ON DECREASING OF MISMATCH-INDUCED STRESS DURING GROWTH OF FILMS DURING MAGNETRON SPUTTERING

E.L. Pankratov

Department of applied mechanics, physics and higher mathematics, Nizhny Novgorod State Agrotechnical University, 97 Gagarin avenue, Nizhny Novgorod

ABSTRACT

In this paper we analyzed mass transfer during the growth of epitaxial layers during magnetron sputtering. During the analysis we obtained modifications of properties of films with variation of several parameters. We analyzed possibility to decrease mismatch -induced stress on the considered films during their growth. An analytical approach for analyzing mass transfer was introduced, which makes it possible to take into account the nonlinearity of processes, as well as changes in parameters in space and time.

KEYWORDS

Mass transfer; magnetrons sputtering; analytical approach for modelling; mismatch-induced stress.

1. INTRODUCTION

Development of solid-state electronics and widespread using of heterostructures for manufacturing of electronic devices leads to the necessity to improve properties of layers of these heterostructures. Different methods are used to manufacture of heterostructures: molecular beam epitaxy, epitaxy from the gas phase, magnetron sputtering. A large number of experimental works have been devoted to the manufacturing and using of heterostructures due to their widespread using [1-12]. At the same time, a relatively small number of works are devoted to predicting the growth of heterostructures [11,12].

In this paper in development of references [13-16] we analyzed processes framework growing films by magnetron sputtering. Structure of the considered magnetron is shown in Fig. 1. In the framework of the structure electrons are emitting from the cathode (line 1). After that under the action of a field between the cathode (line 1) and the anode (line 3) an electron flow is formed between lines 1 and 2. The main purpose of this paper is analysis of mass transfer in magnetrons during growth of films in order to improve their properties. An additional aim of this paper is analysis of possibility to decrease mismatch-induced stress during growth of films. To solve this aim we consider an analytical approach for analysis of mass transfer, which makes a possibility to take into account the nonlinearity of mass transfer, as well as the changing of its parameters in space and time.

DOI:10.5121/ijcax.2024.11402

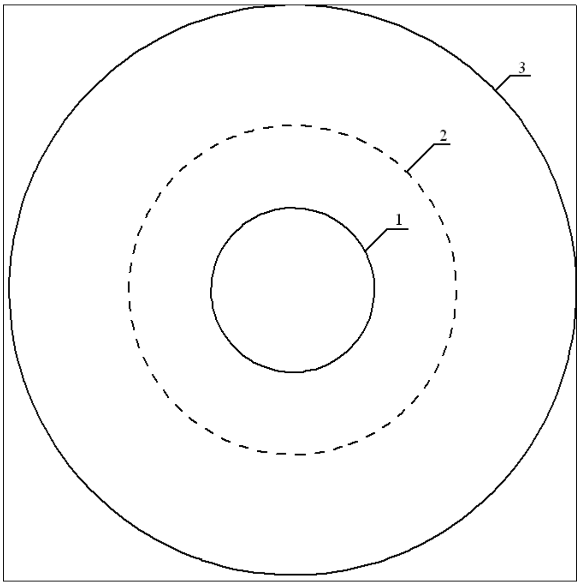

Fig. 1. Structure of magnetron.

In the framework of the structure ions have been emitted from cathode (line 1). After that under influence of field between cathode (line 1) and anode (line 3) one can obtained flow of ions between lines 1 and 2

2. METHOD OF SOLUTION

To solve our aims we determine spatio-temporal distribution of electromagnetic field. We determine the distribution by solving the following boundary problem [17, 18]

$$rot\,\vec{H}(r,\varphi,z,t)=\vec{j}+\frac{\partial\vec{D}(r,\varphi,z,t)}{\partial t},\ rot\,\vec{E}(r,\varphi,z,t)=-\frac{\partial\vec{B}(r,\varphi,z,t)}{\partial t},$$
$$div\vec{D}(r,\varphi,z,t)=\rho,\ div\vec{B}(r,\varphi,z,t)=0,\ \vec{D}=\varepsilon_0\varepsilon\vec{E},\ \vec{B}=\mu_0\mu\vec{H}. \quad (1)$$

Here ε and μ are the dielectric and magnetic constants; $\varepsilon_0=0.886\cdot10^{-11}$ *F/m*; $\mu_0= 1.256\cdot10^{-6}$ *H/m*; $\vec{E}$ and $\vec{H}$ are the electric and magnetic strengths; $\vec{D}$ and $\vec{B}$ are the inductions of electric and magnetic fields; *r*, φ and *z* are the spatial coordinates; *t* is the current time. Boundary and initial conditions for the system of equations could be written as

$$B_z(R,\varphi,z,t)=0,\ D_r(R,\varphi,z,t)=0,\ E_z(R,\varphi,z,t)=0,\ H_r(R,\varphi,z,t)=0,$$
$$\vec{E}(r,\varphi,z,0)=\vec{E}_0,\ \vec{H}(r,\varphi,z,0)=\vec{H}_0.$$

Current density $\vec{j}$ is proportional to speed of ions $\vec{v}$: $\vec{j}=C\cdot\vec{v}$, where *C* is the density of ions. The speed of ions correlated with strengths of electric and magnetic field by the second Newton law

$$m\frac{d\vec{v}}{dt}=e\left(\vec{E}+\vec{v}\times\vec{B}\right). \qquad (3)$$

Scalar form of the Eqs.(1) could be written as

$$Cv_r+\frac{\partial D_r(r,\varphi,z,t)}{\partial t}=\frac{1}{r}\frac{\partial H_z(r,\varphi,z,t)}{\partial\varphi}-\frac{\partial H_\varphi(r,\varphi,z,t)}{\partial z},$$

$$Cv_\varphi+\frac{\partial D_\varphi(r,\varphi,z,t)}{\partial t}=\frac{1}{r}\frac{\partial H_z(r,\varphi,z,t)}{\partial\varphi}-\frac{\partial H_r(r,\varphi,z,t)}{\partial z},$$

$$Cv_z+r\frac{\partial D_z(r,\varphi,z,t)}{\partial t}=\frac{\partial\left[rH_\varphi(r,\varphi,z,t)\right]}{\partial r}-\frac{\partial H_r(r,\varphi,z,t)}{\partial\varphi},$$

$$\frac{\partial E_\varphi(r,\varphi,z,t)}{\partial z}-\frac{1}{r}\frac{\partial E_z(r,\varphi,z,t)}{\partial\varphi}=\frac{\partial B_r(r,\varphi,z,t)}{\partial t},$$

$$\frac{\partial E_r(r,\varphi,z,t)}{\partial z}-\frac{1}{r}\frac{\partial E_z(r,\varphi,z,t)}{\partial\varphi}=\frac{\partial B_\varphi(r,\varphi,z,t)}{\partial t},$$

$$\frac{\partial\left[rE_\varphi(r,\varphi,z,t)\right]}{\partial r}-\frac{\partial E_r(r,\varphi,z,t)}{\partial\varphi}=r\frac{\partial B_z(r,\varphi,z,t)}{\partial t},$$

$$\frac{1}{r}\frac{\partial\left[rD_r(r,\varphi,z,t)\right]}{\partial r}+\frac{1}{r}\frac{\partial D_\varphi(r,\varphi,z,t)}{\partial\varphi}+\frac{\partial D_z(r,\varphi,z,t)}{\partial z}=\rho,$$

$$\frac{1}{r}\frac{\partial\left[rB_r(r,\varphi,z,t)\right]}{\partial r}+\frac{1}{r}\frac{\partial B_\varphi(r,\varphi,z,t)}{\partial\varphi}+\frac{\partial B_z(r,\varphi,z,t)}{\partial z}=0,$$

$$m\frac{dv_r}{dt}=e\left(E_r+v_\varphi B_z\right),\ m\frac{dv_\varphi}{dt}=e\left(E_\varphi+v_r B_z\right),\ m\frac{dv_z}{dt}=e\left(E_z+v_r B_\varphi\right).$$

To solve these equations we use the method of averaging functional corrections [19]. To determine the first-order approximations of the required functions we replace them in the right-hand sides of equations (4) by their not yet known average values α_{1s} in the right-hand side of these equations. As a result of this substitution we obtain the following equations to determine the first-order approximations of components of strength and induction of the considered fields

$$Cv_r+\frac{\partial D_{1r}(r,\varphi,z,t)}{\partial t}=0,\ Cv_\varphi+\frac{\partial D_{1\varphi}(r,\varphi,z,t)}{\partial t}=0,$$

$$Cv_z+\frac{\partial D_{1z}(r,\varphi,z,t)}{\partial t}=\alpha_{1H_\varphi},\ \frac{\partial B_{1r}(r,\varphi,z,t)}{\partial t}=0,\ \frac{\partial B_{1\varphi}(r,\varphi,z,t)}{\partial t}=0,$$

$$\frac{\partial B_{1z}(r,\varphi,z,t)}{\partial t}=\alpha_{1E_\varphi},\ \frac{1}{r}\frac{\partial\left[rD_{1r}(r,\varphi,z,t)\right]}{\partial r}=C,\ \frac{1}{r}\frac{\partial\left[rB_{1r}(r,\varphi,z,t)\right]}{\partial r}=0,$$

$$m\frac{d\,v_{1r}}{d\,t}=e\left(\alpha_{1E_r}+\alpha_{1v_\varphi}\alpha_{1B_z}\right),\ m\frac{d\,v_{1\varphi}}{d\,t}=e\left(\alpha_{1E_\varphi}+\alpha_{1v_r}\alpha_{1B_z}\right),$$

$$m\frac{d\,v_{1z}}{d\,t}=e\left(\alpha_{1E_z}+\alpha_{1v_r}\alpha_{1B_\varphi}\right).$$

Further after integration of left and right sides of these equations on considered variables we obtain the first-order approximations of the considered fields and velocity of ions in the following form

$$D_{1r}\left(r,\varphi,z,t\right)=-\int\limits_0^t C\,v_{1r}\,d\,\tau,\ D_{1\varphi}\left(r,\varphi,z,t\right)=-\int\limits_0^t C\,v_{1\varphi}\,d\,\tau,$$

$$D_{1z}\left(r,\varphi,z,t\right)=\alpha_{1H_\varphi}t-\int\limits_0^t C\,v_{1z}\,d\,\tau+\varepsilon\,\varepsilon_0 E_0,\ m\,v_{1r}=e\left(\alpha_{1E_r}+\alpha_{1v_\varphi}\alpha_{1B_z}\right)t,$$

$$B_{1r}\left(r,\varphi,z,t\right)=0,\ B_{1\varphi}\left(r,\varphi,z,t\right)=\mu\,\mu_0 H_0,\ B_{1z}\left(r,\varphi,z,t\right)=\alpha_{1E_\varphi}t,$$

$$m\,v_{1\varphi}=e\left(\alpha_{1E_\varphi}+\alpha_{1v_r}\alpha_{1B_z}\right)t+m\,v_{\varphi 0},\ m\,v_{1z}=e\left(\alpha_{1E_z}+\alpha_{1v_r}\alpha_{1B_\varphi}\right)t+m\,v_{z0}.$$

Calculation of average values α_{1s} by using the following standard relation [19]

$$\alpha_{1s_q}=\frac{1}{2\pi\,\Theta\,LR^2}\int\limits_0^\Theta\int\limits_0^L\int\limits_0^R r\int\limits_0^{2\pi} S_{1q}\left(r,\varphi,z,t\right)d\,\varphi\,d\,r\,d\,z\,d\,t$$

(Θ is the continuance of growth, L is the length of magnetron) gives a possibility to obtain, that $\alpha_{1v_z}=v_{z0}+e\Theta^2\left(H_0\Theta^2-\alpha_{1v_z}C\,\Theta^2+4\varepsilon\,\varepsilon_0 E_0\right)/4m$, $\alpha_{1E_\varphi}=0$, $\alpha_{1H_\varphi}=H_0$, $\alpha_{1E_z}=2\varepsilon\,\varepsilon_0 E_0+\Theta^2\left(H_0-\alpha_{1v_z}C\right)/2$, $\alpha_{1v_r}=0$, $\alpha_{1E_r}=0$, $\alpha_{1v_\varphi}=v_{\varphi 0}$. Further we obtain the second-order approximations of components of strength and induction of electrical and magnetic fields. To obtain these approximations we replace considered fields in right sides of Eqs. (4) on the following sums S $(r,\varphi,z,t)\to\alpha_{2s}+S_1(r,\varphi,z,t)$. The replacement leads to transformation of Eqs. (4) to the following form

$$C\,v_{2r}+\frac{\partial D_{2r}\left(r,\varphi,z,t\right)}{\partial t}=\frac{1}{r}\frac{\partial H_{1z}\left(r,\varphi,z,t\right)}{\partial\varphi}-\frac{\partial H_{1\varphi}\left(r,\varphi,z,t\right)}{\partial z},$$

$$C\,v_{2\varphi}+\frac{\partial D_{2\varphi}\left(r,\varphi,z,t\right)}{\partial t}=\frac{1}{r}\frac{\partial H_{1z}\left(r,\varphi,z,t\right)}{\partial\varphi}-\frac{\partial H_{1r}\left(r,\varphi,z,t\right)}{\partial z},$$

$$C\,v_{2z}+r\frac{\partial D_{2z}\left(r,\varphi,z,t\right)}{\partial t}=\alpha_{2H_\varphi}+H_{1\varphi}\left(r,\varphi,z,t\right)+r\frac{\partial H_{1\varphi}\left(r,\varphi,z,t\right)}{\partial r}-$$

$$-\frac{\partial H_{1r}\left(r,\varphi,z,t\right)}{\partial\varphi},\ \frac{\partial B_{2r}\left(r,\varphi,z,t\right)}{\partial t}=\frac{\partial E_{1\varphi}\left(r,\varphi,z,t\right)}{\partial z}-\frac{1}{r}\frac{\partial E_{1z}\left(r,\varphi,z,t\right)}{\partial\varphi},$$

$$\frac{\partial B_{2\varphi}(r,\varphi,z,t)}{\partial t}=\frac{\partial E_{1r}(r,\varphi,z,t)}{\partial z}-\frac{1}{r}\frac{\partial E_{1z}(r,\varphi,z,t)}{\partial \varphi},$$

$$r\frac{\partial B_{2z}(r,\varphi,z,t)}{\partial t}=\alpha_{2E_\varphi}+E_{1\varphi}(r,\varphi,z,t)+r\frac{\partial E_{1\varphi}(r,\varphi,z,t)}{\partial r}-\frac{\partial E_{1r}(r,\varphi,z,t)}{\partial \varphi},$$

$$\frac{1}{r}\frac{\partial\left[r\,D_{2r}(r,\varphi,z,t)\right]}{\partial r}=\rho-\frac{1}{r}\frac{\partial D_{1\varphi}(r,\varphi,z,t)}{\partial \varphi}-\frac{\partial D_{1z}(r,\varphi,z,t)}{\partial z},$$

$$\frac{1}{r}\frac{\partial\left[r\,B_{2r}(r,\varphi,z,t)\right]}{\partial r}=-\frac{1}{r}\frac{\partial B_{1\varphi}(r,\varphi,z,t)}{\partial \varphi}-\frac{\partial B_{1z}(r,\varphi,z,t)}{\partial z},$$

$$m\frac{d v_{2r}}{d t}=e\left\{\alpha_{2E_r}+E_{1r}(r,\varphi,z,t)+\left[\alpha_{2v_\varphi}+v_{1\varphi}(r,\varphi,z,t)\right]\left[\alpha_{2B_z}+B_{1z}(r,\varphi,z,t)\right]\right\},$$

$$m\frac{d v_{2\varphi}}{d t}=e\left\{\alpha_{2E_\varphi}+E_{1\varphi}(r,\varphi,z,t)+\left[\alpha_{2v_r}+v_{1r}(r,\varphi,z,t)\right]\left[\alpha_{2B_z}+B_{1z}(r,\varphi,z,t)\right]\right\},$$

$$m\frac{d v_{2z}}{d t}=e\left\{\alpha_{2E_z}+E_{1z}(r,\varphi,z,t)+\left[\alpha_{2v_r}+v_{1r}(r,\varphi,z,t)\right]\left[\alpha_{2B_\varphi}+B_{1\varphi}(r,\varphi,z,t)\right]\right\}.$$

Integration of left and right sides of the above equations on considered variations gives a possibility to obtain the second-order approximations of the considered fields in the following forms

$$D_{2r}(r,\varphi,z,t)=\frac{1}{r}\frac{\partial}{\partial \varphi}\int_0^t H_{1z}(r,\varphi,z,\tau)d\tau-\frac{\partial}{\partial z}\int_0^t H_{1\varphi}(r,\varphi,z,\tau)d\tau-\int_0^t C\,v_{2r}\,d\tau,$$

$$D_{2\varphi}(r,\varphi,z,t)=\frac{1}{r}\frac{\partial}{\partial \varphi}\int_0^t H_{1z}(r,\varphi,z,\tau)d\tau-\frac{\partial}{\partial z}\int_0^t H_{1r}(r,\varphi,z,\tau)d\tau-\int_0^t C\,v_{2\varphi}d\tau,$$

$$r\,D_{2z}(r,\varphi,z,t)=\alpha_{2H_\varphi}t+\int_0^t H_{1\varphi}(r,\varphi,z,\tau)d\tau+r\frac{\partial}{\partial r}\int_0^t H_{1\varphi}(r,\varphi,z,\tau)d\tau+r\,\varepsilon\,\varepsilon_0E_0-$$
$$-\frac{\partial}{\partial \varphi}\int_0^t H_{1r}(r,\varphi,z,\tau)d\tau-C\int_0^t v_{2z}\,d\tau+r\,\varepsilon\,\varepsilon_0E_0,$$

$$r\,B_{2z}(r,\varphi,z,t)=\alpha_{2E_\varphi}t+\int_0^t E_{1\varphi}(r,\varphi,z,\tau)d\tau+r\frac{\partial}{\partial r}\int_0^t E_{1\varphi}(r,\varphi,z,\tau)d\tau-$$
$$-\frac{\partial}{\partial \varphi}\int_0^t E_{1r}(r,\varphi,z,\tau)d\tau,$$

$$B_{2r}(r,\varphi,z,t)=\frac{\partial}{\partial z}\int_0^t E_{1\varphi}(r,\varphi,z,\tau)d\tau-\frac{1}{r}\frac{\partial}{\partial \varphi}\int_0^t E_{1z}(r,\varphi,z,\tau)d\tau,$$

$$B_{2\varphi}(r,\varphi,z,t)=\frac{\partial}{\partial z}\int_0^t E_{1r}(r,\varphi,z,\tau)d\tau-\frac{1}{r}\frac{\partial}{\partial \varphi}\int_0^t E_{1z}(r,\varphi,z,\tau)d\tau+\mu\,\mu_0H_0,$$

$$r\,D_{2r}(r,\varphi,z,t)=C\frac{r^2}{2}-\frac{\partial}{\partial\varphi}\int_0^r D_{1\varphi}(u,\varphi,z,t)\,d\,u-\frac{\partial}{\partial z}\int_0^r u\,D_{1z}(u,\varphi,z,t)\,d\,u\,,$$

$$r\,B_{2r}(r,\varphi,z,t)=-\frac{\partial}{\partial\varphi}\int_0^r B_{1\varphi}(u,\varphi,z,t)\,d\,u-\frac{\partial}{\partial z}\int_0^r u\,B_{1z}(u,\varphi,z,t)\,d\,u\,,$$

$$m v_{2r}=e\left[\alpha_{2E_r}t+\int_0^t E_{1r}(r,\varphi,z,\tau)\,d\,\tau+\alpha_{2v_\varphi}\alpha_{2B_z}t+\alpha_{2v_\varphi}\int_0^t B_{1z}(r,\varphi,z,\tau)\,d\,\tau+\right.$$
$$\left.+\alpha_{2B_z}\int_0^t v_{1\varphi}(r,\varphi,z,\tau)\,d\,\tau+\int_0^t v_{1\varphi}(r,\varphi,z,\tau)B_{1z}(r,\varphi,z,\tau)\,d\,\tau\right],$$

$$m v_{2\varphi}=e\left[\alpha_{2E_\varphi}t+\int_0^t E_{1\varphi}(r,\varphi,z,\tau)\,d\,\tau+\alpha_{2v_r}\alpha_{2B_z}t+\alpha_{2v_r}\int_0^t B_{1z}(r,\varphi,z,\tau)\,d\,\tau+\right.$$
$$\left.+\alpha_{2B_z}\int_0^t v_{1r}(r,\varphi,z,\tau)\,d\,\tau+\int_0^t v_{1r}(r,\varphi,z,\tau)B_{1z}(r,\varphi,z,\tau)\,d\,\tau\right],$$

$$m v_{2z}=e\left[\alpha_{2E_z}t+\int_0^t E_{1z}(r,\varphi,z,\tau)\,d\,\tau+\alpha_{2v_r}\alpha_{2B_\varphi}t+\alpha_{2v_r}\int_0^t B_{1\varphi}(r,\varphi,z,\tau)\,d\,\tau+\right.$$
$$\left.+\alpha_{2B_\varphi}\int_0^t v_{1r}(r,\varphi,z,\tau)\,d\,\tau+\int_0^t v_{1r}(r,\varphi,z,\tau)B_{1\varphi}(r,\varphi,z,\tau)\,d\,\tau\right].$$

Average values of the second-order approximations α_{2s} were calculated by using the following standard relation [19]

$$\alpha_{2s_q}=\frac{1}{2\pi\Theta LR^2}\int_0^\Theta\int_0^L\int_0^R r\int_0^{2\pi}\left[S_{2q}(r,\varphi,z,t)-S_{1q}(r,\varphi,z,t)\right]d\,\varphi\,d\,r\,d\,z\,d\,t\,. \quad (5)$$

Substitution of obtained approximations of strength and induction of considered fields and velocities of movement of ions into relations (5) gives a possibility to obtain relations for the required average values in the following form

$$\alpha_{2D_z}=\left(\alpha_{2H_\varphi}-\alpha_{1H_\varphi}\right)\frac{\Theta^2}{6}+\int_0^\Theta\frac{\Theta-t}{2\pi\Theta LR^2}\int_0^L\int_0^R\int_0^{2\pi}H_{1\varphi}(r,\varphi,z,t)\,d\,\varphi\,d\,r\,d\,z\,d\,t+$$
$$+\int_0^\Theta\frac{\rho(\Theta-t)}{2\pi\Theta LR^2}\int_0^L\int_0^R\int_0^{2\pi}\left(v_{2z}-v_{1z}\right)d\,\varphi\,d\,r\,d\,z\,d\,t\,,\ \alpha_{2D_r}=0,\ \alpha_{2D_\varphi}=0,\ \alpha_{2B_z}=0,$$

$$\alpha_{2B_r}=0,$$

$$\alpha_{2B_\varphi}=\mu\,\mu_0 H_0\,,\ \alpha_{2v_r}=0,\ \alpha_{2v_\varphi}=-v_{\varphi 0}\,,$$

$$\alpha_{2v_z}=\frac{e}{m}\int_0^\Theta\frac{\Theta-t}{2\pi\Theta LR^2}\int_0^L\int_0^R r\int_0^{2\pi}E_{1z}(r,\varphi,z,t)\,d\,\varphi\,d\,r\,d\,z\,d\,t-\Theta\frac{e\,\alpha_{1E_z}}{2m}-v_{z0}\,.$$

In this paper we calculated the second-order approximations of the required strengths and inductions of the considerate fields, as well as the ion velocities by the method of averaging functional corrections. The approximation is usually sufficient to obtain qualitative conclusions and to obtain some quantitative results. The obtained analytical results were checked by comparing them with the results of numerical simulation.

Further let us consider changing of components of displacement vector in the grown multilayer structure with changing of spatial coordinates and time. Equations, which are describe components $u_r(r,\varphi,z,t)$, $u_\varphi(r,\varphi,z,t)$ and $u_z(r,\varphi,z,t)$ of displacement vector $\vec{u}(r,\varphi,z,t)$, could be written as [20,21]

$$\begin{cases} C\frac{\partial^2 u_r(r,\varphi,z,t)}{\partial t^2}=\frac{1}{r}\frac{\partial\left[r\cdot\sigma_{rr}(r,\varphi,z,t)\right]}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{r\varphi}(r,\varphi,z,t)}{\partial\varphi}+\frac{\partial\sigma_{rz}(r,\varphi,z,t)}{\partial z}\\ C\frac{\partial^2 u_\varphi(r,\varphi,z,t)}{\partial t^2}=\frac{1}{r}\frac{\partial\left[r\cdot\sigma_{\varphi r}(r,\varphi,z,t)\right]}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{\varphi\varphi}(r,\varphi,z,t)}{\partial y}+\frac{\partial\sigma_{\varphi z}(r,\varphi,z,t)}{\partial z}\\ C\frac{\partial^2 u_z(r,\varphi,z,t)}{\partial t^2}=\frac{1}{r}\frac{\partial\left[r\cdot\sigma_{zx}(r,\varphi,z,t)\right]}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{zy}(r,\varphi,z,t)}{\partial\varphi}+\frac{\partial\sigma_{zz}(r,\varphi,z,t)}{\partial z}\end{cases}$$

where $\sigma_{ij}=\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial u_i(r,\varphi,z,t)}{\partial x_j}+\frac{\partial u_j(r,\varphi,z,t)}{\partial x_i}-\frac{\delta_{ij}}{3}\frac{\partial u_k(r,\varphi,z,t)}{\partial x_k}\right]+$

$+K(z)\,\delta_{ij}\frac{\partial u_k(r,\varphi,z,t)}{\partial x_k}-\beta(z)\,K(z)\left[T(r,\varphi,z,t)-T_r\right]$; δ_{ij} is the Kronecker symbol.

Accounting of the last relation leads to transformation of the above system of equations to the following form

$$C\frac{\partial^2 u_r(r,\varphi,z,t)}{\partial t^2}=\frac{1}{r}\left\{K(z)+\frac{5E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial}{\partial r}\left[r\frac{\partial u_r(r,\varphi,z,t)}{\partial r}\right]+$$

$$+\frac{1}{r^2}\left\{K(z)-\frac{E(z)}{3[1+\sigma(z)]}\right\}\frac{\partial^2\left[r\cdot u_\varphi(r,\varphi,z,t)\right]}{\partial r\,\partial\varphi}+\left[\frac{1}{r^2}\frac{\partial^2 u_\varphi(r,\varphi,z,t)}{\partial\varphi^2}+\frac{\partial^2 u_z(r,\varphi,z,t)}{\partial z^2}\right]\times$$

$$\times\frac{E(z)}{2[1+\sigma(z)]}+\frac{\partial^2 u_z(r,\varphi,z,t)}{\partial r\,\partial z}\left\{K(z)+\frac{E(z)}{3[1+\sigma(z)]}\right\}-K(z)\beta(z)\frac{1}{r}\frac{\partial\left[r\cdot T(r,\varphi,z,t)\right]}{\partial r}$$

$$C\frac{\partial^2 u_y(r,\varphi,z,t)}{\partial t^2}=\frac{E(z)}{2[1+\sigma(z)]}\left\{\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial u_\varphi(r,\varphi,z,t)}{\partial r}\right]+\frac{\partial}{\partial r}\left[r\frac{\partial u_r(r,\varphi,z,t)}{\partial r}\right]\right\}+$$

$$+\frac{\partial}{\partial z}\left\{\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial u_\varphi(r,\varphi,z,t)}{\partial z}+\frac{1}{r}\frac{\partial u_z(r,\varphi,z,t)}{\partial\varphi}\right]\right\}-K(z)\frac{\beta(z)}{r}\frac{\partial\left[r\cdot T(r,\varphi,z,t)\right]}{\partial\varphi}+$$

$$+\frac{1}{r^2}\left\{\frac{5E(z)}{12[1+\sigma(z)]}+K(z)\right\}\frac{\partial^2 u_\varphi(r,\varphi,z,t)}{\partial\varphi^2}+\frac{K(z)}{r}\frac{\partial^2\left[r\cdot u_\varphi(r,\varphi,z,t)\right]}{\partial r\,\partial\varphi}+$$

$$+\frac{1}{r}\left\{K(z)-\frac{E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial^2 u_y(r,\varphi,z,t)}{\partial\varphi\partial z} \qquad (6)$$

$$C\frac{\partial^2 u_z(r,\varphi,z,t)}{\partial t^2}=\frac{E(z)}{2[1+\sigma(z)]}\left\{\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial u_z(r,\varphi,z,t)}{\partial r}\right]+\frac{1}{r^2}\frac{\partial^2 u_z(r,\varphi,z,t)}{\partial \varphi^2}+\right.$$
$$\left.+\frac{\partial^2[r\cdot u_r(r,\varphi,z,t)]}{\partial r\partial z}+\frac{1}{r}\frac{\partial^2 u_\varphi(r,\varphi,z,t)}{\partial\varphi\partial z}\right\}+\frac{\partial}{\partial z}\left\{K(z)\left[\frac{1}{r}\frac{\partial u_r(r,\varphi,z,t)}{\partial r}+\right.\right.$$
$$\left.\left.+\frac{1}{r}\frac{\partial u_\varphi(r,\varphi,z,t)}{\partial\varphi}+\frac{\partial u_r(r,\varphi,z,t)}{\partial z}\right]\right\}+\frac{1}{6}\frac{\partial}{\partial z}\left\{\frac{E(z)}{1+\sigma(z)}\left[6\frac{\partial u_z(r,\varphi,z,t)}{\partial z}-\right.\right.$$
$$\left.\left.-\frac{1}{r}\frac{\partial[r\cdot u_x(r,\varphi,z,t)]}{\partial x}-\frac{1}{r}\frac{\partial u_\varphi(r,\varphi,z,t)}{\partial y}-\frac{\partial u_z(r,\varphi,z,t)}{\partial z}\right]\right\}-$$
$$-K(z)\beta(z)\frac{\partial T(r,\varphi,z,t)}{\partial z}.$$

where E is the Young modulus; β is the coefficient of thermal expansion; K is the modulus of uniform compression; σ is the Poisson coefficient; $\varepsilon_0=(a_s-a_{EL})/a_{EL}$ is the mismatch parameter; a_s, a_{EL} are the lattice distances of the substrate and the epitaxial layer; $T(r,\varphi,z,t)$ is the temperature of growth with equilibrium value T_r. Conditions for the system of the above equations can be written in the form

$$\left.\frac{\partial\vec{u}(r,\varphi,z,t)}{\partial r}\right|_{r=0}=0;\ \left.\frac{\partial\vec{u}(r,\varphi,z,t)}{\partial r}\right|_{r=R}=0;\ \left.\frac{\partial\vec{u}(r,\varphi,z,t)}{\partial z}\right|_{z=0}=0;$$
$$\left.\frac{\partial\vec{u}(r,\varphi,z,t)}{\partial z}\right|_{z=L}=0;\ \vec{u}(r,0,z,t)=\vec{u}(r,2\pi,z,t);\ \vec{u}(r,\varphi,z,0)=\vec{u}_0;$$
$$\vec{u}(r,\varphi,z,\infty)=\vec{u}_0.$$

Now let us determine solutions of system of equations (6), which describe components of displacement vector. To calculate the first-order approximations of the required components in the framework the method of averaging of function corrections one shall substitute their not yet known average values α_i in the right sides of the equations (6). The substitution leads to the following result

$$C\frac{\partial^2 u_{1r}(r,\varphi,z,t)}{\partial t^2}=-K(z)\frac{\beta(z)}{r}\frac{\partial[r\cdot T(r,\varphi,z,t)]}{\partial r},$$
$$C\frac{\partial^2 u_{1\varphi}(r,\varphi,z,t)}{\partial t^2}=-K(z)\frac{\beta(z)}{r}\frac{\partial T(r,\varphi,z,t)}{\partial\varphi},$$

$$C\frac{\partial^2 u_{1z}(r,\varphi,z,t)}{\partial t^2}=-K(z)\beta(z)\frac{\partial T(r,\varphi,z,t)}{\partial z}.$$

Integration of the right and the left sides of the above relations on time t leads to the following result

$$u_{1r}(r,\varphi,z,t)=K(z)\frac{\beta(z)}{r\cdot C}\frac{\partial}{\partial r}\left[r\cdot\int_0^t\int_0^\vartheta T(r,\varphi,z,\tau)d\tau\, d\vartheta\right]-$$

$$-K(z)\frac{\beta(z)}{r\cdot C}\frac{\partial}{\partial r}\left[r\cdot\int_0^t\int_0^\vartheta T(r,\varphi,z,\tau)d\tau\, d\vartheta\right]+u_{0r},$$

$$u_{1\varphi}(r,\varphi,z,t)=K(z)\frac{\beta(z)}{r\cdot C}\frac{\partial}{\partial\varphi}\int_0^t\int_0^\vartheta T(r,\varphi,z,\tau)d\tau\, d\vartheta-$$

$$-K(z)\frac{\beta(z)}{r\cdot C}\frac{\partial}{\partial\varphi}\int_0^\infty\int_0^\vartheta T(r,\varphi,z,\tau)d\tau\, d\vartheta+u_{0\varphi},$$

$$u_{1z}(r,\varphi,z,t)=K(z)\frac{\beta(z)}{C}\frac{\partial}{\partial z}\int_0^t\int_0^\vartheta T(r,\varphi,z,\tau)\, d\tau\, d\vartheta-$$

$$-K(z)\frac{\beta(z)}{C}\frac{\partial}{\partial z}\int_0^\infty\int_0^\vartheta T(r,\varphi,z,\tau)d\tau\, d\vartheta+u_{0z}.$$

Approximations with the second and higher orders of components of displacement vector could be obtained in the framework of the standard replacement of the required functions in the right sides of equations (6) on the following sum $\alpha_i+u_i(r,\varphi,z,t)$ [16, 19]. The replacement leads to the following result

$$C\frac{\partial^2 u_{2r}(r,\varphi,z,t)}{\partial t^2}=\frac{1}{r}\left\{K(z)+\frac{5E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial}{\partial r}\left[r\frac{\partial u_{1r}(r,\varphi,z,t)}{\partial r}\right]+\frac{E(z)}{2[1+\sigma(z)]}\times$$

$$\times\left[\frac{1}{r^2}\frac{\partial^2 u_{1y}(r,\varphi,z,t)}{\partial\varphi^2}+\frac{\partial^2 u_{1z}(r,\varphi,z,t)}{\partial z^2}\right]+\left\{K(z)-\frac{E(z)}{3[1+\sigma(z)]}\right\}\frac{1}{r^2}\frac{\partial^2[r\cdot u_{1\varphi}(r,\varphi,z,t)]}{\partial r\,\partial\varphi}-$$

$$\times\frac{1}{r^2}\frac{\partial^2[r\cdot u_{1\varphi}(r,\varphi,z,t)]}{\partial r\,\partial\varphi}+\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{1}{r^2}\frac{\partial^2 u_{1y}(r,\varphi,z,t)}{\partial\varphi^2}+\frac{\partial^2 u_{1z}(r,\varphi,z,t)}{\partial z^2}\right]-$$

$$-K(z)\beta(z)\frac{\partial[r\cdot T_{1z}(r,\varphi,z,t)]}{\partial r}+\frac{1}{r}\left\{K(z)+\frac{E(z)}{3[1+\sigma(z)]}\right\}\frac{\partial^2[r\cdot u_{1z}(r,\varphi,z,t)]}{\partial r\,\partial z}$$

$$C\frac{\partial^2 u_{2\varphi}(r,\varphi,z,t)}{\partial t^2}=\frac{E(z)}{2[1+\sigma(z)]}\left\{\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial u_{1\varphi}(r,\varphi,z,t)}{\partial r}\right]+\left[\frac{\partial^2 r\cdot u_{1r}(r,\varphi,z,t)}{\partial r\,\partial\varphi}\right]\right\}-$$

$$-K(z)\beta(z)\frac{\partial T(r,\varphi,z,t)}{\partial y}+\frac{\partial}{\partial z}\left\{\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial u_{1\varphi}(r,\varphi,z,t)}{\partial z}+\frac{1}{r}\frac{\partial u_{1z}(r,\varphi,z,t)}{\partial\varphi}\right]\right\}+$$

$$+\frac{1}{r^2}\left\{\frac{5E(z)}{12[1+\sigma(z)]}+K(z)\right\}\frac{\partial^2 u_{1\varphi}(r,\varphi,z,t)}{\partial\varphi^2}+\frac{1}{r}\left\{K(z)-\frac{E(z)}{6[1+\sigma(z)]}\right\}\times$$
$$\times\frac{\partial^2 u_{1\varphi}(r,\varphi,z,t)}{\partial\varphi\partial z}+\frac{K(z)}{r}\frac{\partial^2 u_{1\varphi}(r,\varphi,z,t)}{\partial r\partial\varphi}$$

$$C\frac{\partial^2 u_{2z}(r,\varphi,z,t)}{\partial t^2}=\frac{E(z)}{2[1+\sigma(z)]}\left\{\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial u_{1z}(r,\varphi,z,t)}{\partial r}\right]+\frac{1}{r^2}\frac{\partial^2 u_{1z}(r,\varphi,z,t)}{\partial\varphi^2}+\right.$$
$$\left.+\frac{\partial^2[r\cdot u_{1r}(r,\varphi,z,t)]}{\partial r\partial z}+\frac{1}{r}\frac{\partial^2 u_{1\varphi}(r,\varphi,z,t)}{\partial\varphi\partial z}\right\}+\frac{\partial}{\partial z}\left(\left\{\frac{1}{r}\frac{\partial[r\cdot u_{1r}(r,\varphi,z,t)]}{\partial r}+\right.\right.$$
$$\left.\left.+\frac{\partial u_{1r}(r,\varphi,z,t)}{\partial z}+\frac{\partial u_{1r}(r,\varphi,z,t)}{\partial z}\right\}\right)+\frac{1}{6}\frac{\partial}{\partial z}\left(\left\{6\frac{\partial u_{1z}(r,\varphi,z,t)}{\partial z}-\frac{1}{r}\frac{\partial u_{1y}(r,\varphi,z,t)}{\partial\varphi}-\right.\right.$$
$$\left.\left.-\frac{1}{r}\frac{\partial[r\cdot u_{1r}(r,\varphi,z,t)]}{\partial r}-\frac{\partial u_{1z}(r,\varphi,z,t)}{\partial z}\right\}\frac{E(z)}{1+\sigma(z)}\right)-K(z)\,\beta(z)\frac{\partial T(r,\varphi,z,t)}{\partial z}.$$

Integration of the right and the left sides of the above relations on time t leads to the following result

$$u_{2r}(r,\varphi,z,t)=\frac{1}{C}\left\{K(z)+\frac{5E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial}{\partial r}\left[r\frac{\partial}{\partial r}\int_0^t\int_0^\vartheta u_{1r}(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right]-\left\{\frac{E(z)}{3[1+\sigma(z)]}-\right.$$
$$\left.-K(z)\right\}\frac{1}{C}\frac{\partial^2}{\partial r\partial\varphi}\left[r\int_0^t\int_0^\vartheta u_{1\varphi}(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right]+\left[\frac{\partial^2}{\partial\varphi^2}\int_0^t\int_0^\vartheta u_{1\varphi}(r,\varphi,z,\tau)\,d\tau\,d\vartheta+\right.$$
$$\left.+\frac{\partial^2}{\partial z^2}\int_0^t\int_0^\vartheta u_{1z}(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right]\frac{E(z)}{2r^2C\,[1+\sigma(z)]}+\frac{1}{C}\frac{\partial^2}{\partial r\partial z}\left[r\int_0^t\int_0^\vartheta u_{1z}(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right]\times$$
$$\times\left\{K(z)+\frac{E(z)}{3\,[1+\sigma(z)]}\right\}-\frac{1}{C}\left\{K(z)+\frac{5E(z)}{6\,[1+\sigma(z)]}\right\}\frac{\partial}{\partial r}\left[r\frac{\partial^2}{\partial r^2}\int_0^\infty\int_0^\vartheta u_{1r}(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right]-$$
$$-K(z)\frac{\beta(z)}{C}\frac{\partial}{\partial r}\left[r\int_0^t\int_0^\vartheta T(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right]-\frac{1}{C}\frac{\partial^2}{\partial r\partial\varphi}\left[r\int_0^\infty\int_0^\vartheta u_{1y}(r,\varphi,z,\tau)d\tau\,d\vartheta\right]\left\{K(z)-\right.$$
$$\left.-\frac{E(z)}{3[1+\sigma(z)]}\right\}-\left[\frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2}\int_0^\infty\int_0^\vartheta u_{1\varphi}(r,\varphi,z,\tau)d\tau\,d\vartheta+\frac{\partial^2}{\partial z^2}\int_0^\infty\int_0^\vartheta u_{1z}(r,\varphi,z,\tau)d\tau\,d\vartheta\right]\times$$
$$\times\frac{E(z)}{2C[1+\sigma(z)]}-\frac{1}{C}\left\{K(z)+\frac{E(z)}{3\,[1+\sigma(z)]}\right\}\frac{\partial^2}{\partial r\partial z}\left[r\int_0^\infty\int_0^\vartheta u_{1z}(r,\varphi,z,\tau)d\tau\,d\vartheta\right]+$$
$$+u_{0r}+K(z)\frac{\beta(z)}{C}\frac{\partial}{\partial r}\left[r\int_0^\infty\int_0^\vartheta T(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right]$$

$$u_{2\varphi}(r,\varphi,z,t)=\left\{\frac{\partial}{\partial r}\left[r\frac{\partial}{\partial r}\int_0^t\int_0^\vartheta u_{1r}(r,\varphi,z,\tau)d\tau d\vartheta\right]+\frac{\partial^2}{\partial r\partial\varphi}\left[r\int_0^t\int_0^\vartheta u_{1r}(r,\varphi,z,\tau)d\tau d\vartheta\right]\right\}\times$$

$$\times\frac{E(z)}{2rC[1+\sigma(z)]}+\frac{1}{r^2C}\left\{\frac{5E(z)}{12[1+\sigma(z)]}+K(z)\right\}\frac{\partial^2}{\partial\varphi^2}\int_0^t\int_0^\vartheta u_{1r}(r,\varphi,z,\tau)d\tau d\vartheta+\frac{K(z)}{rC}\times$$

$$\times\frac{\partial^2}{\partial r\partial\varphi}\left[r\int_0^t\int_0^\vartheta u_{1y}(r,\varphi,z,\tau)d\tau d\vartheta\right]+\frac{1}{2C}\frac{\partial}{\partial z}\left\{\frac{E(z)}{1+\sigma(z)}\left[\frac{\partial}{\partial z}\int_0^t\int_0^\vartheta u_{1\varphi}(r,\varphi,z,\tau)\,d\tau d\vartheta+\right.\right.$$

$$\left.\left.+\frac{1}{r}\frac{\partial}{\partial\varphi}\int_0^t\int_0^\vartheta u_{1z}(r,\varphi,z,\tau)\,d\tau d\vartheta\right]\right\}+\frac{1}{rC}\left\{K(z)-\frac{E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial^2}{\partial\varphi\partial z}\int_0^t\int_0^\vartheta u_{1\varphi}(r,\varphi,z,\tau)d\tau d\vartheta-$$

$$-K(z)\frac{\beta(z)}{C}\int_0^t\int_0^\vartheta T(r,\varphi,z,\tau)d\tau d\vartheta-\frac{E(z)}{2rC[1+\sigma(z)]}\int_0^t\int_0^\vartheta T(r,\varphi,z,\tau)d\tau d\vartheta+$$

$$+\frac{1}{rC}\left\{K(z)-\frac{E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial^2}{\partial\varphi\partial z}\int_0^t\int_0^\vartheta u_{1\varphi}(r,\varphi,z,\tau)d\tau d\vartheta-\frac{E(z)}{2rC[1+\sigma(z)]}\times$$

$$\times\left\{\frac{\partial}{\partial r}\left[r\frac{\partial}{\partial r}\int_0^\infty\int_0^\vartheta u_{1r}(r,\varphi,z,\tau)d\tau d\vartheta\right]+\frac{\partial^2}{\partial r\partial\varphi}\left[r\int_0^\infty\int_0^\vartheta u_{1r}(r,\varphi,z,\tau)d\tau d\vartheta\right]\right\}-K(z)\frac{\beta(z)}{C}\times$$

$$\times\int_0^\infty\int_0^\vartheta T(r,\varphi,z,\tau)d\tau d\vartheta-\frac{1}{r^2C}\frac{\partial^2}{\partial\varphi^2}\int_0^\infty\int_0^\vartheta u_{1r}(r,\varphi,z,\tau)\,d\tau d\vartheta\left\{K(z)+\frac{5E(z)}{12[1+\sigma(z)]}\right\}-$$

$$-\frac{1}{2C}\frac{\partial}{\partial z}\left\{\frac{E(z)}{1+\sigma(z)}\left[\frac{\partial}{\partial z}\int_0^\infty\int_0^\vartheta u_{1\varphi}(r,\varphi,z,\tau)d\tau d\vartheta+\frac{1}{r}\frac{\partial}{\partial\varphi}\int_0^\infty\int_0^\vartheta u_{1z}(r,\varphi,z,\tau)d\tau d\vartheta\right]\right\}+u_{0\varphi}-$$

$$-\frac{K(z)}{rC}\frac{\partial^2}{\partial r\partial\varphi}\left[r\int_0^\infty\int_0^\vartheta u_{1\varphi}(r,\varphi,z,\tau)d\tau d\vartheta\right]+\frac{\partial^2}{\partial\varphi\partial z}\int_0^\infty\int_0^\vartheta u_{1\varphi}(r,\varphi,z,\tau)\,d\tau d\vartheta\times$$

$$\times\left\{\frac{E(z)}{6[1+\sigma(z)]}-K(z)\right\}\frac{1}{rC}$$

$$u_{2z}(r,\varphi,z,t)=\left\{\frac{\partial}{\partial r}\left[r\frac{\partial}{\partial r}\int_0^\infty\int_0^\vartheta u_{1z}(r,\varphi,z,\tau)\,d\tau d\vartheta\right]+\frac{1}{r}\frac{\partial^2}{\partial\varphi^2}\int_0^\infty\int_0^\vartheta u_{1z}(r,\varphi,z,\tau)\,d\tau d\vartheta+\right.$$

$$\left.+\frac{\partial^2}{\partial r\partial z}\left[r\int_0^\infty\int_0^\vartheta u_{1r}(r,\varphi,z,\tau)d\tau d\vartheta\right]+\frac{\partial^2}{\partial\varphi\partial z}\int_0^\infty\int_0^\vartheta u_{1y}(r,\varphi,z,\tau)d\tau d\vartheta\right\}\frac{E(z)}{2rC[1+\sigma(z)]}+$$

$$+\frac{1}{rC}\frac{\partial}{\partial z}\left(\frac{1}{K(z)}\left\{\frac{\partial}{\partial r}\left[r\int_0^\infty\int_0^\vartheta u_{1r}(r,\varphi,z,\tau)\,d\tau d\vartheta\right]+\frac{1}{r}\frac{\partial}{\partial\varphi}\int_0^\infty\int_0^\vartheta u_{1r}(r,\varphi,z,\tau)\,d\tau d\vartheta+\right.\right.$$

$$\left.\left.+\frac{\partial}{\partial z}\int_0^\infty\int_0^\vartheta u_{1r}(r,\varphi,z,\tau)\,d\tau d\vartheta\right\}\right)+\frac{1}{6C}\frac{\partial}{\partial z}\left(\frac{E(z)}{1+\sigma(z)}\left\{6\frac{\partial}{\partial z}\int_0^\infty\int_0^\vartheta u_{1z}(r,\varphi,z,\tau)\,d\tau d\vartheta-\right.\right.$$

$$-\frac{1}{r}\frac{\partial}{\partial r}\left[r\int_0^\infty\int_0^\vartheta u_{1r}(r,\varphi,z,\tau)d\,\tau\,d\,\vartheta\right]-\frac{1}{r}\frac{\partial}{\partial\varphi}\int_0^\infty\int_0^\vartheta u_{1\varphi}(r,\varphi,z,\tau)\,d\,\tau\,d\,\vartheta-$$

$$-\frac{\partial}{\partial z}\int_0^\infty\int_0^\vartheta u_{1z}(r,\varphi,z,\tau)\,d\,\tau\,d\,\vartheta\Bigg\}\Bigg)-K(z)\frac{\beta(z)}{C}\frac{\partial}{\partial z}\int_0^\infty\int_0^\vartheta T(r,\varphi,z,\tau)\,d\,\tau\,d\,\vartheta+u_{0z}\,.$$

In this paper we calculated the second-order approximations of components of displacement vector by using the method of averaging functional corrections. The approximation is usually sufficient to obtain qualitative conclusions and to obtain some quantitative results. The obtained analytical results were checked by comparing them with the results of numerical simulation.

3. Discussion

In this section we analyzed dynamics of mass transport during growth of films by magnetron sputtering to determine conditions to change properties of epitaxial layers. The Fig. 2 shows dependence of concentration of sputtered material on cyclotron frequency ω_c. Increasing of induction of magnetic field B_0 leads to increasing of the cyclotron frequency ω_c and to increase homogeneity of epitaxial layer. The Fig. 3 shows dependence of concentration of sputtered material on electric strengths E_0. Increasing of the strengths leads to increasing of speed of transport of ions to target and their concentration. In this case the opposite effect was obtained with increasing of the ion mass (see Fig. 3), radius (see Fig. 4) and length (see Fig. 5) of the magnetron. During analysis of components of displacement vector we obtained, that increasing of velocity of ions of growing material leads to decreasing of value of the considered vector (see Fig. 6).

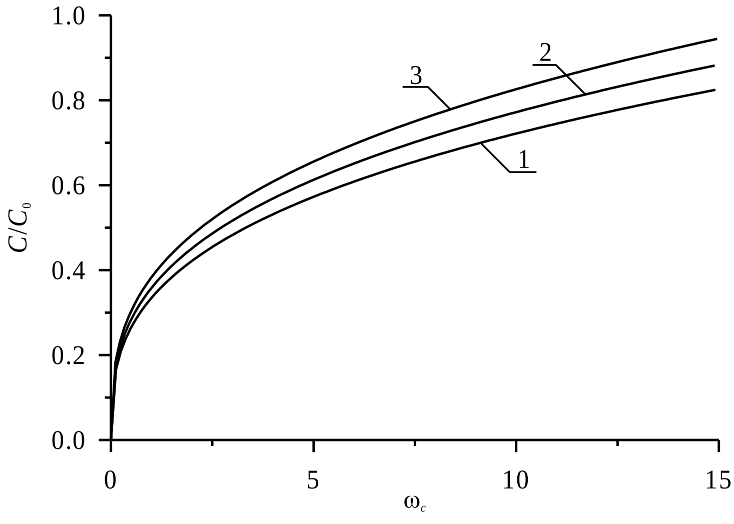

Fig. 2. Typical dependence of concentration of sputtered material on cyclotron frequency ω_c. Increasing of density of curves correspond to increasing of strength of electrical field

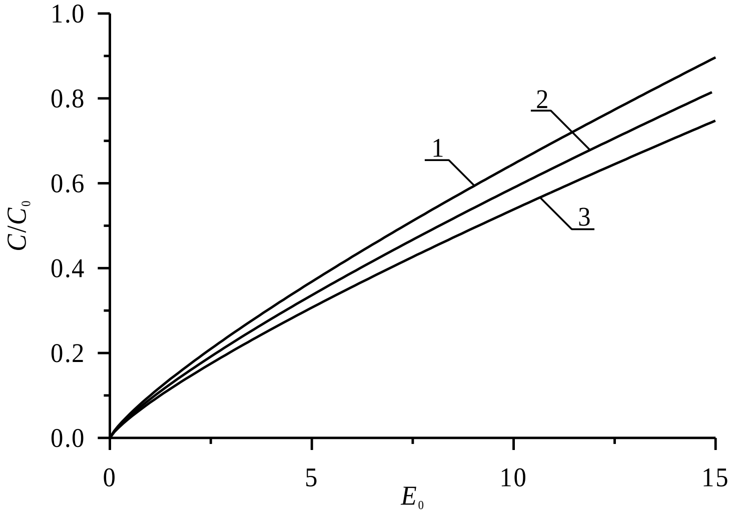

Fig. 3. Typical dependence of concentration of sputtered material on electric strengths E_0. Increasing of density of curves correspond to increasing of mass of ions

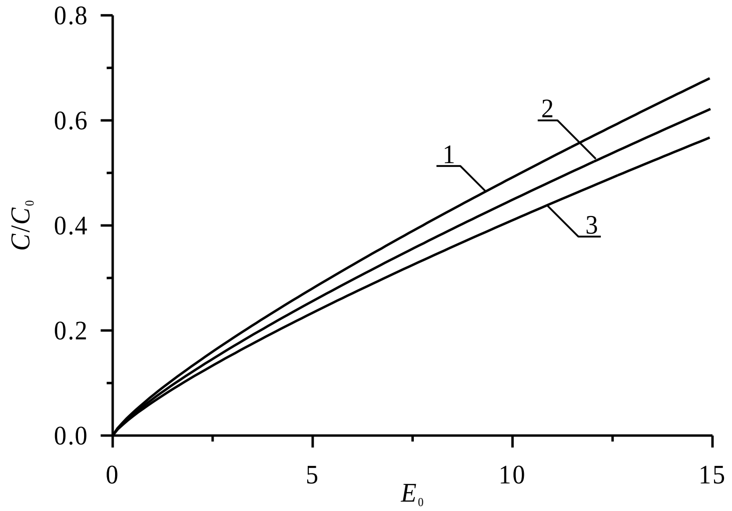

Fig. 4. Typical dependence of concentration of sputtered material on electric strengths E_0. Increasing of density of curves correspond to increasing of radius of magnetron

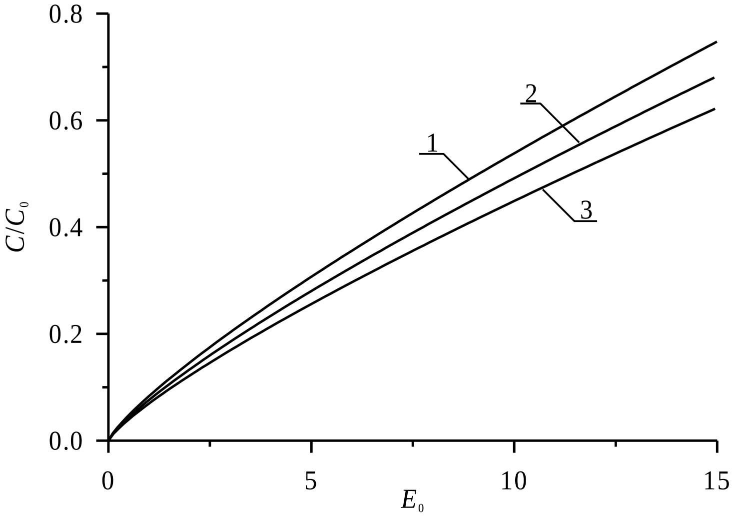

Fig. 5. Typical dependence of concentration of sputtered material on electric strengths E_0. Increasing of density of curves correspond to increasing of length of magnetron

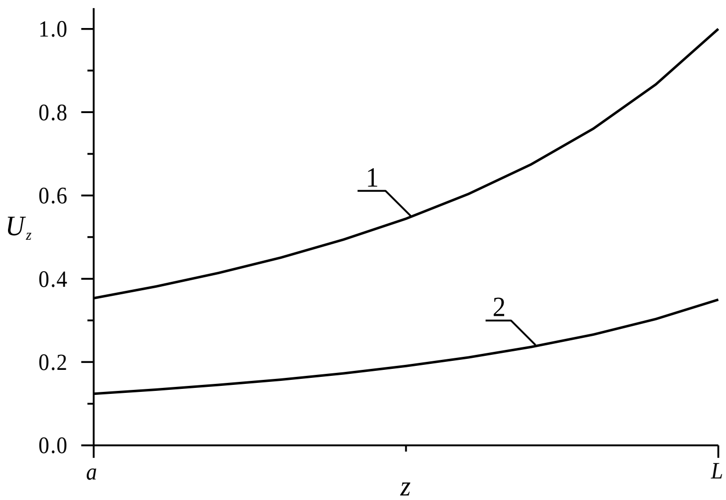

Fig. 6. Normalized dependences of component of displacement vector u_z on coordinate z at small velocity of ions (curve 1) and large velocity of ions (curve 2) of growing material

4. CONCLUSIONS

In the present paper we analyzed mass transport during magnetron sputtering of materials. Based on results of analysis we formulate recommendations to control of properties of epitaxial layers. We analyzed possibility to decrease mismatch-induced stress in multilayer structure during growth. We also introduce an analytical approach for prognosis of mass transport during growth of layers in magnetron.

REFERENCES

[1] Stepanenko, I.P., (1980) *Basis of microelectronics*, Moscow: Soviet radio.

[2] Gusev, V.G. & Gusev, Yu.M., (1991) *Electronics*, Moscow, Higher School.

[3] Lachin, V.I., Savelov & N.S., (2001) *Electronics*, Rostov on Don, Phenics, 2001.

[4] A.A. Vorob'ev, V.V. Korabl'ev, S.Yu. Karpov. Semiconductors. Vol. 37 (7). P. 98 (2003).

[5] Sorokin, L.M., Veselov, N.V., Shcheglov, M.P., Kalmykov, A.E., Sitnikova, A.A., Feoktistov, N.A., Osipov, A.V. & Kukushkin, S.A., (2008) "The use of magnesium to dope gallium nitride obtained by molecular-beam epitaxy from activated nitrogen", Technical Physics Letters. Vol. 34 (22), pp. 838-842.

[6] Lundin, V.V., Sakharov, A.V., Zavarin, E.E., Sinitsin, M.A., Nikolaev, A.E., Mikhailovsky, G.A., Brunkov, P.N., Goncharov, V.V., Ber, B.Ya., Kazantsev, D.Yu. & Tsatsul'nikov, A.F., (2009) "Electron-microscopic investigation of a SiC/Si(111) structure obtained by solid phase epitaxy", Semiconductors. Vol. 43 (7), pp. 812-817.

[7] Li, Y., Antonuk, L.E., El-Mohri, Y., Zhao, Q., Du, H., Sawant, A. & Wang, Yi (2006), "Effects of x-ray irradiation on polycrystalline silicon, thin-film transistors", Journal of applied physics, Vol. 99 (6), pp. 064501.

[8] Chakraborty, A., Xing, H., Craven, M.D., Keller, S., Mates, T., Speck, J.S., Den Baars, S.P. & Mishra, U.K., (2004) "Nonpolar α-plane *p*-type and junction diodes", Journal of applied physics, Vol. 96 (8), pp. 4494-4499.

[9] Mitsuhara, M., Ogasawara, M. & Sugiura, H. (1998) "Beryllium doping of InP during metalorganic molecular beam epitaxy using bismethylcyclopentadienyl-beryllium", Journal of crystal growth,. Vol. 183, pp. 711-716.

[10] Zhukov, V.V., Krivobokov, V.P. & Yanin, S.N., (2006) "Sputtering of the magnetron diode target in the presence of an external ion beam" Technical Physics, Vol. 51 (4). P. 453-458.

[11] Talalaev, R.A., Yakovleva, E.V., Karpova, S.Yu. & Makarov, Yu.N., (2001) "On low temperature kinetic effects in metal–organic vapor phase epitaxy of III-V compounds", Journal of crystal growth, Vol. 230. P. 232-238.

[12] Druzhkov, S.S., (2013) "Mathematical modeling of the magnetron sputtering process", Bulletin of Ufa State Aviation Technical University, Vol. 17 (8). P. 137-142 (2013).

[13] Pankratov, E.L. & Bulaeva, E.A., (2015) "On optimization of regimes of epitaxy from gas phase. some analytical approaches to model physical processes in reactors for epitaxy from gas phase during growth films", Reviews in theoretical science. Vol. 3 (4). P. 365-398.

[14] Pankratov, E.L. & Bulaeva, E.A., (2014) "Analysis of nonlinear model of gas dynamics in a vertical reactor for gas phase epitaxy", International Journal of Computational Materials. Vol. 3 (3). P. 1450013-1450026.

[15] Pankratov, E.L. & Bulaeva, E.A., (2015) "Optimization of pressure in epitaxial growth from gas phase system to decrease energy costs", Quantum Matter. Vol. 4 (5). P. 456-457.

[16] Pankratov, E.L. & Bulaeva, E.A., (2015) "On prognosis of epitaxy from gas phase process to improve properties of epitaxial layers" 3D research. Vol. 6 (4). P. 46-56.

[17] Vorob'ev, G.S., Sokolov, S.V., Pisarenko, L.D. & Zhurba, V.O. (2010) *Theory of electromagnetic field and basis of microwave techniques*. Sumy: State University of Sumy.

[18] Pimenov, Yu.M., Vol'man, V.I. & Muravtsov, A.D. (2010) *Technical electrodynamics*, Moscow: Radio and Communication.

[19] Sokolov, Yu.D. (1955) "About the definition of dynamic forces in the mine lifting" Applied Mechanics. Vol.1 (1). P. 23-35.

[20] Zhang, Y.W. & Bower, A.F. (1999) "Numerical simulations of island formation in a coherent strained epitaxial thin film system", Journal of the Mechanics and Physics of Solids. Vol. 47 (11). P. 2273-2297.

[21] Landau, L.D. & Lifshits, E.M. (2001) *Theoretical physics*. Vol. 7 (*Theory of elasticity*), Moscow: PHYSMATLIT.