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SYNCHRONIZATION AND ANTI-SYNCHRONIZATION OF A MULTISTABLE HYPERCHAOTIC DYNAMICAL SYSTEM WITH APPLICATION TO SECURE COMMUNICATION

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ABSTRACT

With rapid increase in computer networking technology and information security, there is a high demand for a reliable secure communication scheme. Synchronization based on hyperchaotic system have been used for the past decade for effective information encryption. Here, we proposed synchronization and anti-synchronization of a multistable hyperchaotic dynamical system with application to secure communication. We elucidates the dynamical properties through phase attractor, dissipativity, rest points and multistability while the synchronization and anti-synchronization are investigated by adaptive control method. The secure communication scheme is developed using mixed algorithm. Numerical simulation results confirmed that the proposed routine is an ideal candidate for secure communication.

KEYWORDS

Hyperchaotic system, Multistability, Synchronization, Secure communication

1. INTRODUCTION

Chaos theory is one of the greatest field and the most complicated steady-state behavior in nonlinear systems. Many research works have been conducted in the past on the basis of well-known systems with chaotic or hyperchaotic attractors, including Lorenz [1], Chua [2], Chen [3], Sprott[4] systems etc.

Various chaotic applications have been reported in the literature from different disciplines, among them are; sciences [5], electric circuit implementation [6-8], economics and finances [9-13], telecommunication and secure communication [14-18] and steganography [19].

In recent time, hyperchaotic systems with hidden attractors have attract the attention of many researchers due to their potential applications [20]. Chaotic or hyperchaotic systems with hidden attractors are multi-stable in nature.

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Multistability is a unique feature of a chaotic or hyperchaotic systems which is the coexisting of attractors for different set of initial conditions with the same system parameter [21-23]. A multistable system alternate between the stable and unstable systems.

Chaotic oscillations and trajectories could be of benefit in some applications and should be diminished and eliminated if they are undesirable. To improve the systems performance, it is important to control as well as synchronizes the chaotic systems. Chaos control is the stabilization of unstable periodic orbit at the rest point or to track any desire function $f(t)$ at any chosen position and synchronization is the coupling of two or more systems trajectories such that both systems will remains in phase during the signal transmission [24].

Synchronization is found in the dynamics of real-world systems such as; Laser [25], Josephson junction [26], ecological systems[27], among others. However, another interesting phenomenon is the anti-synchronization which is the opposite of synchronization. In this case, the coupling force pulled the oscillations in opposite direction to a common phase in the network. This anti-phase or anti-synchronization was observed by Huygen in pendulum clocks [28]. Other examples are; neuronal networks in brain [29-30], food webs [31] and climatic networks [32-33].

In order to explore the potential applications of chaotic or hyperchaotic synchronization, various approaches have been modelled and adopted in the literature. These includes; adaptive control[34-38]activecontrol[39-41], backstepping technique (refs. [24]), [42-43], sliding mode control [44-45].

It is ideals to consider a hyperchaotic system with uncertain parameters, because in real-world applications, some of the systems parameters are unknown. Therefore, the usual methods of synchronization are not effective in the presence of unknown parameters. As a result, the synchronization approach for uncertain parameters in a chaotic or hyperchaotic systems is an important issue.

The control method employed in this work is the adaptive control technique for synchronization and anti-synchronization on self Chen hyperchaotic system. Adaptive control has advantages over other methods such as robust to uncertain parameter or parameter update law and finite-time convergence.However, this method has some setback such as computational complexity, slow convergence speed in high-order systems, and chattering effect.

In engineering community, chaos synchronization play an important role in secure communication scheme. Since chaos is highly sensitive to initial states and parameters, the encryption technique based on chaos synchronization has better privacy and security then more reliable for chaotic secure communication[46-47]. Presently, the amount of information such as signal, text message, video, image, and other multimedia data needed to be transmitted between parties through the internet are rapidly increase and there are growing concerns about their trust, integrity, security, privacy, storage and confidentiality[48]due to cyber threats.

Chaos-based secure communication faces key challenges, including high sensitivity to parameter mismatches and initial conditions, vulnerability to channel noise, and difficulties in maintaining stability for high-dimensional systems [49]. Alsosecurity vulnerabilities is another limitation of the chaos-based encryption. Despite been chaotic in nature, chaos-based encryption is susceptible to cryptanalysis. Moreover, the authors in [50], reported that the use of chaotic map is not always a guarantee for the highest levels of cryptographic security. In order to overcome the above challenges, a more complex chaotic system called hyperchaotic system is the focal point here. In addition, the multistable hyperchaotic systems with coexisting of attractors are more important in this case than the self-excited attractors [51].

Our concerns here is how to secure and prevent the information from cyber-attacks. Hence, we developed our secure communication scheme by employed adaptive control method on self-synchronization for Chen hyperchaotic system via mixing routine refs. [14-15] and [21]. The feasibility of the above technique wasdemonstrated through numerical results and it shows that the proposed technique is effective for practical implementation.

2. Materials and Methods

2.1. Mathematical Model

The mathematical formulation adopted in this work is the Chen hyperchaotic system with five nonlinear terms ref. [3] given by:

$$\begin{aligned}
\dot{x} &= a(y-x)+w \\
\dot{y} &= dx - xz + cy \\
\dot{z} &= xy - bz \\
\dot{w} &= yz + rw
\end{aligned} \qquad (1)$$

In the system (1) above, x, y, z and w represent the state variables while a, b, c, d and r are the parameters of the system. System (1) is hyperchaoticwith the parameters; $a = 35$, $b = 3$, $c = 12$, $d = 7$ and $r = 0.58$. The strange attractors for system (1) are displayed in Figure 1.

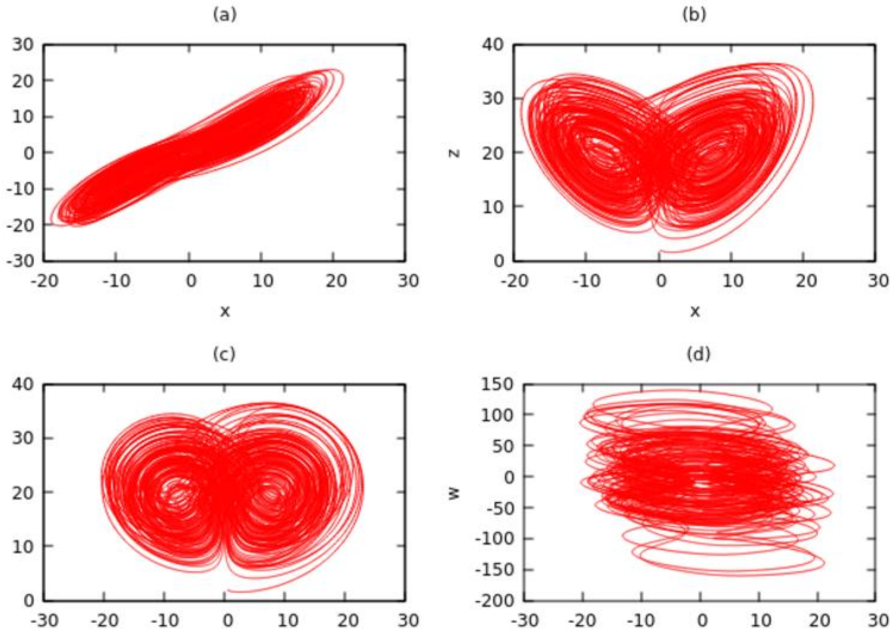


Figure 1.Strange attractors for Chen hyperchaotic system (1)

2.2. Numerical and Adaptive Control Methods

The dynamical behaviors for Chen hyperchaotic system as defined in equation (1) above in terms of attractors, rest points, stability and multistability were evaluated via numerical simulations method and adaptive control method was applied to achieve the self-synchronization and self-anti-synchronization for Chen hyperchaotic system (1)respectivelythroughRunge-kuttafourth-order algorithm.

3. RESULTS AND DISCUSSION

3.1. Dynamical Behaviors for Chen Hyperchaotic System

The dynamical behaviors for Chen hyperchaotic system (1) in terms of dissipativity, rest points, stability and multistability are reported in this section.

3.1.1. Dissipativity

In order to investigate the dissipativity of system (1), there is a need to find the divergence of system (1) along any volume flow via:

$$\nabla V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} + \frac{\partial \dot{w}}{\partial w} \quad (2)$$

Thus;

$$\nabla V = -a + c - b + r = -25.42 < 0$$

The Chen hyperchaotic system (1) is dissipative or non-conservative. All the orbits of system (1) converges to a specific subset of zero as $t \to \infty$ exponentially.

Hence, $\dot{V}(t) = V_0 e^{-(25.42)t}$, that is, for any initial volume, the volume become $V_0 e^{-(25.42)t}$ at instant time through the flow by the system (1). This confirmed the existence of strange attractor in system (1).

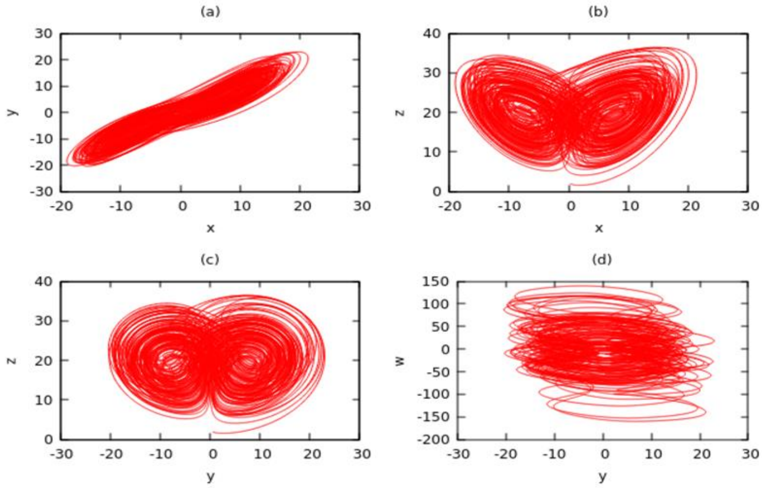
3.1.2. Rest Points And Stability

The rest points and stability of system (1) are examined hereby solving the set of equations in (3).

$$\begin{aligned} & a(y-x) + w = 0 \\ & dx - xz + cy = 0 \\ & xy - bz = 0 \\ & yz + rw = 0 \end{aligned} \quad (3)$$

For all the parameter values; $a = 35, b = 3, c = 12$, $d = 7$ and $r = 0.58$, the Chen system (1) has a unique rest point at the origin.

For the rest points $E_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, the Jacobian matrix ($J_0$) for Chen hyperchaotic system (1) at any rest point is represented as:



$$J_0 = \begin{bmatrix} -a & a & 0 & 1 \\ d & c & 0 & 0 \\ 0 & 0 & -b & 0 \\ 0 & 0 & 0 & r \end{bmatrix} (4)$$

To calculate the eigen values of equation (4) at any rest point, we introduce the law, $|J_0 - \lambda I|$. Then the determinant of the Jacobian matrix (4) at any rest point is written as:

$$\begin{vmatrix} -a-\lambda & a & 0 & 1 \\ d & c-\lambda & 0 & 0 \\ 0 & 0 & -b-\lambda & 0 \\ 0 & 0 & 0 & r-\lambda \end{vmatrix} = 0 \, (5)$$

Where; $a = 35, b = 3, c = 12$, $d = 7$ and $r = 0.58$.

The eigen values ($\lambda_i, i = 1,2,3,4$) of equation (5) evaluated via MATLAB are; $\lambda_1 = -39.7356$, $\lambda_2 = 16.7356$, $\lambda_3 = -3$ and $\lambda_4 = 0.5800$ respectively. These results revealed that λ_1 and λ_3 are node-focus (negative) hence, there are stable while λ_2 and λ_4 are saddle-focus (positive) therefore, there are unstable as presented in table 1. Since the two rest points (λ_2 and λ_4) are unstable saddle points, the Chen hyperchaotic system (1) has a hidden attractor. In addition, the algebraic sum of the eigen values for this system under investigation is negative. This establish that the system (1) is truly a dissipative system.

Table 1. Eigen values and stability

S/N	Eigen values	Type of point	Stability
1	-39.7356	Node-focus	Stable
2	16.17356	Saddle-focus	Unstable
3	-3.0000	Node-focus	Stable

4	0.5800	Saddle-focus	Unstable

3.1.3. Multistability

Multistability is a unique property of a chaotic orhyperchaotic system where two or more coexisting of attractors exist for the same system's parameter but different set of initial conditions. A multistable system is neither stable nor completely unstable but alternate between the two or more exclusive states with time ref. [21].

We demonstrated the multistable state for Chen hyperchaotic system (1) with two coexisting strange attractors for the parameters: $a = 35, b = 3, c = 12$, $d = 7$ and $r = 0.58$ with the following set of initial conditions ; $X_0(0.2,0.5,2.0,0.8)$ plotted in blue color and $Y_0(3.0,0.5,1.0,2.0)$ plotted in red color, in Figure 2.

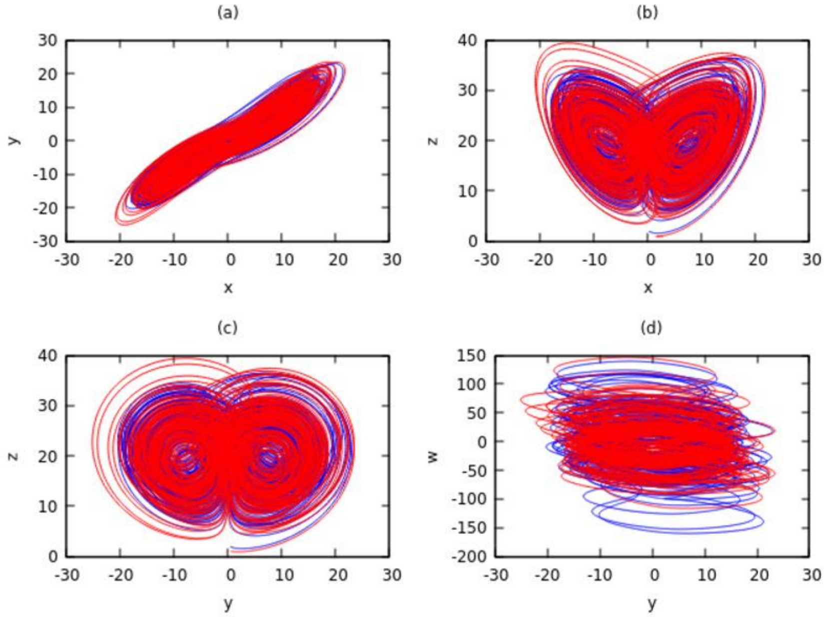


Figure 2. Coexistence of attractor for Chen hyperchaotic system (1)

3.2. Synchronization of Chen Hyperchaotic System

Here, we apply the adaptive control technique for self-synchronization of Chen hyperchaotic systems taken as the master and slave systems with the update law.

Assuming that: $x = x_1$, $y = x_2$, $z = x_3$ and $w = x_4$.

Then, equation (1) will take this form:

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) + x_4 \\ \dot{x}_2 &= dx_1 - x_1x_3 + cx_2 \\ \dot{x}_3 &= x_1x_2 - bx_3 \\ \dot{x}_4 &= x_2x_3 + rx_4 \end{aligned} \quad (6)$$

We regard the system (6) as the master (drive) and system (7) as the slave (response) system.

$$\begin{aligned}
\dot{y}_1 &= a(y_2 - y_1) + y_4 + u_1 \\
\dot{y}_2 &= dy_1 - y_1 y_3 + cy_2 + u_2 \\
\dot{y}_3 &= y_1 y_2 - by_3 + u_3 \\
\dot{y}_4 &= y_2 y_3 + ry_4 + u_4
\end{aligned} \quad (7)$$

Where $u_i(t)$ for each $i = 1,2,3,4$, is the control function needed to be designed with the assumption that the parameters a, b, c, d and r are uncertain.

The synchronization error vector $e_i(t)$ between the master (6) and slave (7) systems is defined in equation (8).

$$\begin{aligned}
e_1 &= y_1 - x_1 \\
e_2 &= y_2 - x_2 \\
e_3 &= y_3 - x_3 \\
e_4 &= y_4 - x_4
\end{aligned} \quad (8)$$

By applying the definition of the error vector $e_i(t)$ in equation (8), the error dynamical variables $\dot{e}_i(t)$ is expressed in equation (9).

$$\begin{aligned}
\dot{e}_1 &= a(e_2 - e_1) + e_4 + u_1 \\
\dot{e}_2 &= de_1 - y_1 y_3 + x_1 x_3 + ce_2 + u_2 \\
\dot{e}_3 &= y_1 y_2 - x_1 x_2 - be_3 + u_3 \\
\dot{e}_4 &= y_2 y_3 - x_2 x_3 + re_4 + u_4
\end{aligned} \quad (9)$$

From equation (9), the control function $u_i(t)$ is obtained in equation (10).

$$\begin{aligned}
u_1 &= -a(e_2 - e_1) - e_4 - k_1 e_1 \\
u_2 &= -de_1 + y_1 y_3 - x_1 x_3 - ce_2 - k_2 e_2 \\
u_3 &= x_1 x_2 - y_1 y_2 + be_3 - k_3 e_3 \\
u_4 &= x_2 x_3 - y_2 y_3 - re_4 - k_4 e_4
\end{aligned} \quad (10)$$

Where k_i for each $i = 1,2,3,4$, is the arbitrary control gain to be determine accordingly.

For asymptotic stability of the error dynamics $\dot{e}_i(t)$ in equation (9), $\dot{V} = -\sum k_i e_i^2 < 0$ must be satisfied.

We then choose the Lyapunov function V as in equation (11).

$$V = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2 + \tilde{a}^2 + \tilde{b}^2 + \tilde{c}^2 + \tilde{d}^2 + \tilde{r}^2\right) \quad (11)$$

Where $\tilde{a} = a - \bar{a}$, $\tilde{b} = b - \bar{b}$, $\tilde{c} = c - \bar{c}$, $\tilde{d} = d - \bar{d}$ and $\tilde{r} = r - \bar{r}$ are the estimated values of the uncertain parameters a, b, c, d and r respectively.

The time derivative of equation (11) along the trajectories of error dynamics and unknown parameters is shown in equation (12) below.

$$\dot{V} = e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4 + a\dot{\tilde{a}} + b\dot{\tilde{b}} + c\dot{\tilde{c}} + d\dot{\tilde{d}} + r\dot{\tilde{r}} \quad (12)$$

We then substitute equation (9) into equation (12) as follows:

$$\begin{aligned}\dot{V} &= e_1[a(e_2 - e_1) + e_4 + u_1] + e_2[de_1 - y_1y_3 + x_1x_3 + ce_2 + u_2] + \\ &e_3[y_1y_2 - x_1x_2 - be_3 + u_3] + e_4[y_2y_3 - x_2x_3 + re_4 + u_4] + \tilde{a}[\dot{\tilde{a}} - (e_2 - e_1)e_1] \quad (13) \\ &+ \tilde{b}[\dot{\tilde{b}} - (e_3^2)] + \tilde{c}[\dot{\tilde{c}} - (-e_2^2)] + \tilde{d}[\dot{\tilde{d}} - (e_1e_2)] + \tilde{r}[\dot{\tilde{r}} - e_4^2]\end{aligned}$$

The parameter update law is estimated from equation (13) as:

$$\begin{aligned}\dot{\bar{a}} &= (e_2 - e_1)e_1 - a \\ \dot{\bar{b}} &= -e_3^2 - b \\ \dot{\bar{c}} &= -e_2^2 - c \qquad (14) \\ \dot{\bar{d}} &= e_1e_2 - d \\ \dot{\bar{r}} &= e_4^2\end{aligned}$$

By substituting equations (10) and (14) respectively into equation (13) yield:

$$\dot{V} = -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 - k_4e_4^2 - a^2 - b^2 - c^2 - d^2 - r^2 < 0 \quad (15)$$

Clearly, $\dot{V} = -\sum_{i=1}^{4} k_ie_i^2 < 0$ for all R[4]. By Lyapunov stability theory [52], we conclude that $e_i(t) \to 0$ as $t \to \infty$. Hence, $e_i(t)$ is globally and exponentially stabilize at the origin when the control input is activated with the parameter estimation update law. Hence, the master and slave systems ((6) and (7)) are globally and exponentially synchronized for the value of the initial states X_i (0) and Y_i (0).

For numerical simulations, Runge Kuta fourth-order algorithm with the time size 10^{-3} is used to solve the control input law $u_i(t)$ in (10) and parameter estimation update law (14) respectively. The parameter values for Chen hyperchaotic system for the master (6) and slave (7) systems are taken as in the hyperchaotic system (1) as; $(a,b,c,d,r) = (35,3,12,7,0.58)$ and control gain k_i for each $i = 1,2,3,4$ as $k_1 = k_2 = k_3 = k_4 = 1.0$. The initial conditions for the master and slave systems (6) and (7) are: $X(0) = (0.2,0.5,2.0,0.8)$ and $Y(0) = (3.0,0.5,1.0,2.0)$ respectively while that of the parameter estimates (14) are taken as $a(0) = 35$, $b(0) = 3$, $c(0) = 12$, $d(0) = 7$ and $r(0) = 0.58$.

Figure 3 show the output of the complete synchronization for the master and slave systems (6) and (7). The time evolution for the synchronization error $e_i(t)$ stabilizes at the origin when the control input $u_i(t)$ is switched on at $t = 10$ as displayed in Figure 4. The parameter estimates: $\bar{a}$, $\bar{b}$, $\bar{c}$, $\bar{d}$ and $\bar{r}$ converges asymptotically to the origin as $t \to \infty$ as shown in Figure 5.

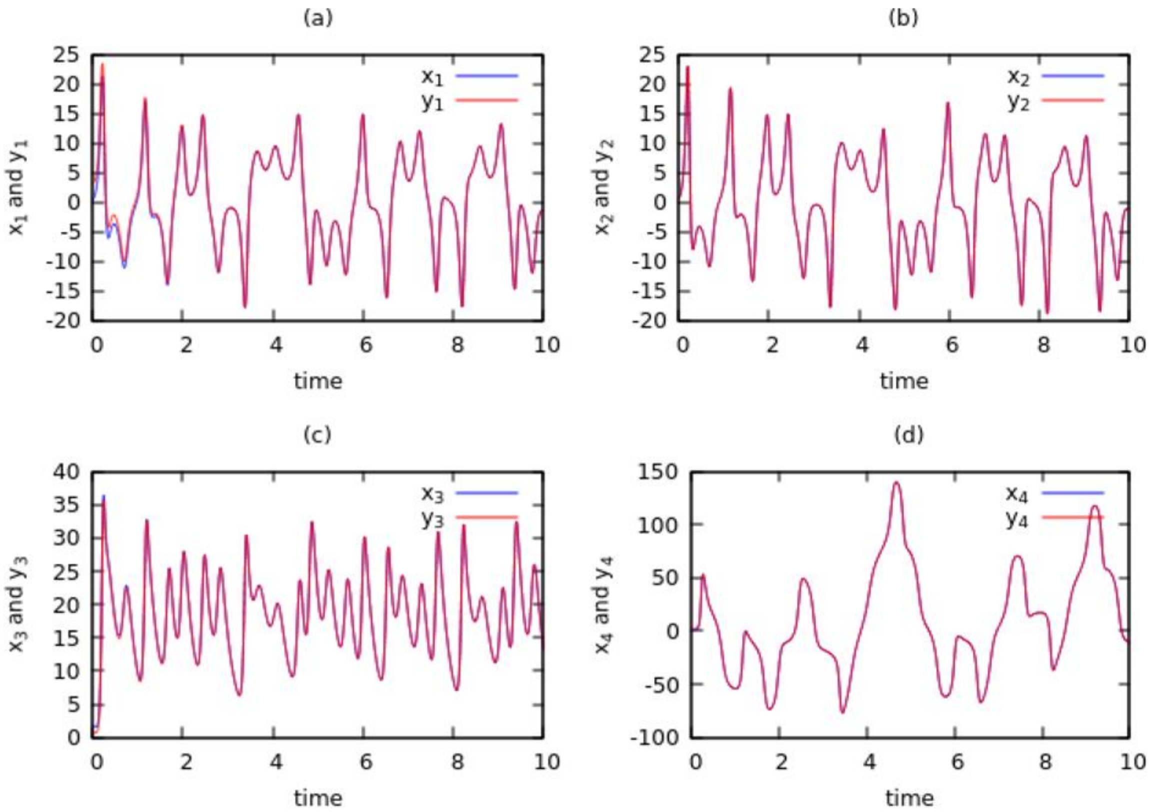


Figure 3. Completete synchronization for the master and slave systems (6) and (7)

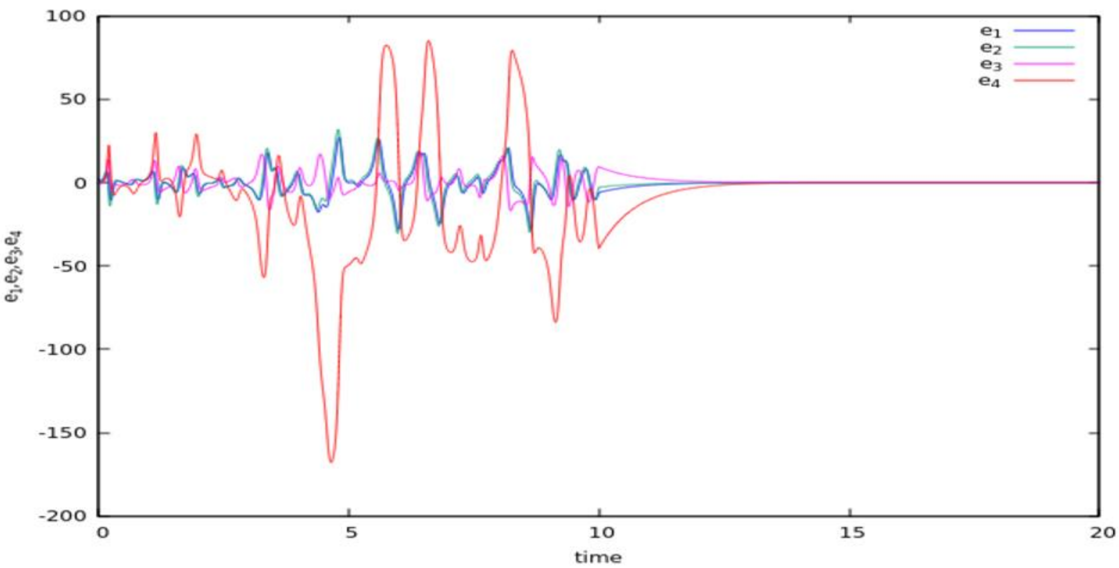


Figure 4. Synchronization error between the the master (6) and slave (7) systems

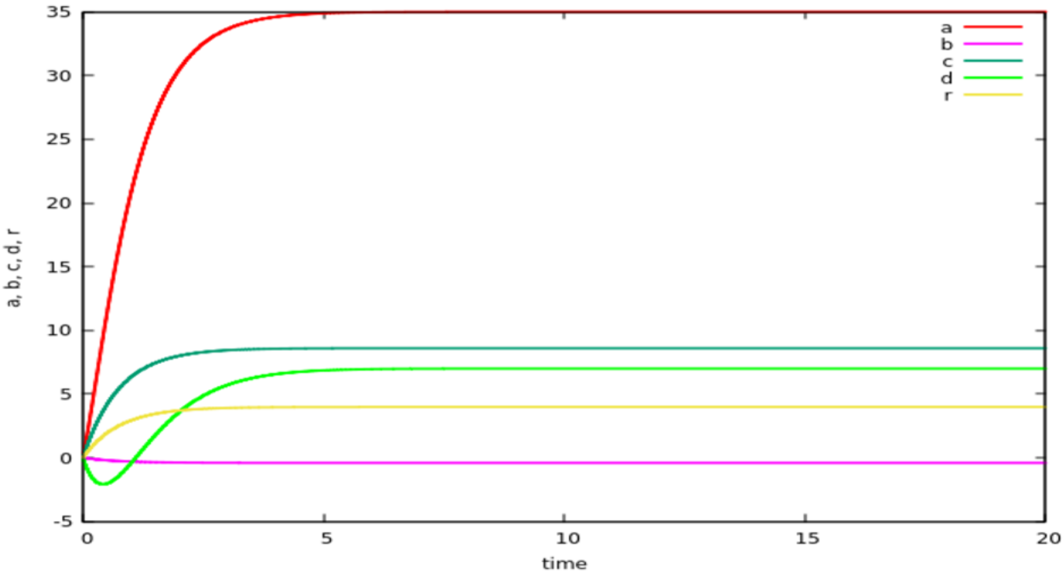


Figure 5. Time response of the parameter update law (14)

3.3. Anti- Synchronization of Chen hyperchaotic system

Similarly, we adopt adaptive control method for anti-synchronization of self-Chen hyperchaotic system (1) by re-defined the error vector in equation (8) as follows:

$$\begin{aligned} e_1 &= y_1 + x_1 \\ e_2 &= y_2 + x_2 \\ e_3 &= y_3 + x_3 \\ e_4 &= y_4 + x_4 \end{aligned} \quad (16)$$

By considering the master and slave systems in equation (6) and (7), the error dynamical variables between them by using equation (16) is presented in equation (17).

$$\begin{aligned} \dot{e}_1 &= a(e_2 - e_1) + e_4 + u_1 \\ \dot{e}_2 &= -de_1 - y_1 y_3 - x_1 x_3 + ce_2 + u_2 \\ \dot{e}_3 &= y_1 y_2 + x_1 x_2 - be_3 + u_3 \\ \dot{e}_4 &= y_2 y_3 + x_2 x_3 + re_4 + u_4 \end{aligned} \quad (17)$$

The nonlinear controller $u_i(t)$ is chosen from equation (17) as follows:

$$\begin{aligned} u_1 &= -a(e_2 - e_1) - e_4 - k_1 e_1 \\ u_2 &= -de_1 + y_1 y_3 + x_1 x_3 - ce_2 - k_2 e_2 \\ u_3 &= -y_1 y_2 - x_1 x_2 + be_3 - k_3 e_3 \\ u_4 &= -y_2 y_3 - x_2 x_3 - re_4 - k_4 e_4 \end{aligned} \quad (18)$$

Where k_i has it usual meaning.

The condition for asymptotic stability of the error function $e_i(t)$ is that; $\dot{V} = -\sum k_i e_i^2 < 0$. Consequently, equations(12)is required here.Then, we substitute equation (17) into equation (12) as follows:

$$\dot{V} = e_1[a(e_2 - e_1) + e_4 + u_1] + e_2[de_1 - y_1y_3 - x_1x_3 + ce_2 + u_2] +$$
$$e_3[y_1y_2 + x_1x_2 - be_3 + u_3] + e_4[y_2y_3 + x_2x_3 + re_4 + u_4] + \tilde{a}[\dot{\tilde{a}} - (e_2 - e_1)e_1] \quad (19)$$
$$+ \tilde{b}[\dot{\tilde{b}} - (e_3^2)] + \tilde{c}[\dot{\tilde{c}} - (-e_2^2)] + \tilde{d}[\dot{\tilde{d}} - (e_1e_2)] + \tilde{r}[\dot{\tilde{r}} - e_4^2]$$

From equation (19), the parameter update law (20) is estimated as in equation (14) earlier.

$$\dot{\bar{a}} = (e_2 - e_1)e_1 - a$$
$$\dot{\bar{b}} = -e_3^2 - b$$
$$\dot{\bar{c}} = -e_2^2 - c \qquad (20)$$
$$\dot{\bar{d}} = e_1e_2 - d$$
$$\dot{\bar{r}} = e_4^2$$

By substituting equations (18) and (20) respectively into equation (19), the necessary condition for asymptotic stability is satisfied as reported by equation (21).

$$\dot{V} = -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 - k_4e_4^2 - a^2 - b^2 - c^2 - d^2 - r^2 < 0 \quad (21)$$

Hence, $\dot{V} = -\sum_{i=1}^{4} k_i e_i^2 < 0$ for all R^4.

According to stability theory ref. [52], $e_i(t) \to 0$, as $t \to 0$ for all $i = 1,2,3,4$, meaning that the $e_i(t)$ is globally and exponentially converge at the origin with the activation of the controller $u_i(t)$ in equation (18) and the parameter estimation update law (20). The master (6) and slave (7) systems are globally anti-synchronized for the chosen value of the initial states X_i (0) and Y_i (0) respectively.

To demonstrate the robustnessdesigned controllers, $u_i(t)$ ($i = 1,2,3,4$)in (18) and update law in (19), the dynamics of the master-slave systems ((6) and (7))are simulatedby applying fourth-order RungeKutaalgorithm with the time step of 10^{-3}. We take the; parameter values for the systems (6) and (7), control gain k_i and initial values of the update estimates as in section 3.2

The initial states for the master (6) and slave (7)systemsare chosen as: $X(0) = (2.0,0.5,6.0,0.8)$ and $Y(0) = (3.0,5.0,2.0,4.0)$.

Figure 6 show the ant-synchronization phase for the master-slave systems (6) and (7). The time-history of the anti-synchronization error $e_i(t)$ converges at the origin asymptotically when the control function is activated at $t = 5$ as revealed in Figure 7. The parameter estimates: $\bar{a}$, $\bar{b}$, $\bar{c}$, $\bar{d}$ and $\bar{r}$ stabilizes at the origin as $t \to \infty$ with the output result in Figure 8.

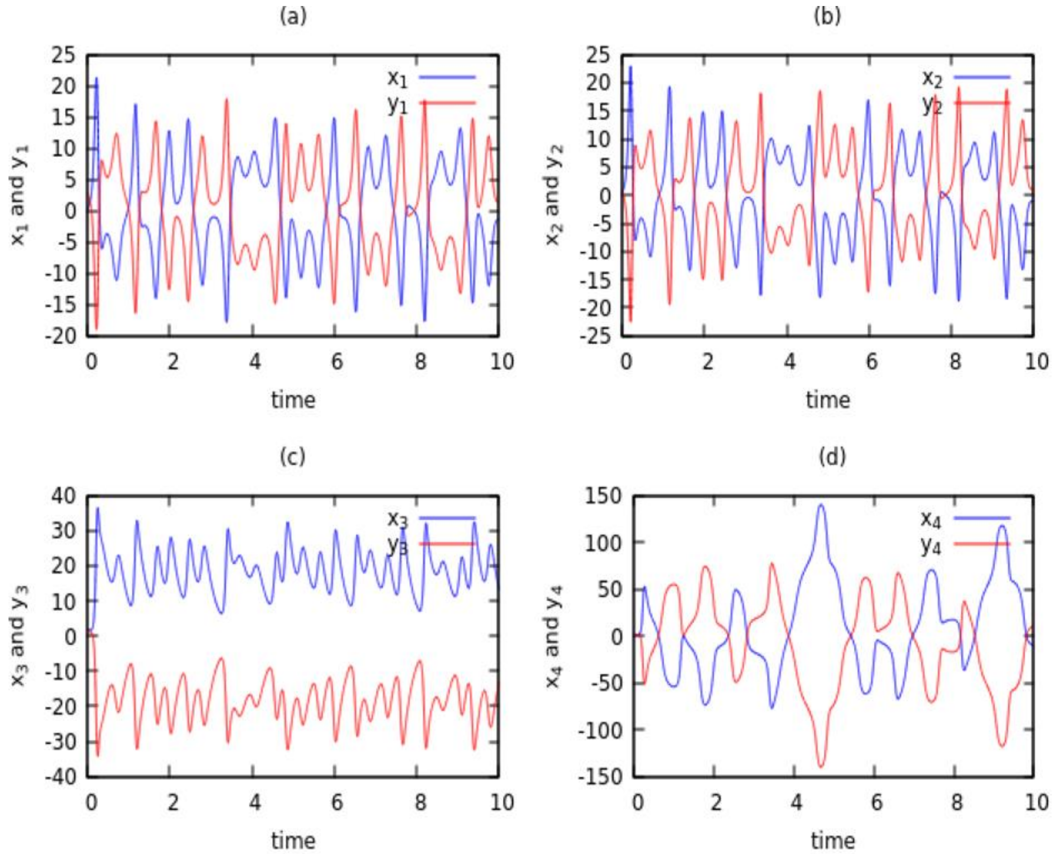


Figure 6. Anti-synchronization of the master-slave systems (6) and (7)

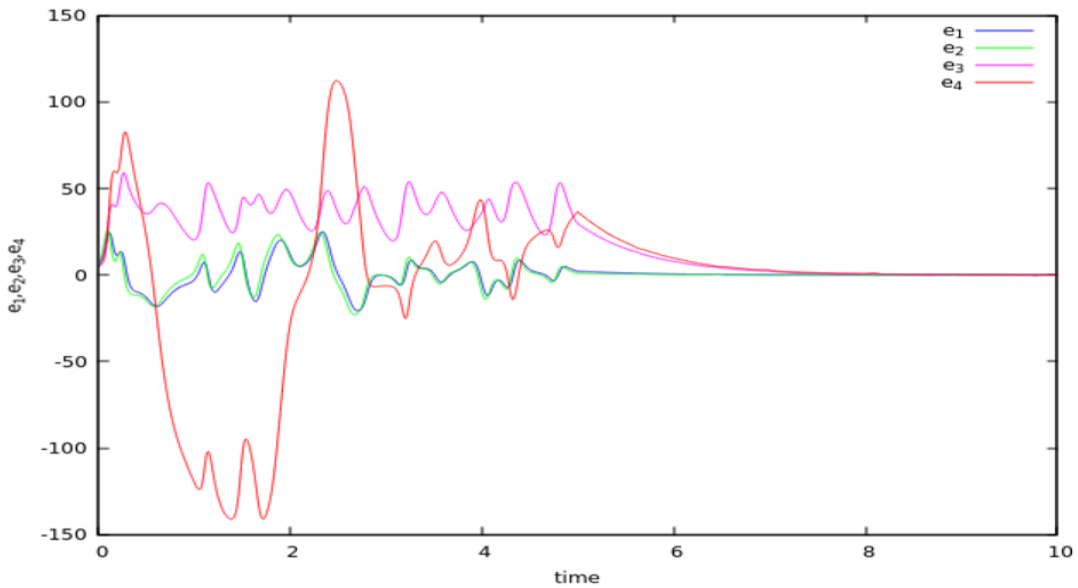


Figure 7. Anti-synchronization error between the systems (6) and (7)

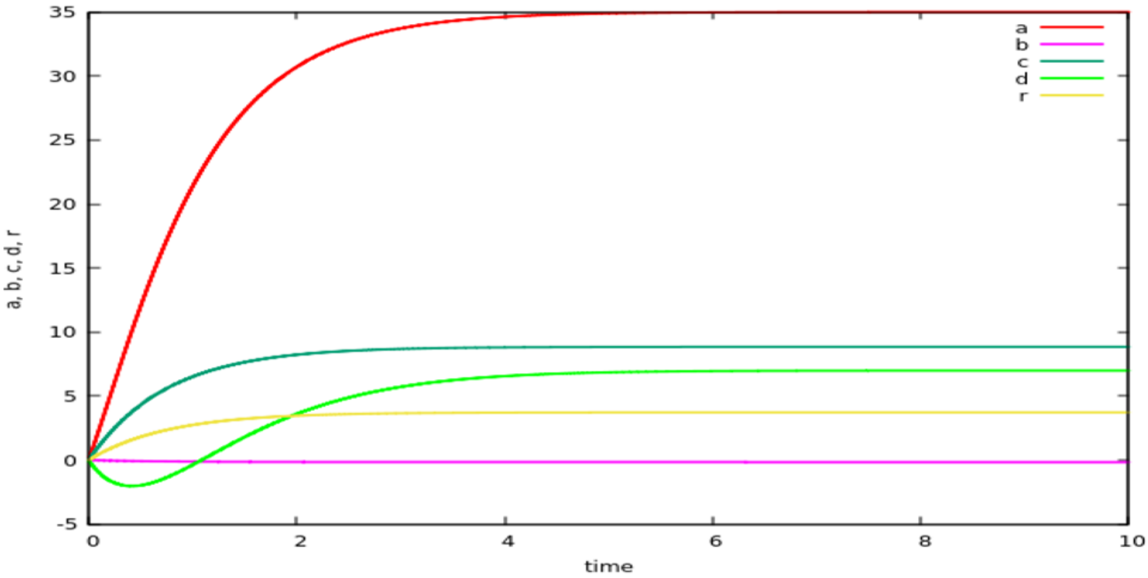


Figure 8. Time response of the parameter update law (20)

3.4. Secure Communication

The success of adaptive self-synchronization for Chen hyperchaotic system is further applied to secure communication for signal encryption via additive algorithm. For instance, the information signal $sm(t)$ in this case is defined as:

$$sm(t) = 2.0\sin(4t) \quad (22)$$

While the encrypted information signal $sc(t)$ is achieved by the combination of information signal $sm(t)$ and hyperchaotic wave form x_i via masking algorithm as presented in equation (23).

$$sc(t) = sm(t) + x_i \quad (23)$$

The encrypted information wave function $sc(t)$ in equation (23) is sent to the receiver via the public channel for synchronization between the master or drive system x_i in equation (6) and slave or response system y_i in equation (7) respectively.

Finally, the encrypted information signal (23) is decrypted via inverse function $sr(t)$ as shown in equation (24).

$$sr(t) = sc(t) - y_i \quad (24)$$

The numerical simulation results for the success of this scheme are displayed in Figure 9.

However, this communication scheme may encounter some challenges such as hardware implementation, complexity and high cost. Hardware implementation require precise components matching, analog circuits are susceptible to temperature, age, and process variations, while FPGA implementations can be resource-intensive and expensive. Designing and implementing secure, high-speed, and robust hyperchaoticsystems is complex and generally more costly to implement.

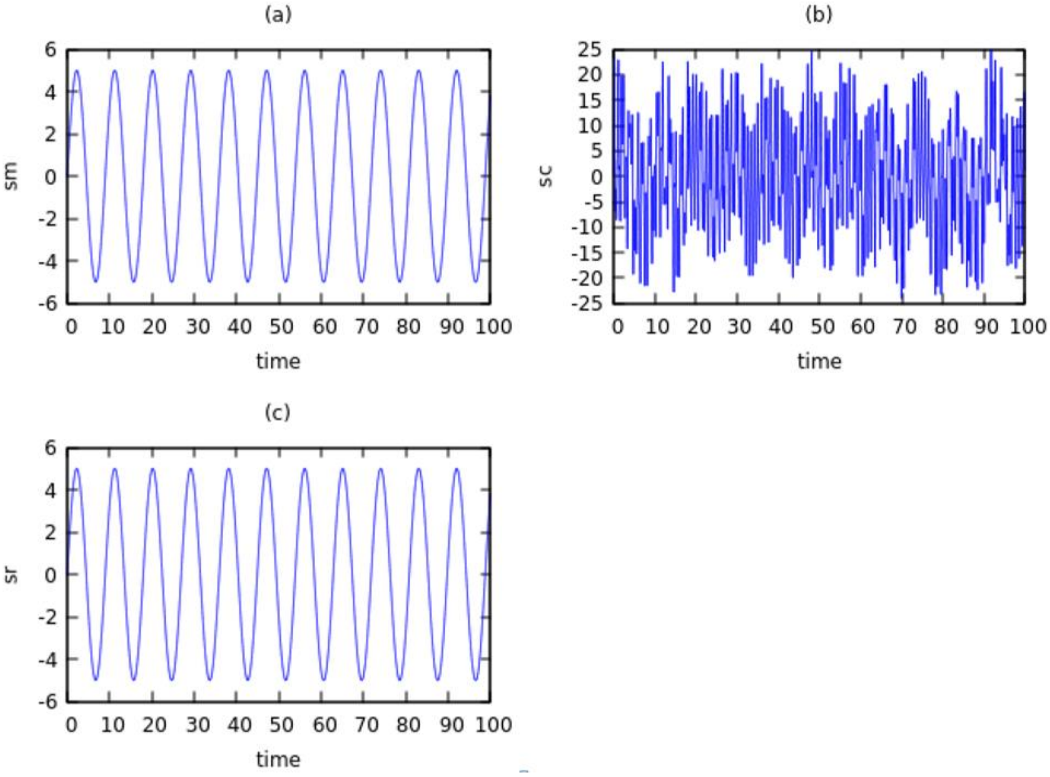


Figure 9. Secure communication scheme for Chen hyperchaotic system (1)

4. CONCLUSION

The Chen hyperchaotic system is a dissipative system with two rest points at the origin and two unique saddle points. These results uncovered the hidden attractors in Chen hyperchaotic system and confirmed that this system is multistable with the coexistence of attractors for different set of initial states. For real-world application, we applied the adaptive control technique to synchronized and anti-synchronized the self hyperchaotic Chen system. Figures 4 and 6 revealed that the synchronization or anti-synchronization problem is not only phase or anti-phase transmissions of signal but also a solution to control unstable periodic orbits to stabilize at the origin at any chosen time. A proposed secure communication scheme via the self-synchronization for Chen hyperchaotic system displayed it robustness for signal encryption. The numerical simulation results presented in this work validate the feasibility of the designed control laws and secure communication scheme. Thus, our findings indicate the potential of Chen hyperchaotic system for varieties of applications including; computer networking, technology, data storage, electronics data transmissions and secure communication.

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