Optimization of Packet Length for Two Way Relaying with Energy Harvesting

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Abstract

In this article, we suggest optimizing packet length for two way relaying with energy harvesting. In the first transmission phase, two source nodes N_1 and N_2 are transmitting data to each others through a selected relay R. In the second phase, the selected relay will amplify the sum of the signals received signals from N_1 and N_2. The selected relay amplifies the received signals using the harvested energy from Radio Frequency (RF) signals transmitted by nodes N_1 and N_2. Finally, N_1 will remove, from the relay's signal, its own signal to be able to decode the symbol of N_2. Similarly, N_2 will remove, from the relay's signal, its own signal to be able to decode the symbol of N_1. We derive the outage probability, packet error probability and throughput at N_1 and N_2. We also optimize packet length to maximize the throughput at N_1 or N_2.

Index Terms : Cooperative systems, Optimal packet length, Rayleigh fading channels.

1 Introduction

In Two-Way Relaying (TWR), two nodes N_1 and N_2 simultaneously transmit data to each other using a selected relay [1-5]. The communication process contains two phases. In the first one, N_1 and N_2 transmit data to some relays. Each relay will receive the sum of signals transmitted by N_1 and N_2. In the second phase, a selected relay amplifies the received signal. Then, N_1 will remove, from the relay's signal, its own signal to be able to decode the symbol of N_2. Similarly, N_2 will remove, from the relay's signal, its own signal to be able to decode the symbol of N_1.

Two way relaying for Multiple Input Multiple Output (MIMO) systems has been considered in [1-5]. Receive and transmit diversity improves the performance of TWR. At the receiver, the best antenna can be selected (Selection Combining SC). The corresponding

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Signal to Noise Ratio (SNR) is the maximum of SNRs over all antennas. It is also possible to combine the signals of all antennas using Maximum Ratio Combining (MRC). The SNR will be the sum of all SNRs [1-5]. TWR with Energy Harvesting (EH) consists to use the Radio Frequency (RF) signal to charge the battery of nodes [6-10]. Relays with EH capabilities has been studied in [6-10]. In order to enhance the throughput especially at low SNRs, channel coding is required in TWR [11-13]. Secure two way relaying has been suggested in [14-20]. Security aspects of TWR should be studied to avoid data recovery by a malicious node.

The main contribution of the paper is to optimize packet length so that the throughput at node N_1 or N_2 is maximized. In all previous studies, a Fixed Packet Length (FPL) is used [1-20]. This is the first paper to suggest an Optimal Packet Length (OPL) for TWR with Energy Harvesting.

The system model is presented in section 2. Section 3 gives the Cumulative Distribution Function (CDF) of SNR. Section 4 derives the PEP while section 5 gives the expression of OPL. Some numerical results are given in section 6. Conclusions are presented in section 7.

2 System model

The system model is shown in Fig. 1. There are two nodes N_1 and N_2 communicating information to each other through a relay R. Node N_1 transmits data to node N_2 and at the same time node N_2 is also communicating data to N_1 through relay R. N_1 and N_2 transmit over the same channel.

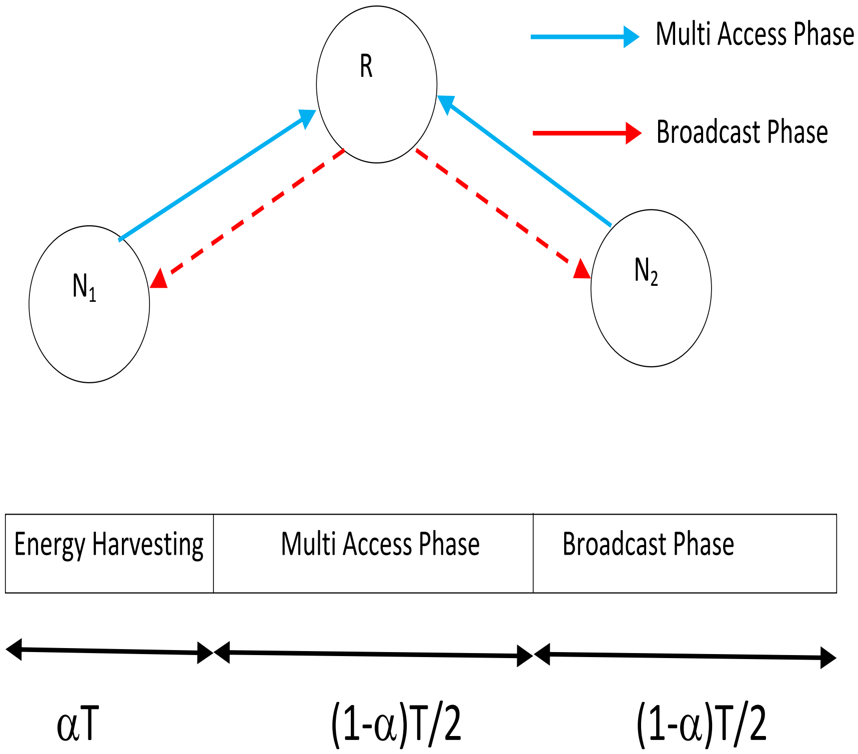

Figure 1: Two way relaying with Energy harvesting.

The frame with duration T is decomposed in three parts :

- The first slot with duration αT is dedicated to energy harvesting. Relay R harvests energy from RF signal transmitted by nodes N_1 and N_2.

The harvested energy is written as

$$E = \beta(P_1|h_{N_1R}|^2 + P_2|h_{N_2R}|^2)\alpha T = \beta(E_1|h_{N_1R}|^2 + E_2|h_{N_2R}|^2)\alpha p, \tag{1}$$

where 0< $\alpha < 1$ is harvesting duration percentage, P_i (resp. E_i) is the transmit power (resp. symbol energy) of node N_i and h_{N_1R} (respectively h_{N_2R}) is channel coefficient between nodes N_1 (respectively N_2) and R. $p = T/T_s$ is the number of symbols per frame T. We have $E_X = T_s P_X$

- During the second time slot with duration $(1-\alpha)T/2$, N_1 and N_2 transmit data to node R over the same channel. This is the multiple access phase. The received signal at R is written as

$$y_R(j) = \sqrt{E_1}x_1(j)h_{N_1R} + \sqrt{E_2}x_2(j)h_{N_2R} + n_R(j) \tag{2}$$

where E_i is the transmitted energy per symbol of node i with 1$\leq i \leq 2$, $x_i(j)$ is the j-th transmitted symbol by node N_i and $n_R(j)$ is an Addivite White Gaussian Noise (AWGN) with variance N_0. A Rayleigh block fading channel is assumed where the channel remains constant over all the time frame with duration T.

- During the third time slot with duration $(1-\alpha)T/2$, R transmits amplifies the received signal to nodes N_1 and N_2. This is the broadcast phase.

Relay R uses the harvested energy E to amplify the received signal $y_R(j)$ to N_1 and N_2. The transmit symbol energy of R is equal to the harvested energy E devided by the number of transmitted symbols during $(1-\alpha)T/2$ seconds i.e. $(1-\alpha)T/(2T_s) = (1-\alpha)p/2$ with $p = T_s/T$:

$$\begin{aligned} E_R &= \frac{E}{(1-\alpha)p/2} = \frac{\beta(E_1|h_{N_1R}|^2 + E_2|h_{N_2R}|^2)\alpha p}{(1-\alpha)p/2} \\ &= 2\frac{\alpha\beta}{(1-\alpha)}(E_1|h_{N_1R}|^2 + E_2|h_{N_2R}|^2) \end{aligned} \quad (3)$$

Using (43), the amplification factor G used by relay R is written as

$$G = \sqrt{\frac{E_R}{E_1|h_{N_1R}|^2 + E_2|h_{N_2R}|^2 + N_0}} \quad (4)$$

2.1 SNR at node N_1

The received signal at N_1 is written as

$$y_1(j) = Gh_{RN_1}y_R(j) + n_1(j), \quad (5)$$

where $n_1(j)$ is an AWGN with variance N_0.

Using (43), we deduce

$$\begin{aligned} y_1(j) &= Gh_{RN_1}[\sqrt{E_1}x_1(j)h_{N_1R} + \sqrt{E_2}x_2(j)h_{N_2R} + n_R(j)] + n_1(j), \\ &= \sqrt{E_1}Gh_{RN_1}x_1(j)h_{N_1R} + \sqrt{E_2}Gh_{RN_1}x_2(j)h_{N_2R} \\ &\quad +Gh_{RN_1}n_R(j) + n_1(j). \end{aligned} \quad (6)$$

Node N_1 removes the self interference, $\sqrt{E_1}Gh_{RN_1}x_1(j)h_{N_1R}$, since it knows the value of symbol $x_1(j)$. After removing self interference, we obtain

$$y_1(j) = \sqrt{E_2}Gh_{RN_1}x_2(j)h_{N_2R} + Gh_{RN_1}n_R(j) + n_1(j). \quad (7)$$

The SNR at N_1 is written as

$$\Gamma_1 = \frac{E_2G^2|h_{RN_1}|^2|h_{N_2R}|^2}{N_0 + N_0G^2|h_{RN_1}|^2}. \quad (8)$$

Using the expression of amplification factor G (45), we deduce

$$\Gamma_1 = \frac{E_2|h_{RN_1}|^2|h_{N_2R}|^2}{\frac{N_0}{G^2} + N_0|h_{RN_1}|^2} = \frac{|h_{RN_1}|^2E_2|h_{N_2R}|^2}{\frac{N_0}{E_R}[N_0 + E_1|h_{N_1R}|^2 + E_2|h_{N_2R}|^2] + N_0|h_{RN_1}|^2}. \quad (9)$$

We assume that channels are reciprocal i.e. $h_{N_1R} = h_{RN_1}$. By neglecting the term in N_0^2 and using (44), the SNR at node N_1 lower bounded by

$$\Gamma_1 > \frac{2\frac{\alpha\beta}{N_0(1-\alpha)}E_2|h_{RN_1}|^2|h_{N_2R}|^2}{1+2\frac{\alpha\beta}{(1-\alpha)}|h_{RN_1}|^2} \tag{10}$$

This upper bound is tight at high average SNR as the term N_0^2 can be neglected.
We can write

$$\Gamma_1 > \frac{a_1X_1X_2}{1+a_2X_1}, \tag{11}$$

where

$$a_1 = 2\frac{\alpha\beta}{N_0(1-\alpha)}E_2, \tag{12}$$

$$a_2 = 2\frac{\alpha\beta}{(1-\alpha)}, \tag{13}$$

$$X_1 = |h_{RN_1}|^2, \tag{14}$$

and

$$X_2 = |h_{N_2R}|^2. \tag{15}$$

2.2 SNR at node N$_2$

The received signal at N_2 is written as

$$y_2(j) = Gh_{RN_2}y_R(j) + n_2(j), \tag{16}$$

where $n_2(j)$ is an AWGN with variance N_0.
Using (43), we deduce

$$\begin{aligned} y_2(j) &= Gh_{RN_2}[\sqrt{E_1}x_1(j)h_{N_1R} + \sqrt{E_2}x_2(j)h_{N_2R} + n_R(j)] + n_2(j), \\ &= \sqrt{E_1}Gh_{RN_2}x_1(j)h_{N_1R} + \sqrt{E_2}Gh_{RN_2}x_2(j)h_{N_2R} \\ &\quad +Gh_{RN_2}n_R(j) + n_2(j). \end{aligned} \tag{17}$$

Node N_2 removes the self interference, $\sqrt{E_2}Gh_{RN_2}x_2(j)h_{N_2R}$, since it knows the value of symbol $x_2(j)$. After removing self interference, we obtain

$$y_2(j) = \sqrt{E_1}Gh_{RN_2}x_1(j)h_{N_1R} + Gh_{RN_2}n_R(j) + n_2(j). \tag{18}$$

The SNR at N_2 is written as

$$\Gamma_2 = \frac{E_1G^2|h_{RN_2}|^2|h_{N_1R}|^2}{N_0 + N_0G^2|h_{RN_2}|^2}. \tag{19}$$

Using the expression of amplification factor G (45), we deduce

$$\Gamma_2 = \frac{E_1|h_{RN_2}|^2|h_{N_1R}|^2}{\frac{N_0}{G^2} + N_0|h_{RN_2}|^2} = \frac{|h_{RN_2}|^2E_1|h_{N_1R}|^2}{\frac{N_0}{E_R}[N_0 + E_1|h_{N_1R}|^2 + E_2|h_{N_2R}|^2] + N_0|h_{RN_2}|^2}. \quad (20)$$

We assume that channels are reciprocal i.e. $h_{N_2R} = h_{RN_2}$. By neglecting the term in N_0^2 and using (44), the SNR at node N_1 lower bounded by

$$\Gamma_2 > \Gamma_2^{low} = \frac{2\frac{\alpha\beta}{N_0(1-\alpha)}E_1|h_{RN_2}|^2|h_{N_1R}|^2}{1 + 2\frac{\alpha\beta}{(1-\alpha)}|h_{RN_2}|^2} \quad (21)$$

This upper bound is tight at high average SNR as the term N_0^2 can be neglected.
We can write

$$\Gamma_2^{low} = \frac{a_1X_1X_2}{1 + a_2X_2}, \quad (22)$$

where

$$a_1 = 2\frac{\alpha\beta}{N_0(1-\alpha)}E_2 \quad (23)$$

$$a_2 = 2\frac{\alpha\beta}{(1-\alpha)} \quad (24)$$

$$X_1 = |h_{RN_1}|^2 \quad (25)$$

and

$$X_2 = |h_{N_2R}|^2 \quad (26)$$

2.3 Two way relaying in the presence of multiple relays

Fig. 2 shows the principle of TWR in the presence of K relays. The selected relay offers the largest SNR at node N_1 or N_2. When the selected relay maximizes the SNR at node N_1, the CDF of SNR is the products of CDF of SNRs of different relays

$$F_{\Gamma_1}(x) = \prod_{k=1}^{K} F_{\Gamma_1^k}(x) \quad (27)$$

where Γ_1^k is the SNR at node N_1 when relay R_k is the active relay. Γ_1^k is given in (9).

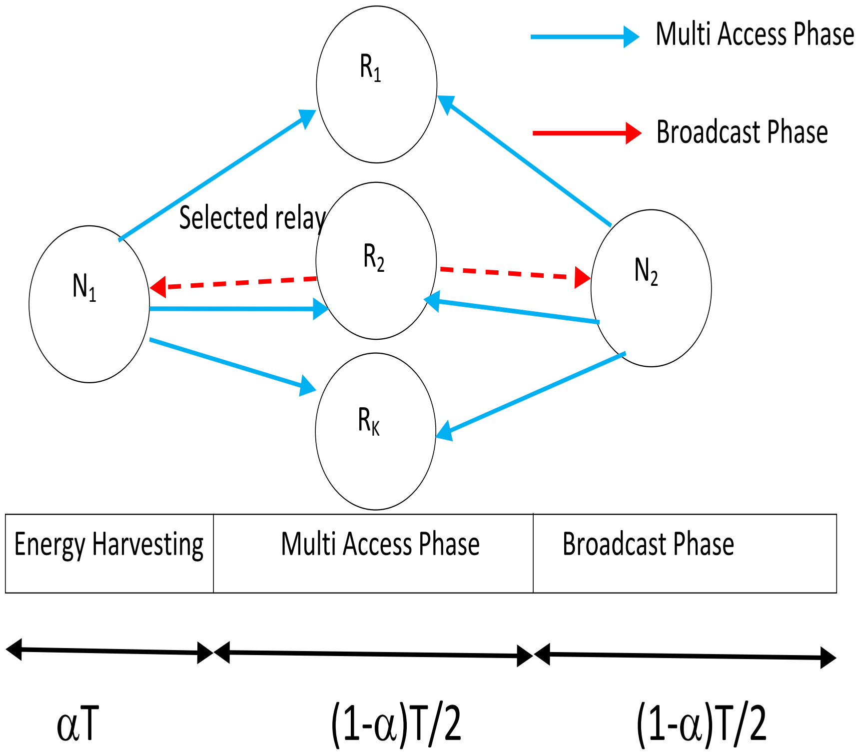

Figure 2: Two way relaying with Energy harvesting in the presence of K relays.

3 CDF of SNR

3.1 CDF of SNR at node N_1

The SNR at node N_1 is lower bounded by

$$\Gamma_1 > \Gamma_1^{low} = \frac{a_1 X_1 X_2}{1 + a_2 X_1}. \tag{28}$$

The CDF of SNR is upper bounded by

$$F_{\Gamma_1}(x) < F_{\Gamma_1^{low}}(x) = P(\Gamma_1^{low} \leq x). \tag{29}$$

We have

$$\begin{aligned} P(\Gamma_1^{low} &\leq x) = \int_0^{+\infty} [1 - P(\Gamma_1^{low} > x | X_1 = u)] f_{X_1}(u) du \\ &= \int_0^{+\infty} [1 - P(\frac{a_1 u X_2}{1 + a_2 u} > x)] f_{X_1}(u) du \end{aligned} \tag{30}$$

where $f_{X_1}(u)$ is the Probability Density Function (PDF) of X_1.
For Rayliegh fading channels, X_1 is exponentially distributed with mean

$$\frac{1}{\lambda_1} = E(X_1) = E(|h_{RN_1}|^2). \tag{31}$$

We deduce

$$P(\Gamma_1^{low} \leq x) = \int_0^{+\infty} [1 - P(X_2 > \frac{x(1+a_2u)}{a_1u})]\lambda_1 e^{-u\lambda_1} du \tag{32}$$

$$= \int_0^{+\infty} [1 - e^{-\frac{x(1+a_2u)\lambda_2}{a_1u}}]\lambda_1 e^{-u\lambda_1} du \tag{33}$$

We use the following result grad

$$\int_0^{+\infty} e^{-\frac{b}{4v} - av} dv = \sqrt{\frac{b}{a}} K_1(ab), \tag{34}$$

where $K_1(x)$ is the modified Bessel function of first order and second kind.
We finally obtain

$$F_{\Gamma_1}(x) < P(\Gamma_1^{low} \leq x) = 1 - 2\lambda_1 e^{-\frac{a_2x\lambda_2}{a_1}} \sqrt{\lambda_2 \frac{x}{a_1\lambda_1}} K_1(2\sqrt{\lambda_1 \frac{x\lambda_2}{a_1}}). \tag{35}$$

3.2 CDF of SNR at node N$_2$

The SNR at node N_2 is lower bounded by

$$\Gamma_2 > \Gamma_2^{low} = \frac{a_1X_1X_2}{1 + a_2X_2}. \tag{36}$$

The CDF of SNR is upper bounded by

$$F_{\Gamma_2}(x) < F_{\Gamma_2^{low}}(x) = P(\Gamma_2^{low} \leq x). \tag{37}$$

We have

$$P(\Gamma_2^{low} \leq x) = \int_0^{+\infty} [1 - P(\Gamma_2^{low} > x|X_2 = u)] f_{X_2}(u) du$$

$$= \int_0^{+\infty} [1 - P(\frac{a_1X_1u}{1+a_2u} > x)] f_{X_2}(u) du \tag{38}$$

We deduce

$$P(\Gamma_2^{low} \leq x) = \int_0^{+\infty} [1 - P(X_1 > \frac{x(1+a_2u)}{a_1u})]\lambda_2 e^{-u\lambda_2} du$$

$$= \int_0^{+\infty} \left[1 - e^{-\lambda_1 \frac{x(1+a_2u)}{a_1u}}\right] \lambda_2 e^{-u\lambda_2} du \tag{39}$$

We use (34), to deduce

$$F_{\Gamma_2}(x) < P(\Gamma_2^{low} \leq x) = 1 - 2\lambda_2 e^{-\frac{a_2 x \lambda_1}{a_1}} \sqrt{\lambda_1 \frac{x}{a_1 \lambda_2}} K_1(2\sqrt{\lambda_1 \frac{x\lambda_2}{a_1}}). \tag{40}$$

4 PEP

In this section, we derive the expression of the average Packet Error Probability (PEP). The PEP can be tightly upper bounded by [21]

$$\text{PEP} \leq \int_0^{w_0} f_\Gamma(\gamma) d\gamma \tag{41}$$

where $f_\Gamma(\gamma)$ is the Probability Density Function (PDF) of SNR Γ and w_0 is a waterfall threshold.

Equation (13) shows that the PEP for a given instantaneous SNR, $\gamma \leq w_0$, can be approximated to 1. However, the PEP for a given instantaneous SNR, $\gamma > w_0$ can be approximated to 0 [21].

Hence,

$$\text{PEP} \leq F_\Gamma(w_0), \tag{42}$$

where $F_\Gamma(x)$ is the Cumulative Distribution Function (CDF) of the received SNR. We denote $\overline{\Gamma} = \frac{E_b}{N_0}$ as the average SNR, where E_b is the transmitted energy per bit, N_0 is the noise Power Spectral Density (PSD) and w_0 is a waterfall threshold written as [21],

$$w_0 = \int_0^{+\infty} g(\gamma) d\gamma \tag{43}$$

$g(\gamma)$ is the PEP for a given instantaneous SNR, $\gamma = \overline{\Gamma}|h|^2$ and h is the channel coefficient.

4.1 PEP for uncoded transmission

For uncoded M-QAM modulation, we have

$$g(\gamma) = 1 - (1 - P_{es}(\gamma))^{\frac{N+n_d}{\log_2(M)}}, \tag{44}$$

where N is the number of useful information bits per packet, n_d is the number of parity bits per packet and P_{es} is the Symbol Error Probability (SEP) given as [22]

$$P_{es}(\gamma) \simeq 2(1 - \frac{1}{\sqrt{M}})\text{erfc}\left(\sqrt{\frac{\log_2(M)3\gamma}{(M-1)2}}\right). \tag{45}$$

erfc(x) is the complementary error function,

$$\text{erfc}(x) \leq e^{-x^2} \tag{46}$$

Using (45) and (46), the SEP is approximated by

$$P_{es} \simeq a_1 e^{-c_1\gamma} \tag{47}$$

where,

$$a_1 = 2\left(1 - \frac{1}{\sqrt{M}}\right), \tag{48}$$

$$c_1 = \frac{3\log_2(M)}{2(M-1)} \tag{49}$$

4.2 PEP with Channel Coding

If a convolutional encoding is used, $g(\gamma)$ can expressed as,

$$g(\gamma) = 1 - (1 - P_E(\gamma))^{\frac{N+n_d}{\log_2(M)}}, \tag{50}$$

where

$$P_E(\gamma) \leq \sum_{d=d_f}^{+\infty} a_d P_d(\gamma) \tag{51}$$

d_f and a_d are respectively the free distance and the number of trellis with Hamming weight d. Further,

$$P_d(\gamma) \simeq 2\left(1 - \frac{1}{\sqrt{M}}\right) \operatorname{erfc}\left(\sqrt{\frac{3R_c d\gamma \log_2(M)}{2(M-1)}}\right). \tag{52}$$

where R_c is the rate of convolutional encoding.

Using the approximation in (46) and keeping only the first term of (23), we have

$$P_E(\gamma) \simeq a_2 e^{-c_2\gamma} \tag{53}$$

where

$$a_2 = a_{d_f} 2\left(1 - \frac{1}{\sqrt{M}}\right), \tag{54}$$

$$c_2 = \frac{3R_c d_f \log_2(M)}{2(M-1)}. \tag{55}$$

Hence, we can generalize $g(\gamma)$ as follow,

$$g(\gamma) \simeq 1 - (1 - a_i e^{-c_i\gamma})^{\frac{N+n_d}{\log_2(M)}}, \tag{56}$$

where $i = 1$ in the absence of any channel coding and $i = 2$ for convolutional coding.

4.3 Waterfall Threshold

Using (43), the waterfall threshold is given by

$$w_0 \simeq k_1 \ln\left(\frac{N+n_d}{\log_2(M)}\right) + k_2 \tag{57}$$

where the Proof is provided in Appendix A.

$$k_1 = \frac{1}{c_i}, \tag{58}$$

$$k_2 = \frac{E+\ln(a_i)}{c_i}, \tag{59}$$

$E \simeq 0.577$ is the Euler constant.

5 Optimal Packet Length for TWR

The average number of attempts of HARQ protocols is equal to

$$T_r = \sum_{i=1}^{+\infty} PEP^{i-1}(1-PEP) = \frac{1}{1-PEP} \tag{60}$$

Therefore, the throughput in bit/s/Hz is expressed as

$$\begin{aligned} Thr &= \frac{\log_2(M)N}{(N+n_d)T_s B T_r} = \frac{\log_2(M)N}{(N+n_d)}(1-PEP) \\ &\geq \frac{\log_2(M)N}{(N+n_d)}\left[1-F_\Gamma\left(w_0\right)\right] \end{aligned} \tag{61}$$

where $B = 1/T_s$ is the used bandwidth and T_s is the symbol period.

The optimal packet length maximizing the throughput can be obtained using the Gradient algorithm.

$$N(i+1) = N(i) + \mu\frac{\partial Thr(N=N(i))}{\partial N} \tag{62}$$

We can write

$$\frac{\partial Thr}{\partial N} = \frac{log_2(M)nd}{(N+n_d)^2}\left[1-F_\Gamma\left(w_0\right)\right] - \frac{log_2(M)N}{(N+n_d)} f_\Gamma\left(w_0\right)\frac{k_1}{N+n_d} \tag{63}$$

OPL can be applied to maximize the throughput at node N_1 or N_2.

6 Theoretical and simulation results

Simulation results were obtained using MATLAB as a simulation environment.

Simulation results were performed by measuring the Packet Error Rate (PER) to deduce the throughput. The packet error rate is the number of erroneous packets/number of transmitted packets. We made simulation until 1000 packets are erroneously received.

Fig. 3 and 4 show the throughput at node N_1 for $\alpha = 1/3$, a QPSK modulation for average SNR 10 and 20 dB. The distance between all nodes is equal to 1. We notice that we can maximize the throughput by choosing the packet length. Also, the throughput increases as the number of relays increase due to cooperative diversity. In fact, we always select the relay with the largest SNR. Finally, by comparing Fig. 3 and 4, we observe that packet length should be increased as the average SNR increases. There is good accordance between theoretical and simulation results.

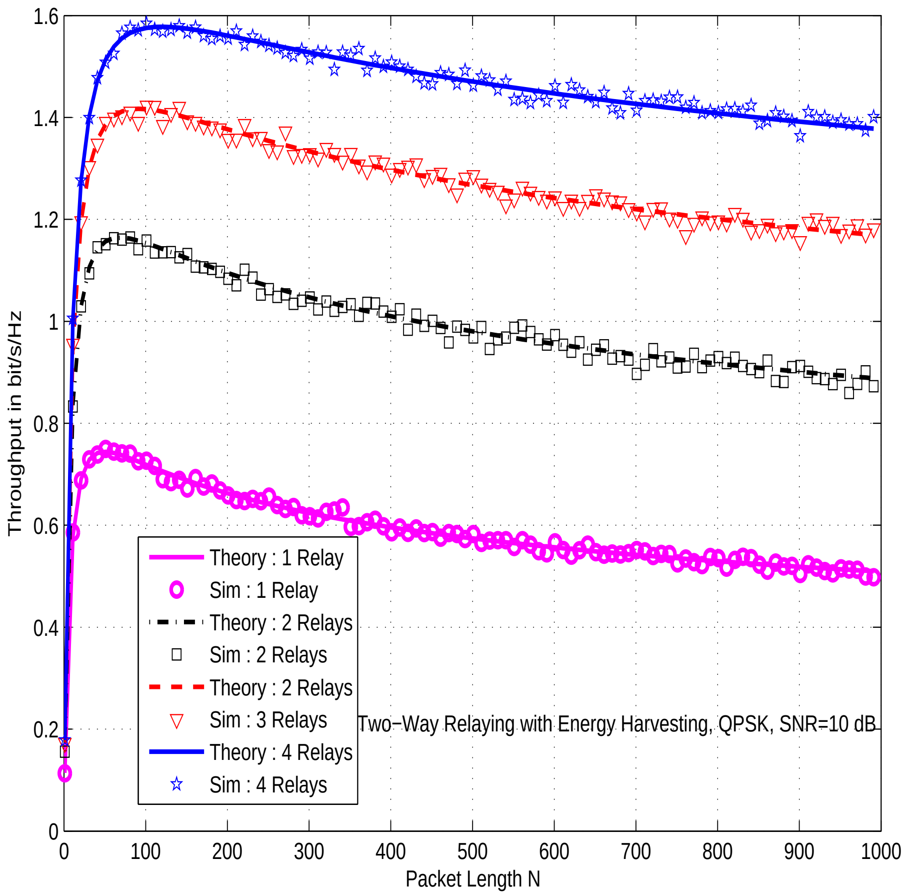

Figure 3: Throughput at node N_1 versus packet length at SNR=10 dB : 64 QAM modualtion.

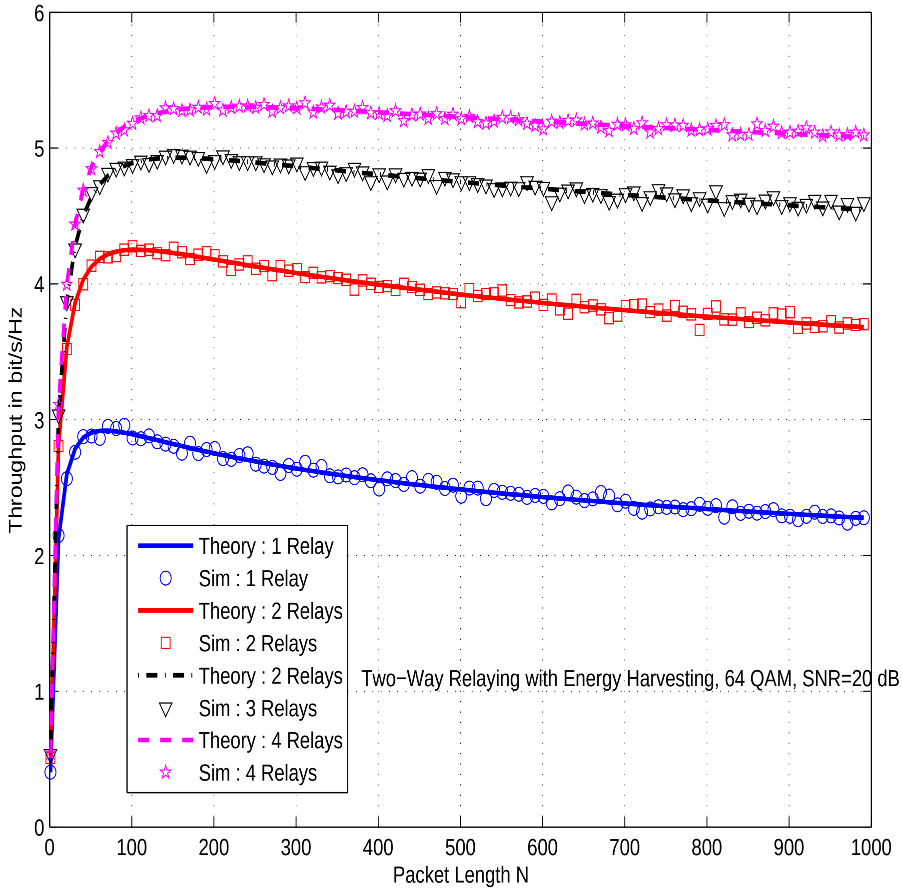

Figure 4: Throughput at node N_1 versus packet length at SNR=20 dB: 64 QAM modualtion.

Fig. 5 shows that OPL offers higher throughput than Fixed Packet Length (FPL) as studied in [1-20]. These results correspond to throughput of N_1 for $\alpha = 1/3$. They were obtained using MATLAB for a 64 QAM modulation. In fact, the proposed optimal packet length allows maximizing the throughput. If the SNR is low, the packet length is decreased. However, at high SNR, we can increase packet length.

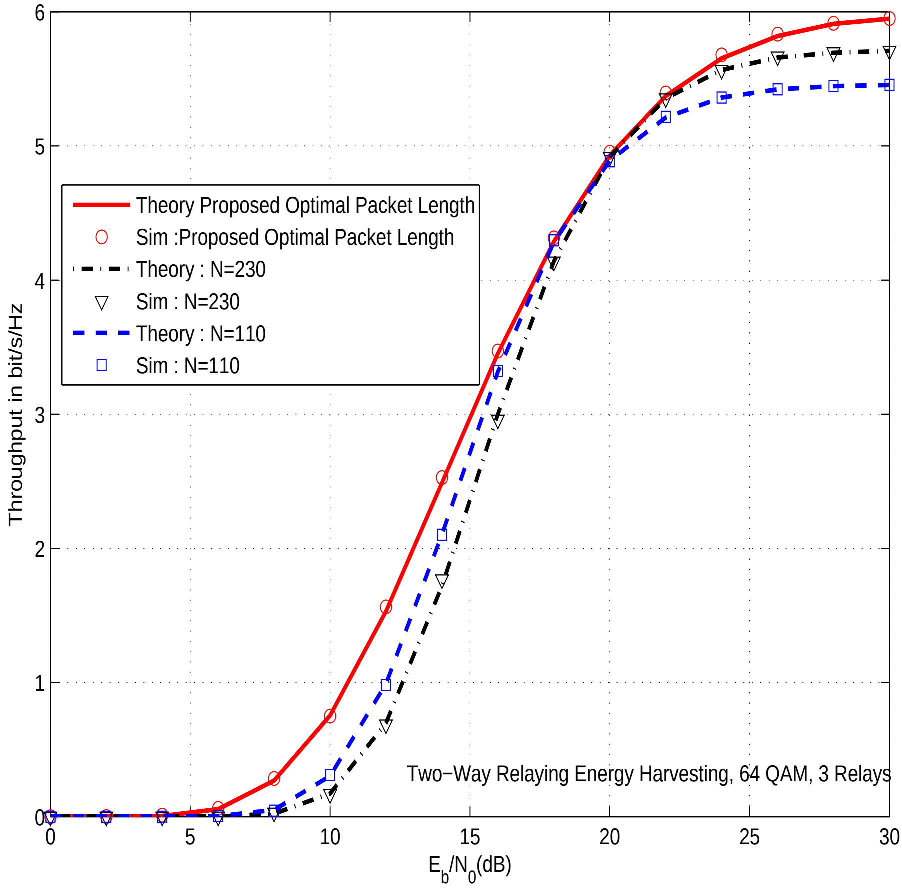

Figure 5: Throughput at node N_1 for OPL and FPL :64 QAM modualtion.

Fig. 6 shows the OPL for QPSK, 16 QAM and 64 QAM modulation. We observe that packet length should be increases when we use a small modulation such as QPSK. When 64 QAM modulation is used, packet length should be reduced since the PEP is high. Also packet length should be increased (respectively decreased) at high (respectively low) SNR.

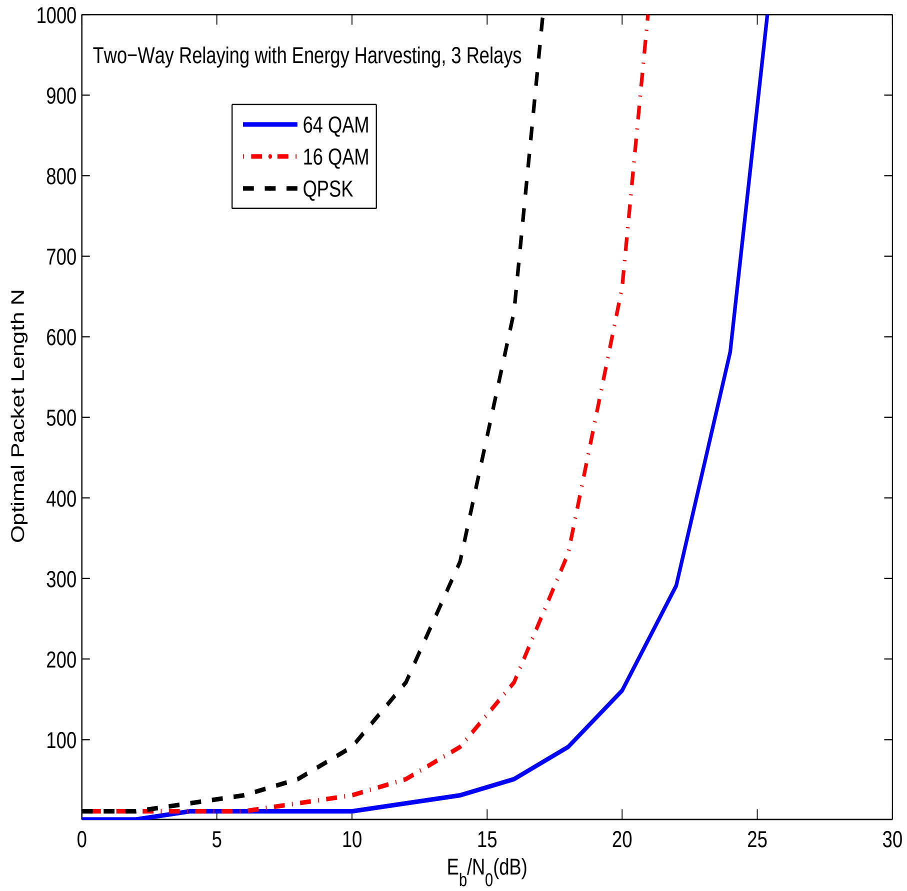

Figure 6: Optimal packet length versus SNR.

7 Conclusion

In this paper, we suggested enhancing the throughput of Two Way Relaying (TWR) with energy harvesting. We derive the best packet length that yields the largest throughput at node N_1 or N_2. Our study is valid for energy harvesting systems where the relay harvest energy from RF signals transmitted by source nodes N_1 and N_2. We have shown that the proposed TWR with best packet length offers better throughput than previous studies. Also, the throughput can be enhanced by increasing the number of relays. The proposed optimal packet length can be used in Wireless Sensor Networks (WSN) with two way relaying.

Appendix A : We have

$$w_0 = \int_0^{+\infty} [1 - (1 - a_i e^{-c_i u})^{\frac{N+n_d}{log_2(M)}}] du \quad (64)$$

We deduce

$$w_0 = \frac{1}{c_i} \int_0^{a_i} [1 - (1 - y)^{\frac{N+n_d}{log_2(M)}}] \frac{dy}{y} \quad (65)$$

Therefore, we have

$$w_0 = \frac{1}{c_i} \int_{1-a_i}^{1} \frac{1}{1-x} [1 - x^{\frac{N+n_d}{log_2(M)}}] dx \quad (66)$$

We obtain

$$w_0 = \frac{1}{c_i} \int_{1-a_i}^{1} \sum_{k=0}^{\frac{N+n_d}{log_2(M)}-1} x^k dx. \quad (67)$$

We deduce

$$w_0 = \frac{1}{c_i} \sum_{k=1}^{\frac{N+n_d}{log_2(M)}} (\frac{1}{k} - \frac{(1-a_i)^k}{k}). \quad (68)$$

For $\frac{N+n_d}{log_2(M)} >> 1$, we can write

$$\sum_{k=1}^{\frac{N+n_d}{log_2(M)}} \frac{1}{k} = ln(\frac{N+n_d}{log_2(M)}) + E, \quad (69)$$

and

$$\sum_{k=1}^{\frac{N+n_d}{log_2(M)}} \frac{(1-a_i)^k}{k} \simeq \sum_{k=1}^{+\infty} \frac{(1-a_i)^k}{k} = -ln(a_i). \quad (70)$$

Combining (68), (69) and (70), we obtain (57).

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