

USING CUCKOO ALGORITHM FOR ESTIMATING TWO GLSD PARAMETERS AND COMPARING IT WITH OTHER ALGORITHMS

Jane Jaleel Stephan, Haitham Sabah Hasan and Alaa Hamza Omran

University of Information Technology & Communications, Iraq

ABSTRACT

This study introduces and compares different methods for estimating the two parameters of generalized logarithmic series distribution. These methods are the cuckoo search optimization, maximum likelihood estimation, and method of moments algorithms. All the required derivations and basic steps of each algorithm are explained. The applications for these algorithms are implemented through simulations using different sample sizes ($n = 15, 25, 50, 100$). Results are compared using the statistical measure mean square error.

KEYWORDS

Cuckoo search optimization (CSO) algorithm, maximum likelihood estimation (MLE) algorithm, method of moments (MOM) algorithm, mean square error (MSE).

1. INTRODUCTION

The process of modifying a system to present several new features that cooperate in enhancing the system and work more efficiently is known as optimization process. Also the Optimization process can be defined as the process of finding alternative solution to increase the performance of the system under specific constraints such as increasing the desired parameters and decrease the undesired parameters in the system which has a problem to solve it [1]. The increasing means trying to get additional good results without additional cost such as the optimization which occurs on computer or any android phone will results in increasing the speed of processing which makes them run faster with less memory requirements. There are many algorithms in solving optimization problems such as cuckoo search algorithm which introduced for the first time by Yang and Deb [2]. Many researchers work on testing this algorithm on some benchmark functions and compare the results with other algorithms like PSO, GA; the obtained results show that the cuckoo algorithm is better than the others. One of the popular meta heuristic, combinatorial search optimization techniques is ACO (Ant Colony Optimization) which is developed from natural ant behavior ACO was used along with Rough Sets and Fuzzy Rough Sets in feature selection in [3], [4], [5] also it is used for optimizing of firewall rules in [6].

Today the Cuckoo search algorithm became as the one of the most optimization algorithm which used in every domain like scheduling planning, forecasting, image processing, feature selection and engineering optimization [7]. This paper presents a Comparing of the Cuckoo Algorithm with Other Algorithms for Estimating Two GLSD Parameters. Some important functions are defined as follows:

The discrete random variable (x) exhibits the generalized logarithmic series distribution (GLSD) with two parameters (α and β), where (α) is a scale parameter and (β) is a shape parameter. Let

$$\theta = -\frac{1}{\log(1-\alpha)}$$

Where Θ is a function from α . The positive matrix factorization (p.m.f.) of GLSD is defined by Eq. (1) as follows:

$$p(X = x) = \frac{\theta \Gamma(\beta x) \alpha^x (1-\alpha)^{\beta x - x}}{x! \Gamma(\beta x - x + 1)}, \quad (1)$$

Where $x = 1, 2, \dots, \infty$, $\beta \geq 1$, $0 \leq \alpha \leq 1/\beta$, $0 < \theta < 1$.

The distribution in Eq. (1) depends on the zero-truncated generalized negative binomial defined by Eq. (2):

$$pr(X = x, \alpha, \beta, k) = \frac{k}{k+\beta x} \frac{\binom{k+x}{x} \alpha^x (1-\alpha)^{k+\beta x - x}}{1-(1-\alpha)^k}, \quad (2)$$

Where $x = 1, 2, \dots, \infty$, $k > 0$, $1 \leq \beta \leq \frac{1}{\alpha}$.

When limit ($k \rightarrow 0$) is considered for Eq. (2), we obtain the studied distribution in Eq. (1).

The mean of GLSD is defined in Eq. (1), and the variance obtained from the general formula of the (k^{th}) moments about the mean is as follows:

$$\begin{aligned} \mu_k &= E(x - \mu)^k, & \theta &= [\ln(1 - \alpha)]^{-1}, \\ \mu_1 &= E(x) = \frac{\alpha\theta}{(1-\alpha\beta)}, \end{aligned} \quad (3)$$

$$\sigma^2 = \mu_2 - \mu_1^2 = \frac{(1-\alpha)[2\alpha\beta - \alpha - \alpha^2\beta]}{(1-\alpha\beta)^2}. \quad (4)$$

2. ESTIMATING PARAMETERS

We apply different methods for the *p.m.f.* parameters in Eq. (1).

2.1 Maximum Likelihood Estimation (MLE)

The Maximum Likelihood Estimation (MLE) [8], [9] that corresponds to Eq. (1) is given by:

$$\begin{aligned} L(x; \alpha, \beta) &= \prod_{i=1}^n p(X_i = x_i) \\ \log L &= \\ n \log \theta &+ n\bar{x} \log \alpha + \sum_{x=1}^k \sum_{j=1}^{x-1} f(x) \log(\beta x - j) + n\bar{x} (\beta - 1) \log(1 - \alpha) - \sum_{i=1}^n f(x_i) \log x_i! \end{aligned} \quad (5)$$

$$\frac{\partial \log L}{\partial \alpha} = \frac{n\bar{x}}{\alpha} + \frac{n\bar{x}(\beta-1)}{(1-\alpha)} - \frac{n\theta}{(1-\alpha)} = 0,$$

$$\text{And } \frac{\partial \log L}{\partial \beta} \Rightarrow -\frac{n\bar{x}}{\beta} + \sum_{x=1}^k \sum_{j=1}^{x-1} \left[\frac{x f(x)}{(\beta x - j)} \right].$$

This equation derives:

$$\hat{\beta}_{MLE} = \frac{1}{\hat{\alpha}} - \frac{\theta}{\bar{x}}. \quad (6)$$

From $\frac{\partial \log L}{\partial \alpha} = 0$, we obtain $\hat{\alpha}_{MLE}$.

2.2 Method of Moments (MOM) Estimator for GLSD Parameters

Method of Moments (MOM) Estimator for GLSD Parameters [10], these estimators $(\hat{\alpha}_{mom}, \hat{\beta}_{mom})$ are obtained by solving the following:

$$\begin{aligned} \mu'_r &= m'_r, \\ m'_r &= \sum_{x=1}^{\infty} x^r p(X=x) \end{aligned}$$

When $r = 1$,

$$\begin{aligned} m'_1 &= \sum_{x=1}^{\infty} \frac{x\theta\Gamma(\beta x)\alpha^x(1-\alpha)^{\beta x-x}}{x!\Gamma(\beta x-x+1)}, \\ m'_1 &= \bar{x} = \frac{1-\alpha}{(1-\alpha\beta)^2}. \end{aligned} \tag{7}$$

When $r = 2$,

$$\begin{aligned} m'_2 &= \sum_{x=1}^{\infty} \frac{x^2\theta\Gamma(\beta x)\alpha^x(1-\alpha)^{\beta x-x}}{x!\Gamma(\beta x-x+1)}, \\ m'_2 &= \theta(1-\alpha\beta)^{-3}\alpha(1-\alpha). \end{aligned} \tag{8}$$

Given that:

$$\frac{1}{\bar{x}^2} = \frac{(1-\alpha)^2}{(1-\alpha\beta)^4}$$

and

$$\sigma^2 = E(x^2) - [E(x)]^2;$$

then,

$$S^2 = m'_2 - (m'_1)^2 = \frac{(1-\alpha)(2\alpha\beta - \alpha - \alpha^2\beta)}{(1-\alpha\beta)^4}.$$

We have

$$m'_1 = \theta(1-\alpha\beta)^{-1}\alpha, \tag{9}$$

$$m'_2 = \theta(1-\alpha\beta)^{-3}\alpha(1-\alpha), \tag{10}$$

and

$$S^2 = m'_2 - (m'_1)^2,$$

which is simplified as follows:

$$S^2 = \frac{(1-\alpha)(2\alpha\beta - \alpha - \alpha^2\beta)}{(1-\alpha\beta)^4}.$$

We also obtain the following:

$$\frac{1}{\bar{x}^2} = \frac{(1-\alpha)^2}{(1-\alpha\beta)^4}, \tag{11}$$

Then,

$$\frac{S^2}{\bar{x}^2} = \frac{(2\alpha\beta - \alpha - \alpha^2\beta)}{(1-\alpha)}. \tag{12}$$

We derive the first three non-central moments obtained from $[m'_r = E(x^r)]$.
Then,

$$\begin{aligned} m'_1 &= \theta(1 - \alpha\beta)^{-1} \alpha, \\ m'_2 &= \theta(1 - \alpha\beta)^{-2} \alpha(1 - \alpha), \\ m'_3 &= \frac{\theta\alpha(1-\alpha)(1-2\alpha+2\alpha\beta-\alpha^2\beta)}{(1-\alpha\beta)^3}. \end{aligned} \tag{13}$$

Given that:

$$m'_r = \frac{\sum x_i^r}{n},$$

We obtain the following based on the preceding relation:

$$\begin{aligned} \frac{\sum x_i^3}{\sum x_i^2} &= \frac{[(1 - \alpha) + \frac{s^2}{\bar{x}^2} (1 - \alpha)]}{(1 - \alpha\beta)^2}, \\ \frac{\sum x_i^3}{\sum x_i^2} &= \frac{(1-\alpha)\left(1+\frac{s^2}{\bar{x}^2}\right)}{(1-\alpha\beta)^2}. \end{aligned} \tag{14}$$

Eq. (14) can be simplified into

$$\frac{\bar{x}^2 \sum x_i^3}{\sum x_i^2} \left(1 + \frac{s^2}{\bar{x}^2}\right) = \frac{\alpha^2 \theta^2}{(1-\alpha)}. \tag{15}$$

Given that

$$\theta = - \frac{1}{\log(1-\alpha)},$$

Eq. (15) can be written as follows:

$$\frac{\bar{x}^2 \sum x_i^3}{\sum x_i^2} \left(1 + \frac{s^2}{\bar{x}^2}\right) = \frac{\alpha^2 \log(1-\alpha)^{-2}}{(1-\alpha)}, \tag{16}$$

which is an implicit function that can be solved numerically to determine $(\hat{\alpha}_{mom})$ based on observation. We then obtain $(\hat{\beta}_{mom})$ by using $(\hat{\alpha}_{mom})$ and solving Eq. (11).

2.3 Cuckoo Search Optimization (CSO) Algorithm

This algorithm is based on the breeding behaviour of the cuckoo bird. It has three basic optimization rules [11], [12].

1. Each bird lays one egg at a time, and each egg is placed randomly in a selected nest.
2. The best nest with the highest fitness value will be carried over to the next generation.
3. The number of available host nests is fixed. $(p_a \in [0,1])$ represents the probability of the host bird discovering egg/s in its nest. The host bird can either throw away the egg/s or abandon the nest to build a new one [13][14].

The latter scenario can be regarded as the new best solution.

Let:

$X_i^{(t)}$ Be the nest where the cuckoo bird initially lives, and
 $X_i^{(t+1)}$ Be the new nest with the highest fitness value.

When random walk is used, the performance of the cuckoo (i) that applies levy flight is expressed as [5, 7]:

$$X_i^{(t+1)} = X_i^{(t)} + S \text{levy } \lambda$$

Levy flight was first introduced by the French mathematician Paul Pierre.

$$S \sim \text{Normal} (\mu_{10}, \sigma^2 = 1)$$

The probability ($p_{\alpha} \in [0,1]$) indicates that the egg/s in the nest may be from another bird, and thus, the cuckoo bird may leave this host nest and build another one. The n hosts may be changed to new hosts with random positions (probability p_{α} of change). Thus, the objective function belongs to the maximization type and the objective must be fitted into this type. The most important algorithm that can be applied is one used to solve nonlinear equation problems or one used in neural networks because these objects allow the algorithm to be transformed from state to state to reach the optimal solution. Given that GLSD has two parameters (θ and β), then the algorithm implements the following steps.

Each bird lays one egg at a time in a randomly selected nest. The number of selected nests is equal to the number of parameters to be estimated. The number of nests is determined from the following equation:

$$\text{Number of nests} = LB + (UB - LB) \times \text{random number} (0, 1).$$

Let $X_i^{(t)}$ be the nest where the cuckoo bird initially lives.

$X_i^{(t+1)}$ is the new nest with the highest fitness value.

$$X_i^{(t+1)} = X_i^{(t)} + S t^{-\lambda}$$

Each nest contains the parameters to be estimated, and the number of nests is also determined based on these parameters.

Step (1):

$$\text{Number of nests} = LB + (UB - LB) \times \text{random number} (0, 1)$$

Step (2):

The objective function for each nest is calculated as follows:

$$O = \sum_{i=1}^m \left[F(x_{(i)}, \hat{\theta}, \hat{\beta}) - \frac{n_{x_{(i)}}}{n} \right]$$

Step (3):

The best values of the parameters determine the best nest with respect to the eggs.

Step (4):

The repetition begins. Let

$X_i^{(t+1)}$ be the nest in which the cuckoo bird initially lives, and

$X_i^{(t+1)} = X_i^{(t)} + S t^{-\lambda}$ be the new nest with the highest fitness value.

Step (5):

A new nest is generated for the cuckoo from k , as follows:

$$k = \left[\Gamma(1 + \beta) \frac{\sin\left(\pi \frac{\beta}{2}\right)}{\Gamma\left(\frac{1+\beta}{2}\right) \Gamma\left(\frac{\beta-1}{2}\right)} \right]^{1/\beta},$$

$U = rand(1,2)k$,

$L = rand(1,2)$,

$step = \frac{U}{|L|^{1/\beta}}$

$step\ size = (0.01)step (nest - best)$,

$new\ nest = n\ set + step\ size \times rand(1,2)$.

Step (6):

The objective function for each new nest is computed.

Step (7):

The solution is continued until the stopping rule ends with the total frequency. The best solution determined is then printed.

The CSO algorithm, which represents a meta-heuristic algorithm, is adopted to estimate $(\hat{\theta}, \hat{\beta})$. Then, $(\hat{\theta})$ provides the estimate of (α) . More details on this algorithm are explained in detail in [15].

3. SIMULATION

The three estimators of $(\alpha$ and $\beta)$, i.e., the CSO, MLE, and MOM algorithms, are compared through MATLAB: A11 program. Different sample sizes ($n = 15, 25, 50, 100$) are considered, and the results are compared using the statistical measure mean square error (MSE) and run of each experiment ($R = 1000$).

TABLE 1: Comparison of the Different Estimators When $\beta = 1.5$ and $\alpha = 0.3$

n	Method	$\beta = 1.5$	$\alpha = 0.3$	Kurtosis	Skewness
15	mle	1.0512	0.3056	1	0
	mse_mle	0.6889	0.0019		
	mom	1.0961	0.7347		
	mse_mom	0.5226	0.1897		
	cuckoo	1.3536	0.3010		
	mse_cuckoo	0.0313	0.0014		
	best	cuckoo	cuckoo		
25	mle	1.3894	0.2991	1.7500	0
	mse_mle	0.0271	0.0018		
	mom	1.0121	0.5908		
	mse_mom	0.2558	0.0869		
	cuckoo	1.3689	0.3292		
	mse_cuckoo	0.0364	8.6762e-004		
	best	cuckoo	cuckoo		
50	mle	1.4254	0.3277	1.7000	0
	mse_mle	0.0479	0.0048226		
	mom	1.2055	0.7163		
	mse_mom	0.0937	0.1765		
	cuckoo	1.3983	0.2991		
	mse_cuckoo	0.0357	8.2262e-004		
	best	mle	mle		
100	mle	1.4910	0.3034	1.7877	0
	mse_mle	0.0220	7.4992e-004		
	mom	1.2194	0.6353		
	mse_mom	0.0788	0.1139		
	cuckoo	1.4032	0.2994		
	mse_cuckoo	0.0343	0.0022		
	best	mle	mle		

TABLE 2: Comparison of the Different Estimators When $\beta = 2$ and $\alpha = 0.2$

n	Method	$\beta = 2$	$\alpha = 0.2$	Kurtosis	Skewness
15	mle	1.8177	0.2233	1	0
	mse_mle	0.0673	0.0017		
	mom	1.0430	0.6648		
	mse_mom	0.9581	0.2182		
	cuckoo	1.9709	0.2107		
	mse_cuckoo	0.0422	0.0015		
	best	cuckoo	cuckoo		
25	mle	1.9701	0.2077	1.7314	0
	mse_mle	0.0407	8.8789e-004		
	mom	1.1209	0.7815		
	mse_mom	0.7887	0.3383		
	cuckoo	1.9801	0.2193		
	mse_cuckoo	0.0375	0.0016		
	best	cuckoo	cuckoo		
50	mle	1.9914	0.2036	1.7982	0
	mse_mle	9.1436e-004	5.7535e-004		
	mom	1.2229	0.7853		
	mse_mom	0.6041	0.3470		
	cuckoo	1.8305	0.2102		
	mse_cuckoo	0.0425	0.0014		
	best	mle	mle		
100	mle	2.0130	0.2198	1.7997	0
	mse_mle	7.8137e-004	4.2334e-004		
	mom	1.2277	0.6513		
	mse_mom	0.6014	0.2035		
	cuckoo	1.9860	0.1971		
	mse_cuckoo	0.0161	0.0012		
	best	mle	mle		

TABLE 3: Comparison of the Different Estimators When $\beta = 2.2$ and $\alpha = 0.4$

n	Method	$\beta = 2.2$	$\alpha = 0.4$	Kurtosis	Skewness
15	mle	1.9464	0.6294	1.7832	0
	mse_mle	0.1197	0.1129		
	mom	1.2001	0.7940		
	mse_mom	0.9998	0.1553		
	cuckoo	2.4615	0.3879		
	mse_cuckoo	0.8312	0.0031		
	best	cuckoo	cuckoo		
25	mle	1.9705	0.3929	1.7955	0
	mse_mle	0.0982	0.0030		
	mom	1.2136	0.8431		
	mse_mom	0.9731	0.1968		
	cuckoo	2.1559	0.3919		
	mse_cuckoo	0.0480	0.0029		
	best	cuckoo	cuckoo		
50	mle	2.0000	0.4325	1.7990	0
	mse_mle	0.0804	0.0022		
	mom	1.2003	0.8801		
	mse_mom	0.9432	0.2326		
	cuckoo	2.3042	0.3823		
	mse_cuckoo	0.0110	0.0050		
	best	Cuckoo	mle		
100	mle	2.1386	0.4386	1.7998	0
	mse_mle	0.0063	0.0021		
	mom	1.5161	0.7435		
	mse_mom	0.4677	0.1184		
	Cuckoo	2.0096	0.4441		
	mse_Cuckoo	0.0071	0.0022		
	best	mle	mle		

TABLE 4: Comparison of the Different Estimators When $\beta = 3$ and $\alpha = 0.33$

n	method	$\beta = 3$	$\alpha = 0.33$	Kurtosis	Skewness
15	mle	3.6985	0.8433	1.7997	0
	mse mle	0.6205	0.2643		
	mom	2.1156	0.8854		
	mse mom	0.7822	0.3097		
	cuckoo	2.6793	0.3326		
	mse cuckoo	0.1882	0.0022		
	best	cuckoo	cuckoo		
25	mle	3.5721	0.6693	1.7990	0
	mse mle	0.3273	0.1835		
	mom	2.1598	0.8443		
	mse mom	0.7059	0.2646		
	cuckoo	2.7307	0.3326		
	mse cuckoo	0.1482	0.0022		
	best	cuckoo	cuckoo		
50	mle	3.1131	0.3312	1.7945	0
	mse mle	0.0128	1.4400e-006		
	mom	2.3598	0.6443		
	mse mom	0.4099	0.0988		
	cuckoo	2.8307	0.3316		
	mse cuckoo	0.0287	2.5600e-006		
	best	mle	mle		
100	mle	3.0030	0.3310	1.7800	0
	mse mle	9.0000e-006	1.0000e-006		
	mom	2.5598	0.4443		
	mse mom	0.1938	0.0131		
	cuckoo	2.9307	0.3313		
	mse cuckoo	0.0048	1.6900e-006		
	best	mle	mle		

TABLE 5: Comparison of the Different Estimators When $\beta = 1.8$ and $\alpha = 0.5$

n	Method	$\beta = 1.8$	$\alpha = 0.5$	Kurtosis	Skewness
15	mle	2.4976	0.8870	1.7998	0
	mse mle	0.4867	0.1498		
	mom	1.1168	0.8875		
	mse mom	0.4668	0.1502		
	cuckoo	1.6294	0.4769		
	mse cuckoo	0.0626	0.0046		
	best	cuckoo	cuckoo		
25	mle	2.2176	0.8095	1.7990	0
	mse mle	0.1744	0.0959		
	mom	1.1642	0.8400		
	mse mom	0.4045	0.1159		
	cuckoo	1.7142	0.4684		
	mse cuckoo	0.0074	0.0052		
	best	cuckoo	cuckoo		
50	mle	1.8522	0.6278	1.7958	0
	mse mle	0.0027	0.0311		
	mom	1.2056	0.7846		
	mse mom	0.3534	0.0815		
	cuckoo	1.7278	0.4649		
	mse cuckoo	0.0052	0.0057		
	best	mle	cuckoo		
100	mle	1.8022	0.5349	1.7832	0
	mse mle	4.8400e-006	0.0014		
	mom	1.2154	0.7430		
	mse mom	0.3418	0.0595		
	cuckoo	1.7663	0.4899		
	mse cuckoo	0.0011	0.0039		
	best	mle	mle		

3. CONCLUSION

After estimating (α and β) using the three different methods (i.e., MOM, CSO, and MLE) with different sample sizes ($n = 15, 25, 50, 100$), we determined that the best estimator for small sample sizes ($n = 15, 25$) based on MSE was the CSO estimator, as shown in Tables 1 to 5. By contrast, MLE was the best estimator for large sample sizes ($n = 50, 100$). However, we conclude that the CSO estimator is the best type for small sample sizes ($n = 15, 25$) because the CSO algorithm depends on the number of eggs in the host nest, which is limited.

REFERENCES

- [1] Azizah Binti Mohamad, Azlan Mohd Zain & Nor Erne Nazira Bazin, (2014), "Cuckoo Search Algorithm for Optimization Problems—A Literature Review and its Applications", Applied Artificial Intelligence An International Journal Volume 28, Issue 5.
- [2] Xin She Yang and Sush Deb, "Nature & Biologically Inspired Computing," in IEEE, University of Cambridge, Trumpinton Street, CB2 1PZ, UK, 2010.
- [3] Ravi Kiran Varma P, Valli Kumari V, and Srinivas Kumar S, "A novel intelligent attribute reduction technique based on Ant Colony Optimization," International Journal of Intelligent Systems Technologies and Applications, vol. 1, no. 1, pp. 23-45, 2015.
- [4] Ravi Kiran Varma P, Valli Kumari V, and Srinivas Kumar S, "Feature selection using relative fuzzy entropy and ant colony optimization applied to real-time intrusion detection system," Procedia Computer Science, vol. 85, no. 2016, pp. 503-510, 2016.

- [5] Ravi Kiran Varma P, Valli Kumari V, and Srinivas Kumar S, "Application of Rough Sets and Ant Colony Optimization in feature selection for Network Intrusion Detection," *International Journal of Applied Engineering Research*, vol. 10, no. 22, pp. 43156-43163, 2015.
- [6] Ravi Kiran Varma P, Valli Kumari V, and Srinivas Kumar S, "Ant Colony Optimization Based Anomaly Mitigation Engine," *Springerplus*, vol. 5, no. 1, pp. 1-32, 2016.
- [7] Xin-She Yang and Suash, ""Engineering optimisation by cuckoo search"," *International Journal of Mathematical Modelling and Numerical Optimisation*, vol. 1, no. 4, pp. 330-343, 2010.
- [8] D. S. Bunch, "Maximum Likelihood Estimation (MLE) of probabilistic choice models", *SIAM Journal on Scientific and Statistical Computing*, 8(1):56-70.
- [9] M.S.Prasad Babu et al, (2012), "Development of Maize Expert System using Ada-Boost Algorithm and Navie Bayesian Classifier", *International journal of computer Applications technology and research*, volume 1-issue 3, 89-93.
- [10] Persi D., "Application of the Method of Moments in Probability and Statistics", *Auspices national science foundation grant DMS86-00235*, Nov. 1986.
- [11] Xin She Yang and Suash, "A brief literature review: Cuckoo Search and Firefly Algorithm," *Studies in Computational Intelligence*, vol. 516, pp. 49-62, 2014.
- [12] Hongqing Zheng and Yongquan Zhou,(2013), "A Cooperative Coevolutionary Cuckoo Search Algorithm for Optimization Problem", *Journal of Applied Mathematics, J. Appl. Math. Volume 2013, Special Issue (2013)*.
- [13] Najla Akram AL-Saati, Marwa Abd-ALKareem, (2013), "The Use of Cuckoo Search in Estimating the Parameters of Software Reliability Growth Models", *International Journal of Computer Science and Information Security*, Vol. 11, No. 6.
- [14] Manjeet Kumar, Tarun Kumar Rawat,(2015), "Optimal design of FIR fractional order differentiator using cuckoo search algorithm", *Expert Systems with Applications*, volume 42, Issue 7, Pages 3433–3449.
- [15] Prasad Babu, B.Jyothsna, (2015), "Implementation of Cuckoo Search Optimization Algorithm using Semantic Web – Based Coconut Expert System", *International Journal of Advanced research in Computer Science and Software Engineering*, Vol.5, Issue 9.