

FUZZY ROUGH INFORMATION MEASURES AND THEIR APPLICATIONS

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ABSTRACT

The degree of roughness characterizes the uncertainty contained in a rough set. The rough entropy was defined to measure the roughness of a rough set. Though, it was effective and useful, but not accurate enough. Some authors use information measure in place of entropy for better understanding which measures the amount of uncertainty contained in fuzzy rough set. In this paper three new fuzzy rough information measures are proposed and their validity is verified. The application of these proposed information measures in decision making problems is studied and also compared with other existing information measures.

KEYWORDS AND PHRASES

Fuzzy Rough Set, Similarity Information Measure, Logarithmic Information Measure, Weighted Information Measure and Decision Making Problem

1. INTRODUCTION

The extension of crisp set theory to fuzzy set and rough set theories was developed by Zadeh [26] and Pawlak [16] respectively. People use to compare rough set with that of fuzzy set, but both the notions in aims and objectives are different. However, there is no sense to compare which one is better or more useful than other. Rough set theory has its own importance in artificial intelligence and in cognitive sciences, particularly in the areas of pattern recognition, machine learning, inductive reasoning, knowledge acquisition, etc.

The concept of rough set theory sometimes overlaps with Dempster-Shafer theory [23] of evidence. But the main difference between these is that the main tool in Dempster-Shafer theory is a belief function, where in rough set theory, lower and upper approximations sets are used. No preliminary and additional information is needed in rough set theory like membership grade in fuzzy set theory, probability distribution in statistics and basic probability assignment in Dempster-Shafer theory.

A rough set deals with incomplete information where a fuzzy set deals with vagueness, so it is interesting to know how to handle the real life problems having both incomplete information as well as vagueness in data. Thus, to handle such kind of situation Nakamura [13] and Dubois and Prade [4] introduced the concept of fuzzy rough set, which was called as a hybrid model of fuzzy and rough sets. Thereafter, Nanda and Majumdar [14] widely used this concept in the development of their research. Banerjee and Pal [1] studied the roughness of fuzzy set in 1996 and Verbiest [25] worked on fuzzy rough and evolutionary approaches to instance selection.

Jabar and Rashid [9] combined fuzzy rough set with salient features for human resource management classification.

Some trigonometric information measures for fuzzy rough set and their applications in medical diagnosis were studied by Sharma and Gupta [21]. Similarity and distance information measures on fuzzy rough set with their applications were described by Sharma et al. [19]. Recently, a new approach to rough set based on remote system was introduced by Sun et al. [24]. Zhan et al. [28] also studied covering based multi-granulation (I,T)- fuzzy rough set models and their applications in multi-attribute group decision-making problems.

The word “entropy” was first used to measure an amount of uncertainty in probability distribution of a random variable in an experiment by Shannon [18]. Later, non-probabilistic entropy of a fuzzy set was proposed and described by Zadeh [27]. Fuzziness of a fuzzy set due to ambiguity, impreciseness and vagueness can be measured by using fuzzy entropy which was defined and characterized by De Luca and Termini [3].

Several other researchers, like Kapur [10], Liu [12], Pal and Pal [15], Kosko [11] and Gupta and Sheoran [5] used Shannon’s entropy axioms of characterization to measure uncertainty in fuzzy rough set. A similarity information measure between fuzzy rough set and fuzzy rough values was defined by Chengyi et al. [2] and that was characterized by Qi & Chengyi [17] in 2008.

Logarithmic entropy for fuzzy rough set and its application in decision making was proposed by Sharma [22]. The word entropy is a tedious word usually not understood easily, so fuzzy entropy was replaced by fuzzy information measure. Thus, some authors have called fuzzy entropy as fuzzy information measure. Hooda and Jain [6] in 2009 introduced three sub additive measures of fuzzy information and studied their applications in medical and social sciences. A new information measure of a fuzzy set was suggested and characterized by Hooda and Bajaj [7] and called it as “useful” fuzzy information measure. Hooda and Raich [8] unified existing work of various authors and described various generalizations of fuzzy information measures with their applications.

In the present paper, some new logarithmic information measures for fuzzy rough values and fuzzy rough set are proposed and their applications are studied. Basic concepts and definitions used in the later development of the paper are described in section 2. In section 3, a new logarithmic information measure for fuzzy rough values is defined and its validity is proved. Another information measure for fuzzy rough set and its application with illustrations are studied in section 4. In section 5, a weighted information measure for fuzzy rough set is discussed with its application. Comparison of the information measure with other existing information measures is studied in section 6. Conclusion is given in section 7 with references at the end of paper.

2. PRELIMINARIES

In this section some basic concepts and definitions used in development of the later part of the paper are described and illustrated with examples.

Definition 2.1[26]

Let X be a non-empty universal set and A is a subset of X , then a function $\mu_A(x): X \rightarrow [0,1]$, defines fuzzy set on X and is usually written as

$$A = \{(x_i, \mu_A(x_i)) : \mu_A(x_i) \in [0,1]; \forall x_i \in X\}$$

where $\mu_A(x)$ is called membership function from X to $[0, 1]$ with the following properties:

$$\mu_A(x) = \begin{cases} 0, & \text{if } x \notin A \text{ there is no ambiguity} \\ 1, & \text{if } x \in A \text{ there is no ambiguity} \\ 0.5, & \text{whether } x \in A \text{ or } x \notin A \text{ there is} \\ & \text{maximum ambiguity} \end{cases}$$

Definition 2.2[4]

Let U be the universal set and R be an equivalent relation on $U \times U$ which is also known as indistinguishable relation, then $W = (U, R)$ is known as Pawlak approximation space. The set of equivalent class generated by R on U i.e. $U/R = \{X_1, X_2, \dots, X_m\}$ is called knowledge or equivalence class, $[x]_R$ is equivalent class of elements x and the set of all fuzzy sets on U is denoted by $F(U)$. Now suppose $\forall X \subseteq U, \underline{X} = \{x \in U \mid [x] \subseteq X\}$ and $\overline{X} = \{x \in U \mid [x] \cap X \neq \emptyset\}$, then $X = (\underline{X}, \overline{X})$ is called rough set in W and \underline{X} and \overline{X} are called lower and upper approximation of X on W respectively.

Example2.1

Let $U = \{1, 2, 3, 4\}$ be a universal set, $B = \{\{1, 2\}, \{3, 4\}\}$ and $X = \{1, 2, 3\} (X \subset U)$. Let $B(X)$ be the rough set of X , then $B(X) = \{\{1, 2\}, \{1, 2, 3, 4\}\} = \{\{1, 2, 3\}, \{1, 2, 4\}\}$, where $\{1, 2\}$ is the lower approximation of X and $\{1, 2, 3, 4\}$ is upper approximation of X . $\{\{1, 2, 3\}, \{1, 2, 4\}\}$ is family of all sets containing $\{1, 2\}$ and $\{1, 2, 3, 4\}$ as their lower and upper approximations. Thus, $(\{1, 2\}, \{1, 2, 3, 4\})$ is the rough set of $\{1, 2, 3\}$.

Definition 2.3[4]

Let U be universe of discourse, R be a fuzzy relation on $U \times U$ and (U, R) is fuzzy approximation space. For any set $A \in F(U)$, the lower and upper approximations of A namely $\underline{R}(A)$ and $\overline{R}(A)$ with respect to approximation space (U, R) are called fuzzy sets of U whose membership functions are defined by

$$\begin{aligned} \underline{R}(A) &= \bigvee_{y \in U} [R(x, y) \wedge A(y)], \quad x \in U \text{ and} \\ \overline{R}(A) &= \bigwedge_{y \in U} [(1 - R(x, y)) \vee A(y)], \quad x \in U. \end{aligned}$$

Hence, the pair $(\underline{R}(A), \overline{R}(A))$ is defined as fuzzy rough set.

Example2.2.

Suppose $R = \{r_1, r_2, r_3, r_4, r_5\}$ be the set of objects and $C = \{c_1, c_2, c_3, c_4, c_5, c_6\}$ be the set of parameters, then fuzzy rough set (R, C) is given below:

Table 2.1: Fuzzy Rough Set

$\begin{matrix} C \\ R \end{matrix}$	c_1	c_2	c_3	c_4	c_5	c_6
r_1	(.1,.5)	(.2,.6)	(.4,.8)	(.7,.9)	(.6,.8)	(.4,.6)
r_2	(.2,.3)	(.1,.3)	(.4,.9)	(.6,.8)	(.5,.6)	(.8,.8)
r_3	(.1,.5)	(.3,.5)	(.5,.7)	(.3,.8)	(.6,.6)	(.4,.9)
r_4	(.5,.7)	(.1,.5)	(.3,.8)	(.3,.7)	(.1,.3)	(.5,.8)
r_5	(.4,.6)	(.5,.7)	(.3,.5)	(.7,.9)	(.4,.5)	(.5,.6)

Definition 2.4[19]

A real-valued function that enumerates the similarity between two objects is defined as similarity information measure. Actually, there is no particular definition of similarity information measures, in general these information measures are some implication of inverse of distance measures. Scores are given for similar quality, high score for more similar objects and low or negative scores for dissimilar quality.

Let S be the similarity measure on H , then the entropy corresponding to S is as follows:

$$e(F) = S(F, F^C), \forall F \in H, \quad (2.1)$$

where ‘ e ’, the entropy on H is the entropy generated by similarity information measure S and denoted by $e(F)$.

3. INFORMATION MEASURE FOR FUZZY ROUGH VALUES

In this section a new information measure for fuzzy rough values is proposed on the lines of other existing similarity information measures.

Definition 3.1[3]

A real valued function $e: A \rightarrow [0, +\infty)$, is a fuzzy information measure on A if e satisfies the following four axioms $\forall x \& y \in A$:

- $e(x) = 0$, if $x = [0, 0]$ or $x = [1, 1]$ i.e. $\underline{x} \& \bar{x} = 0$ or 1.
- $e(x) = e(x^C)$.
- Fuzzy Information measure e assumes a unique value that is $e(x) = 1$ for $x = [0.5, 0.5]$.
- $e(x) \geq e(y)$, if y is crisper than x , i.e. $x \geq y$ for $x \leq 0.5$ ($\bar{x} \leq 0.5$) and $y \geq x$ for $x \geq 0.5$ ($\underline{x} \geq 0.5$).

In 2004 Chengyi et al. [2] defined a similarity information measure between two fuzzy rough values which is as follows:

Let A be a fuzzy rough set and x, y are the fuzzy rough values in A , then the degree of similarity between fuzzy rough values x and y is given by M_Z as

$$M_Z(x, y) = 1 - \frac{1}{2} (|\underline{x} - \underline{y}| + |\bar{x} - \bar{y}|). \quad (3.1)$$

A similarity information measure between fuzzy rough sets and its elements was defined by Qi and Chengyi [17] in 2008 as given below:

Let A be a fuzzy rough and x, y are the fuzzy rough values in A , then similarity degree between elements x and y can be evaluated as

$$M(x, y) = 1 - \frac{1}{2}(\rho_{xy} + \sigma_{xy}), \quad (3.2)$$

where $\rho_{xy} = |\rho_x - \rho_y|$, $\sigma_{xy} = |\sigma_x - \sigma_y|$ and $\tau_x = \bar{x} - \underline{x}$ is called the degree of indeterminacy of element $x \in A$. $\rho_x = \underline{x} + \tau_x \underline{x} = (1 + \tau_x)\underline{x}$ and $\sigma_x = 1 - \bar{x} + \tau_x(1 - \bar{x})$ are called degree of favour $x \in A$.

Sharma et al. [20] defined entropy for fuzzy rough values corresponding to similarity information measure (3.1) as follows:

$$e(x) = 1 - \frac{1}{2}(|2\underline{x} - 1| + |2\bar{x} - 1|), \forall x \in A, \quad (3.3)$$

where A is a fuzzy rough set and $x = (\underline{x}, \bar{x})$.

Corresponding to the similarity information measure (3.3), logarithmic information measure for fuzzy rough values is proposed as given below:

$$e_{\log}(x) = \log_2(2 - \frac{1}{2}(|2\underline{x} - 1| + |2\bar{x} - 1|)), \forall x \in A,$$

where A is a fuzzy rough set and $x = (\underline{x}, \bar{x})$. (3.4)

(3.4) is called a similarity fuzzy rough information measure which must satisfy the four axioms given in definition (3.1) for its validity. Thus, these are verified by as lemmas 1 to 4 and explained below:

Lemma1

Let A be a fuzzy rough set then $\forall x \in A$ and $x = (\underline{x}, \bar{x})$, the fuzzy rough information measure defined in (3.4) is equal to zero for $x = [0, 0]$ or $x = [1, 1]$ i.e. $\underline{x} = 0$ or 1 and $\bar{x} = 0$ or 1 .

Proof

Putting $x = [0, 0]$ i.e. $\underline{x} = 0$ and $\bar{x} = 0$ in (3.4) we get

$$\begin{aligned} e_{\log}(x) &= \log_2\left(2 - \frac{1}{2}(|2 \times 0 - 1| + |2 \times 0 - 1|)\right) \\ &\Rightarrow e_{\log}(x) = \log_2\left(2 - \frac{1}{2}(|0 - 1| + |0 - 1|)\right) \\ &\Rightarrow e_{\log}(x) = \log_2\left(2 - \frac{1}{2} \times 2\right) \\ &\Rightarrow e_{\log}(x) = \log_2(2 - 1) = \log_2 1 = 0. \end{aligned}$$

Again putting $x = [1, 1]$ i.e. $\underline{x} = 1$ and $\bar{x} = 1$ in (3.4) we get

$$\begin{aligned} e_{\log}(x) &= \log_2\left(2 - \frac{1}{2}(|2 \times 1 - 1| + |2 \times 1 - 1|)\right) \\ &\Rightarrow e_{\log}(x) = \log_2\left(2 - \frac{1}{2}(|2 - 1| + |2 - 1|)\right) \end{aligned}$$

$$\begin{aligned} \Rightarrow e_{log}(x) &= \log_2 \left(2 - \frac{1}{2} \times 2 \right) \\ \Rightarrow e_{log}(x) &= \log_2(2 - 1) = \log_2 1 = 0. \end{aligned}$$

Hence, for $x = [0, 0]$ or $x = [1, 1]$ the fuzzy rough information measure (3.4) is zero.

Lemma2

Let A be a fuzzy rough set then $\forall x \in A$ and $x = (\underline{x}, \bar{x})$, the fuzzy rough information measure defined in (3.4) is equal to its compliment.

Proof

The measured (3.4) is equal to its compliment when $e_{log}(x^c) = e_{log}(x)$, for this put $\underline{x} = 1 - \underline{x}$ and $\bar{x} = 1 - \bar{x}$ in equation (3.4), we get

$$\begin{aligned} e_{log}(x^c) &= \log_2 \left(2 - \frac{1}{2} (|2(1 - \underline{x}) - 1| + |2(1 - \bar{x}) - 1|) \right) \\ \Rightarrow e_{log}(x^c) &= \log_2 \left(2 - \frac{1}{2} (|2 - 2\underline{x} - 1| + |2 - 2\bar{x} - 1|) \right), \\ \Rightarrow e_{log}(x^c) &= \log_2 \left(2 - \frac{1}{2} (|1 - 2\underline{x}| + |1 - 2\bar{x}|) \right), \\ \Rightarrow e_{log}(x^c) &= \log_2 \left(2 - \frac{1}{2} (|2\underline{x} - 1| + |2\bar{x} - 1|) \right). \end{aligned}$$

Clearly, $e_{log}(x^c) = e_{log}(x)$, hence information measure (3.4) is equal to its compliment. Thus, second condition is also satisfied.

Lemma3

Let A be a fuzzy rough set, then $\forall x \in A$ and $x = (\underline{x}, \bar{x})$, the fuzzy rough information measure defined in (3.4) is equal to one i.e. it assumes a unique maximum value for $x = [0.5, 0.5]$.

Proof

Let us put $\underline{x} = 0.5$ and $\bar{x} = 0.5$ in equation (3.4), then

$$\begin{aligned} e_{log}(x) &= \log_2 \left(2 - \frac{1}{2} (|2 \times 0.5 - 1| + |2 \times 0.5 - 1|) \right), \\ \Rightarrow e_{log}(x) &= \log_2 \left(2 - \frac{1}{2} (|1.0 - 1| + |1.0 - 1|) \right), \\ \Rightarrow e_{log}(x) &= \log_2 \left(2 - \frac{1}{2} (|0| + |0|) \right), \\ \Rightarrow e_{log}(x) &= \log_2(2 - 0) = \log_2(2) = 1. \end{aligned}$$

Thus, for $x = [0.5, 0.5]$ fuzzy rough information measure (3.4) assumes a unique maximum value 1. Hence third property is satisfied.

Lemma 4

Let A be a fuzzy rough set, then $\forall x \in A$ and $x = (\underline{x}, \bar{x})$, we have $e_{log}(x) \geq e_{log}(y)$ if y is sharper or crisper than x i.e. $y \leq x$ for $x \leq 0.5$ ($\bar{x} \leq 0.5$) and $y \geq x$ for $x \geq 0.5$ ($\underline{x} \geq 0.5$).

Proof

For the first case let $y \leq x$ for $x \leq 0.5$ ($\bar{x} \leq 0.5$),

$$\begin{aligned} &\Rightarrow \underline{y} \leq \bar{y} \leq \bar{x} \leq 0.5 \text{ \& } \underline{y} \leq \underline{x} \leq \bar{x} \leq 0.5, \\ &\Rightarrow |2\bar{x} - 1| \leq |2\bar{y} - 1| \text{ and } |2\underline{x} - 1| \leq |2\underline{y} - 1|, \\ &\Rightarrow \frac{1}{2}(|2\bar{x} - 1| + |2\underline{x} - 1|) \leq \frac{1}{2}(|2\bar{y} - 1| + |2\underline{y} - 1|). \end{aligned}$$

Subtracting the equation on both side from '2' and taking binary logarithm we get

$$\log_2 \left(2 - \frac{1}{2}(|2\bar{x} - 1| + |2\underline{x} - 1|) \right) \geq \log_2 \left(2 - \frac{1}{2}(|2\bar{y} - 1| + |2\underline{y} - 1|) \right),$$

$$\Rightarrow e_{log}(x) \geq e_{log}(y).$$

Similarly, second case can be proved. Thus, all the four axioms given in definition (3.1) are satisfied by fuzzy rough information measure (3.4). Hence, it is a valid information measure.

4. INFORMATION MEASURE FOR FUZZY ROUGH SETS

In the previous section we have defined an information measure for fuzzy rough values. Next, corresponding to equation (3.4) another information measure for fuzzy rough set is proposed. Let us consider a fuzzy rough set ' A ', then information measure for fuzzy rough set ' A ' is proposed as given below:

$$E_{log}(A) = \frac{1}{n} \sum_{i=1}^n \log_2 \left(2 - \frac{1}{2}(|2\underline{x}_i - 1| + |2\bar{x}_i - 1|) \right), \forall x_i \in A \text{ and } x_i = (\underline{x}_i, \bar{x}_i) \quad (4.1)$$

(4.1) is called as a fuzzy rough information measure and it lies in the interval $[0, 1]$. Larger value of the fuzzy rough information measure indicates more uncertainty in ' A '. Obviously, four lemmas for the validity of the information measure hold.

Next, some propositions are enumerated as

Proposition 4.1

Let A be a fuzzy rough set, then $\forall x_i \in A$ value of information measure (4.1) is equal to zero for $\underline{x}_i = 0$ or 1 and $\bar{x}_i = 0$ or 1 i.e. for A to be a crisp set.

Proposition 4.2

Let A be a fuzzy rough set, then $\forall x_i \in A$ value of information measure (4.1) is equal to the value of its complement, where $A^c = (1 - \underline{x}_i, 1 - \bar{x}_i)$.

Proposition 4.3

Let A be a fuzzy rough set, then $\forall x_i \in A$ value of information measure (4.1) is equal to one i.e. it assumes a unique maximum value for $\underline{x}_i = 0.5$ and $\overline{x}_i = 0.5$.

Proposition 4.4

Let A, B be fuzzy rough sets, then $\forall x_i \in A$ and $x_j \in B$, value of information measure for A is greater than or equal to the value of information measure for B when B is sharper or crisper than A that is $B \leq A$ for $x_i \leq 0.5$ ($\overline{x}_i \leq 0.5$) and $B \geq A$ for $x_i \geq 0.5$ ($\underline{x}_i \geq 0.5$).

These propositions can be proved by following the same procedure as described in previous section 3.

4.1. Application and Illustration

In this section application of fuzzy rough information measures (3.4) and (4.1) is illustrated with two examples.

Example 4.1.

Let us consider the case of choice of restaurants in a particular city by the customers due to several factors such as quality of food, service provide, behaviour of staff etc. It is clear from customer's behaviour that they are not happy with any one restaurant. They prefer different restaurants on the basis of their taste. This whole situation is represented in the form of fuzzy rough set as follows:

Suppose $R = \{r_1, r_2, r_3, r_4, r_5\}$ be the set of restaurants in a particular city and $C = \{c_1, c_2, c_3, c_4, c_5, c_6\}$ be the set of attributes on basis of the taste of customers as $c_1 =$ interior decoration, $c_2 =$ best possible value of customers money, $c_3 =$ location, $c_4 =$ behaviour of staff, $c_5 =$ quality of food and $c_6 =$ offer available.

The aim is to find out which restaurant is best in term of which attribute by using the information given by customers. The information available is model in the form of fuzzy rough set and represented in tabular form as given below:

Table 4.1: Fuzzy Rough Set

$R \backslash C$	c_1	c_2	c_3	c_4	c_5	c_6
r_1	(.1, .5)	(.2, .6)	(.4, .8)	(.7, .9)	(.6, .8)	(.4, .6)
r_2	(.2, .3)	(.1, .3)	(.4, .9)	(.6, .8)	(.5, .6)	(.8, .8)
r_3	(.1, .5)	(.3, .5)	(.5, .7)	(.3, .8)	(.6, .6)	(.4, .9)
r_4	(.5, .7)	(.1, .5)	(.3, .8)	(.3, .7)	(.1, .3)	(.5, .8)
r_5	(.4, .6)	(.5, .7)	(.3, .5)	(.7, .9)	(.4, .5)	(.5, .6)

From above table we find the uncertainty among restaurants and given attribute by using information measure for fuzzy rough set (4.1) as $E_{log}(R: C) = 0.626786$.

Now we find out the uncertainties among restaurants and attributes by using information measure (3.4) for fuzzy rough values which are given below in table 4.2:

Table 4.2: Uncertainties among Restaurants and Attributes

$\begin{matrix} C \\ R \end{matrix}$	c_1	c_2	c_3	c_4	c_5	c_6
$e_{log}(r_1)$	0.678072	0.678072	0.678072	0.485427	0.678072	0.847997
$e_{log}(r_2)$	0.584963	0.584963	0.584963	0.678072	0.926	0.485427
$e_{log}(r_3)$	0.678072	0.847997	0.847997	0.584963	0.847997	0.584963
$e_{log}(r_4)$	0.847997	0.678072	0.584963	0.678072	0.485427	0.765535
$e_{log}(r_5)$	0.847997	0.847997	0.847997	0.485427	0.926	0.926

Comparing the uncertainty $E_{log}(R:C)$ with those in table 4.2 and choosing the uncertainty less than or equal to $E_{log}(R:C)$ we can conclude that restaurants 1 and 5 are good in terms of attribute c_4 i.e. behaviour of staff of restaurants 2 and 3 are good in terms of attribute c_6 i.e. offer available and restaurant 4 is good in terms of attribute c_5 i.e. quality of food.

Example 4.2.

Covid-19 pandemic has transformed individual lives as well as social life on global scale. The most effected sector by this virus is healthcare. Healthcare personnel have faced a significant higher risk of infection, particularly in the early stages of the outbreak. On an average it takes 5-6 days for symptoms to appear when someone is infected with the corona virus. Symptoms of covid-19 are divided into three categories as

Most Common Symptoms

- Dry Cough
- Tiredness
- Fever

Less Common Symptoms

- Headache
- Diarrhoea
- Sore throat
- Rash on Skin
- Discolouration of Finger
- Aches and Pain
- Conjunctivitis
- Loss of taste or Smell

Serious Symptoms

- Chest Pain
- Loss of Movement or Speech
- Shortness of breath or Difficulty in Breathing

If you have serious type of symptoms then immediate medical attention is required. For most common and mild symptoms, people should stay at home and consult doctor for precaution and cure.

Here we make use of fuzzy rough information measure to detect the type of symptoms in patient for covid-19 and suggest whether patient requires immediate medical attention or not. To see the

application of the fuzzy rough information measure a hypothetical case study is framed as follows:

Let $A = \{A_1, A_2, A_3, A_4\}$ is the set of patients under study and

$T = \{T_1 = \text{Fever}, T_2 = \text{Headache}, T_3 = \text{Chest pain}, T_4 = \text{Difficulty in breathing}, T_5 = \text{Loss of speech}\}$ are the symptoms of the patients. Symptoms T_3, T_4, T_5 are the serious type of symptoms and if they occur then immediate treatment is required. The data for the relation between patients and symptoms is given hypothetically as

Table 4.3: Fuzzy Rough Set for Patients and Symptoms

$\begin{matrix} T \\ A \end{matrix}$	T_1	T_2	T_3	T_4	T_5
A_1	[0.0,0.4]	[0.6,0.9]	[0.1,0.5]	[0.3,0.7]	[0.5,0.7]
A_2	[0.3,0.5]	[0.2,0.6]	[0.2,0.3]	[0.5,0.9]	[0.6,0.9]
A_3	[0.7,0.8]	[0.0,0.9]	[0.2,0.3]	[0.1,0.2]	[0.7,1]
A_4	[0.9,1]	[0.3,0.8]	[0.3,0.4]	[0.2,0.6]	[0.5,0.5]

Using fuzzy rough information measure (3.4), the uncertainties for patients and symptoms are given below in table 4.4:

Table 4.4: Uncertainties among Patients and Symptoms

$\begin{matrix} T \\ A \end{matrix}$	T_1	T_2	T_3	T_4	T_5
A_1	0.4855	0.585	0.6781	0.6781	0.848
A_2	0.848	0.6781	0.585	0.6781	0.585
A_3	0.585	0.1375	0.585	0.3785	0.3785
A_4	0.1375	0.585	0.7656	0.6781	1

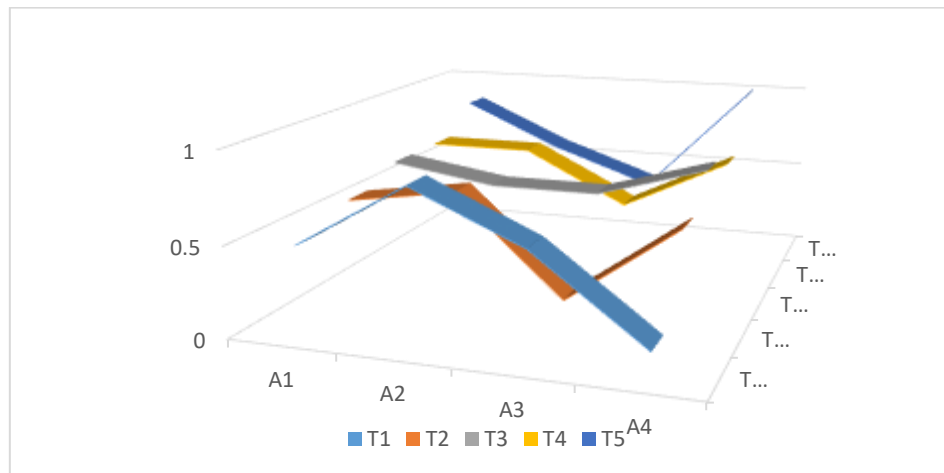


Fig 1: Uncertainties among Patients and Symptoms

The uncertainty for complete set using fuzzy rough information measure (4.1) is $E_{\log}(A;T) = 0.593980$. Now, compare the uncertainty $E_{\log}(A;T)$ with those in table 4.4, if uncertainties for symptoms T_3, T_4, T_5 together are greater than $E_{\log}(A;T)$, then the patient is suffering from serious symptoms of covid-19. From table 4.4 it is clear that patients A_1 and A_4 are suffering from serious symptoms and need immediate medical treatment, while patients A_2 and A_3 are less effected by virus and can take home remedies to cure themselves.

5. WEIGHTED INFORMATION MEASURE FOR FUZZY ROUGH SET

In this section weighted information measure for fuzzy rough set is proposed corresponding to measure (3.4) and an application is presented.

Let us consider a fuzzy rough set $A, \forall x_i \in A$ and $w_i \in [0,1]$ is the weight for element x_i of A , then weighted fuzzy rough information measure for A is given below:

$$H_{log}(A) = \frac{\sum_{i=1}^n w_i \log_2 \left(2 - \frac{1}{2} (|2\underline{x}_i - 1| + |2\overline{x}_i - 1|) \right)}{\sum_{i=1}^n w_i},$$

$$\forall x_i \in A, w_i \in [0,1] \text{ and } x_i = (\underline{x}_i, \overline{x}_i). \quad (5.1)$$

(5.1) is called weighted fuzzy rough information measure. Next, we state some propositions for the validity of (5.1).

Proposition 5.1

Let A be a fuzzy rough set, then $\forall x_i \in A$ value of weighted information measure (5.1) is equal to zero for $\underline{x}_i = 0$ or 1 and $\overline{x}_i = 0$ or 1 i.e. for A to be a crisp set.

Proposition 5.2

Let A be a fuzzy rough set, then $\forall x_i \in A$ the value of weighted information measure (5.1) is equal to the value of its complement, where $A^C = (1 - \underline{x}_i, 1 - \overline{x}_i)$.

Proposition 5.3

Let A be a fuzzy rough set, then $\forall x_i \in A$ the value of weighted information measure (5.1) is equal to one i.e. it assumes a unique maximum value for $\underline{x}_i = 0.5$ and $\overline{x}_i = 0.5$.

Proposition 5.4

Let A, B be fuzzy rough sets, then $\forall x_i \in A$ and $x_j \in B$, the value of weighted information measure for A is greater than or equal to the value of weighted information measure for B when B is sharper or crisper than A that is $B \leq A$ for $x_i \leq 0.5 (\overline{x}_i \leq 0.5)$ and $B \geq A$ for $x_i \geq 0.5 (\underline{x}_i \geq 0.5)$.

It may noted that the above proposition hold good obviously, so the weighted fuzzy information measure is a valid measure.

5.1. Application for Illustration

For illustration purpose the data set of example 4.1 is considered in the following table:

Table 5.1: Fuzzy Rough Set

$\begin{matrix} C \\ R \end{matrix}$	c_1	c_2	c_3	c_4	c_5	c_6
r_1	(.1,.5)	(.2,.6)	(.4,.8)	(.7,.9)	(.6,.8)	(.4,.6)
r_2	(.2,.3)	(.1,.3)	(.4,.9)	(.6,.8)	(.5,.6)	(.8,.8)
r_3	(.1,.5)	(.3,.5)	(.5,.7)	(.3,.8)	(.6,.6)	(.4,.9)
r_4	(.5,.7)	(.1,.5)	(.3,.8)	(.3,.7)	(.1,.3)	(.5,.8)
r_5	(.4,.6)	(.5,.7)	(.3,.5)	(.7,.9)	(.4,.5)	(.5,.6)

The uncertainties calculated on applying fuzzy rough information measure (3.4) are given in table 5.2.

Table 5.2: Uncertainties among Restaurants and Attributes

$\begin{matrix} C \\ R \end{matrix}$	c_1	c_2	c_3	c_4	c_5	c_6
$e_{log}(r_1)$	0.678072	0.678072	0.678072	0.485427	0.678072	0.847997
$e_{log}(r_2)$	0.584963	0.584963	0.584963	0.678072	0.926	0.485427
$e_{log}(r_3)$	0.678072	0.847997	0.847997	0.584963	0.847997	0.584963
$e_{log}(r_4)$	0.847997	0.678072	0.584963	0.678072	0.485427	0.765535
$e_{log}(r_5)$	0.847997	0.847997	0.847997	0.485427	0.926	0.926

Now, the weights corresponding to each element of table 5.1 are given below:

Table 5.3: Weights Corresponding to Each Element of Fuzzy Rough Set

0.53	0.62	0.55	0.82	0.92	0.87
0.84	0.80	0.66	0.45	0.33	0.23
0.16	0.25	0.39	0.64	0.87	0.33
0.29	0.36	0.46	0.47	0.78	0.63
0.13	0.85	0.74	0.53	0.24	0.82

Using weighted fuzzy rough information measure (5.1), the weighted uncertainties for fuzzy rough values are

Table 5.4: Uncertainties Using Weighted Fuzzy Rough Information Measure (5.1)

$\begin{matrix} C \\ R \end{matrix}$	c_1	c_2	c_3	c_4	c_5	c_6
$h_{log}(r_1)$	0.3594	0.4204	0.3730	0.3980	0.6239	0.7378
$h_{log}(r_2)$	0.4914	0.468	0.3861	0.3051	0.3056	0.1116
$h_{log}(r_3)$	0.1085	0.212	0.3307	0.3744	0.7378	0.1931
$h_{log}(r_4)$	0.2459	0.2441	0.2691	0.3187	0.3786	0.4823
$h_{log}(r_5)$	0.1102	0.7208	0.6275	0.2573	0.2222	0.7593

Weighted uncertainty for complete fuzzy rough set is $H_{\log}(A;T) = 0.6988$. After assigning the weights it can be noted that the decision regarding restaurants is changed. All the restaurants are up to the expectations of customers except restaurants r_1, r_3 and r_5 , particularly for attributes c_6 of restaurant r_1 , c_5 of restaurant r_3 and c_2, c_6 of restaurant r_5 . Since, the uncertainties corresponding to these attributes are greater than $H_{\log}(A;T)$.

6. COMPARISON WITH OTHER FUZZY ROUGH INFORMATION MEASURES

In this section information measure (3.4) and (4.1) are compared with the existing fuzzy rough information measures defined and studied by Sharma and Gupta [21].

Let 'A' be a rough set then, sine trigonometric information measures for fuzzy rough values of set 'A' and for fuzzy rough set 'A' are written respectively as given below:

$$e_{\sin}(x) = \sin\left[\frac{\pi}{2}\left(1 - \frac{1}{2}(|2\underline{x} - 1| + |2\overline{x} - 1|)\right)\right], \text{ for every element } x \in A, \quad (6.1)$$

$$\text{and } E_{\sin}(x) = \frac{1}{n} \sum_{i=1}^n \sin\left[\frac{\pi}{2}\left(1 - \frac{1}{2}(|2\underline{x}_i - 1| + |2\overline{x}_i - 1|)\right)\right], \text{ for the whole set A.} \quad (6.2)$$

Similarly, the cosine and tangent trigonometric information measures for fuzzy rough values and for fuzzy rough set respectively are

$$e_{\cos}(x) = \cos\left[\frac{\pi}{4}(|2\underline{x} - 1| + |2\overline{x} - 1|)\right], \text{ for every element } x \in A, \quad (6.3)$$

$$E_{\cos}(x) = \frac{1}{n} \sum_{i=1}^n \cos\left[\frac{\pi}{4}(|2\underline{x}_i - 1| + |2\overline{x}_i - 1|)\right], \text{ for the whole set A.} \quad (6.4)$$

$$e_{\tan}(x) = \tan\left[\frac{\pi}{4}\left(1 - \frac{1}{2}(|2\underline{x} - 1| + |2\overline{x} - 1|)\right)\right], \text{ for every element } x \in A, \quad (6.5)$$

and

$$E_{\tan}(x) = \frac{1}{n} \sum_{i=1}^n \tan\left[\frac{\pi}{4}\left(1 - \frac{1}{2}(|2\underline{x}_i - 1| + |2\overline{x}_i - 1|)\right)\right], \text{ for the whole set A.} \quad (6.6)$$

The uncertainties for fuzzy rough values in example 4.1 are computed using the above trigonometric fuzzy rough information measures as given in table 6.1.

Table 6.1: Uncertainties using Trigonometric FRI Measures

$\begin{matrix} \text{R} \\ \text{C} \end{matrix}$	c_1	c_2	c_3	c_4	c_5	c_6
$e_{sin}(r_1)$.8090	.8090	.8090	.5878	.8090	.9511
$e_{cos}(r_1)$.8090	.8090	.8090	.5878	.8090	.9511
$e_{tan}(r_1)$.5095	.5095	.5095	.3249	.5095	.7265
$e_{sin}(r_2)$.7071	.5878	.7071	.8090	.9877	.5878
$e_{cos}(r_2)$.7071	.5878	.7071	.8090	.9877	.5878
$e_{tan}(r_2)$.4142	.3249	.4142	.5095	.8541	.3249
$e_{sin}(r_3)$.8090	.9511	.9511	.7071	.9511	.7071
$e_{cos}(r_3)$.8090	.9511	.9511	.7071	.9511	.7071
$e_{tan}(r_3)$.5095	.7265	.7265	.4242	.7265	.4142
$e_{sin}(r_4)$.9511	.8090	.7071	.8090	.5878	.8910
$e_{cos}(r_4)$.9511	.8090	.7071	.8090	.5878	.8910
$e_{tan}(r_4)$.7265	.5095	.4142	.5095	.3249	.6128
$e_{sin}(r_5)$.9511	.9511	.9511	.5878	.9877	.9877
$e_{cos}(r_5)$.9511	.9511	.9511	.5878	.9877	.9877
$e_{tan}(r_5)$.7265	.7265	.7265	.3249	.8541	.8541

The uncertainties calculated using the trigonometric fuzzy rough information measures for fuzzy rough set are $E_{sin}(R) = 0.8136$, $E_{cos}(R) = 0.8136$ and $E_{tan}(R) = 0.55862$.

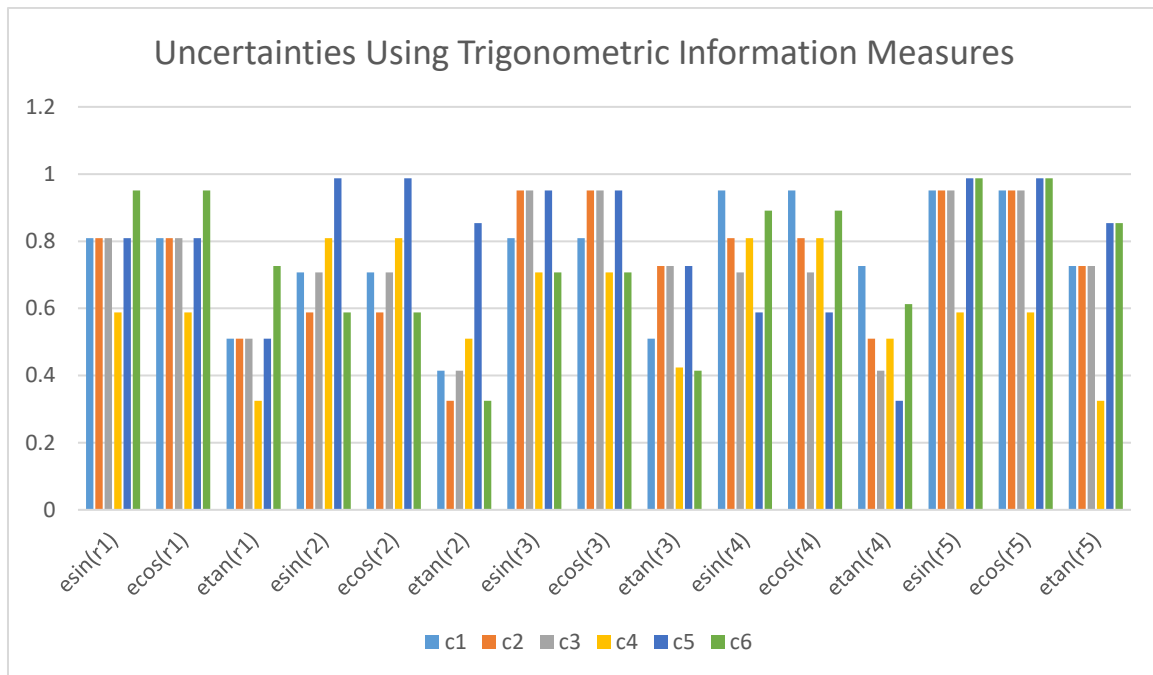


Fig 2. Uncertainties Using Trigonometric FRI measures

Next, the uncertainties calculated using information measure (3.4) for fuzzy rough values of example 4.1 are given below:

Table 6.2: Uncertainty Using FRI Measure (3.4)

$R \backslash C$	c_1	c_2	c_3	c_4	c_5	c_6
$e_{log}(r_1)$	0.678072	0.678072	0.678072	0.485427	0.678072	0.847997
$e_{log}(r_2)$	0.584963	0.584963	0.584963	0.678072	0.926	0.485427
$e_{log}(r_3)$	0.678072	0.847997	0.847997	0.584963	0.847997	0.584963
$e_{log}(r_4)$	0.847997	0.678072	0.584963	0.678072	0.485427	0.765535
$e_{log}(r_5)$	0.847997	0.847997	0.847997	0.485427	0.926	0.926

Now, uncertainty computed by information measure (4.1) for fuzzy rough set is $E_{log}(R:C) = 0.626786$.

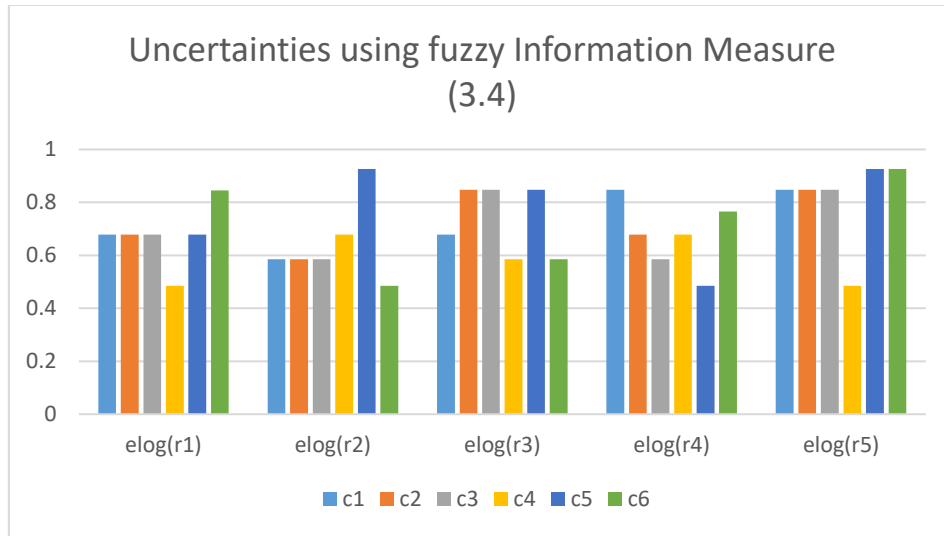


Fig 3: Uncertainties Using fuzzy rough Information Measure (3.4)

From the table 6.1 and 6.2 and from the graphs it may be noted that the uncertainties calculated on applying information measure (3.4) are much less than that of sine and cosine trigonometric information measures studied by Sharma and Gupta [21]. Thus, our information measure is more useful and simple than the trigonometric information measures. However, the uncertainty calculated using sine and cosine measures are almost same.

7. CONCLUSION

As we know a right decision can change one's life, so it is important to study decision making methods for solving the problems of our daily life and fuzzy rough set theory is one of the choices. Fuzzy rough set is the hybridization of rough set and fuzzy set which had been widely used to deals with complex data containing different types of uncertainties. In this paper a logarithmic information measure for fuzzy rough values is proposed and verified axiomatically. Logarithmic information measure for fuzzy rough set and weighted logarithmic fuzzy rough information measure are also defined with their application in decision making problem. The Proposed fuzzy rough information measure is compared with other existing trigonometric fuzzy rough information measures and it is proved that our information measure is better and simple.

The research on application of fuzzy rough set theory can be extended in studying data dimension reduction technique and decision making in design of experiment. A new form of rough set that is hyper rough set can be defined. Hyper rough set is completely a new idea in the area of rough set theory. In future researchers can put their attention towards defining new information measures by combining logarithmic and trigonometric functions as combination of two functions overcome the shortcoming of one another and provide better result than single one.

CONFLICT OF INTEREST

There is no conflict of Interest among the authors.

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