being an S5 system because it does not satisfy the following property:

B: $\Diamond\Box\phi \Rightarrow \phi$

If we consider the propositions formed from the Present-at relations, we can argue that axioms K, T and 4 hold. For example note that it is the case that if l is regionally part of l_1, then any individual that is present in the location l is also present at location l_1.

$$\forall x, l, l_1, t\ (NTPP(l, l_1) \vee TPP(l, l_1) \vee l_1 = l) \Leftrightarrow$$
$$(Present\text{-}at(x, l, t) \Rightarrow Present\text{-}at(x, l_1, t))$$

Thus, the following clearly hold:

$$KP1 \quad \forall x, l, l_1, t.\ \Box(Present\_at(x, l, t) \Rightarrow Present\_at(x, l_1, t)) \Rightarrow$$
$$(\Box Present\_at(x, l, t) \Rightarrow \Box Present\_at(x, l_1, t))$$

Similarly, note that x_1 is always collocated with x if and only if x_1 is part of x. This axiom is stated as:

$$\forall x, x_1\ Part\text{-}of(x_1, x) \Leftrightarrow \forall l, t\ (Present\text{-}at(x, l, t) \Rightarrow Present\_at(x_1, l, t))$$

Therefore, it is the case that:

$$KP2 \quad \forall x, x_1, l, t.\ \Box(Present\_at(x, l, t) \Rightarrow Present\_at(x_1, l, t)) \Rightarrow$$
$$(\Box Present\_at(x, l, t) \Rightarrow \Box Present\_at(x_1, l, t))$$

In another vein, the fact that a body is in a certain location at time t can imply that the same body is in a different location at a later time, if the body is in some kind of constant and predictable motion such as the case of planetary bodies, that is if its trajectory is fixed. As such:

$$\forall x, x_1, l, l_1, t.\ Fixed\_Trajectory(x)$$
$$\Leftrightarrow \quad Not\_PP(l, l_1) \wedge Not\_PP(l_1, l) \wedge$$
$$Present\_at(x, l, t) \Rightarrow Present\_at(x, l_1, t)$$

Thus, if a body x is always in a fixed trajectory, it must be the case that:

$$\forall x, x_1, l, l_1, t.\ \Box(Present\_at(x, l, t) \Rightarrow Present\_at(x, l_1, t))$$

$$\Rightarrow (\Box Present\_at(x, l, t) \Rightarrow \Box Present\_at(x_1, l, t))$$

Axioms *KP1* and *KP2* show that our system conforms to the properties of the Kripke minimal system. In another vein the only way a particular presence log i.e. the fact that *x* is present at a location *l* at time *t*, can occur in all possible worlds reachable from the current world if that presence log already occurs in the current world.

$$TP \qquad \forall x, l, t.\ \Box Present\_at(x, l, t) \Rightarrow Present\_at(x, l, t)$$

Similarly, the fact that a presence log holds in all the worlds accessible from the current world implies it will be true in all worlds accessible from those worlds accessible from the current world.

$$4P \qquad \forall x, l, t.\ \Box Present\_at(x, l, t) \Rightarrow \Box\Box Present\_at(x, l, t)$$

Axioms *KP1, KP2, TP* and *4P* all show that the logic of presence we describe here constitutes an **S4** system of axioms.

4. CONCLUSION

The modalities introduced to our statements make assertion about the mode of truth of the statement about where or how the statements are true or the circumstances under which the statements may be true but not when the statements are true. Time is explicitly expressed in our model.

The issue of vagueness of space occupied by an individual and object is ignored in this paper mainly because the individuals or object whose spatial qualification we reason about occupy very little space compared with region of space that we are interested in. As such, Galton and Hood's anchoring relations may only be useful here when we need to make inferences about relations among regions.

In the definition of our SQM, we noticed that possible world *W* has all the properties of History, *H* in it with the presence of the accessibility relation and valuation function as a plus. It is the presence of the valuation function in our model that allows us to determine the possibility and the impossibility of an agent's presence in space at a certain time via the accessibility relations based on the historic set of possible worlds. The possibility of being present at a location remains valid even when a state at some time points, seen to be possibly true is not actually true. Our model of time is a branching model of time. It is linear in the past and branches into the future. Within each world there is a linear model of time that branches into different accessible worlds in the future.

There are two major applications that have been identified for spatial qualification reasoning in this paper. One is alibi reasoning which involves reasoning about the possibility of an agent's presence at a crime scene given what we know about the agent's antecedents. The other application has to do with plan reasoning. A proof system for the SQM system is being developed. One possible extension of the current work is towards collaborative spatial qualification reasoning, so that a reasoner can depend on other agents to help reach its conclusion.

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