DESIGN ANALYSIS OF MATHEMATICAL MODEL FOR COLLISION AFFECTED ENVIRONMENT IN WIRELESS SENSOR NETWORK

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ABSTRACT

In this paper, a mathematical model for collision affected Medium Access Control (MAC) environment has been presented to improve the network lifetime (NL). In this model, a network has been divided into layers and each layer is assigned different probabilities to form clusters of unequal sizes which provide an effective method in order to investigate the hot spot problem (HSP) in multi-hop sensor networks. Also clusters closest to sink node have no intra data traffic and with small amount of energy, they devote to heavy data relaying. In addition, distribution of sensor nodes is done according to energy balancing technique. The proposed mathematical model not only provides insight into the MAC layer collision impacts on the energy balancing (EB) routing algorithm but also indicates the sufficient number of CHs to be presented in the first layer for the minimum energy consumption so that heavy data traffic can be relayed and also determine the optimum number of network layers resulting in minimum energy consumption.

KEYWORDS

WSN, HSP, EB, NL, MAC

1. INTRODUCTION

Wireless sensor networks (WSN) consist of small battery powered sensor nodes with limited power. After deployment, the small sensor nodes are usually inaccessible to the user, and thus replacement of energy source is not possible [1]. Therefore, energy efficiency is the key design issue which has to be enhanced so that the network lifetime improves. Sensor nodes can be organized into clusters in order to achieve low energy consumption and hence increase the lifetime of the network. Two routing communication models viz; single hop and multi-hop communication are possible. In single hop communication model, every node can directly access the sink node, whereas in multi hop communication, nodes are restricted with transmission range and therefore are forced to route their data with multiple hops until the data packet reaches the

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sink node. Better power efficiency can be achieved with multi-hop communication, as this reduces the requirement for long- range transmission since signal path loss is an inverse exponent with range or distance. In both the models, there is a problem of unbalanced energy consumption among different nodes and hence after some rounds of lifetime, nodes start to lose their energy at a higher rate and die much faster than others. In single hop communication, nodes which are furthest away from the sink node have the highest energy consumption and hence are the critical nodes, whereas in multi-hop communication, the nodes which are closest to the sink node are the most critical nodes due to heavy relaying data traffic load, and hence die faster, leaving areas of the network uncovered and causing network partitions, that is the hot spot problem [2]. Low Energy Adaptive Clustering Hierarchy Algorithm (LEACH) [4] is a hierarchical routing algorithm, in which CH is randomly selected amongst the nodes and it allows only single hop clusters to be formed but does not consider the hot spot problem. Energy-Efficient Unequal Clustering (EEUC) [3] algorithm is a self organized competition based algorithm, where cluster heads are selected based on the residual energy of neighboring nodes but this technique does not consider density of sensors which causes too much load on cluster heads. Energy-Balancing Unequal Clustering Protocol (EB-UCP) derives the reasonable ratios of distribution density among layers according to the energy- balancing principle, so that the energy consumption in every layer is nearly equal after such a deployment and the hot spot problem can be alleviated. But this mechanism does not consider the effect of collision on the energy consumption of a layer and on the total energy consumption of the network.

2. THEORETICAL MODEL

In this model, following assumptions have been made:

i. All the nodes are homogenous and deployed in a circular area with radius R.
ii. The Sink node is located at the center.
iii. Each sensor node is generating and transmitting at l bps.
iv. A free space propagation channel model for multi-hop forwarding scheme.
v. Single level Intra cluster collisions and no inter-Cluster Head collision are considered.
vi. No collision between the clusters of different layers.
vii. Energy consumed during back off is negligible as compared to energy consumed during retransmission

In this model, a layered network is considered where every layer contains a particular number of clusters, as shown in Figure1. The model of energy dissipation is taken from LEACH and is shown in Figure 2. The maximum layer number is n and the width of each layer is d. Nodes belonging to layer $\{L_i | i \neq n\}$ will forward both the data generated by them and the data generated by nodes from layers $\{L_j | (i+1) \leq j \leq n\}$[4]. The nodes in the outermost layer, L_n, need not forward any data. The energy spent for transmission of a l-bit packet over distance d is:

$$E_{tx}(l, d) = l(e_1 + e_2 d^2) \tag{1}$$

and the energy spent on receiving a l-bit packet is:

$$E_{rx}(l, d) = l\, e_1 \tag{2}$$

The electronics energy, e_1, depends on factors such as the digital coding, modulation, filtering and spreading of the signal. The amplifier energy, e_2, depends on the distance to the receiver and the acceptable bit-error rate.

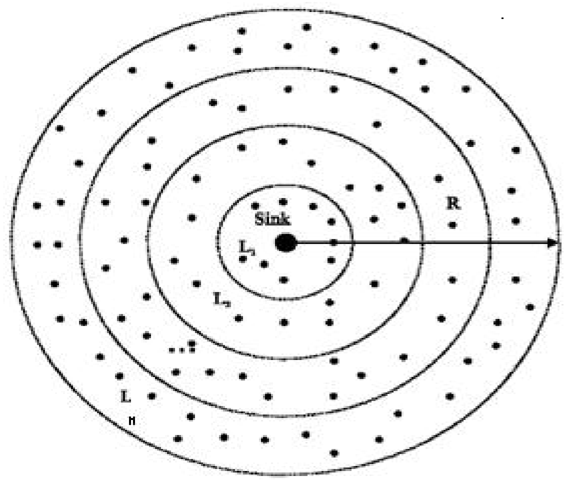

Figure1. A circular area network model

In order to alleviate the hot spot problem, clusters of unequal sizes are utilized. The probability of a node in a layer to be a CH is different in different layers with the probability being maximum for first layer and minimum for last layer. In this model, nodes in the first layer all become cluster heads as they have direct transmission to sink node with small amount of energy for relaying heavy data traffic. Also low probability indicates a small number of densely populated and huge clusters.

Primarily the CH selection in each layer is based on the residual energy of tentative cluster heads. But in order to maintain energy consumption among all layers equal, the node i's probability of becoming a final CH p_i will be computed as follows:

$$p_i = p_{\min} + \frac{n-j}{n-1} \times \left(p_{\max} - p_{\min} \right) \tag{3}$$

where p_{max} is the node`s probability of becoming a CH in first layer, $p_{min}$ is the node`s probability in the nth layer, j represents node i`s layer number [4].

The table for a node becoming cluster head in each layer when *n* varies from 2 to 15 is shown in Table 1.

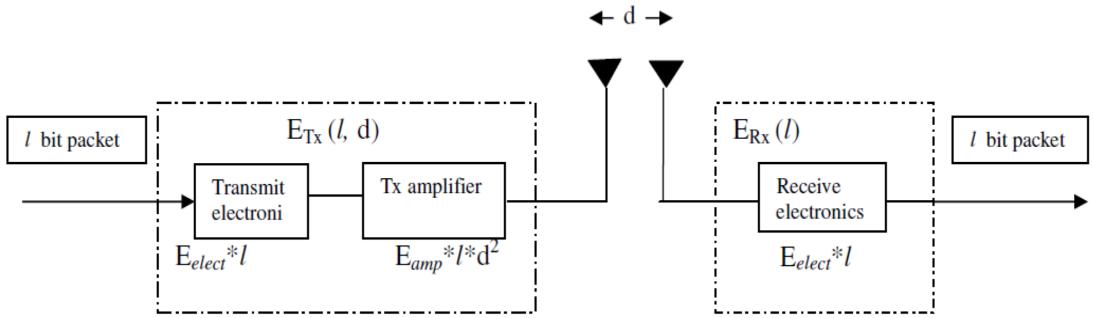

Figure2. Radio Energy Dissipation Model

Table1. Probabilities of a node becoming final CH in their respective layers for different values of n, network layers

n	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}	p_{11}	p_{12}	p_{13}	p_{14}	p_{15}
2	1	.05	-	-	-	-	-	-	-	-	-	-	-	-	-
3	1	.525	.05	-	-	-	-	-	-	-	-	-	-	-	-
4	1	.683	.367	.05	-	-	-	-	-	-	-	-	-	-	-
5	1	.763	.525	.288	.05	-	-	-	-	-	-	-	-	-	-
6	1	.81	.62	.43	.24	.05	-	-	-	-	-	-	-	-	-
7	1	.842	.683	.525	.367	.208	.05	-	-	-	-	-	-	-	-
8	1	.864	.729	.593	.457	.321	.186	.05	-	-	-	-	-	-	-
9	1	.881	.763	.644	.525	.406	.288	.169	.05	-	-	-	-	-	-
10	1	.894	.789	.683	.578	.472	.367	.261	.156	.05	-	-	-	-	-
11	1	.905	.81	.715	.62	.525	.43	.335	.24	.145	.05	-	-	-	-
12	1	.914	.827	.741	.655	.568	.482	.395	.309	.223	.136	.05	-	-	-
13	1	.921	.842	.7625	.683	.604	.525	.446	.367	.288	.208	.129	.05	-	-
14	1	.927	.854	.781	.708	.635	.562	.488	.415	.342	.269	.196	.123	.05	-
15	1	.932	.864	.796	.729	.661	.593	.525	.457	.389	.321	.254	.186	.118	.05

Unequal clusters of different layers are associated with different probabilities; also clusters nearest to the sink node have smaller sizes than those far away from the sink node. Similarly, clusters of different layers are associated with different collision probabilities, p_{c_i} because of unequal sizes of clusters and this will be computed as follows:

$$P_{c\_i} = 1 - \left(1 - ptr\right)^{\frac{1}{pi} - 1}$$

Where, p_{tr} is the probability of transmission.

The result shows that the optimum value of p_{tr} is 0.5 at which the probability of collision starts reducing for different network layers, *n*. The collision probabilities for each layer when *n* varies from 2 to 15 are shown in Table 2.

Table2. Probabilities of Collision in their respective layers for different values of n, network layers

n	p_{c_1}	p_{c_2}	p_{c_3}	p_{c_4}	p_{c_5}	p_{c_6}	p_{c_7}	p_{c_8}	p_{c_8}	p_{c_10}	p_{c_11}	p_{c_12}	$p_{c\ 13}$	p_{c_14}	p_{c_15}
2	0	. 99	-	-	-	-	-	-	-	-	-	-	-	-	-
3	0	.466	. 99		-	-	-	-	-	-	-	-	-	-	-
4	0	.275	.697	. 99	-	-	-	-	-	-	-	-	-	-	-
5	0	.194	.466	.8197	. 99	-	-	-	-	-	-	-	-	-	-
6	0	.150	.346	.601	.889	. 99	-	-	-	-	-	-	-	-	-
7	0	.122	.275	.466	.697	.929	. 99	-	-	-	-	-	-	-	-
8	0	.103	.227	.379	.561	.769	.951	. 99	-	-	-	-	-	-	-
9	0	.0893	.194	.318	.466	.637	.819	.967	. 99	-	-	-	-	-	-
10	0	.0789	.169	.275	.397	.539	.697	.859	.976	. 99	-	-	-	-	-
11	0	.070	.150	.241	.346	.466	.601	.747	.889	.983	. 99	-	-	-	-
12	0	.631	.135	.215	.306	.409	.525	.654	.788	.911	.988	. 99	-	-	-
13	0	.058	.122	.194	.275	.365	.466	.577	.698	.819	.929	.99	.99		-
14	0	.053	.112	.177	.249	.329	.417	.517	.624	.736	.848	.942	.99	.99	-
15	0	.0493	.103	.163	.227	.299	.379	.466	.561	.663	.769	.869	.95	.99	.99

3. PROPOSED MATHEMATICAL MODEL

In this subsection, considering energy balancing in each layer, it is assumed that all CHs equally compress the data, within a cluster, with aggregation coefficient α and energy dissipation is e_3 Joules/bit. Let N_n and E_n denote the number of nodes and energy consumption of outermost layer L_n respectively. The outermost layer L_n only needs to forward the data generated by them. Therefore the energy dissipation per unit time of each CH in Layer L_n is:

$$E_{ch\_n} = \left(\frac{1}{p_{\min}} - 1\right) l e_1 + \frac{1}{p_{\min}} l e_3 + \frac{1}{p_{\min}} \alpha l \left(e_1 + e_2 d^2\right) \tag{4}$$

Above equation shows that if the aggregation coefficient, α is smallest, highest efficiency of data aggregation of CH is possible. As in this configuration, more CHs are closer to the sink node, hence minimum power consumption of each layer is possible and the hot spot problem will be mitigated. In our model, we kept α = 0.1, so that CH forward only 10% of the total cluster load.

When the effect of collision is considered, the energy dissipation per unit time of each non-CH in outermost layer L_n is given by:

$$E_{nonch\_n} = l\left(e_1 + e_2 d^2\right) + p_{c\_n} l \left(e_1 + e_2 d^2\right) \tag{5}$$

Where, p_{c_n} is the probability of collision for a cluster in n^{th} layer.

The second term on the RHS of the (5) equation devotes the energy consumed during retransmission after collision is considered.

Hence, the total energy consumption of both the CHs nodes and non CHs nodes per unit time of in L_n is given by:

$$E_n = p_{\min} N_n E_{ch\_n} + (1 - p_{\min}) N_n E_{nonch\_n} \tag{6}$$

$$= N_n l \left\{ e_1 - e_1 p_{\min} + e_3 + \alpha e_1 + \alpha e_2 d^2 \right\} + (1 - p_{\min}) N_n l \left(e_1 + e_2 d^2 \right)\left(1 + p_{c\_n}\right) \tag{7}$$

$$= N_n l \left\{ e_1 - e_1 p_{\min} + e_3 + \alpha e_1 + \alpha e_2 d^2 + (1 - p_{\min}) \left[e_1 + e_1 p_{c\_n} + e_2 d^2 + e_2 d^2 p_{c\_n} \right] \right\} \tag{8}$$

$$E_n = N_n l \left\{ \left[2(1 - p_{\min}) + \alpha + p_{c\_n}(1 - p_{\min}) \right] e_1 + e_2 d^2 \left(1 + \alpha - p_{\min} + p_{c\_n} - p_{\min} p_{c\_n}\right) + e_3 \right\} \tag{9}$$

As the nodes belonging to layer $\{L_i | i \neq n\}$ will forward both the data generated by them and the data generated by nodes from layers $\{L_j | (i+1) \leq j \leq n\}$.Therefore the energy consumption per unit time of all nodes in layer L_i $(1 \leq i \leq k)$ is expressed as;

$$E_i = N_i l \left\{ \left[2(1 - p_i) + \alpha + p_{c\_i}(1 - p_i) \right] e_1 + e_2 d^2 \left(1 + \alpha - p_i + p_{c\_i} - p_i p_{c\_i}\right) + e_3 \right\} + \sum_{t=i+1}^{k} \alpha l N_t \left(2e_1 + e_2 d^2\right) \tag{10}$$

Similarly, we can obtain,

$$E_{n-1} = N_{n-1} l \left\{ \left[2(1 - p_{n-1}) + \alpha + p_{c\_(n-1)}(1 - p_{n-1}) \right] e_1 + e_2 d^2 \left(1 + \alpha - p_{n-1} + p_{c\_(n-1)} - p_{n-1} p_{c\_(n-1)}\right) + e_3 \right\} + \alpha l N_n \left(2e_1 + e_2 d^2\right) \tag{11}$$

In this model, the energy consumption of all the layers is equal, so that the energy efficiency can be maximized. Such that,

$E_1 = E_2 = E_3 \ldots\ldots.. = E_i \ldots\ldots = E_{k-1} = E_k$

Therefore,

$$E_n = E_{n-1}$$

Now, also we can obtain the ratio of the nodes deployed in n^{th} layer and $(n-1)^{th}$ layer respectively, that is,

$$\frac{N_{n-1}}{N_n} = \frac{\left\{ \left[2(1 - P_{\min}) - \alpha + p_{c\_n}(1 - p_{\min}) \right] e_1 + e_2 d^2 \left(1 - p_{\min} + p_{c\_n} - p_{\min} p_{c\_n}\right) + e_3 \right\}}{\left\{ \left[2(1 - p_{n-1}) + \alpha + p_{c\_(n-1)}(1 - p_{n-1}) \right] e_1 + e_2 d^2 \left(1 + \alpha - p_{n-1} + p_{c\_(n-1)} - p_{n-1} p_{c\_(n-1)}\right) + e_3 \right\}} \tag{12}$$

Similarly, we can have

$$E_{n-2} = N_{n-2} l \left\{ \left[2(1 - p_{n-2}) + \alpha + p_{c\_(n-2)}(1 - p_{n-2}) \right] e_1 + e_2 d^2 \left(1 + \alpha - p_{n-2} + p_{c\_(n-2)} - p_{n-2} p_{c\_(n-2)}\right) + e_3 \right\} + N_n \alpha l \left(2e_1 + e_2 d^2\right) + \frac{N_n N_{n-1}}{N_n} \alpha l \left(2e_1 + e_2 d^2\right) \tag{13}$$

Put, $\frac{N_{n-1}}{N_n} = m$

Also, $E_n = E_{n-2}$

Similarly, we can obtain the ratio of the nodes deployed in n[th] layer and (n-2) [th] layer respectively, that is,

$$\frac{N_{n-2}}{N_n} = \frac{\left\{\left[2(1-p_{\min})-\alpha+p_{c\_n}(1-p_{\min})-2\alpha m\right]e_1+e_2d^2\left(1-\alpha m-p_{\min}+p_{c\_n}-p_{\min}p_{c\_n}\right)+e_3\right\}}{\left\{\left[2(1-p_{n-2})+\alpha+p_{c(n-2)}(1-p_{n-2})\right]e_1+e_2d^2\left(1+\alpha-p_{n-2}+p_{c\_(n-2)}-p_{n-2}p_{c\_(n-2)}\right)+e_3\right\}} \quad (14)$$

After iterative calculations, we can obtain the values of N_n, N_{n-1}…N_i….N_2, N_1 of whole network when the effect of collision under energy- balancing approach is considered. For obtaining the above values of nodes in each layer for different values of n, the various analytical parameters are set to the values as shown in the Table3 [4].

Table3. Parameter's Value [4]

Parameter	Value
Network Size	R=200m
Sink Location	(0,0)
Data Packet Size	500(bytes)
No. of nodes, N	500-2000
e1	50(nJb-1)
e2	10(pJ/b/m2)
e3	5(nJ/b/signal)
α	0.1
pmax	1
Pmin	0.05
n	2 to 15

4. RESULTS AND DISCUSSION

Here, the performance parameters are evaluated. The values for analytical parameters are given in Table 3. Energy consumption is one of the criteria for evaluating the performance of WSNs. For N=500, the energy consumption of a layer i.e, E_i, has been evaluated for varying number of CHs in the first layer. The Characteristics of the scheme are observed and analyzed. Figure3 shows the distribution of nodes in a layered network.

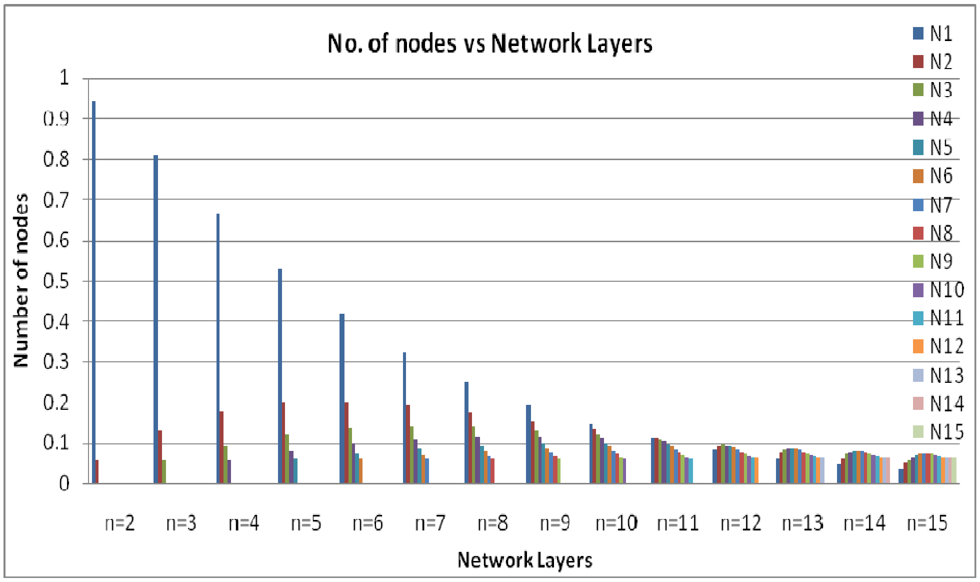

Figure3. Distribution of the number of nodes in an n- layered network

It is clear from Figure 4, the optimum value of p_{tr} is 0.5 at which the p_c, that is probability of collision starts reducing for different network layers. The magnified plot in Figure5 provides a better insight.

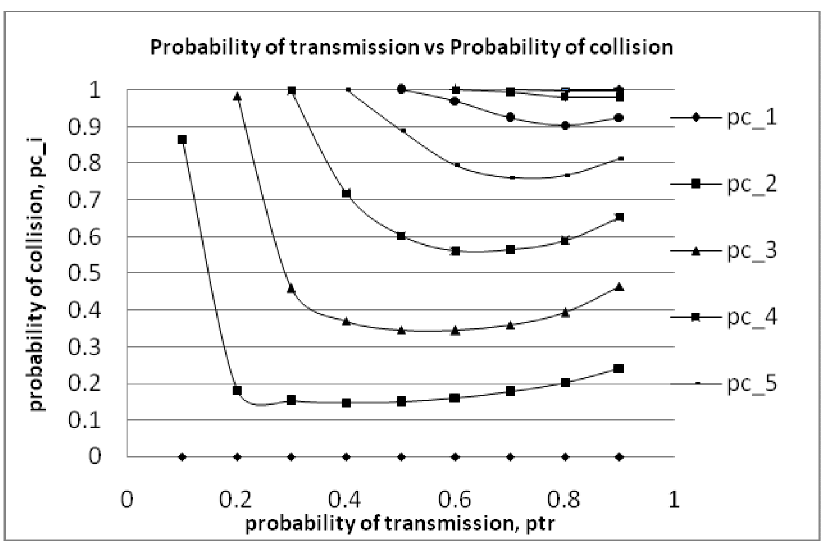

Figure4. Impact of probability of collision on the probability of transmission

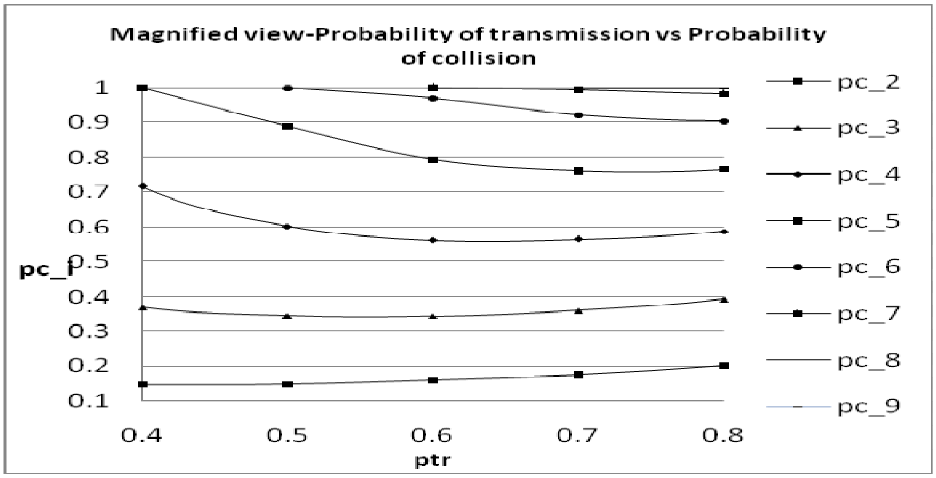

Figure5. Magnified view of Figure4

It is evident from Figure 6, that as the value of *n* increases, the number of CHs in first layer or number of nodes nearest to the sink node decreases. Too large a value of *n* causes hot spot problem, because the number of nodes in first layer reduces drastically, thus an optimum value of *n* is required in order to have reasonable number of nodes in first layer.

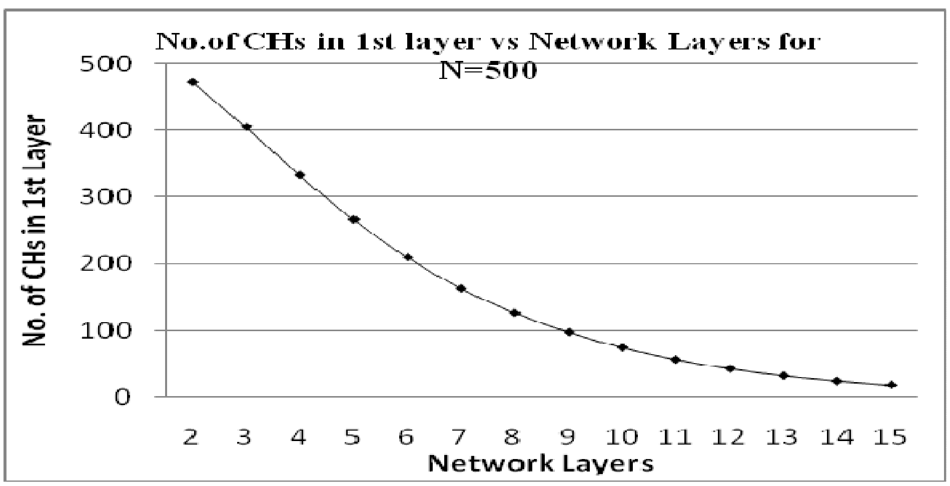

Figure6. Impact of Network layers on the number of CHs in first layer

Thus, we can infer that optimum value of *n* depends upon reasonable number of CHs in first layer and the energy consumption of a layer. As shown in Figure7, E_i has been plotted against the number of cluster heads, calculated for different *n* for N = 500. It is also shown that the reasonable value for the number of nodes in first layer is 163, for energy consumption to be 21.65mj in order to avoid hot spot problem and this corresponds to n=7. It is more clear from the magnified plot given in Figure8 and Table 4.

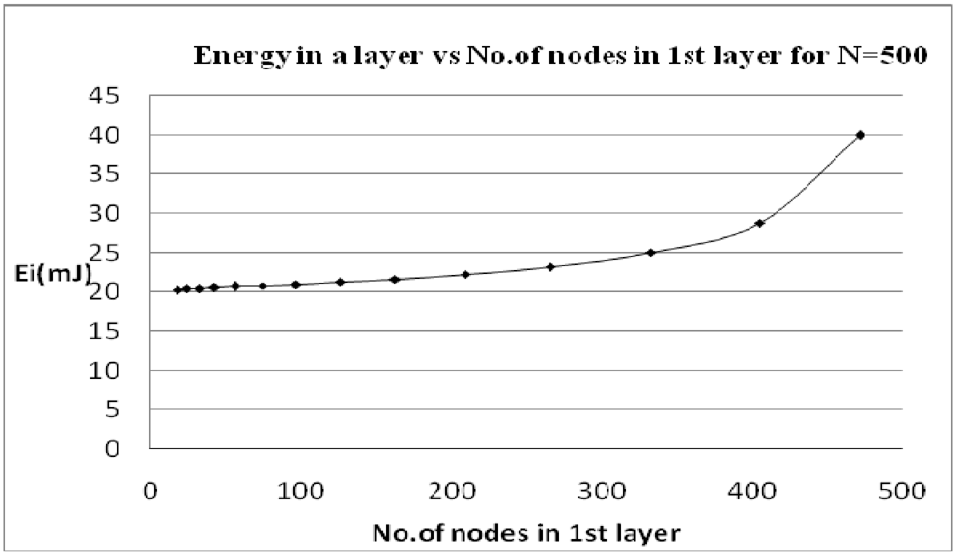

Figure7. Impact of CHs of the first layer

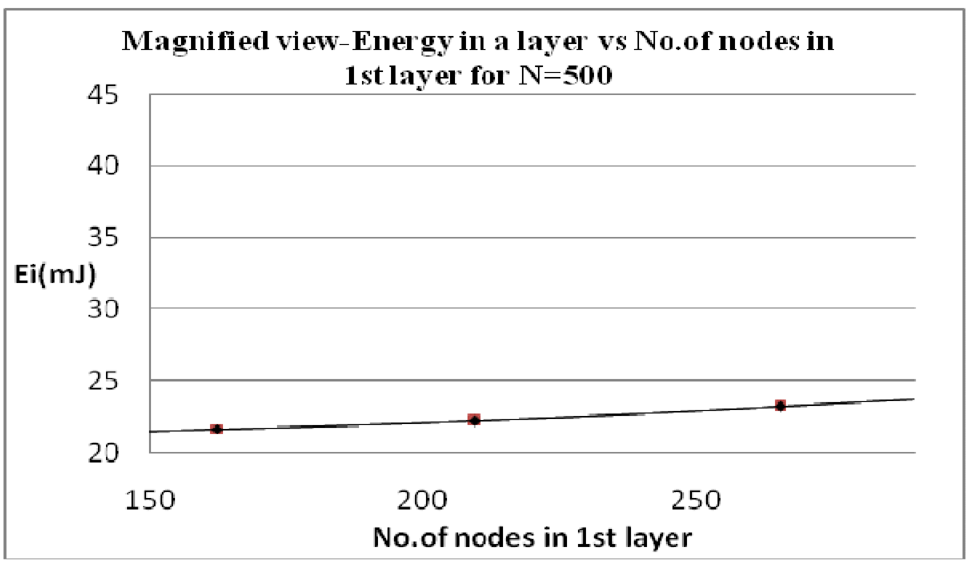

Figure8. Magnified view of Figure7

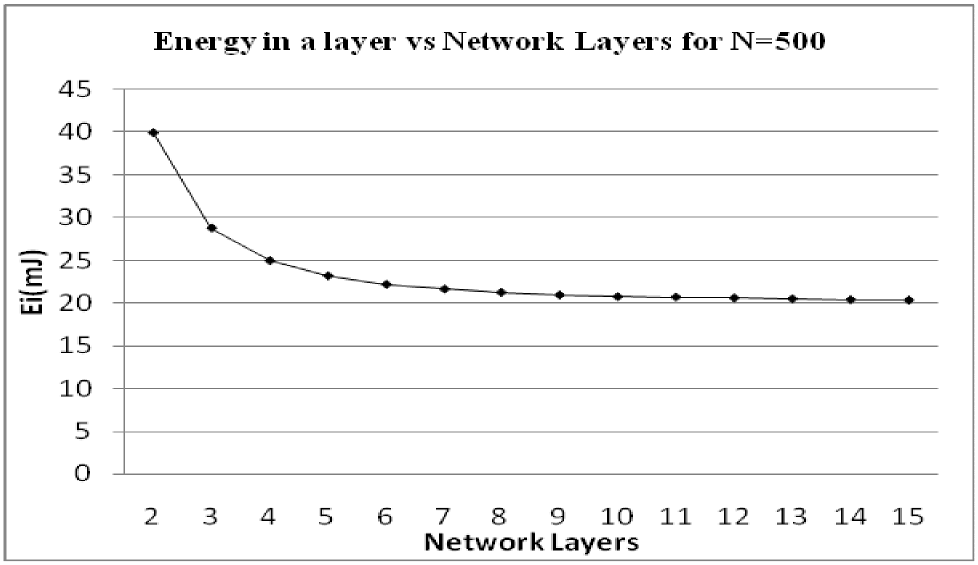

Figure9. Impact of Network layers

It is clear from Figure 9 that there is a continuous fall of energy consumption of a layer from 39.95mJ to 22.2mJ, when number of network layers changing from 2 to 6, respectively. But for *n* greater than 7, the energy consumption is almost constant and it ranges from 21.65mJ to 20.35mJ for network layers, 7 to 15 respectively. It is more clear from Figures10, 11 and from Table 4.

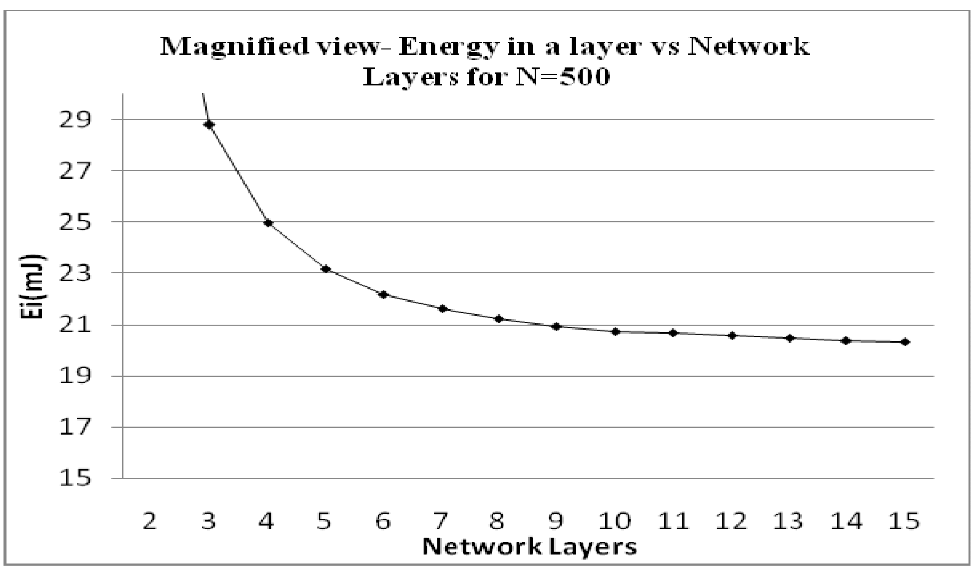

Figure10. Magnified view of Figure 9.

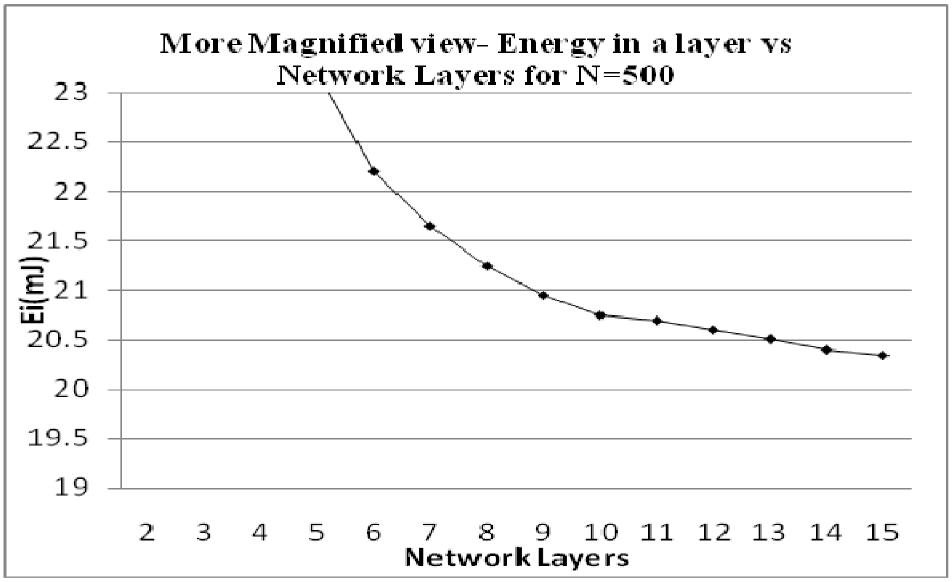

Figure11. More Magnified view of Figure 9.

Table4. Optimum number of CHs for N=500

(Network layers), n	No. of CHs in first Layer	E_i (mJ)
2	472	39.95
3	405	28.8
4	332	24.975
5	266	23.2
6	210	22.2
7	**163**	**21.65**
8	126	21.25
9	97	20.95
10	74	20.75
11	56	20.7
12	43	20.6
13	32	20.5
14	24	20.4
15	18	20.35

There will always be tradeoff between the reasonable number of nodes in first layer which have heavy data relaying traffic and minimum energy consumption of a layer, when the effect of collision is considered.

5. CONCLUSIONS

To address the hot spot problem, a mathematical model has been developed. This model considers the effect of collision in MAC Layer for energy balancing, layering and clustering technique in order to relay data traffic to the sink node. Unequal clusters of different layers are associated with different probabilities; clusters nearest to the sink node have smaller sizes than those far away. Analytical results show that for *n* greater than 7, the energy consumption is increasing slowly but the number of CHs starts reducing drastically. In order to address the hot spot problem, sufficient number of CHs is must in the first layer to relay the heavy data traffic for minimum energy consumption of a layer.

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