A New Hyperchaotic System with Coexisting Attractors: Its Control, Synchronization and Secure Communication

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Abstract

A new hyperchaotic system with coexisting attractors based on Sprott B chaotic system is proposed in this work. A novel feature of this new hyperchaotic system under investigation is that it has two-wing and four-wing coexisting attractors for two sets of different initial conditions. Thus, the new hyperchaotic system has hidden attractors. Interestingly, the proposed designed control function $u_i(t)$ *using adaptive control method was able to control and globally synchronizes two identical new hyperchaotic systems evolving from different initial conditions with uncertain parameters. The adaptive synchronization scheme was applied to secure communication. Finally, the numerical simulation results presented demonstrated the effectiveness of the analytical results of the designed scheme.*

Keywords

hyperchaotic system, coexisting attractors, adaptive control, uncertain parameters, synchronization, secure communication.

1. Introduction

Recently, chaos theory has becomes a focal point of discussion among the expert and researcher due to its potential applications in: physics, chemical and biological sciences [1], finances [2-3], economic [4-6], telecommunication and secure communication [7-11], high performance electric circuit design [12-14].

Historically, the first hyperchaotic system popularly known as four-dimensional hyperchaotic Rossler system was reported in 1979 [15]. Hyperchaotic system is more prominent over the chaotic system, because chaotic system has only one positive Lyapunov exponent while hyperchaotic system has at least two positive Lyapunov exponents. This feature make it more complex and unpredictable than chaotic system, hence give room to wide range of potential applications compared to 3D chaotic system [16-17].

Numerous techniques have been developed and reported in the literature to achieve chaos control and synchronization. Some of these methods are: active control [18-20], adaptive control [21-27], backstepping technique [28-30], sliding mode control [31-32].

The main focus in chaotic or hyperchaotic synchronization is to design the effective control feedback function $u_i(t)$ that will force the state variables of the response (slave) system to track the corresponding trajectories of the state variables of the drive (master) system asymptotically with time. In most practical applications, the unknown parameters in the drive or response state or both states at time usually destroyed the desired synchronization. Therefore, the convectional synchronization techniques are not effective in such situation [33]. Thus, the synchronization technique for chaotic or hyperchaotic systems with uncertain parameter is an interesting challenge that has attracted great attention in a recent time. As the results, the synchronization method for unknown parameter in chaotic systems remains a significant point among the researchers.

The synchronization of chaotic system is motivated by its potential applications in secure communication, information security and privacy protection. To improve the security of the aforementioned applications, more complex chaotic dynamical behaviors are used. Consequently, coexisting attractors with more complex dynamical behaviors are more important compared to generated chaotic attractor. To improve the information security and reduce the probability of information being decoded, coexisting attractors are more reliable [34-35].

Coexistence of attractors also known as multistability refer to the systems that neither stable nor totally unstable but alternate between two or more mutually exclusive attractor with time [36].
Coexistence (multistability) is a unique property of a chaotic and hyperchaotic system indicated by the presence of two or more coexisting attractors for the same set of system parameter but different sets of initial conditions [37].

The most important application of chaos synchronization in engineering is in secure communication. The basic idea is to use a chaotic oscillator as a broadband signal generation. The chaotic signal is mask (encrypt) the information signal to produce unpredictable signal which is transmitted from the drive to the response (see refs. [30] and [8]), [38]. At the response, the pseudo-random is generated through the inverse operation and the original signal is retrieved.

In this paper, a new hyperchaotic system with two-wing and four-wing attrctors that displayed multistability for two different sets of initial conditions is discussed. The next task is to design a control function $u_i(t)$ to control as well as to synchronize the drive and response systems; design parameter update law to identify the unknown system parameters and to apply the synchronization scheme to secure communication.

To the best of our knowledge, adaptive control and synchronization with application to secure communication for Sprott B-based hyperchaotic system is reported here for the first time.

2. NUMERICAL DESCRIPTION OF THE MODELS

The mathematical formulation investigated in this paper is the modified Sprott B chaotic system constructed by adding a state-feedback controller on the Sprott B chaotic system and is given in equation (1).

$$\begin{aligned}\dot{x} &= a(y - x) \\ \dot{y} &= xz + w \\ \dot{z} &= b - xy \\ \dot{w} &= yz - cw\end{aligned} \qquad (1)$$

In equation (1), x, y, z and w are the state-variables of the system, where a, b and c are the real positive constant system parameters. System (1) displayed hyperchaotic behavior with the real positive constant parameters; $a = 6$, $b = 11$ and $c = 5$ via numerical simulation. The strange attractors of the Sprott B-based hyperchaotic system (1) are displayed in figure 1. Figure 1 (a, b and c) displayed 2-wing attractor and figure 1(d) shown 4-wing attractor at the same time.

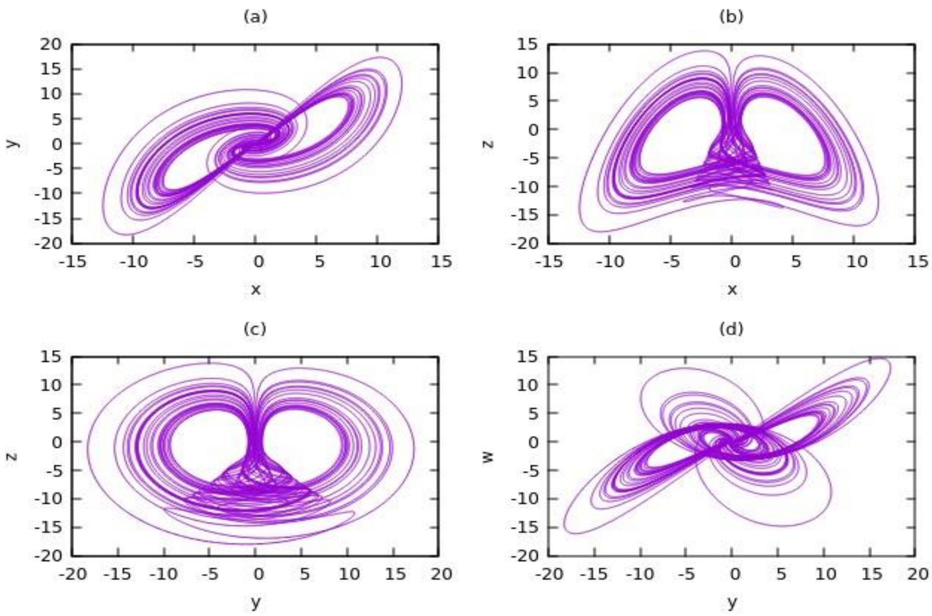

Figure 1: The two-wing and four-wing attractors for hyperchaotic system (1)

3. COEXISTENCE OF ATTRACTORS

System (1) is invariant under the transformation $S : (x, y, z, w) \mapsto (-x, -y, z, -w)$. Hence, any projection of the attractor has rotational symmetry in the z-axis. Thus, system (1) may likely display coexisting attractors.

It is cleared from figure 2 that system (1) exhibited coexisting attractors with respect to two sets of different initial conditions; $(x, y, z, w) = (0.5, 0.6, 0.6, 0.8)$ plotted in blue color and $(x, y, z, w) = (-9.0, 1.0, 0.2, 9.0)$ plotted in red color through numerical simulation. Thus, system (1) has hidden attractors.

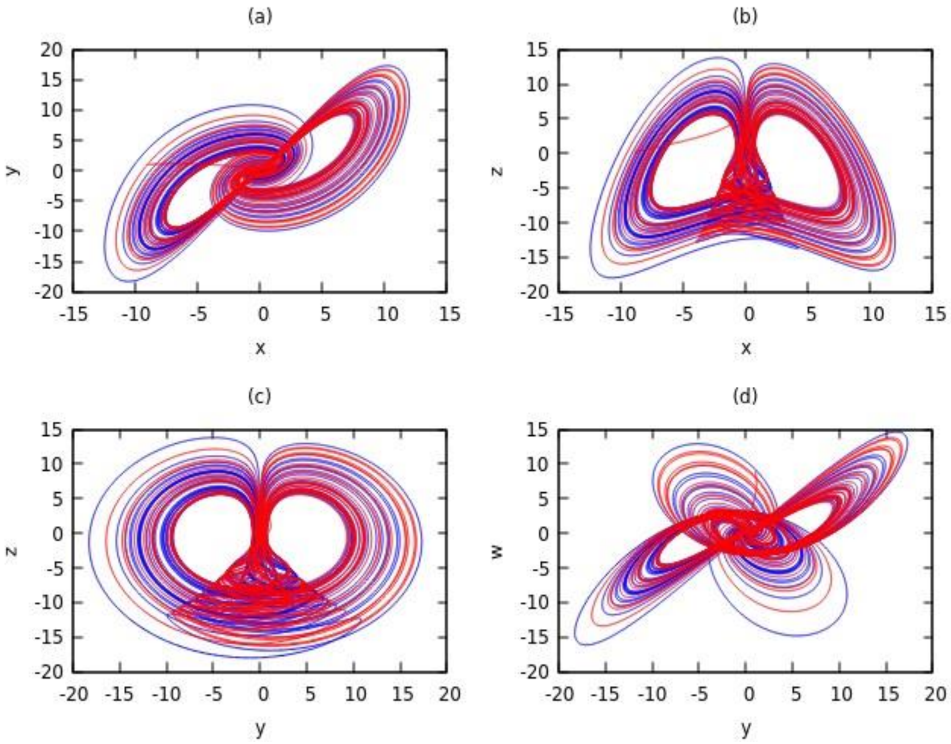

Figure 2: Two-wing and four-wing coexistence of attractors of the hyperchaotic system (1) with two sets of initial conditions.

4. Adaptive Control for the New Hyperchaotic System

In this section, we applied the adaptive control method to designed the control function $u_i(t)$ that converge the state variables (x, y, z, w) asymptotically to the origin with at time according to Lyapunov stability theory [39].

4.1. Design of Adaptive control input $u_i(t)$ for system (1)

The assumption here is that the positive real parameters of the system; a , b and c are uncertain. Therefore, adaptive control technique is used to design the control input $u_i(t)$ as well as the parameter update law to identify the unknown system parameters.

Then, the controlled system is considered as follows:

$$\begin{aligned}
\dot{x} &= a(y-x)+u_1 \\
\dot{y} &= xz+w+u_2 \\
\dot{z} &= b-xy+u_3 \\
\dot{w} &= yz-cw+u_4
\end{aligned} \tag{2}$$

Where $u_i(t)$ ($i=1,2,3,4$) are the control functions to design appropriately.

The Lyapunov stability theory (ref. [39]) is used to validate the result of system (2) by selecting a Lyapunov function as:

$$V = \frac{1}{2}\left(x^2 + y^2 + z^2 + w^2 + \tilde{a}^2 + \tilde{b}^2 + \tilde{c}^2\right) \quad (3)$$

Where $\tilde{a} = a - \bar{a}$, $\tilde{b} = b - \bar{b}$ and $\tilde{c} = c - \bar{c}$ are the estimated values of the assumed unknown parameters a, b and c respectively. The time derivative of equation (3) above is given in equation (4) below.

$$\dot{V} = x\dot{x} + y\dot{y} + z\dot{z} + w\dot{w} + \tilde{a}\dot{\tilde{a}} + \tilde{b}\dot{\tilde{b}} + \tilde{c}\dot{\tilde{c}} \quad (4)$$

In order to ensure that the control function $u_i(t)$ in equation (2) converge the state variables of system (1) to the origin asymptotically, the control input $u_i(t)$ is selected from equation (2) as follows:

$$\begin{aligned} u_1 &= -a(y - x) - x \\ u_2 &= -xz - w - y \\ u_3 &= -b + xy - z \\ u_4 &= -yz + cw - w \end{aligned} \quad (5)$$

The Substitution of equation (2) into equation (4) yielded equation (6).

$$\begin{aligned} \dot{V} &= x[a(y - x) + u_1] + y[xz + w + u_2] + z[b - xy + u_3] + w[yz - cw + u_4] + \\ &\tilde{a}[-\dot{\bar{a}} - x^2 + xy] + \tilde{b}[-\dot{\bar{b}} + z] + \tilde{c}[-\dot{\bar{c}} - w^2] \end{aligned} \quad (6)$$

The parameter update laws are estimated from equation (6) and presented in equation (7).

$$\begin{aligned} \dot{\bar{a}} &= -x^2 + xy \\ \dot{\bar{b}} &= z \\ \dot{\bar{c}} &= -w^2 \end{aligned} \quad (7)$$

Substituting equations (5) and (7) respectively into equation (4) give:

$$\dot{V} = -x^2 - y^2 - z^2 - w^2 - \tilde{a}^2 - \tilde{b}^2 - \tilde{c}^2 < 0 \quad (8)$$

Hence, V is a quadratic positive definite Lyapunov function (see equation (3)) and its time derivative $(\dot{V})$ is a quadratic negative definite as reflected in equation (8).

According to the Lyapunov stability theory, system (2) can converge to the origin asymptotically with the control input $u_i(t)$ ($i = 1,2,3,4$) as defined in equation (5) and the parameter estimated update laws in equation (7).

4.2. Numerical Simulation Results

To studies the time response of the new Sprott B-based hyperchaotic system with coexisting attractors as described in system (1), classical fourth-order RungeKuta routine with time step $h = 0.001$is adopted in the numerical simulation.

Fixing the parameters value $[a,b,c] = [6.0,11.0,5.0]$ in that order and the initial conditions $(x, y, z, w) = (0.0,0.5,0.6,0.1)$, the state variables move hyperchaotically with the control function $u_i(t)$ deactivated and converges asymptotically to the origin when the control function $u_i(t)$ is activated at $t = 50$according to the Lyapunov stability theory.

Figure 3 show the results for the time responses of the state variables (x, y, z, w) of the new hyperchaotic system (1).

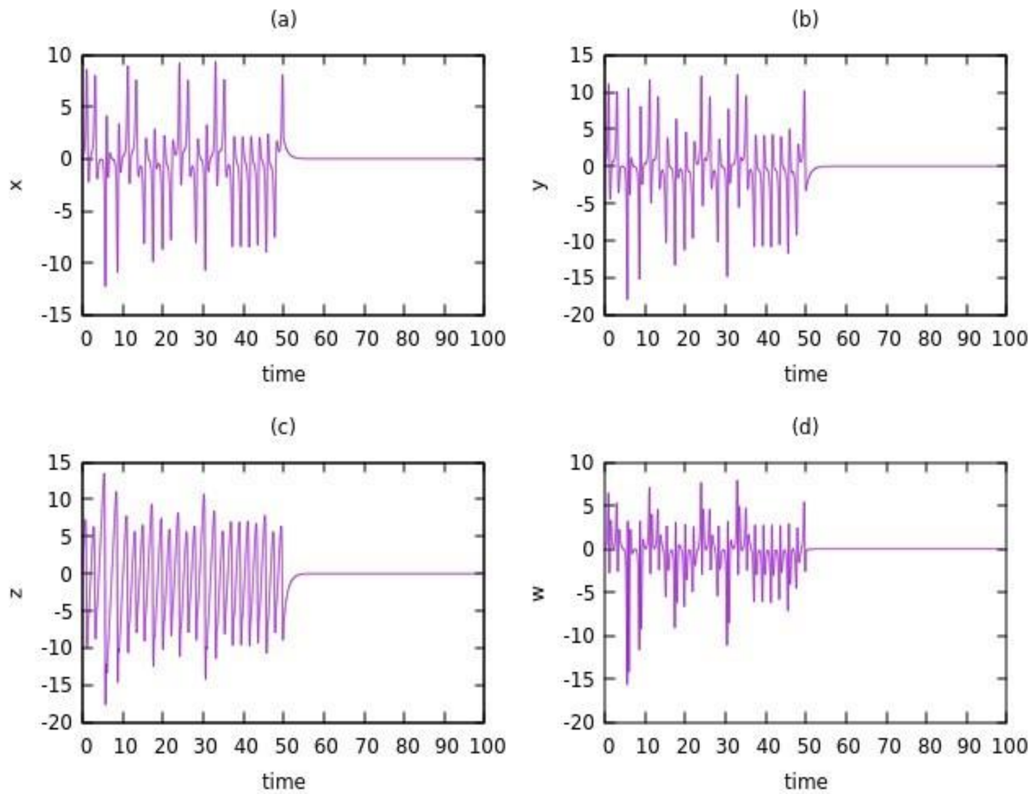

Figure 3: Time responses of the state variables (x, y, z, w) for new hyperchaotic system (1) via adaptive control.

5. ADAPTIVE SYNCHRONIZATION FOR THE NEW HYPERCHAOTIC SYSTEM

Here, we employed adaptive control techniques base on Lyapunov stability theory (ref. [39]) to achieved complete synchronization of two identical hyperchaotic systems.

5.1. Design of Adaptive control input $u_i(t)$ for system (1)

In this section, the adaptive control method is used to synchronize two identical hyperchaotic systems emanating from different initial condition.

From equation (1), let; $x = x_1$, $y = x_2$, $z = x_3$ and $w = x_4$.

Then,

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) \\ \dot{x}_2 &= x_1 x_3 + x_4 \\ \dot{x}_3 &= b - x_1 x_2 \\ \dot{x}_4 &= x_2 x_3 - c x_4 \end{aligned} \tag{9}$$

The equation (9) above is called the master or drive system while equation (10) below is designated as slave or response system.

$$\begin{aligned} \dot{y}_1 &= a(y_2 - y_1) + u_1 \\ \dot{y}_2 &= y_1 y_3 + y_4 + u_2 \\ \dot{y}_3 &= b - y_1 y_2 + u_3 \\ \dot{y}_4 &= y_2 y_3 - c y_4 + u_4 \end{aligned} \tag{10}$$

Where $u_i(t)$ ($i = 1,2,3,4$) are the control input to determines.

The synchronization error vector between the master (9) and the slave (10) is defined by:

$$\begin{aligned} e_1 &= y_1 - x_1 \\ e_2 &= y_2 - x_2 \\ e_3 &= y_3 - x_3 \\ e_4 &= y_4 - x_4 \end{aligned} \tag{11}$$

Hence, using the definition of the error vector in equation (11), the error dynamic is calculated as follows:

$$\begin{aligned} \dot{e}_1 &= a(e_2 - e_1) + u_1 \\ \dot{e}_2 &= x_1 e_3 + x_3 e_1 + e_1 e_3 + e_4 + u_2 \\ \dot{e}_3 &= -(x_1 e_2 + x_2 e \ \ + e_1 e_2) + u_3 \\ \dot{e}_4 &= x_2 e_3 + x_3 e_2 + e_2 e_3 - c e_4 + u_4 \end{aligned} \tag{12}$$

Choosing the Lyapunov function; $V = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2 + \tilde{a}^2 + \tilde{b}^2 + \tilde{c}^2\right)$ and differentiating it with respect to time result in equation (13).

$$\dot{V} = e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4 + a\dot{\tilde{a}} + b\dot{\tilde{b}} + c\dot{\tilde{c}} \quad (13)$$

Where $\tilde{a} = a - \bar{a}$, $\tilde{b} = b - \bar{b}$, and $\tilde{c} = c - \bar{c}$ are the estimated values of the unknown parameter a, b and c respectively.

Then, equation (14) is obtained by substituted equation (12) into equation (13).

$$\begin{aligned}\dot{V} = & e_1[a(e_2 - e_1) + u_1] + e_2[x_1e_3 + x_3e_1 + e_1e_3 + e_4 + u_2] + \\ & e_3[-(x_1e_2 + x_2e_1 + e_1e_2) + u_3] + e_4[x_2e_3 + x_3e_2 + e_2e_3 - ce_4 + u_4] + \\ & \tilde{a}[\dot{\tilde{a}} - e_1(e_2 - e_1)] + \tilde{b}\left[\dot{\tilde{b}}\right] + \tilde{c}[\dot{\tilde{c}} - (-e_4^2)]\end{aligned} \quad (14)$$

From equation (12), the control input $u_i(t)$ is chosen as:

$$\begin{aligned} u_1 &= -a(e_2 - e_1) - e_1 \\ u_2 &= -(x_1e_3 + x_3e_1 + e_1e_3 + e_4) - e_2 \\ u_3 &= x_1e_2 + x_2e_1 + e_1e_2 - e_3 \\ u_4 &= ce_4 - (x_2e_3 + x_3e_2 + e_2e_3) - e_4 \end{aligned} \quad (15)$$

And the estimated parameter update law is chosen from equation (14) as follows:

$$\begin{aligned} \dot{\bar{a}} &= e_1(e_2 - e_1) - a \\ \dot{\bar{b}} &= -b \\ \bar{c} &= -e_4^2 - c \end{aligned} \quad (16)$$

Substituting equations (15) and (16) respectively into equation (14) gives equation (17).

$$\dot{V} = -e_1^2 - e_2^2 - e_3^2 - e_4^2 - a^2 - b^2 - c^2 < 0 \quad (17)$$

The Lyapunov function V is positively definite with it derivative $\dot{V}$ is negatively definite as confirmed by equation (17) above. Hence, the error dynamic variable in equation (12) can converge to the origin asymptotically in line with the Lyapunov stability theory (ref. [39]) and one can conclude that system (12) is globally and exponentially stable. Also the master (drive) and the slave (response) systems (equations (9) and (10)) are globally and exponentially synchronized for all the initial conditions $x_i(0)$ and $y_i(0)$, and the estimated update law (16).

5.2. Numerical Simulation Results

The main objective of adaptive synchronization is to design an approximate control function $u_i(t)$ to force the state variables of the response (slave) system to track the trajectories of the drive (master) state variables such that both systems will remain in step throughout the transmission of signal with the parameter update law as well as to stabilize the error function $e_i(t)$ between the drive (master) and the response (master) systems at the origin $(0,0,0,0)$ at any chosen time.

To achieve the stated objective, fourth-order RungeKuta algorithm is used to solve the control law (15) and the estimated parameter update law (16) by fixing the parameter values of the system (1) $[a,b,c]=[6.0,11.0,5.0]$ with the time step $h=0.001$. The initial conditions for the drive (master) (x_i) and the response (slave) (y_i) systems are respectively choosing as $(x_1,x_2,x_3,x_4)=(0.5,0.6,0.6,0.8)$ and $(y_1,y_2,y_3,y_4)=(-9.0,1.0,0.2,9.0)$. The reports of the numerical simulation are: the state variables of the response (slave) system track the dynamics of the drive (master) system when the control function $u_i(t)$ is activated at $t=50$ as shown in figure 4; the error function $e_i(t)$ converges asymptotically to the origin in line with the Lyapunov stability theory, when the controllers are switched on at $t=50$ as depicted in figure 5 and the synchronization norm $e=\sqrt{e_1^2+e_2^2+e_3^2+e_4^2}$ is displayed in figure 6.

For parameter updating, the initial values of the parameter update law (16) are selected as $a_1(0)=6.0$, $b_1(0)=11.0$ and $c_1(0)=5.0$. The parameters estimated value $\bar{a}$, $\bar{b}$ and $\bar{c}$ updated to $a=10.0$, $b=13.0$ and $c=8.0$ respectively as shown in figure 7 as $t\to\infty$.

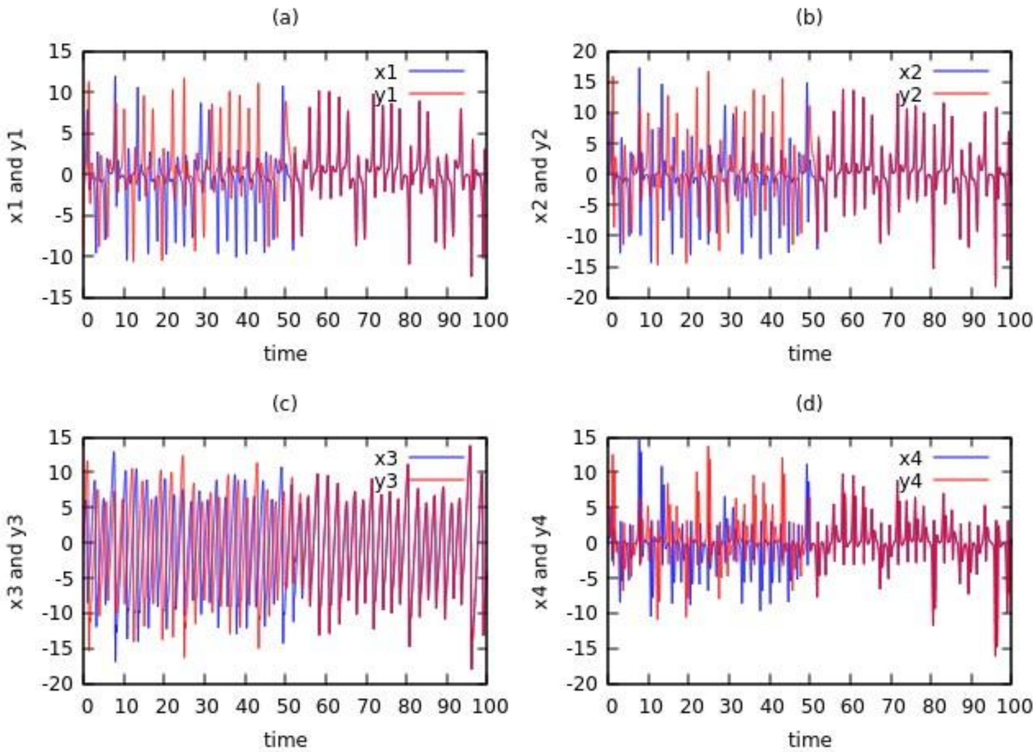

Figure 4: Time responses for the state variables; drive (master) (x_1,x_2,x_3,x_4) and response (slave) (y_i,y_2,y_3,y_4) systems for new hyperchaotic system (1) with the controller activated.

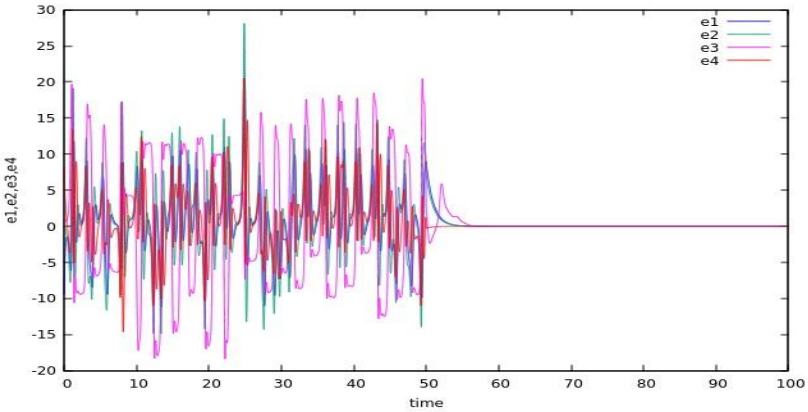

Figure 5: Error dynamic between the drive (master) and the response (slave) systems for new hyperchaotic system (1) with the controllers activated.

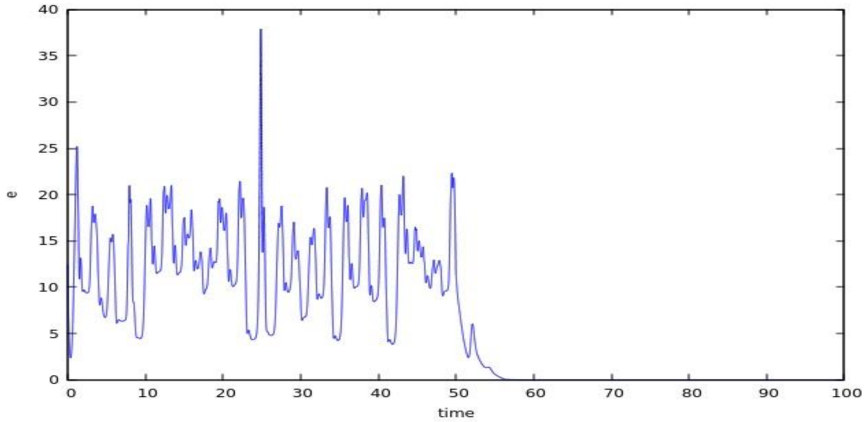

Figure 6: Synchronization norm between the drive (master) and the response (slave) systems for new hyperchaotic system (1) with the controllers activated.

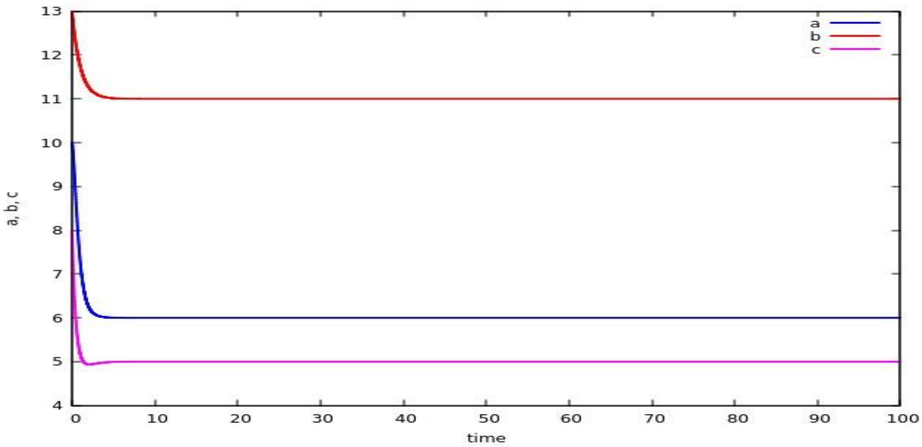

Figure 7: Time response of the parameter update law for new hyperchaotic system (1).

6. SECURE COMMUNICATION

The adaptive synchronization scheme in section 5 is applied for secure communication here. The major components considering in this scheme are: information signal (Sm), encryption signal (Sc), decryption signal (Sr) and decryption error signal (er).

In this secure communication, the information signal is masked with the hyperchaotic wave signal carrier using mixing algorithm. The information signal is given by:

$$Sm = 4.0\cos 0.5t \tag{17}$$

The encrypted information with the hyperchaotic wave signal x_i remains hyperchaotic throughout the signal transmission as illustrated in equation (18).

$$Sc = x_i + Sm \tag{18}$$

The information is later decrypted by the inverse function at $t = 50$ when the controllers are activated for decryption. The decrypted information is given by equation (19).

$$Sr = Sc - y_i \tag{19}$$

The hyperchaotic wave signal carrier x_i of the drive (master) system is transmitted to the response (slave) system y_i via coupling channel for synchronization between the drive (master) and response (slave) systems. The difference between the information signal and the decrypted signal approaches zero at $t \to \infty$ with the controllers activated at $t = 50$, implied that the information is recovered. The error function $er(t)$ between the information signal and the decrypted signal is given by equation (20).

$$er = Sm - Sr \tag{20}$$

The numerical simulation results of the secure communication scheme are shown in figure 8.

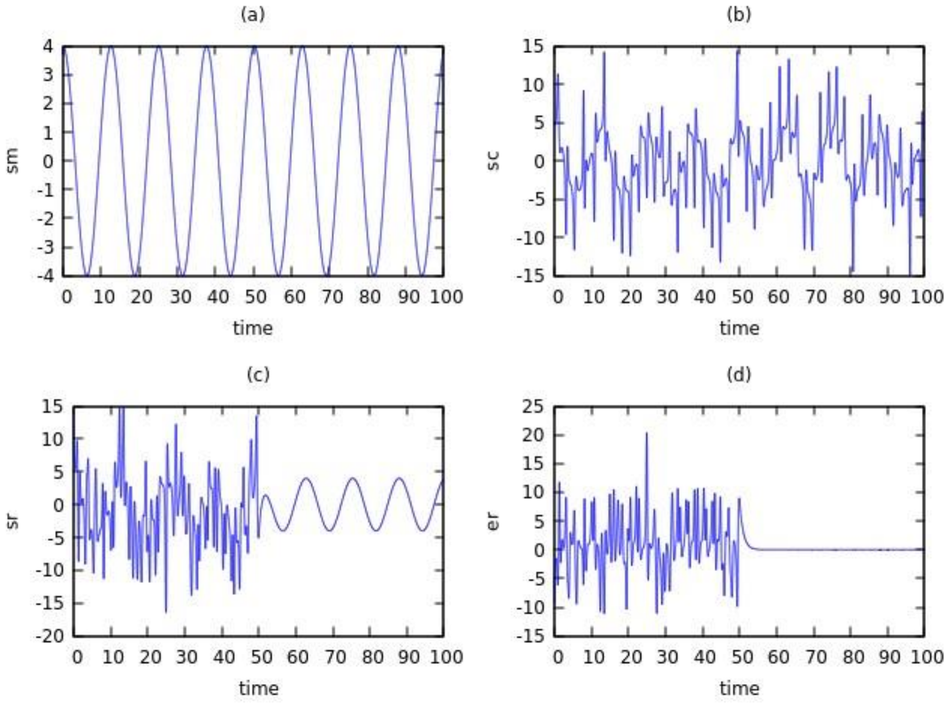

Figure 8: Secure communication scheme for new hyperchaotic system (1).

7. CONCLUSION

Coexisting attractors, control, synchronization and secure communication of a new hyperchaotic system with two-wing and four-wing were studied in this paper. We established that this new hyperchaotic system belong to a family of hidden attractor. Adaptive control method base on the Lyapunov stability theory was used to designed the control function $u_i(t)$ with the parameter update law to control as well as to synchronizes two identical hyperchaotic systems emanating from two sets of different initial conditions $x_i(0)$ and $y_i(0)$ as drive (master) and response (slave) systems respectively. The results of the synchronization were applied to secure communication. The success of secure communication scheme demonstrated here shows the potential of two-wing and four-wing hyperchaotic system (1) in voice encryption, image encryption and pseudo-random number generation.

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