

Hybrid Fuzzy Sliding Mode Controller for Time-delay System

N. K. Yadav¹ and R. K. Singh²

¹Research Scholar Department of Electrical Engineering, Motilal Nehru National Institute of Technology (M.N.N.I.T.)

Allahabad-211004, India

EmailId:nyadav24@gmail.com

² Prof. Department of Electrical Engineering, Motilal Nehru National Institute of Technology (M.N.N.I.T.)

Allahabad-211004, India

ABSTRACT

This paper is concerned with the problems of stability analysis and stabilization control design for a class of discrete-time T-S fuzzy systems with state-delay for multi-input and multi-output. The nonlinear fuzzy controller helps to overcome the problems of the ill - defined model of the systems, which are creating the undesirable performance. . Here sliding surface is being designed for error function of nonlinear system and sliding mode control is being designed here. The switching surface is being proven for its asymptotic stability. The generated error signal and change of error signal will be utilized for application heuristic knowledge to design the rule base in the fuzzy logic control and fuzzy logic controller is designed here. The proposed technique also brings in a systematic approach to the fuzzy logic control, thus overcoming lots of heuristics that were in vogue with earlier fuzzy logic applications. Fuzzy logic control has been applied to a second order model of a roll autopilot. It has been found that the proposed scheme is robust and works satisfactorily even when parameters are perturbed as much as fifteen percent of their geometric mean value. This designed algorithm will be more effective for highly unstable nonlinear systems such as aerospace system.

KEYWORDS:

Discrete-time system, fuzzy control, sliding surface, sliding mode control, nonlinear time-delay nonlinear, stability.

1. INTRODUCTION

Nonlinear uncertain systems with time-delays are existing in various engineering networks-based control systems, mechanical system, chemical systems, communication systems, applications [1-3]. Mechatronics system is such kind of systems which consist of electro-subsystem and mechanical subsystem in cascade form. Possible sources of time-delays are the process may involve the transportation of materials or fluids over long distances; the measuring devices may be subject to the long delay in providing a measurement; and the final control element may need some time to develop the actuating signal. Its existence is frequently a source of instability and poor performance. The time-delay systems have received considerable attention over recent years. In recent years, the study of sliding mode control (SMC) technique is more suitable for time-delay system stability analysis. The delays present in a system can be broadly categorized into three categories; input delay, state delay and output delay. The input delay is caused by the transmission of control signal over a long distance. State-delay is a result of transmission or transport delay among interacting elements in a dynamic system. The output delay is the delay resulting from sensors.

Many papers have considered the problem of SMC for classes of uncertain system with time-delay [4-8]. As a general design tool for robust control system design, SMC has been well established for a wide range of nonlinear systems in both continuous-time and discrete-time. It has some attractive features such as a fast response and good transient performance. The silent advantages include: (i) robust against a large class of perturbations or model uncertainties; and (ii) control of certain classes of complex nonlinear systems that are otherwise difficult to achieve [9]. A discrete version of the SMC is important when the implementation of the control is reached digitally using a relatively low sampling period. It is worth pointing out that discrete SMC cannot be obtained from the continuous counterpart by means of sampling equivalence.

Zadeh introduced the fuzzy set theory in 1965 (*Zadeh, 1965*) [23], it has received much attention from various fields and has also demonstrated nice performance in various applications. One of those successful fuzzy applications is to model unknown nonlinear systems by a set of fuzzy rules. On the other hand, most industrial plants have severe nonlinearities, which lead to additional difficulties in the analysis and design of control systems. In the past few years, the control technique based on the so-called K. Takagi-M. Sugeno (T-S) fuzzy model [10] has attracted lots of attention [11-16], since it is regarded as a powerful solution to bridge the gap between the fruitful linear control and the fuzzy logic control targeting complex nonlinear systems. The common practice is as follows, first, the T-S fuzzy model is employed to represent or approximate a nonlinear system. This fuzzy model is described by a family of fuzzy IF-THEN rules that represent local linear input-output relations of the system. The overall fuzzy model of the system is achieved by smoothly blending these local linear models together through membership functions. Then, based on this fuzzy model, a control design is developed to achieve stability and performance for the nonlinear system. The stability issue fuzzy control systems has been discussed in an extensive literature, i.e. [11-13], [15-16], [20]. Most of the existing results were usually derived by using a single Lyapunov function (SLF) method, i.e. [11-13]. However, the main drawback associated to this method is that an SLF must work for all linear models, which in general leads to conservative results. To relax this conservatism, recently, the piecewise Lyapunov function approach [13] and the fuzzy Lyapunov function approach [14] have been proposed.

In recent years, based on the T-S fuzzy model, an intensive study on the stability issue of nonlinear time-delay systems have been made, and several approaches have been proposed, i.e. The Lyapunov-Krasovskii functional (LKF) [17-22]. The stability analysis and stabilization problems for discrete-time T-S fuzzy systems with state delay are dealt with by using a fuzzy LKF approach. A new fuzzy LKF is constructed from a delay dependent stability analysis of open-loop systems, which can reduce the conservatism of using a non-fuzzy LKF. The paper has been organized as follows. The problem statement has been described in Section II. In Section III, sliding surface is being constructed. Section IV illustrates the designing of the fuzzy rule base and simulation results are given in Section V to illustrate the effectiveness of the proposed method. Finally, conclusions are drawn in Section VI.

2. PROBLEM STATEMENTS AND DESCRIPTION:

Consider the nonlinear state delay system as follows

$$\begin{aligned} x(k+1) &= f(x(k), u(k), k, h) \\ x(k+1) &= Ax(k) + A_d x(k-h) + Bu(k) \end{aligned} \quad (1)$$

where $x(t) \in \mathfrak{R}^n$, $x(t-h) \in \mathfrak{R}^n$ and $u(t) \in \mathfrak{R}^m$ are the system state and control input vector.

$A \in \mathfrak{R}^{n \times n}$, $A_d \in \mathfrak{R}^{n \times n}$ and $B \in \mathfrak{R}^{m \times n}$ are the state matrix and control vector respectively and they are with appropriate dimensions and parameter $h > 0$ is a constant and time-delay.

Assumption: Pair (A, B) is controllable.

3. DESIGN OF SWITCHING SURFACE

In general, the design of discrete-time SMC consists of two steps. The first step is to design a switching surface so that in the quasi-sliding mode system response acts like the desired dynamics. The second step is to design the control law in order that the quasi-sliding mode is reached and stays for all time. In the article, the switching surface is designed as follows:

If the desirable trajectory is $x_d(k)$, then the error dynamics can be generated as below

$$e(k) = x(k) - x_d(k) \quad (2)$$

$$S(k) = Ge(k) = G[x(k) - x_d(k)] \quad (3)$$

Sliding mode condition

$$S(k+1) = S(k) = 0 \quad (4)$$

$$S(k+1) = S(k) = Ge(k) = 0$$

$$S(k+1) = Ge(k+1) = 0$$

$$e(k) = [e_1(k) \quad e_2(k)] \quad (5)$$

$$S(k) = G_1 e_1(k) + G_2 e_2(k) = 0$$

$$e_2(k) = -G_2^{-1} G_1 e_1(k) \quad (6)$$

$$K = G_2^{-1} G_1$$

$$e_2(k) = -K e_1(k)$$

where $G_2^{-1} G_1$ are the designing parameters.

$$e_1(k+1) = Ae(k) + A_d e(k-h) + B_2 u(k)$$

$$e_1(k+1) = A_{11} e_1(k) + A_{12} e_2(k) + A_{d11} e_1(k) + A_{d12} e_2(k) \quad (7)$$

$$e_2(k+1) = A_{11} e_1(k) + A_{12} e_2(k) + A_{d11} e_1(k) + A_{d12} e_2(k) + B_2 u(k)$$

$$e(k+1) = x(k+1) - x_d(k+1) \quad (8)$$

$$S(k+1) = Ge(k+1) = G[x(k+1) - x_d(k+1)] = 0$$

Substitute the (6) in (7)

$$e_1(k+1) = A_{11} e_1(k) - KA_{12} e_1(k) + A_{d11} e_1(k) - KA_{d12} e_1(k-h)$$

$$e_2(k+1) = A_{11} e_1(k) - A_{12} K e_1(k) + A_{d11} e_1(k) - KA_{d12} e_1(k-h) \quad (9)$$

$$+ B_2 u(k)$$

$$e_1(k+1) = (A_{11} - A_{12} K) e_1(k) + (A_{d11} - A_{d12} K) e_1(k-h)$$

where

$$M = (A_{11} - A_{12} K) \quad (10)$$

$$M_d = (A_{d11} - A_{d12} K)$$

For stability of the system all the eigenvalues of $M = (A_{11} - A_{12} K)$ and $M_d = (A_{d11} - A_{d12} K)$ must lie inside the unit disc.

$$q(k) = A_{11} x_{1d}(k) + A_{12} x_{2d}(k) - x_{1d}(k+1) \quad (11)$$

equation (11) can be written as zero because

$$A_{11} x_{1d}(k) + A_{12} x_{2d}(k) - x_{1d}(k+1) = 0 \quad (12)$$

therefore $q(k) = 0$

$$G[Ax(k) + A_d x(k-h) + Bu(k) - 0] = 0 \quad (13)$$

$$u(k) = -(GB)^{-1}[GAx(k) + GA_d x(k-h)] \quad (14)$$

4. FUZZY LOGIC CONTROLLER

Although fuzzy control is very successful, especially for control of nonlinear systems. There is a drawback in the designs of such controllers with respect to performance and stability. The success of fuzzy controlled plants stems from the fact that they are similar to the sliding mode control, which is an appropriate robust control method for a specific class of nonlinear systems. The fuzzy logic system performs a mapping from $U \subset \mathfrak{R}^n$ to $V \subset \mathfrak{R}$. Let $U = U_1 \times \dots \times U_n$ where $U_i \subset \mathfrak{R}$, $i = 1, 2, 3, \dots, n$. The fuzzifier maps a crisp point in U into a fuzzy set in U . The fuzzy rule base consists of a collection of fuzzy IF-THEN rules:

$R^{(l)}$: IF $e_1(k)$ is F_1^l , and $e_2(k)$ is F_2^l , and \dots , $e_n(k)$ is F_n^l ,
 THEN y_j is G_j^l , $j = 1, 2, \dots, p$

4.1 Design of proposed fuzzy controller

The error between model states and plant state is one of the input variables and is partitioned into Fuzzy sets NL, NS, PS and PL. Similarly the error between second state of plant and model is another input variable, which is also partitioned into fuzzy sets. The rule bases for the reference model with fuzzy controller can be thought to be a two dimensional matrix as indicated in figures. The rows represent various linguistic values that can be assigned to the first input variable i.e. error between first state of plant and desired state. Similarly the columns represent linguistic values assigned to difference between second state of plant and desired state. The entries in the matrix are linguistic variables that represent control action. However, if error is PB and change in error is NS, then the control needs to be large and additive to the earlier output so that operating point moves closer to the reference point (i.e. U is PB). In another case when error is zero and error derivative is both PB or PS it indicates plant state is moving away from reference state and control output needs to be positive to arrest this trend and make plant state move towards reference state. On the other hand if error is zero and change in error is NB or NS, then a negative change is required in the control output to arrest the trend and force plant state towards reference trajectory state. All the fuzzy rules for above controller can be designed by extrapolating this logic to all possible combinations of linguistic variables which are represented by various entries in the matrix. Thus we obtain fuzzy rule base as stated below for a second order system. The inferencing method used is Mamdani and membership function for input and output variable is as shown in Fig 1. Fuzzy rules are used to formulate control laws in a transparent human oriented fashion, using the linguistic variables defined by the membership functions. For each Mamdani controller, $4 \times 4 = 16$ fuzzy rules need to be defined to cover all possible combinations of the linguistic variables of the two inputs.

If (Process state) then (control output)

4.2 Structure of Fuzzy Controller

There are specific components characteristic of a fuzzy controller to support a design procedure, In the block diag. The controllers between a pre-processing block and a post-processing block and The basic function of the rule base is to represent in a structured way the control policy of an experienced process operator.

If (Process state) then (Control Output)

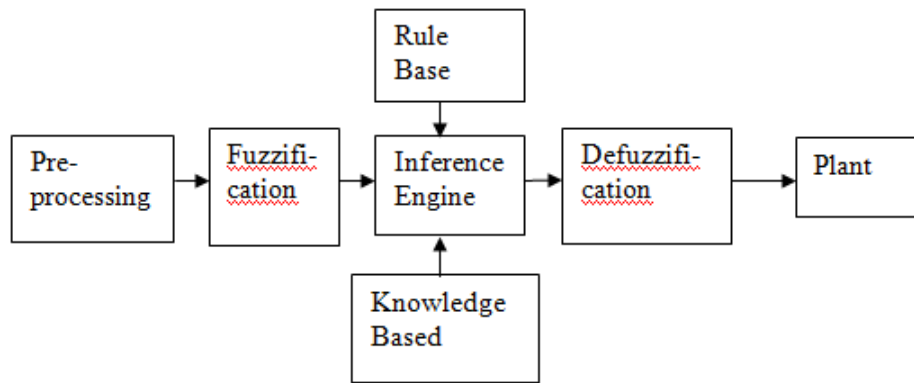


Fig. 1 Blocks Structure of Fuzzy Logic Controller

4.3 Table No. 1 for Rule Base of Inputs and Output

E / CE	NB	NS	PS	PB
NB	NB	NB	NS	NS
NS	NB	NB	NS	PS
PB	NS	PS	PS	PB
PB	NS	PS	PB	PB

4.4 Design of membership functions

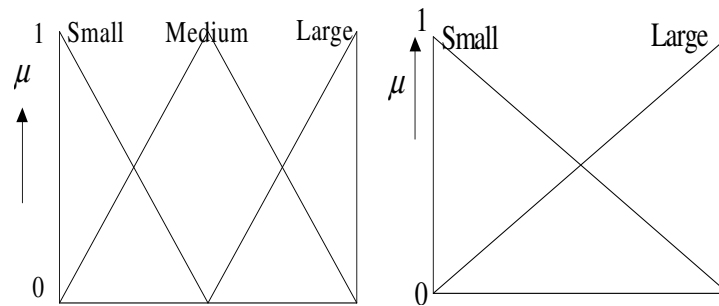


Fig. 2 Membership function

Input / output variables

5. SIMULATOR RESULTS

5.1 Error signal

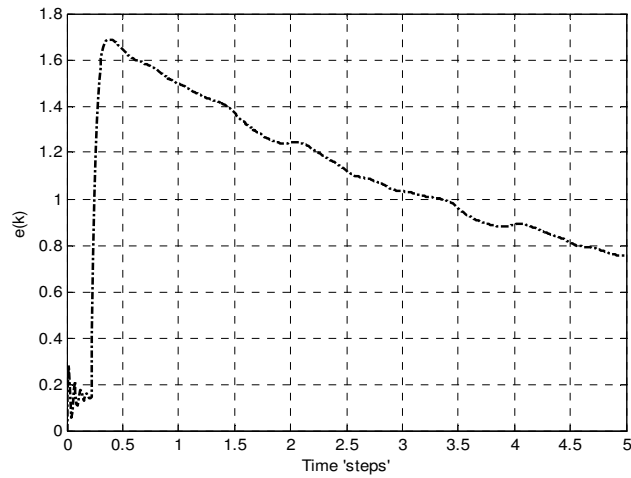


Fig. 3 Error signal of the system

5.2 Change in error signal

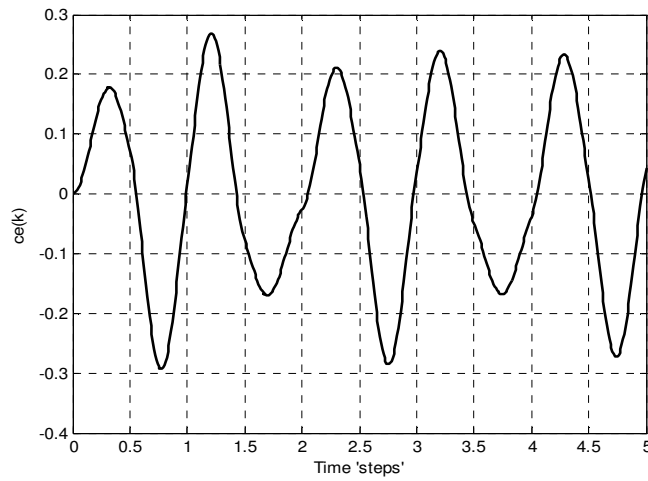


Fig.4 Change in Error signal of the system

5.3 Stable Sliding Surface

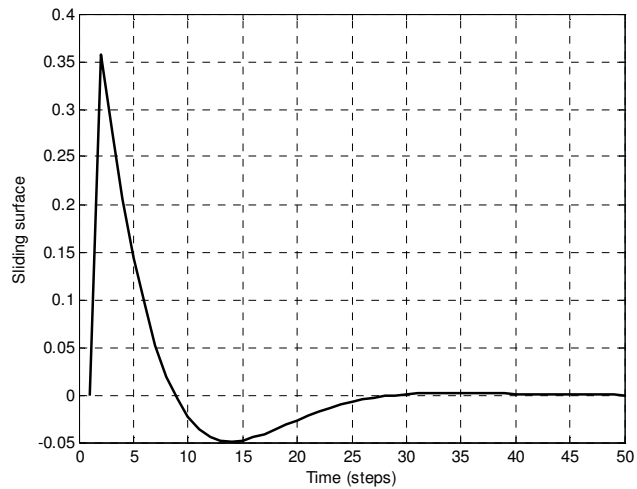


Figure.5 Stable Sliding surface

5.4 Stable output response by fuzzy controller

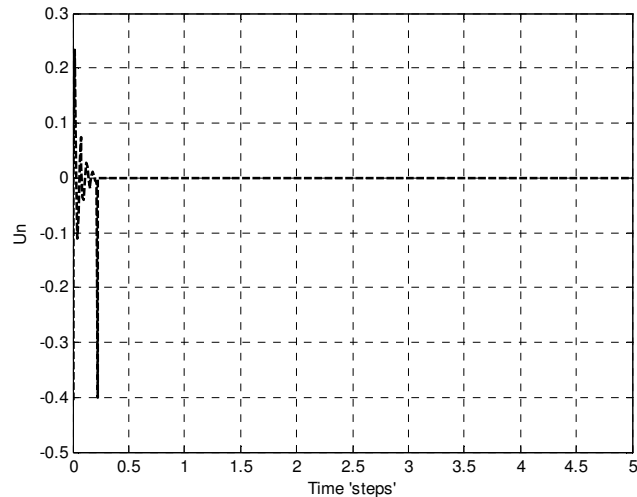


Fig. 6 Stable output response by proposed robust controller

6. CONCLUSIONS

This paper proposes the new fuzzy based sliding mode controller technique for nonlinear control systems, which are highly unstable in practice. Therefore stability problem of nonlinear systems become, the major issues. Here the sliding surface is designed on error dynamics with reference trajectory, which is a robust control technique and drawbacks of sliding mode controller are being reduced by designing of fuzzy sliding mode controller. This drives the state trajectory in the stable form on a sliding surface. The real world systems are quite complex nature and they are ill defined model and the classical methods are quite cumbersome to generate the desirable stable performances by the system. This new hybrid controller is made on the intelligence heuristic

method. The proposed algorithm is more compatible even if in the variation of environmental conditions or any other type of uncertainties. The simulation results are showing stable output responses when the parameters are perturbed. This newly designed controller will be more effective for highly unstable systems like aerospace systems.

ACKNOWLEDGEMENTS

The author would like to thanks to Motilal Nehru National Institute of Technology (M.N.N.I.T.) Allahabad, which provided the highly conducive research environment.

REFERENCES

- [1] J. K. Hale and S. M. Luned Verduyn, "Introduction to functional differential equations", Springer, New York, 1993.
- [2] D. Yue, Q. L. Han and C. Peng, "State feedback controller design of networked control systems", IEEE Trans. Circuits System II Express brief, 2004, 51, (11), pp. 640-644.
- [3] M. S. Mahmoud, "Robust control and filtering for time delay systems", Marcel-Dekker, New York, NY, 2000.
- [4] Shyu K. and Yan J. , "Robust stability of uncertain time-delay systems and its stabilization by variable structure control", Int. Jour. Control, vol. 57, pp. 237-246, 1993.
- [5] Xia Y., and Jia Y., "Robust sliding mode control of uncertain time-delay systems: an LMI approach", IEEE Transaction Automatic Control, vol. 46, no.6, pp. 1086-1092, 2003.
- [6] Shi P., Xia Y., Liu G. P., and Rees D., "On designing of sliding mode control for stochastic jump systems" IEEE Transaction Automatic Control, vol. 51, no. 1, pp. 97-103, 2006.
- [7] Gao W., Wang Y., and Homaifa A, "Discrete-time variable structure control systems", IEEE Transactions Industrial Electronics, vol. 42, no. 2, pp. 117-122, 1995.
- [8] Utkin V. I., "Variable structure system with sliding modes", IEEE Transaction Automatic Control, AC-22, pp. 212-222, 1977.
- [9] Itkis U. "Control systems of variable structure", John Willey and Sons, New York, 1976.
- [10] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," IEEE Trans. Syst., Man, Cybern., vol. SMC-15, no. 1, pp. 116-132, Jan./Feb. 1985.
- [11] K. Tanaka and H. O. Wang, Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach. New York: Wiley, 2001.
- [12] H. N. .Wu, "Reliable LQ fuzzy control for nonlinear discrete-time systems via LMIs," IEEE Trans. Syst., Man, Cybern. B, Cybern., vol. 34, no. 2, pp. 1270-1275, Apr. 2004.
- [13] G. Feng, "Stability analysis of discrete-time fuzzy dynamic systems based on piecewise Lyapunov functions," IEEE Trans. Fuzzy Syst., vol. 12, no. 1, pp. 22-28, Feb. 2004.
- [14] K. Tanaka, T. Hori, and H. O. Wang, "A multiple Lyapunov function approach to stabilization of fuzzy control systems," IEEE Trans. Fuzzy Syst., vol. 11, no. 4, pp. 582-589, Aug. 2003.
- [15] T. M. Guerra and L. Vermeiren, "LMI-based relaxed non-quadratic stabilization conditions for nonlinear systems in the Takagi-Sugeno's form," Automatica, vol. 40, no. 5, pp. 823-829, 2004.
- [16] W. J. Wang and C.-H. Sun, "A relaxed stability criterion for T-S fuzzy discrete systems," IEEE Trans. Syst., Man, Cybern. B, Cybern., vol. 34, no. 5, pp. 2155-2158, Oct. 2004.
- [17] P. Park, S. S. Lee, and D. J. Choi, "State-feedback stabilization for nonlinear time-delay systems: A new fuzzy weighting-dependent Lyapunov-Krasovskii functional approach," in Proc. IEEE Conf. Decision and Control, Maui, HI, 2003, pp. 5233-5238.
- [18] Y.-Y. Cao and P. M. Frank, "Stability analysis and synthesis of nonlinear time-delay systems via linear Takagi-Sugeno (T-S) fuzzy models," Fuzzy Sets Syst., vol. 124, no. 2, pp. 213-229, Dec. 2001.
- [19] R. J. Wang, W. W. Lin, and W. J. Wang, "Stability of linear quadratic state feedback for uncertain fuzzy time-delay system," IEEE Transaction Syst., Man, Cybern. B, Cybern., vol. 34, no. 2, pp. 1288-1292, Apr. 2004.
- [20] Z. Yi and P. A. Heng, "Stability of fuzzy control systems with bounded uncertain delays," IEEE Trans. Fuzzy Syst., vol. 10, no. 1, pp. 92-97, Feb. 2002.
- [21] S. Zhou and T. Li, "Robust stabilization for delayed discrete-time fuzzy systems via basis-dependent Lyapunov-Krasovskii function," Fuzzy Sets Syst., vol. 151, no. 1, pp. 139-153, Apr. 2005.

- [22] Y.-Y. Cao and P. M. Frank, "Analysis and synthesis of nonlinear time-delay systems via fuzzy control approach," IEEE Trans. Fuzzy Syst., vol. 8, no. 2, pp. 200–211, Apr. 2000.
- [23] L. Zadeh, "Fuzzy sets, Information Control", vol. 8, no. 1, pp. 338-353, 1965.

AUTHORS

Biographies

N. K. Yadav (S'09) was born in Etawah, Uttar Pradesh, India, on April 02, 1969. He was graduated with degree in Electronics and Telecommunication Engineering from Institution of Engineers (India), Calcutta, India in 1992, MBA (Marketing & Personnel) from University of Pune, India 1997 & 1999 respectively, M. Tech. degree in Electrical Engineering with specialization in Control and Instrumentation (Control & Instrumentation) from Motilal Nehru National Institute of Technology (MNNIT), (Deemed University), Allahabad (UP), in 2003. He is currently working towards the Ph. D. degree, from M.N.N.I.T., Allahabad (UP), and India. He is Chartered Engineer (India) (C.Eng. I), Elected Executive Committee member of Institution of Engineers (India), Faridabad, Local Centre and member of professional bodies LMISTE, IEEE and MIE. He is having rich experience of 29 years in teaching, research and Industry.



He is also an Assistant Professor with Electrical and Electronics Engineering Department, Lingaya's University, Faridabad, Haryana, India. His current research interests are in areas of variable structure control, nonlinear system dynamics and control, and robust control techniques. Presently he is an executive committee member of IEI, Faridabad LC, India.

Dr. Ravindra K Singh, is a Professor in the Department of Electrical Engineering, Motilal Nehru National Institute of Technology, Allahabad (M.N.N.I.T.) , India. He is having more than 20 years of teaching and research experience. His area of interest is dc-dc converters, electrical vehicle and distributed generation application. He is reviewer of national and international journal/conferences and working on research projects of government of India. Presently he is chairing IEEE local centre at MN National Institute of Technology, Allahabad. He is also a student's counselor of IET local chapter of M. N. National Institute of Technology Allahabad (UP) India. . He is a Fellow of IE (India) and member of IEEE.

