SHORTCOMINGS AND FLAWS IN THE MATHEMATICAL DERIVATION OF THE FUNDAMENTAL MATRIX EQUATION

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ABSTRACT

In stereo vision, the epipolar geometry is the intrinsic projective geometry between the two views. The essential and fundamental matrices relate corresponding points in stereo images. The essential matrix describes the geometry when the used cameras are calibrated, and the fundamental matrix expresses the geometry when the cameras are uncalibrated. Since the nineties, researchers devoted a lot of effort to estimate the fundamental matrix. Although it is a landmark of computer vision, in the current work, three derivations of the essential and fundamental matrices have been revised. The Longuet-Higgins' derivation of the essential matrix where he draws a mapping between the position vectors of a 3D point; however, the one-to-one feature of that mapping is lost when he changed it to a relation between the image points. In the two other derivations, we demonstrate that the authors established a mapping between the image points through the misuse of mathematics.

KEYWORDS

Fundamental Matrix, Essential Matrix, Stereo Vision, 3D Reconstruction.

1. INTRODUCTION

In computer stereo vision, the reconstruction of 3D object shape from two 2d images can be defined as follows:

The object to be reconstructed is a set of 3D points M, it is depicted by two cameras from two different standpoints. Left and right coordinate systems are defined in each of these standpoints. And every 3D point is projected on the left and right images as a pair of 2D points m_l and m_r, respectively. They are called corresponding points.

The epipolar geometry is the intrinsic projective geometry between the two views. It is independent of scene structure, and only depends on the cameras' internal parameters and relative pose. The fundamental matrix F encapsulates this intrinsic geometry [1].

A 3D point M is represented in the left and right coordinate systems by two position vectors $M_l = [X_l \quad Y_l \quad Z_l]^T$ and$M_r = [X_r \quad Y_r \quad Z_r]^T$. And$m_l = [x_l \quad y_l]^T$ and$m_r = [x_r \quad y_r]^T$ are the position vectors of the projective points m_l and m_r in the left and right coordinate systems, respectively, as depicted in Figure 1.

3D shape reconstruction is performed in the following three steps [1]

1. Compute the fundamental matrix from point correspondences.
2. Compute the camera matrices from the fundamental matrix.

3. For each point correspondence $m_l \leftrightarrow m_r$, compute the point in space that projects to these two image points.

Thus, the first step is to compute the fundamental matrix. The eight-point algorithm is the most used method to do so. In practice the number of image points is large; so, the fundamental matrix can only be estimated rather calculated. Researchers keep developing methods that overcome previously devised ones in terms of accuracy and mitigating noise effects. Only few researchers thought that the bad performance of the eight-point algorithm would requires the revision of the projective geometry approach itself.

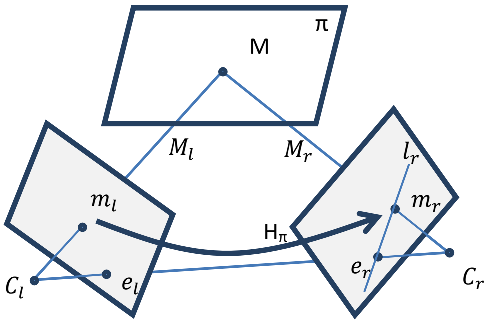

Figure 1. The epipolar geometry. A point m_l in one image is transferred via the plane π to a matching point m_r in the second image. The epipolar line l_r through m_r is obtained by joining m_r to the epipole e_r.

This article elaborates on the work of [2], where the author showed that the equation of the essential and fundamental matrix is a linear equation in two variables. Thus, it has an infinite number of solutions. Which means that any image points can have an infinite number of corresponding points. On top of that, the author discloses mathematical flaws of two well known derivations of the essential and fundamental matrix equations. Thus, clarify the reason behind the bad performance of the projective geometry application to 3D reconstruction from 2D views.

The rest of the paper is organized as follows: Section 2 introduces the motivation of addressing a classic problem like the fundamental matrix of stereo vision. Sections 3 exposes some related work. Section 4 demonstrates the shortcoming of the essential matrix equation. Section 5 shows the mathematical flaws of two derivations of the fundamental matrix. Finally, the paper concludes in section 6.

2. Why Should we Address Such A Classic Problem?

The epipolar geometry application in computer stereo vision represented by the fundamental matrix is still part of computer vision courses in most universities around the world. On top of that, researchers are still spending time to develop methods to estimating the fundamental matrix [3, 4, 5, 6, 7]. Table 1 shows a sample of outstanding universities with links to their computer vision courses that include at least a chapter on epipolar geometry and the fundamental matrix.

Table 1 Sample universities teaching the epipolar geometry to reconstruct 3D shape from two views.

University	Course Title	Course Link
Stanford University, USA	Computer Vision, From 3D Reconstruction to Recognition	web.stanford.edu/class/cs231a/syllabus.html
The University of	Computer Vision	courses.cs.washington.edu/courses/cse455/

Washington, USA		
MIT, USA	Computer Vision and Applications	www.ai.mit.edu/courses/6.891/lectnotes/lect8/lect8-slides.pdf
University College London, UK	Machine Vision	www.ucl.ac.uk/module-catalogue/modules/machine-vision/COMP0137
University of Toronto, Canada	Foundations of Computational Vision	www.cs.toronto.edu/~kyros/courses/2503
Tokyo Institute of Technology, Japan	Computer Vision	www.ocw.titech.ac.jp/index.php?module=General&action=T0300&JWC=201804591&lang=EN&vid=03
Sorbonne Université - Télécom Paris	Master Informatique - Parcours IMA	https//perso.telecom-paristech.fr/bloch/P6Image/VISION.html

3. RELATED WORK

Though the fundamental matrix theory is considered a landmark achievement in computer vision, certain researchers called it into question.

In [8], Zisserman, et al. showed that it is not possible to recover the epipolar geometry for several configurations. Three years later after introducing the essential matrix, Longuet-Higgins discovered configurations that defeat the eight-point algorithm [9]. The work of Hartley [10] is an attempt to present excuses for the bad performance of the eight-point algorithm. In [11], Luong,et al. discovered that the general methods to compute the fundamental matrix are unstable when the points lie near planes. Most of these reviews attribute the failure of the fundamental matrix theory to the performance of the eight-point algorithm.

Marill [12] went too far and gave an example that, as he claimed, should cause scientists to consider recovering the three-dimensional scene as a theory that is subject to empirical verification or falsification. He further argued that if it held up under further examination, the example would be evidence that the projective geometry to recover 3D shapes is false. In his unpublished work [13], Horn regarded the use of projective geometry as harmful and less accurate compared to perspective geometry. In [14], Basta considered the fundamental matrix equation $m_r^T F m_l = 0$ as an invalid mathematical expression. He stressed the fact that F is a 3×3 matrix of rank 2 and in $\mathbb{R}^3$ the rank of rotation transformation matrix is 3. We elaborate further on this reasoning in section 5.3. below. In [15], Basta argued that the derivation based on a line passing through two points is flawed because of mixing up operations in Cartesian space and homogeneous space. The author of [16] argued that 3D scenes typically include prominent parts that make some 3D points visible to one camera and invisible to then other. A fact that contradicts the existence of Homography between the two images of a scene as claimed in the geometrical derivation of the fundamental matrix of [1], we also elaborate on this in section 5.5.

In [17, 18], the author uses the estimation methods of the fundamental matrix of [19] to show that the eight-point algorithm performs poorly when the scene is composed of parts with different depths. In [20], Basta presented a mathematical analysis of the fundamental matrix equation based on the facts that the matrix F depends only on the rotation and translation of the second camera with respect to the first one ($F \triangleq [t]_\times R$), to assert that it is a matrix with constant coefficients. The vectors m_l and m_r are not orthogonal for every 3D point M. And because the dot product $m_r^T \cdot F m_l$ is equal to zero if and only if the two vectors m_r and $F m_l$ are orthogonal, the equation $m_r^T F m_l = 0$ is not always true.

In [18] and [21], the author presented extensive experimental results of two real images of a building captured from two standpoints. The building (Figure 2) is composed of two parts with different depths with respect to the camera lens. In [18], the author used a MATLAB Toolbox [19] that contains several methods for estimating the fundamental matrix using the eight-point algorithm. In [21], he implemented the solution in Python and used the findFundamentalMat() function of the cvonline package to estimate the fundamental matrix.

Figure 2. The building image used to estimate the fundamental matrix in [18] and [21].

In both works [18] and [21], the author estimated the fundamental matrix that satisfies the equation $m_r^T F m_l = 0$. Then, he calculated the values of the expression $m_r^T F m_l$ for several pairs of corresponding points (m_l, m_r). Such values are supposed to be equal to zero. The matrix F is calculated from different regions of the images (whole images, back part of the images, and front side of the images) and the pairs of corresponding points are selected arbitrarily from the images. Table 2 shows that the values of $m_r^T F m_l$ are sometimes very far away from 0: greater than 10 for some cases.

Table 2 the values of the expression $m_r^T F m_l$ calculated for selected points from the whole images, the back side, and the front side of the images. As it is apparent the image is composed of components with different depth with respect to the camera lens. This result is published in [21].

F matrix calculated from		
Whole	Back	Front
0.322	0.121	-0.504
0.084	1.496	0.557
-0.026	0.545	0.684
0.234	3.978	0.748
0.328	7.314	-0.726
0.135	16.158	-0.508
-0.165	9.001	-0.784
0.184	13.800	2.989
0.070	12.401	-0.109
0.135	10.794	-1.970

In the current work, three main publications where the essential and fundamental matrices are derived as a product of a skew matrix and a rotation transformation matrix are scrutinized. One of these is where the first time the essential matrix introduced to the computer vision community by Longuet-Higgins [22]. Next section shows how Longuet-Higgins succeeded in securing a one-to-one mapping between the position vectors of world points of a scene and that mapping is lost when he transformed it to a relation between the image points. In the other two derivations, the authors try to directly establish a one-to-one relation between the image points. Such a relation is represented by the fundamental matrix. The current work shows the mathematical flaws in these two derivations.

4. Longuet-Higgins' Derivation of the Essential Matrix

4.1. The Equation Derivation

In [22], Longuet-Higgins created a matrix $Q = RS$ where $S = \begin{bmatrix} 0 & t_3 & -t_2 \\ -t_3 & 0 & t_1 \\ t_2 & -t_1 & 0 \end{bmatrix}$. The matrix R and the vector t are the rotation and translation of the right coordinate system with respect to the left coordinate system. M_l and M_r are the position vectors of a world point M on the left and right coordinate systems, respectively. The author formed the expression $M_r^T\, QM_l$ and after some arithmetic manipulations he found out that

$$M_r^T Q M_l = 0 \tag{1}$$

For every 3D point there are exactly two position vectors; one represents that point in the left coordinate system and the other represents the point in the right coordinate system. Thus, Q in (1) is a one-to-one mapping between M_l and M_r.

In terms of coordinates, $M_l = (X_l, Y_l, Z_l)$ and $M_r = (X_r, Y_r, Z_r)$. And the coordinates of the projective points m_l and m_r of the point M in the left and right coordinate systems, respectively are

$$\begin{aligned} m_l &= (X_l/Z_l\,, Y_l/Z_l\,, 1) \\ m_r &= (X_r/Z_r\,, Y_r/Z_r\,, 1) \end{aligned} \tag{2}$$

Finally, the author divided the left-hand side of (1) by $Z_l Z_r$ to conclude the essential matrix equation

$$m_r^T E m_l = 0 \tag{3}$$

4.2. Shortcoming of Longuet-Higgins's derivation

Longuet-Higgins approached the problem from an algebraic perspective, he used matrix product as the main operation to derive the essential matrix equation. He formed the expression $M_r^T\, QM_l$. And because the matrix product is an associative operation, the expression $M_r^T\, QM_l$ is the product of 1×3 row matrix and a 3×3 matrix and a 3×1 column matrix which led to equation (1).

The problem of Longuet-Higgins' derivation started when he divided equation (1) by $\mathrm{Z}_l\mathrm{Z}_r$. As it is known, the position vector of a point is the unique vector from the origin of the coordinate system to the point itself. So, for every point M, equation (1) holds for exactly two position vectors M_l and M_r in the left and right coordinate systems, respectively. Dividing (1) by $\mathrm{Z}_l\mathrm{Z}_r$ results in the following equation

$$\frac{M_r^T}{Z_r} \cdot Q \cdot \frac{M_l}{Z_l} = 0 \tag{4}$$

Where $m_l = \frac{M_l}{Z_l}$ and $m_r = \frac{M_r^T}{Z_r}$ are the projection of the vectors M_l and M_r on the left and right camera planes, respectively.

In projective geometry, m_l could be the projection of a single world point or multiple world points (Figure 3). It is the projection of all world points laying on the ray drawn from the camera lens centre to the point M.

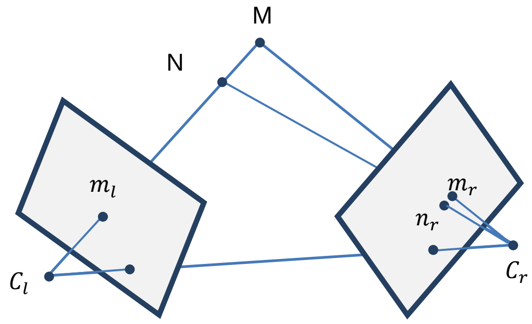

Figure 3. The image point m_l is the projection of two world points M and N. m_l is a corresponding point to two image points m_r and n_r.

Furthermore, there are world points visible to one camera and invisible to the other. This could be because these points are hidden by 3D objects in the scene. This is one of the characteristics of 3D scenes. So, these world points are projected on the first camera plane and does not have an image on the other camera. However, when you plug this image point into m_l or m_r and solve equation (3), you get a false corresponding point.

Recall the 3D shape reconstruction as described in [1] is accomplished through the following steps:

1. Compute the fundamental (essential) matrix from point correspondences.
2. Compute the camera matrices from the fundamental matrix.
3. For each point correspondence $m_l \leftrightarrow m_r$, compute the point in space that projects to these two image points.

Assuming the point $p = (1,2,1)$ is on the left camera plane (image). And the matrix E is already calculated or estimated. To compute the corresponding point of p, we plug the value of p into (3).

$$[x_r \quad y_r \quad 1]E[1 \quad 2 \quad 1]^T = 0, E = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \tag{5}$$

Substituting for the matrix E, we get the following equation

$$[x_r \quad y_r \quad 1]\begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} = 0 \text{ , where} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} = \begin{bmatrix} a_{11} + 2a_{12} + a_{13} \\ a_{21} + 2a_{22} + a_{23} \\ a_{31} + 2a_{32} + a_{33} \end{bmatrix} \tag{6}$$

which leads to the following equation

$$A_1 x_r + A_2 y_r + A_3 = 0 \tag{7}$$

Equation (7) is a linear equation in two variables, it has infinitely many solutions. There are infinite values of (x_r, y_r) satisfying equation (7). Geometrically, this means that any point p has many corresponding points. Which is incorrect; the certainty is each image point has at most one corresponding point in each other image except the case of occlusion when two different points have the same corresponding point.

Consequently, the essential (fundamental) matrix equation does not ensure the recovery of the right shapes of 3D scenes.

4. ESTABLISHING A DIRECT MAPPING BETWEEN THE IMAGE POINTS

Because the above essential matrix derivation suffers from the drawback of an image point can have unlimited number of corresponding points, computer vision researchers try to directly draw a mapping between the image points without passing through position vectors of the 3D point. The next sections explore the flaws of two well-known derivations of the essential and fundamental matrices equations.

4.1. Vectors transformation and operations

Is it correct to perform a dot product or cross product of two vectors defined in two different coordinate systems? Let us explore the case through the example depicted in Figure 4 below. The vectors u and v are defined in the coordinate system (X, Y, Z), and the vector w is defined in the coordinate system (X', Y', Z'). Let $u = [3,3,0]^T$ and $v = [2,2,3]^T$ in (X, Y, Z), and $w = [2,2,3]^T$ in (X', Y', Z').

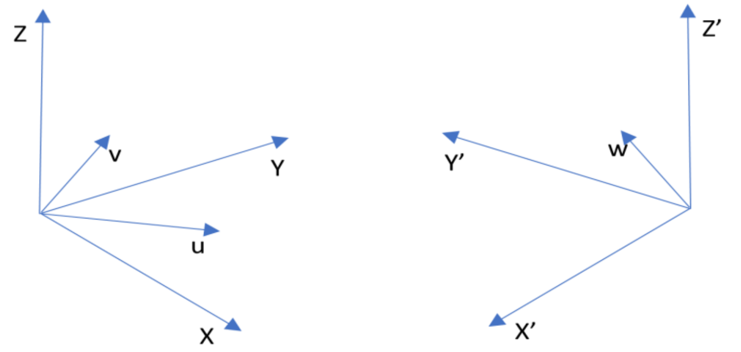

Figure 4. The coordinate system (X´, Y´, Z´) is obtained by translating (X, Y, Z) to the right and rotating it around the Z axis counterclockwise by an angle of 180°.

We have $u.w = [3,3,0].\begin{bmatrix}2\\2\\3\end{bmatrix}$, and at the same time, we have $u.v = [3,3,0].\begin{bmatrix}2\\2\\3\end{bmatrix}$, and of course $v \neq w$. A vector is a quantity with both magnitude and direction [23]. The vectors v and w have the same magnitude but their directions are different.

The vector $v = 2i + 2j + 3k$ and $w = 2i' + 2j' + 3k'$, where (i, j, k) are the unit vectors of (X, Y, Z) and (i', j', k') are the unit vectors of (X', Y', Z').

The dot product of unit vector by itself is equal to 1, and the dot product of two different unit vectors is equal to 0 [24].

$$u.v = 3i * 2i + 3j * 2j + 0k * 3k = 6 + 6 + 0 = 12$$
$$u.w = 3i * 2i' + 3j * 2j' + 0k * 3k' = ?$$

Thus, the operation $u.w$ is invalid unless the two vectors u and w are transformed to the same coordinate system.

The cross product of unit vectors of any two of the unit vectors i, j, k is equal to positive or negative of the remaining third unit vector [25].

In the contrary to unit vectors of a given coordinate system, no rules are available to calculate the cross product of unit vectors of two different coordinate systems.

4.2. Luong-Faugeras derivation of the essential matrix

In [26], Luong et al. assert that because the vector from the first camera optical centre to the first imaged point m_l, the vector from the second optical centre to the second imaged point m_r, and the vector from one optical center to the other t are all coplanar. In normalized coordinates, this constraint can be expressed simply as

$$m_r^T\ (t \times Rm_l) = 0 \tag{8}$$

where R and t capture the rotation and translation of the right cameras coordinate system with respect to the left one. In [27], Birchfield explicitly stated that the multiplication by R is necessary to transform m_l into the second camera's coordinate system. The authors [26] defined $[t]_\times$ as the matrix such that $[t]_\times\, y = t \times y$ for any vector y, and they rewrite equation (8) as a linear equation

$$m_r^T\ ([t]_\times Rm_l\ ) = m_r^T E m_l = 0, \tag{9}$$

Where $E = [t]_\times R$ is called the Essential matrix and $[t]_\times = \begin{bmatrix} 0 & -t_3 & t_2 \\ t_3 & 0 & -t_1 \\ -t_2 & t_1 & 0 \end{bmatrix}$.

4.3. The flaw in Luong-Faugeras derivation

Let us examine equation (8), $m_r^T\ (t \times Rm_l) = 0$.

We have the following facts. The point m_l is on the left image, so the position vector m_l is defined in the left coordinate system and not defined in the right one. The point m_r is on the right image, then the vector m_r is defined in the right coordinate system and not defined in the left one. And the vector t, the translation of the origin of the right coordinate system with respect to the left coordinate system; so, t is defined in the left coordinate system and not defined in the right one.

The left hand-side of (8) consists of three vector operations. The term inside the parenthesis is evaluated first which includes a vector product and a matrix product.

Let assume that Rm_l is to be evaluated first; it is the product of a rotation transformation matrix and a vector defined in the left coordinate system. So, $v = Rm_l$ is the vector m_l expressed in the right coordinate system. Therefore $t \times Rm_l = t \times v$ is the cross product of t defined in the left coordinate system and v defined in the right coordinate system. Thus, $t \times Rm_l$ is the cross product of two vectors not defined in the same coordinate systems; so, it is invalid.

Now, let us consider that the cross-product operation $t \times R$ is to be evaluated first.

DEFINITION

The cross product (or vector product) of two vectors

$x = \langle x_1, x_2, x_3 \rangle$ and $y = \langle y_1, y_2, y_3 \rangle$ in $\mathbb{R}^3$ is the vector $x \times y = \langle x_2y_3 - x_3y_2 , x_3y_1 - x_1y_3 , x_1y_2 - x_2y_1 \rangle$.

The cross product of two vectors x and y in $\mathbb{R}^3$ is a vector orthogonal to both x and y [23].

The cross product of a 3D vector and a 3×3 matrix is undefined [23].

Therefore, there is no operation called cross product of a vector and a matrix; therefore, the term $t \times R$ is undefined. Thus, equation (8) that is the premise of the current derivation of the essential matrix is invalid. And the current derivation of the essential matrix is flawed.

Algebraically, researchers consider $[t]_\times = \begin{bmatrix} 0 & -t_3 & t_2 \\ t_3 & 0 & -t_1 \\ -t_2 & t_1 & 0 \end{bmatrix}$ as the matrix such that $[t]_\times y = t \times y$ for any vector y to conclude $E = [t]_\times R$, the Essential matrix.

1. The first flaw here is the term $[t]_\times y$ is defined for a vector y and not for a 3×3 matrix R.
2. The second, regardless of whether the term $E = t \times R$ is undefined. Let us assume that it is evaluated first. Substitute for E in the expression $m_r^T (t \times Rm_l)$, we obtain$m_r^T Em_l$, where E is 3×3 matrix of rank 2. E cannot be a transformation matrix in $\mathbb{R}^3$. Thus, Em_l is a vector in the left coordinate system as is m_l. The vector m_r is defined in the right coordinate system. Therefore, the dot product $m_r^T \cdot Em_l$ is an undefined operation.

One could claim that Rm_l is a product of a matrix and a vector defined in the left coordinate system, which produces a vector defined in the same coordinate system. Then the cross-product$t \times Rm_l$ is a vector defined in the left coordinate system as well. In this case, $m_r^T \cdot (t \times Rm_l)$ is a dot product of two vectors, m_r from the right coordinate system and $t \times Rm_l$ from the left coordinate system. As demonstrated above, it is an undefined operation again.

4.4. Hartley-Zisserman derivation of the fundamental matrix

In the geometric derivation of the fundamental matrix equation, the authors [1] assert the existence of 2D homography H_π mapping each point m_l from the left image to a point m_r on the right image, because the set of all such points m_l in the left image and the corresponding points m_r in the right image are projectively equivalent, since they are each projectively equivalent to the planar point set M (Figure 1). Thus, $F = [\mathrm{e}_r]_\times H_\pi$ that is a matrix product of a skew matrix and a transformation from left to right.

4.5. The flaw in Hartley-Zisserman derivation

The points M in the above statement are the world points of the 3D scene to be reconstructed from a pair of its images. If the 3D scene is planar, why are we constructing a planar scene from two of its planar images in the first place. Thus, the existence of a homography mapping points of the left image to points on the right image is on condition that the 3D scene is planar. And because typical 3D scenes might contain objects with different depths (i.e., distance from the camera centre), some points on these objects can be visible to one camera and hidden from the other. Therefore, some image points on the left camera plane will not have corresponding points on the right camera plane and points on the right image will not have corresponding points on the left image. Furthermore, researchers recognize the existence of occlusion problem [28] where two 3D points or more are projected onto the same image point on one view as in Figure 3. At the same time, they assert the existence of a homography between points of the left image and those on the right image. These facts, confirm that points on the left and right images are not projectively equivalent and no homography exists between them. In conclusion, the expression $F = [\mathrm{e}_r]_\times H_\pi$ where $H_\pi$is a homography is irrational.

5. CONCLUSION

In this work, we demonstrated that the first ever derivation of the essential matrix that has been introduced to the computer vision community is free of flaws; however, it does not ensure a one-to-one mapping between the corresponding points of the two views. Later, researchers tried to address such shortcoming through deriving the essential and fundamental matrices equation as a direct mapping between the image points. We showed that two of the well-known of these derivations are mathematically flawed. The main discovered flaw consists of performing dot and cross multiplications on vectors not defined in the same coordinate system.

The current work establishes a rigorous scrutiny of a theory that claims to be mathematically founded to conclude that such a theory that is still taught in universities around the world is flawed. The trend for solving computer vision problems shifted from mathematically based theory to machine learning tools to obtain good solutions.

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