IMPLICIT SET BLENDS WITH BOUNDED PRIMITIVES FOR ZERO IMPLICIT SURFACE

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ABSTRACT

In Zero implicit surface modeling, a complex implicit surface is constructed like block building from primitive surfaces via set blends including union, intersection and difference. Set blends can smooth out sharp corners or edges by automatically generated transition surfaces from the level surfaces in the blending regions of blended primitive. However, level surfaces of primitives in existing set blends always have similar shape and size in the blending regions and are proportional to those of blended primitives. To solve this problem, this paper develops new union, intersection and difference blends with bounded primitives for zero implicit surface modeling. In the proposed set blends, each blended primitive is assigned a bounding solid respectively to adjust the size and shape of the level surfaces of the primitive. Via these new set blends with chosen bounding solids, some modelling functions are provided, such as (a) bulge eliminations on sequential unions of intersecting and connecting cylinders, (b) changing the shape of the contour of a blend's transition, and (c) creating smooth sequential blends with overlapping blending regions.

KEYWORD

Zero implicit surfaces, Implicit set blends, The scale method, The displacement method

1. INTRODUCTION

In implicit surface modeling [5, 8], an zero implicit surface is represented as a level surface $f_i(x,y,z)=0$ using a defining function $f_i(x,y,z)$. Furthermore, a complex zero implicit surface is constructed from basic primitive implicit surfaces $f_i=0$, $i=1$ to k, and defined by an implicit blend $B_k(f_1,...,f_k)=0$ via a blending operator $B_k(f_1,...,f_k)$. $B_k(f_1,...,f_k)$ can automatically generate transitions for smoothly connecting blended primitives $f_1=0,...,$ and $f_k=0$ and avoiding sharp edges and corners.

In the literature, a lot of defining functions with different shapes were developed, including one-branch plane [14], planes [6], LP distance metrics [4], Super-quadrics [2], generalized distance functions [1], Skeleton-based primitives (Circular cylinders) [6], Sweep objects [7], spherical product functions [14]. In addition, a lot of set blends for blending zero implicit surfaces were also proposed and is described as follows:

- Blends with C^0 continuity: *Min* and *Max* provide pure union and intersection in [17].
- Blends with blending range parameters for generating local blends: Conic blends in [11, 12, 18] generate smooth transitions only within regions specified by range parameters. That is, they are local blends and deform primitives locally.
- Blends with blend range parameters and C^1 continuity everywhere: binary elliptic blends [3] and high-dimensional blends from the displacement method [18] and the scale method

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[13, 15] have C^1 continuity everywhere. As a result, they can generate smooth sequential blends even containing overlapping blending regions.

- Sequential blends can be organized and represented into a CSG (constructive solid geometry) tree [16, 19].
- Blends in [3] especially can generate transitions with a free-form profile.
- In addition, for eliminating unwanted bulges caused by blending, gradient-based methods [9, 13, 18] were also developed by varying the value of range parameters via gradient-based functions.

Among existing set blends, the shape and size of the level surfaces of blended primitives are always similar and proportional to those of blended surfaces. This incurs that the shape of the contour of the transition of blends is similar to the shape of blended surfaces, too. To solve this problem, this paper develops new union, intersection and difference blends with bounded primitives for zero implicit surface modelling. More precisely, these newly proposed blends can assign each primitive an individual bounding solid for changing each primitive's blending region such that the shapes of the level surfaces of a blended primitive in the blending trigon are varied from the shape of the blended primitive to the shape of the bounding solid. As a result, by choosing suitable bounding solids for primitives, some special modeling functions are obtained for Zero implicit surface modeling and described below:

(1). Bulge elimination of intersecting and connecting cylinders and toroids.
(2). Shape adjustment on the contour of the transition of a union blend via a chosen solid.
(3). Creating smooth sequential blends with overlapping blending regions.

The remainder of this paper is organized as follows: Section 2 reviews zero implicit surface. Section 3 introduces set blend with bounded primitives, whose applications are presented in Section 4. Conclusion is given in Section 5.

2. ZERO IMPLICIT SURFACE

In this section, defining functions and implicit set blends for zero implicit surface are reviewed.

2.1. Definition of Primitive Implicit Surface

Given a defining function $f_i(x,y,z){:}R^3 \to R$, a primitive implicit surface is defined by

$$f_i(x,y,z)=0$$

, where $f_i(x,y,z)<0$ is the inside of the surface. Primitive defining function f_i is obtained from a non-negative ray-linear functions $f_{ri}(x,y,z){:}R^3 \to [0, \infty]$ by:

$$f_i(x,y,z)=1- f_{ri}(x,y,z).$$

Existing non-negative ray-linear functions f_{ri} include one-branch plane [14], parallel planes [6], LP distance metrics [4], Super-quadrics [2], generalized distance functions [1], skeleton-based primitives (Circular cylinders) [6], Sweep objects [7], spherical product functions [14]. Some of their shapes $f_{ri}(x,y,z)=1$ are displayed in Figure 1.

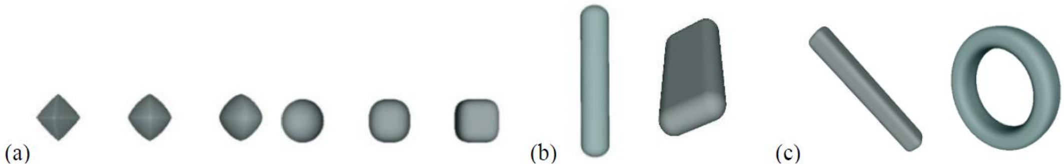

Figure 1. Shaped of non-negative ray-linear functions, including (a). LP distance metrics, $(|x|^n+|y|^n+|z|^n)^{1/n}=1$, where n varies from 1.1, 1.3, 1.5, 2, 3, and 5. (b). Line and polygon skeleton primitives. (c). spherical cross-product functions.

2.2. Implicit Set Blends

Moreover, a complex implicit surface is constructed from primitive surfaces $f_1=0$,..., and $f_k=0$ and is written through blending operator $B_k(x_1,...,x_k)$:$R^k \rightarrow R$ by:

$$B_k(f_1,...,f_k)=0.$$

$B_k(f_1,...,f_k)$ maps R^3 to R, and the boundary $B_k(f_1,...,f_k)$ is also called a blending surface. Basically, operator $B_k(x_1,...,x_k)$ offer union, intersection and difference operations, which are denoted by B_{Uk} , B_{Uk} and B_{Uk} in the following. When performing a union, intersection or difference operation, a blend is capable of erasing and smoothing out sharp edges and corners shown in Figure 2(a), by means of automatically generated surface tangent to blended primitives such as rounding and filleting as shown in Figure 2(b). Furthermore, sequential blends are allowed to be depicted as a constructive solid geometry (CSG) tree, for example, a wheel written by $B_{D2}(B_{U2}(B_{U2}(f_1,f_2),f_3),f_4)$ is depicted in Figure 2(c).

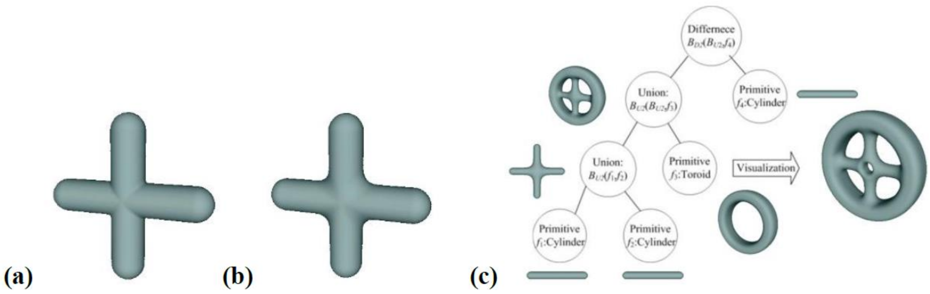

Figure 2. (a). A pure union of cylinders with sharp edges. (b). A union of cylinders with smooth transition. (c). CSG tree of sequential blends BD2(BU2(BU2(f1,f2),f3),f4) for defining a wheel.

2.3. Binary Blends with Blend Range Parameters

Min and *Max* [17] offer pure union and intersection with C^0 continuity only and they always generate sharp edges and corners as in Figure 2(a).

To smooth out sharp edges and corners, binary blends (f_1, f_2) with blend range parameters r_1 and r_2 for f_1 and f_2 were developed in [11, 12, 13]. These blends generate smooth transitions which are limited within primitives' blend regions specified by parameters r_1 and r_2. Precisely:

- The transition of a unión $B_{U2}(f_1, f_2)=0$ is composed of the set:

$$\{(x,y,z)\in R^3 |\ f_1(x,y,z)=l_1 \cap f_2(x,y,z)=l_2 \text{ for } 0\le l_1\le r_1 \text{ and } 0\le l_2\le r_2\}.$$

- The transition of an intersection $B_{I2}(f_1, f_2)=0$ is composed of the set :

$$\{(x,y,z)\in R^3 |\ f_1(x,y,z)=l_1 \cap f_2(x,y,z)=l_2 \text{ for } -r_1\le l_1\le 0 \text{ and } -r_2\le l_2\le 0\}.$$

These indicate that the sizes of the resulting transitions are limited and specified by parameters r_1 and r_2. As a result, blended primitives can deform locally after blending as shown in Figure 2(b).

2.4. High-Dimensional Blends with Blend Range Parameters and C^1 Continuity Everywhere

Since binary blends [11, 12] are suitable for generating sequential blends with overlapping blending regions as shown in Figure 3, blends with C^1 continuity everywhere were proposed and are presented in this section.

2.4.1. The Displacement Method

As in [18], based on an existing union operator $H_k(x_1,\ldots,x_k)= 1-\sum_{i=1}^{k}[(1-x_i/r_i]_+^{p_i}$, on $f_i\le 0$ with range parameters r_i, $i=1,\ldots,k$, whose 2D curve is like the thick and blue curve in Figure 4(a), a new union operator $B_{Uk}(x_1,\ldots,x_k): R^k \to R$ with C^1 continuity everywhere on $f_i\le 0$ is obtained by

- Union:

$$B_{Uk}(x_1,\ldots,x_k)=h_p, \qquad (1)$$

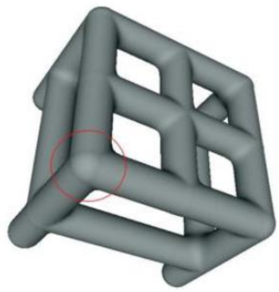

Figure 3. Intersecting cylinders with bulge elimination.

where $h_p\in T^{-1}(0)$ and $T(h)=$

$$H_k(x_1-h,\ldots,x_k-h)=1-\sum_{i=1}^{k}[(1-(x_i-h)/r_i]_+^{p_i}=0$$

with $r_i>0$ and $p_i>1$, $i=1,\ldots,k$.

In addition, its due forms also provide new intersection and difference B_{Ik} and $B_{Dk}: R^k \to R$ by

- Intersection:

$$B_{Ik}(x_1,\ldots,x_k)= -B_{Uk}(-x_1,\ldots,-x_k) \qquad (2)$$

- Difference:

$$B_{Dk}(x_1,\ldots,x_k)= - B_{Uk}(-x_1,x_2\ldots,x_k) \quad (3)$$

All level surfaces $B_{Uk}(x_1,\ldots,x_k)=h$, $h\in R$, in Eq. (1) are smooth surfaces with the same ranges r_i as in Figure 4(a). As a result, set blends in Eq. (1)-(3) can be applied for generating and creating smooth sequential blends, especially with overlapping blending regions.

2.4.2. The Scale Method's Extension

As stated in [13, 15], based on an existing union operator $H_k(x_1,\ldots,x_k)= \sum_{i=1}^{k}[1-x_i/r_i]_+^{p_i}$ -1=0 on $f_i\leq 0$ with range parameters r_i, i=1,...,k, a new family of set operators $B_k(x_1,\ldots,x_k)$: $R^k\rightarrow R$ with C^1 continuity except at (0,..,0) are given by :

- Union:

$$B_{Uk}(x_1,\ldots,x_k)= B_{ASk}(x_1+1,\ldots,x_k+1)-1, \quad (4)$$

- Intersection:

$$B_{Ik}(x_1,\ldots,x_k)=B_{SAk}(x_1+1,\ldots,x_k+1)-1, \quad (5)$$

- Difference:

$$B_{Dk}(x_1,\ldots,x_k)= B_{Ik}(x_1, -x_2,\ldots,-x_k), \quad (6)$$

where

$$B_{ASk}(x_1,\ldots,x_k)=\begin{cases} B_{Ak}(x_1,\ldots,x_k) & Min(x_1,\ldots,x_k)>0 \\ -B_{Sk}(-x_1,\ldots,-x_k) & otherwise \end{cases},$$

$$B_{SAk}(x_1,\ldots,x_k)=\begin{cases} B_{Sk}(x_1,\ldots,x_k) & Max(x_1,\ldots,x_k)>0 \\ -B_{Ak}(-x_1,\ldots,-x_k) & otherwise \end{cases},$$

$$B_{Ak}(x_1,\ldots,x_k)=\begin{cases} h_p & Min(x_1,\ldots,x_k)>0 \\ Min(x_1,\ldots,x_k) & otherwise \end{cases},$$

where $h_p\in T^{-1}(0)$ and $T(h)=H_k(x_1/h-1,\ \ldots,\ x_k/h-1)= \sum_{i=1}^{k}[(1+r_i-x_i/h)/r_i]_+^{p_i}$ -1 with $r_i>0$ and $p_i>1$, i=1,...,k and

$$B_{Sk}(x_1,\ldots,x_k)=\begin{cases} h_p & Max(x_1,\ldots,x_k)>0 \\ Max(x_1,\ldots,x_k) & otherwise \end{cases},$$

where $h_p\in T^{-1}(0)$, and $T(h)=H_k(1-x_1/h,\ \ldots,\ 1-x_k/h)= \sum_{i=1}^{k}[(x_i/h-1+r_i)/r_i]_+^{p_i}$-1=0 and $0<r_i\leq 1$ and $p_i>1$, i=1,...,k.

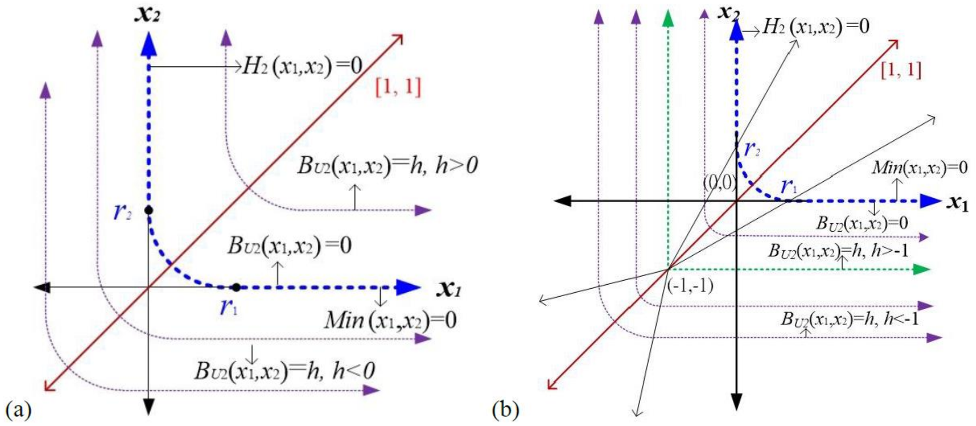

Figure 4. (a). The shapes of H2(x1,x2)=0 (thick) and level curves of union BU2(x1,x2)=h, h∈R, in Eq. (1), which have the same ranges (thin). (b). Level curves of union BU2(x1,x2)=h, h∈R, in Eq. (4), which have increasing ranges (thin).

Level surfaces of Eq. (4) have increasing ranges r_i and all are smooth curves, as shown in Figure 4(b). This implies that Eqs. (4)-(6) can be used to create smooth sequential blends with overlapping blending regions, as shown from the region around the circle in Figure 3.

2.5. Blends with Bulge Elimination

In fact, unwanted bulges usually occur on a union blend of intersecting or connecting cylinders or toroids as in Figure 5(a). To eliminate unwanted bulges, gradient-based functions $R(x,y,z)$ [13, 18] were proposed to vary the values of range parameters r_1 and r_2 in a binary union from Eqs. (1)-(6). That is, parameters r_1 and r_2 are replaced with functions $R_1(x,y,z)$ and $R_2(x,y,z)$:[0, π]→[0, 2]:

$$R_1(x,y,z)=r_1*(1-Cos(\theta(x,y,z))+\omega) \text{ and } \qquad R_2(x,y,z)=r_2*(1-Cos(\theta(x,y,z))+\omega),$$

where $\omega\cong 0$, such as 0.001, and $\theta(x,y,z)$ is the angle of the gradients of f_1 and f_2 at (x,y,z). Bulges can therefore be eliminated like the left object in Figure 5(b).

(a) (b)

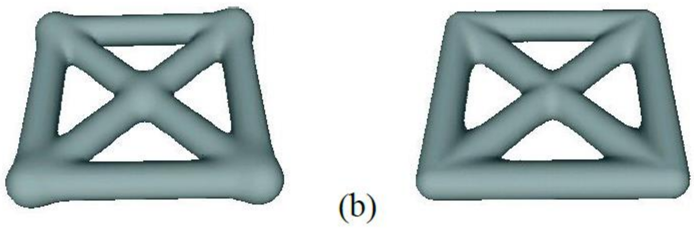

Figure 5. (a). Chair with three cylinders overlapping each other and connecting together around the circled region. (b). Intersecting cylinders with bulges.

3. SET BLENDS WITH BOUNDED PRIMITIVES

3.1. Requirements for Set Blends With Bounded Primitives

From the composing sets of the transitions of existing set blends stated in Section 2.3, it is found that the transitions are composed of the intersection curves of the level surfaces of blended primitives in blending regions. However, the shapes and sizes of the level surfaces of primitives are always similar and proportional to the shapes and sizes of blended primitives. To break this limitation, new set blends $B_{bk}(f_1,\ldots,f_k)$ with bounded primitives are developed in this section. In a $B_{bk}(f_1,\ldots,f_k)$, each primitive of $f_1=0,\ldots,$ and $f_k=0$ is assigned an individual bounding soild $f_{ci}\leq 0$, $i=$ 1 to k, and the following requirements are satisfied:

- Requirement for a union blend $B_{Ubk}(f_1,\ldots,f_k)$ with range parameters r_i and bounding solid $f_{ci}\leq 0$:

  The blending region of each f_i changes from $\{(x,y,z)\in R^3|f_i(x,y,z)\geq 0$ and $f_i(x,y,z)\leq r_i\}$ to $\{(x,y,z)\in R^3|\ f_i(x,y,z)\geq 0$ and $f_{ci}(x,y,z)\leq 0\}$ as in Figure 6(a).

- Requirement for an intersection blend $B_{Ibk}(f_1,\ldots,f_k)$ with range parameters r_i and bounding solid $f_{ci}\leq 0$:

  The blending region of each f_i changes from $\{(x,y,z)\in R^3|\ f_i(x,y,z)\geq -r_i$ and $f_i(x,y,z)\leq 0\}$ to $\{(x,y,z)\in R^3|\ f_{ci}(x,y,z)\geq 0$ and $f_i(x,y,z)\leq 0\}$ as in Figure 6(b).

- Requirement for a difference blend $B_{Dbk}(f_1,\ldots,f_k)$ with range parameters r_i and bounding solid $f_{ci}\leq 0$:

The blending region of f_1 changes from $\{(x,y,z)\in R^3|f_1(x,y,z)\geq 0$ and $f_1(x,y,z)\leq r_1\}$ to $\{(x,y,z)\in R^3|f_1(x,y,z)\geq 0$ and $f_{c1}(x,y,z)\leq 0\}$ as in Figure 6(a); the blending region of f_i, $i=2$ to k, changes from $\{(x,y,z)\in R^3|\ f_i(x,y,z)\geq -r_i$ and $f_i(x,y,z)\leq 0\}$ to $\{(x,y,z)\in R^3|f_{ci}(x,y,z)\geq 0$ and $f_i(x,y,z)\leq 0\}$ as in Figure 6(b).

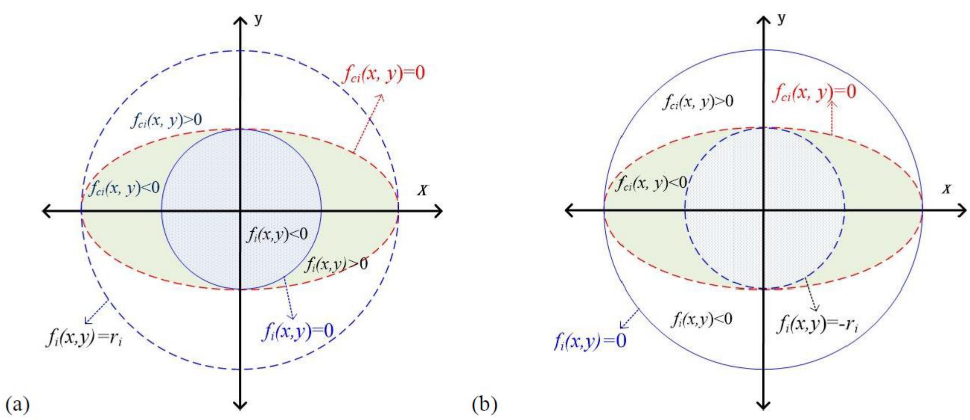

Figure 6. (a). New blending regions, in green, of primitives of a union bounded by $fci\leq 0$. (b) New blending regions, in green, of primitives of an intersection bounded by $fci\leq 0$.

3.2. Set Blends with Bounded Primitives Extended from the Scale Method

This section develops new set blend that can satisfy the requirements in Section 3.1 and these blends are extended from Eqs. (4)-(6) of the scale method as described below:

- Union blend $B_{Ubk}(f_1,\dots,f_k)$ with bounded primitives $f_{ci}\leq 0$, $i=1,\dots,k$:

$$B_{Ubk}(f_1,\dots,f_k)=B_{ASk}(f_1+1,\dots,f_k+1)-1 \text{ in Eq. (4),} \qquad (7)$$

where primitive f_i and bounding solid f_{ci}, $i=1,\dots,k$, are given by

$$f_i(x,y,z)=f_{ri}(x,y,z)-1 \text{ and } f_{ci}(x,y,z)=f_{cri}(x,y,z)-1,$$

and parameters r_i , $i=1,\dots, k$, in Eq. (4) are replaced with function $R_i(x,y,z)$:

$$R_i(x,y,z)=(f_i(x,y,z)+1)/(f_{ci}(x,y,z)+1)-1 \text{ for } f_{ci}>-1, \text{ otherwise } \omega$$

where $\omega>0$ and $\omega\cong 0$.

In Eq. (7), f_{ri} and f_{cri} must belong to the same kind of non-negative ray-linear functions and $f_i(x,y,z)\leq 0$ must be included inside $f_{ci}(x,y,z)\leq 0$. Thus, the blending region of f_i in $B_{Ubk}(f_1,\dots,f_k)$ is transformed form $0\leq f_i(x,y,z) \leq r_i$ to $\{(x,y,z)\in R^3 | f_i(x,y,z)\geq 0 \text{ and } f_{ci}(x,y,z) \leq 0\}$

- Intersection blend $B_{Ibk}(f_1,\dots,f_k)$ with bounded primitives $f_{ci}\leq 0$, $i=1,\dots,k$:

$$B_{Ibk}=B_{SAk}(x_1+1,\dots,x_k+1)-1 \text{in Eq. (5),} \qquad (8)$$

where primitive f_i and bounding solid f_{ci}, $i=1,\dots,k$, are written by

$$f_i(x,y,z)=f_{ri}(x,y,z)-1 \text{ and } f_{ci}(x,y,z)=f_{cri}(x,y,z)-1$$

and parameters r_i , $i=1,\dots, k$, in Eq. (5) are replaced with function $R_i(x,y,z)$:

$$R_i(x,y,z)=1-(f_i(x,y,z)+1)/(f_{ci}(x,y,z)+1) \text{ for } f_{ci}>-1, \text{ otherwise } \omega,$$

where $\omega>0$ and $\omega\cong 0$.

In Eq. (8), f_{ri} and f_{cri} must belong to the same kind of non-negative ray-linear functions and $f_{ci}(x,y,z)\leq 0$ must be included inside $f_i(x,y,z)\leq 0$. Thus, the blending region of f_i in $B_{Ibk}(f_1,\dots,f_k)$ is transformed from $-r_i\leq f_i(x,y,z) \leq 0$ to $\{(x,y,z)\in R^3 | f_{ci}(x,y,z)\geq 0 \text{ and } f_i(x,y,z)\leq 0\}$.

- Difference blend $B_{Dbk}(f_1,\dots,f_k)$ with bounded primitives $f_{ci}\leq 0$, $i=1,\dots,k$:

B_{Dbk} is extended from Eq. (6) but parameter r_1 of f_1 is replaced with R_1 in Eq. (8) and parameters r_i of $f_{2,}$,,,,and f_k are replaced with R_i in Eq. (7).

4. Applications of Blends with Bounded Primitives

Based on the proposed blend with bounded primitive in Eqs. (7) and (8), modeling functions are obtained as shown below:

- Bulge elimination on sequential unions of intersecting or connecting cylinders and toroids is achieved by choosing bounding solids that do not contain the region where unwanted bulges are located.

Figure 7(a) shows a stool made of intersecting cylinders and a plate, and it contains unwanted bulges on the joint regions because the plate's blending region, as in Figure 7(b), covers the region where bulges sit. However, as in Figure 7(c) the bulges are eliminated by adopting the proposed blends and using the bounding solid in Figure 7(d) for the plate to avoid the blending region of the plate from covering the bulged region.

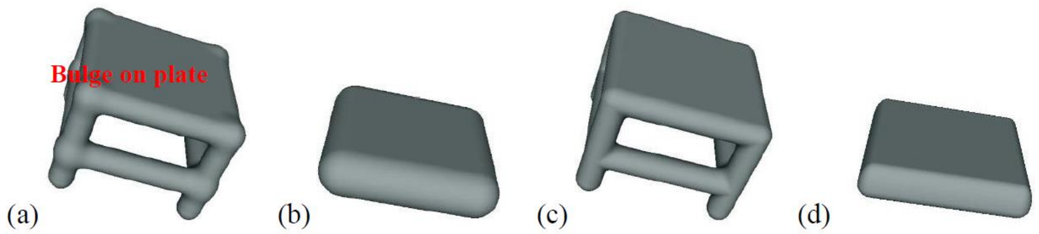

Figure 7. (a). Stool, unions of cylinders and a plate, containing bulges. (b). Blending region of the plate in (a) which causes the bulges around the corners of the plate. (c). Stool without bulges by using the object in (d) as the bounding solid for the plate.

- Shape adjustment on the contour of the transition of a union.

This is achieved by selecting a bounding solid to adjust the shape of the contour. As in Figure 8, using the bounding solids on the center columns for the unions have changed the shape of the contours of the resulting transitions as shown on the right columns.

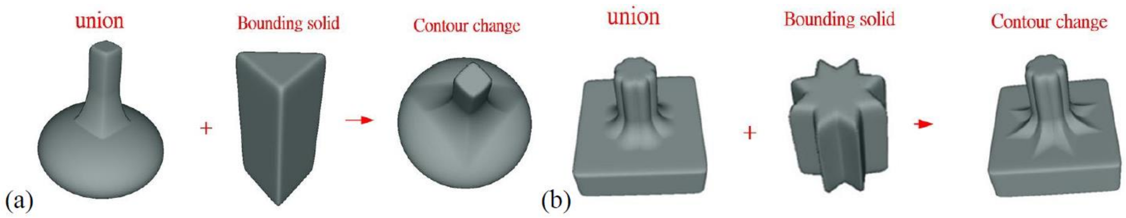

Figure 8. The shapes of the contours of the transitions of unions in (a) and (b) are changed by the bounding solids on the center columns.

5. Conclusion

This paper has extended the scale method and successfully developed new set blends with bounded primitives for zero implicit surface. In the proposed blends with bounded primitives, each primitive can be assigned an individual bounding solid to change its blending region. Thus, by choosing suitable bounding solids for primitives, set blends with bounded primitives can be used to:

(1). Generate smooth sequential blends with overlapping blending regions.
(2). Eliminate bulges on intersecting and connecting cylinders and toroids.
(3). Change and adjust the contour's shape of the transition of a union through the chosen bounding solid.

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