FREQUENCY RESPONSE ANALYSIS OF 3-DOF HUMAN LOWER LIMBS

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ABSTRACT

Frequent and prolonged expose of human body to vibrations can induce back pain and physical disorder and degeneration of tissue. The biomechanical model of human lower limbs are modeled as a three degree of freedom linear spring-mass-damper system to estimate forces and frequencies. Then three degree of freedom system was analysed using state space method to find natural frequency and mode shape. A program was develop to solve simplified equations and results were plotted and discussed in detail. The mass, stiffness and damping coefficient of various segments are taken from references. The optimal values of the damping ratios of the body segments are estimated, for the three degrees of freedom model. At last resonance frequencies are found to avoid expose of lower limbs to such environment for optimum comfort.

KEYWORDS

Frequency Response Analysis, Biomechanics, Human Limbs, Dynamics

1. INTRODUCTION

Most of the time vibrations are undesirable and when people are exposed to vibration it may causing back pain, fatigue stresses and disorder. So it is very much essential to know frequency at various joints of lower limbs at particularly frequencies near the principal resonance. Several models capable of undergoing vibratory motion have been described in the literature. The model, consisting of a mass, spring and damper, was developed by Y. Matsumoto and M.J. Griffin [1]. They simulated the standing subjects exposed to vertical whole-body vibration. A model with two-degree of freedom discrete system was analysed for damped vibration analysis by Z. Oniszczuk [2]. More details of the human body were imitated in the three-dimensional biomechanical model for simulating the response of the human body to vibration stress was developed by M. Fritz [3]. In the study carried out by Tae-Hyeong Kim at. el. [4], vibration transmissibility in the vertical direction was measured for a biomechanical model of the human body in a sitting posture. S. Kitazaki and M. J. Griffin [5] analysed a whole-body vertical vibration, using a finite element model of the human body.

2. DEVELOPMENT OF THE EQUATION OF MOTION

In all literature mentioned before no extensive study of human lower limbs were found. Using basic theory of vibration [6], a three-degree of freedom model consists of masses, springs and dampers is developed simulating the lower limb in preceding work presented by K P Hirpara [7].

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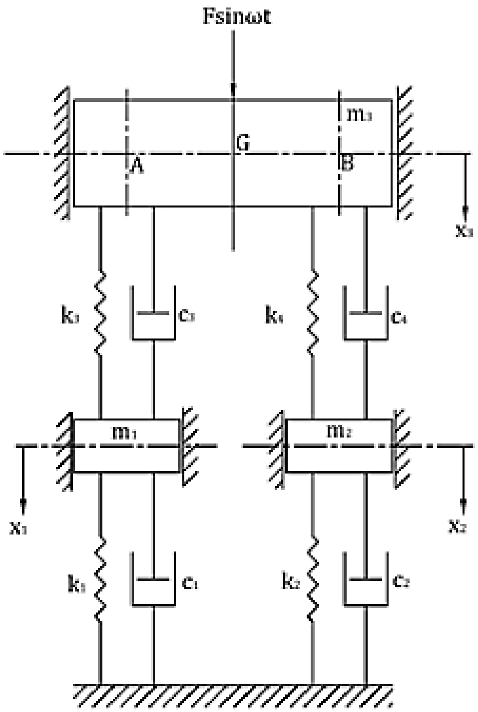

Figure 1. 3-dof linear vibration model under consideration

Figure 1 shows a three degree of freedom with three masses m_1, m_2 andm_3. m_1 and m_2 tied to ground through spring k_1 and k_2 and damper c_1 and c_2 similarly both are connected to mass m_3 through spring k_3 and k_4 and damper c_3 and c_4. The system is assumed to be free in executing oscillations in the vertical direction only, also clearance between mass and guide is negligible, the springs and dampers are assumed mass less and deformation of spring and damper is linear.Measuring the displacement, velocity and acceleration quantities (downwards positive), and applying Lagrangian equation to this system,

$$m_1\ddot{x_1} + (c_1 + c_3)\dot{x_1} + (k_1 + k_3)x_1 - c_3\dot{x_3} - k_3x_3 = F1$$
$$m_2\ddot{x_2} + (c_2 + c_4)\dot{x_2} + (k_2 + k_4)x_2 - c_4\dot{x_3} - k_4x_3 = F2$$
$$m_3\ddot{x_3} + (c_3 + c_4)\dot{x_3} + (k_3 + k_4)x_3 - c_3\dot{x}_1 - k_3x_1 - c_4\dot{x}_2 - k_4x_2 = F3$$

In order to solve time domain problems using a computer, it is desirable to change the form of the equations for the 3-dof system with three second order differential equations to six first order differential equations which is known as state space form.

Considering forces are acting on each masses are F1, F2 and F3 respectively. Equations of motion for 3-dof model can be written in state space form as,

$$\dot{z_1} = z_2$$
$$\dot{z_2} = [F_1 - (c_1 + c_3)z_2 - (k_1 + k_3)z_1 + c_3z_6 + k_3z_5]/m_1$$
$$\dot{z_3} = z_4$$
$$\dot{z_4} = [F_2 - (c_2 + c_4)z_4 - (k_2 + k_4)z_3 + c_4z_6 + k_4z_5]/m_2$$
$$\dot{z_5} = z_6$$
$$\dot{z_6} = [F_3 - (c_3 + c_4)z_6 - (k_3 + k_4)z_5 + c_3z_2 + k_3z_1 + c_4z_4 + k_4z_3]/m_3$$

Eiganvalue can be written in state space form,

$$(\lambda I - A) = \lambda I - \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{-(k_1+k_3)}{m_1} & \frac{-(c_1+c_3)}{m_1} & 0 & 0 & \frac{k_3}{m_1} & \frac{c_3}{m_1} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{-(k_2+k_4)}{m_2} & \frac{-(c_2+c_4)}{m_2} & \frac{k_4}{m_2} & \frac{c_4}{m_2} \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{k_3}{m_3} & \frac{c_3}{m_3} & \frac{k_4}{m_3} & \frac{c_4}{m_3} & \frac{-(k_3+k_4)}{m_3} & \frac{-(c_3+c_4)}{m_3} \end{bmatrix}$$

3. DYNAMIC ANALYSIS OF THE 3-DOF MODEL

To solve original damped system equation for complex Eigenvalues and Eigenvectors it is Normalize to unity. Than magnitude and phase angle of each of the Eigenvector entries are calculated. Also the percentage of critical damping (damping ratio) for each mode and the motions of the three masses for all three modes are calculated. Results were plotted the real and imaginary displacements of each of the degrees of freedom separately.

For the 3-dof damped system matrix, taking the closed form determinant is far too complicated, so MATLAB was used to solve the Eigenvalue problem numerically. MATLAB code was develop to determine eigenvalue, eigenvector, magnitude and phase angle, critical damping ratio and frequency response using specific values of m, c and k and which are taken from research paper published by Devendra P. Garg et al and T. C. Gupta [8,9].

Table 1: Values of Mass, Damping coefficients stiffness and force at each joints

Mass (kg)	Stiffness (N m^{-1})	Damping coefficient (N sm^{-1})	Force (N)
m1= 8.26	k_1= 3.590 X 10^5	c_1= 963.2	F_1= 100
m2= 8.26	k_2= 3.590 X 10^5	c_2= 963.2	F_2= 100
m3= 59.10	k_3= 3.590 X 10^5	c_3= 963.2	F_3= 100
--	k_4= 3.590 X 10^5	c_4= 963.2	--

4. RESULT AND DISCUSSIONS

4.1. Eigen values and Eigenvectors

The six Frequencies derived from the program are listed below,

Table 2: Natural frequencies at each joints

	Eigenvalues in complex form	Natural frequency (rad/sec)
Mass 1	-1.2901 + 2.8200i -1.2901 - 2.8200i	3.101
Mass 2	-1.1661 + 2.7079i -1.1661 - 2.7079i	2.948
Mass 3	-0.1022 + 0.8670i -0.1022 - 0.8670i	0.873

Note that the two Eigenvalues which correspond to each of the three modes are complex conjugates of each other, and that the real parts of the all third mode Eigenvalues are negative. Here it shows that mode 1 takes highest time to reach at equilibrium stage as compare to mode 2, and mode 3 having highest damping ratio so it will take lesser time to reach on equilibrium stage.

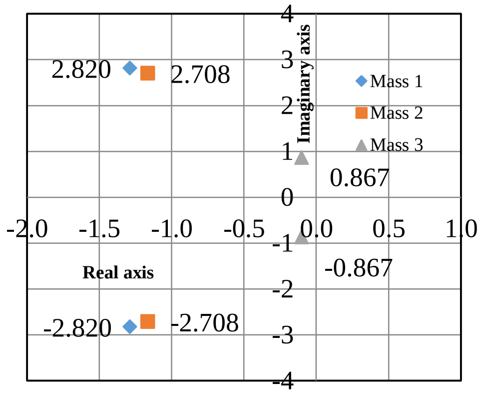

Figure 2: Eigenvalues in complex form

The Eigenvectors than normalizes from low to high frequency by dividing each Eigenvector by its position state for mass 1, the first term in each Eigenvector.

Table 3: Eigenvectors

	Displacement states	Velocity states	Displacement states	Velocity states	Displacement states	Velocity states
Mode 1	0.0100 -0.1022 +0.8670i	0.0100 -0.1022 -0.8670i	0.0100 -1.1661 +2.7079i	0.0100 -1.1661 -2.7079i	0.0100 -1.2901 +2.8200i	0.0100 -1.2901 -2.8200i
Mode 2	0.0100 + 0.0000i	0.0100 - 0.0000i	-0.0100 + 0.0000i	-0.0100 - 0.0000i	0.0100 - 0.0000i	0.0100 + 0.0000i
	-0.1022 + 0.8670i	-0.1022 - 0.8670i	1.1661 - 2.7079i	1.1661 + 2.7079i	-1.2901 + 2.8200i	-1.2901 - 2.8200i
Mode 3	0.0182 + 0.0000i	0.0182 - 0.0000i	-0.0000 - 0.0000i	-0.0000 + 0.0000i	-0.0021 + 0.0000i	-0.0021 - 0.0000i
	-0.1865 + 1.5820i	-0.1865 - 1.5820i	0.0000 - 0.0000i	0.0000 + 0.0000i	0.2743 - 0.5996i	0.2743 + 0.5996i

The six rows of each Eigenvector (mode shapes) are related to the six states, where even rows are the displacement states and odd rows are the velocity states. Each velocity row is equal to the displacement row associated with it times.

The first two columns of the Eigenvector matrix define mode 1, the third and fourth define mode 2 and the fifth and sixth columns define mode 3. Like the two complex conjugate Eigenvalues for each mode, the two Eigenvector columns for each of the modes are complex conjugates of each other.

The percentage of critical damping for each of the three modes is than calculated,

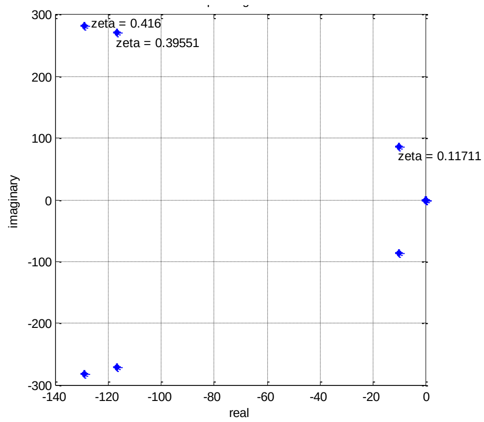

Figure 3: Damped Natural frequencies

Note that the damping ratios are 0.1171, 0.3955 and 0.416 for modes 1 and 2 and 3 respectively which is shown in the figure 3.

4.2Initial Condition Responses

The motion in that mode is defined as the sum of the motions due to the two conjugate Eigenvalues/Eigenvectors for that mode. Initial condition transient responses for the three modes, illustrating the cancelling of the imaginary components and the doubling of the real components are plotted

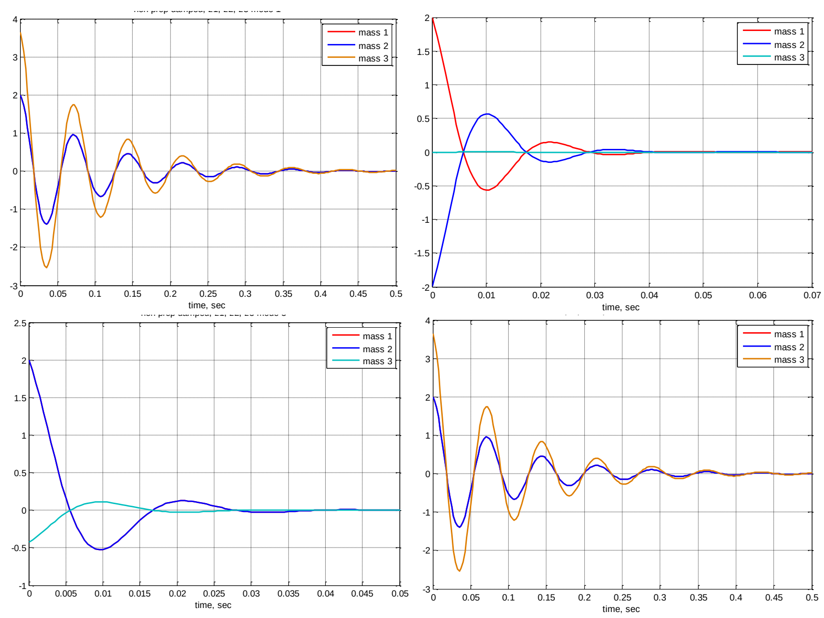

Figure 4: Non proportional damped vibration for mode 1; (a) real and imaginary for m1 (a) real and imaginary for m2 (a) real and imaginary for m3 (d) combined for m1, m2 and m3

The figure 4 show the motions of the masses decreasing due to the damping. The imaginary components are out of phase and cancel each other, leaving only twice the real component as the final motion. Unlike the undamped case, the three masses do not reach their maximum or minimum positions at the same time. Since the damping is quite small, it is hard to see on the plots the small differences in times at which the maxima and minima are reached. Also it can be seen from the Figure 4(d), mass 1 and mass 2 are moving in a same phase and mass 3 is in different phase. Response time for this mode is 0.5 seconds.

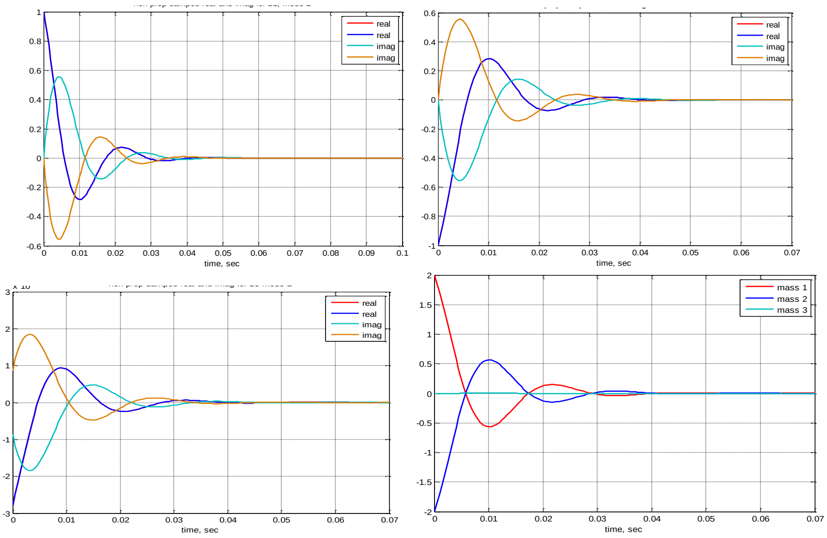

Figure 1: Non proportional damped vibration for mode 2; (a) real and imaginary for m1 (a) real and imaginary for m2 (a) real and imaginary for m3 (d) combined for m1, m2 and m3

Compared to the responses for the mode 1 in Figure 5, the response for mode 2 damps out faster for two reasons, first, it has higher damping. Secondly, even if zeta were the same for the two modes, the higher frequency of mode 2 will create higher velocities, hence higher damping from the velocity-dependent damping term. Note that the equal damping values for the dampers make the center mass have a small motion. Response time for this mode is 0.07 seconds. Also from Figure 5(d) mass 1 and 2 are in opposite phase and having almost same magnitude but mass 3 is almost in steady condition.

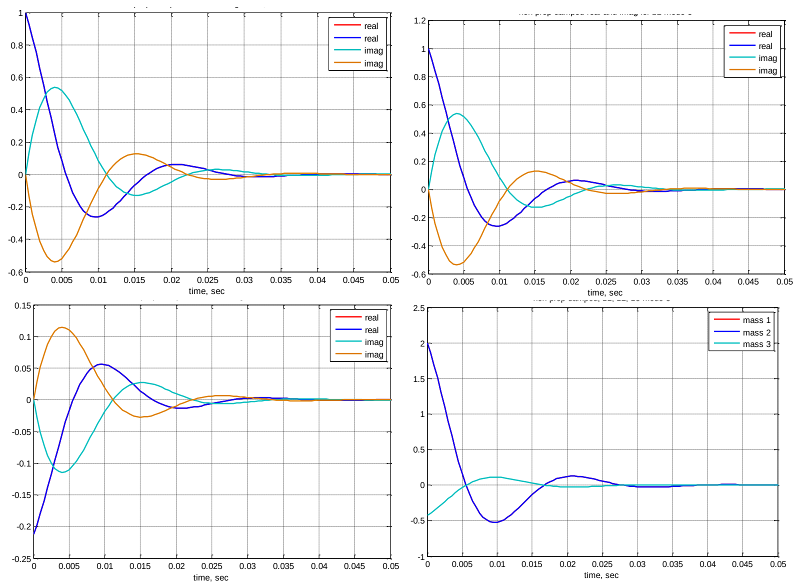

Figure 6: Non proportional damped vibration for mode 3; (a) real and imaginary for m1 (a) real and imaginary for m2 (a) real and imaginary for m3 (d) combined for m1, m2 and m3

Compared to the responses for the mode 2 in Figure6, the response for mode 3 damps out faster because it has higher damping as mode 2 has 0.3955 damping ratio and mode 3 has 0.416 damping ratio. Also, if zeta were the same for the two modes, the higher frequency of mode 3 will create higher velocities, hence higher damping from the velocity-dependent damping term. Response time for this mode is 0.05 seconds. Also from Figure 6(d) we can see that mass 1 and 2 are in same phase but mass 3 is in different phase.

4.3Frequency response

The four distinct transfer functions for the default values of m, k and c are plotted using MATLAB. Frequency response curves are also determined from analysis and plots are displayed in Figure 7.

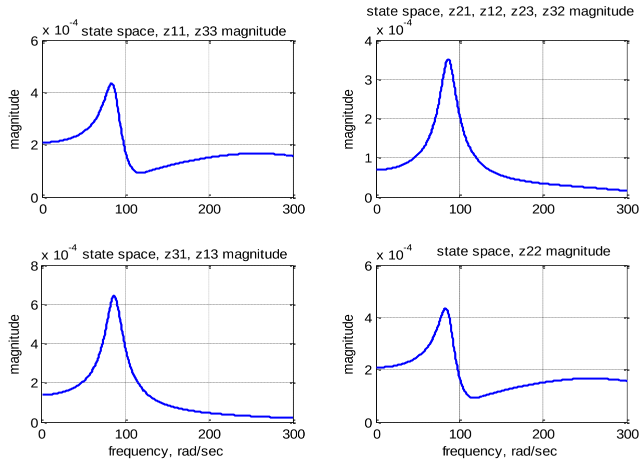

Figure 7: Frequency response curves, magnitude v/s frequency

From the above graph it can be written that at frequency 83 rad/sec magnitude reaches maximum at 0.434 mm in state Z11 and Z33. In state Z21, Z12, Z23, Z32 at 86.5 rad/sec frequency, maximum magnitude reached was 0.335 mm. In third graph at same frequency max magnitude reached was 0.665 mm. From forth graph it is predicted that 85 rad/sec frequency magnitude reaches at 0.434 mm and get down.

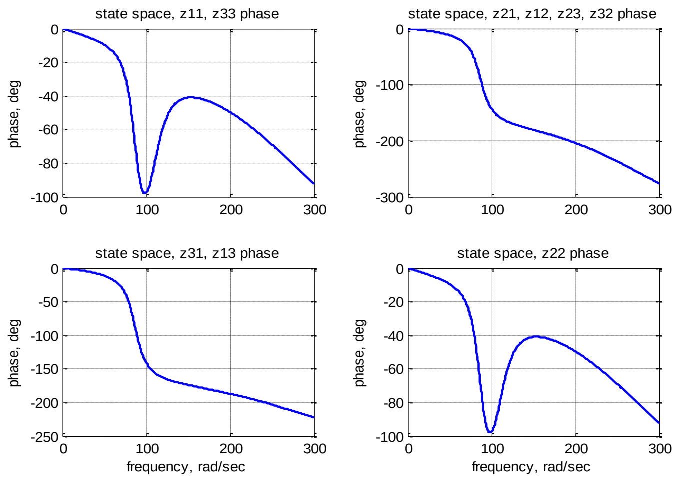

Figure 8: Frequency response curves, magnitude v/s frequency

Phase difference at frequency 98 rad/sec is -98.05 in first graph, -138 in second graph and -136.3 in third graph. In last graph having same behaviour as first graph. Also from graph it can be easily predicted that with the increase in frequency phase angle is decreases from 0 to up to 300.

5. CONCLUSIONS

Initially 3-dof model is developed using Lagrange's considering three linear motions. Three Eigenvalues was found in complex form using MATLAB and natural frequencies obtained, which are 3.101, 2.948 and 0.873 rad/sec. Also different mode shapes are found. The damping ratio obtained for mode 1 is 0.117, for mode 2 is 0.3955 and for mode 3 is 0.416. Initial condition response graphs were plotted for each mode in Figure 4, Figure 5 and Figure 6. Further frequency response curves were depicted in the study. At frequency of 85 rad/sec each state variables were having highest magnitude.

With the aid of the model it is possible to hypothesize the causes of various resonance peaks. It is seen from the result that at joint of one knee joint resonant frequency is approximately 0.49 Hz for both legs and the mass of the whole body on the stiffness of the legs results in a resonance near 0.13 Hz.

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