ON PROGNOSIS OF PATIENT SERVICE IN A MEDICAL ORGANIZATION

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ABSTRACT

In this paper we introduce a model for prognosis of service intensity of patient service in a medical organization. The model gives a possibility to do prognosis of service intensity of patient with account changing of conditions of the service. We intro-duce an analytical approach of analysis of service intensity of patient.

KEYWORDS

service intensity of patient; prognosis of service of patient; analytical approach for prognosis.

1. INTRODUCTION

One of the most important factors, which determines increasing of competi-tiveness of medical organizations and improvement of quality of patient ser-vice, is development of all spheres of their activity [1,2]. In this situation medical organizations should develop effective methods and adhere to the strategic concept of innovation development [3-5]. In this paper we intro-duce a model for prognosis of patient service in medical organizations with account possible changing of their regime of functioning. The model based on solution of ordinary differential equations. In the framework of this paper we consider a hospital with several positions for patients. It is known inten-sities of input flows of medical service and requests on medical service. We introduce an analytical approach to solve the considered system of ordinary differential equations.

2. METHOD OF SOLUTION

We analyzed activity of medical organization by solution of the following initial problem

$$\begin{cases} \frac{d\,P_0(t)}{d\,t} = -a(t)P_0(t) - b_1(t)P_1(t) - b_2(t)P_2(t) \\ \frac{d\,P_1(t)}{d\,t} = -a(t)P_0(t) - b_1(t)P_1(t) - b_2(t)P_2(t) \\ \frac{d\,P_2(t)}{d\,t} = -a(t)P_0(t) - b_1(t)P_1(t) - b_2(t)P_2(t) \end{cases} \quad (1)$$

Here $P_0(t)$ is the probability of freeing up of position of patient; $P_1(t)$ is the probability of a queue of one patient; $P_2(t)$ is the probability of a queue of two patients; $a(t)$ is intensity of input flow on medical service; $b_i(t)$ is intensity of service in a hospital at appropriate length of queue. Consideration of queue with higher length is possible by appropriate addition of system of equations (1). Initial conditions for the considered probabilities could be presented in the following form

$$P_0(0)=1,\ P_1(0)=0,\ P_2(0)=0. \tag{2}$$

We solved equations (1) by method of averaged of function corrections [6,7]. In the framework of the approach to determine the first-order approximations of the considered probabilities we replace them on their not yet known average values $P_i(t) \rightarrow \alpha_{1i}$ in the right sides of Eqs. (1). The above substitution and integration of left and right sides of the obtained equations we determine equations to determine the first-order approximations of the considered probabilities in the following form

$$\begin{cases} P_{10}(t) = -\alpha_{10}\int\limits_0^t a(\tau)\,d\tau - \alpha_{11}\int\limits_0^t b_1(\tau)\,d\tau - \alpha_{12}\int\limits_0^t b_2(\tau)\,d\tau + 1 \\ P_{11}(t) = -\alpha_{10}\int\limits_0^t a(\tau)\,d\tau - \alpha_{11}\int\limits_0^t b_1(\tau)\,d\tau - \alpha_{12}\int\limits_0^t b_2(\tau)\,d\tau \\ P_{12}(t) = -\alpha_{10}\int\limits_0^t a(\tau)\,d\tau - \alpha_{11}\int\limits_0^t b_1(\tau)\,d\tau - \alpha_{12}\int\limits_0^t b_2(\tau)\,d\tau \end{cases} \tag{3}$$

Average values α_{1i} were calculated by the following standard relation [6, 7]

$$\alpha_{1i} = \frac{1}{\Theta}\int\limits_0^\Theta P_{1i}(t)\,dt\,. \tag{4}$$

Here Θ is the continuance of medical service. Substitution of equations (3) into relation (4) gives a possibility to obtain the following system of equations to calculate average values α_{1i}

$$\begin{cases} \Theta\,\alpha_{10} = -\alpha_{10}\int\limits_0^\Theta (\Theta - t)a(t)\,dt - \alpha_{11}\int\limits_0^\Theta (\Theta - t)b_1(t)\,dt - \alpha_{12}\int\limits_0^\Theta (\Theta - t)b_2(t)\,dt + 1 \\ \Theta\,\alpha_{11} = -\alpha_{10}\int\limits_0^\Theta (\Theta - t)a(t)\,dt - \alpha_{11}\int\limits_0^\Theta (\Theta - t)b_1(t)\,dt - \alpha_{12}\int\limits_0^\Theta (\Theta - t)b_2(t)\,dt \\ \Theta\,\alpha_{12} = -\alpha_{10}\int\limits_0^\Theta (\Theta - t)a(t)\,dt - \alpha_{11}\int\limits_0^\Theta (\Theta - t)b_1(t)\,dt - \alpha_{12}\int\limits_0^\Theta (\Theta - t)b_2(t)\,dt \end{cases} \tag{5}$$

Solution of systems of equations (5) could be obtained by using standard approaches [8] and presented in the following form

$$\alpha_{10}=\Delta_1/\Delta,\ \alpha_{11}=\Delta_2/\Delta,\ \alpha_{12}=\Delta_3/\Delta. \tag{6}$$

Here

$$\Delta=\left\{\left[\Theta+\int_0^\Theta(\Theta-t)b_1(t)dt\right]\left[\Theta+\int_0^\Theta(\Theta-t)b_2(t)dt\right]-\int_0^\Theta(\Theta-t)a(t)dt\int_0^\Theta(\Theta-t)\times\right.$$
$$\left.\times b_2(t)\,dt\right\}\left[\Theta+\int_0^\Theta(\Theta-t)a(t)dt\right]-\left\{\Theta+\int_0^\Theta(\Theta-t)[b_2(t)-b_1(t)]dt\right\}\int_0^\Theta(\Theta-t)a(t)dt\times$$
$$\times\int_0^\Theta(\Theta-t)\,b_1(t)dt+\int_0^\Theta(\Theta-t)\,a(t)dt\left\{\int_0^\Theta(\Theta-t)b_1(t)dt-\left[\Theta+\int_0^\Theta(\Theta-t)b_1(t)dt\right]\right\}\times$$
$$\times\int_0^\Theta(\Theta-t)\,b_2(t)dt;$$
$$\Delta_1=\left[\Theta+\int_0^\Theta(\Theta-t)b_1(t)dt\right]\left[\Theta+\int_0^\Theta(\Theta-t)b_2(t)dt\right]-\int_0^\Theta(\Theta-t)\times$$
$$\times a(t)dt\int_0^\Theta(\Theta-t)\,b_2(t)dt;$$
$$\Delta_2=\int_0^\Theta(\Theta-t)a(t)\,dt\left\{\Theta+\int_0^\Theta(\Theta-t)[b_2(t)-b_1(t)]dt\right\}\times$$
$$\times\int_0^\Theta(\Theta-t)b_1(t)\,dt;\ \Delta_3=\left\{\int_0^\Theta(\Theta-t)b_1(t)dt-\left[\Theta+\int_0^\Theta(\Theta-t)b_1(t)dt\right]\right\}\int_0^\Theta(\Theta-t)\times$$
$$\times a(t)dt\int_0^\Theta(\Theta-t)\,b_2(t)\,dt.$$

The second-order approximations of the required probabilities were determined in the framework standard procedure of method of averaged of function corrections [6, 7], i.e. by the following replacement $P_i(t)\to\alpha_{2i}+P_{1i}(t)$. Here α_{2i} are the average values of the required the second-order approximations $P_{2i}(t)$. Equations to determine required approximations after integration on time could be presented in the following form

$$\begin{cases}P_{20}(t)=-\int_0^t a(\tau)[\alpha_{20}+P_{10}(\tau)]d\tau-\int_0^t b_1(\tau)[\alpha_{21}+P_{11}(\tau)]d\tau-\\ \qquad -\int_0^t b_2(\tau)[\alpha_{22}+P_{12}(\tau)]d\tau+1\\ P_{21}(t)=-\int_0^t a(\tau)[\alpha_{20}+P_{10}(\tau)]d\tau-\int_0^t b_1(\tau)[\alpha_{21}+P_{11}(\tau)]d\tau-\\ \qquad -\int_0^t b_2(\tau)[\alpha_{22}+P_{12}(\tau)]d\tau\\ P_{22}(t)=-\int_0^t a(\tau)[\alpha_{20}+P_{10}(\tau)]d\tau-\int_0^t b_1(\tau)[\alpha_{21}+P_{11}(\tau)]d\tau-\\ \qquad -\int_0^t b_2(\tau)[\alpha_{22}+P_{12}(\tau)]d\tau\end{cases}\qquad(7)$$

We calculate average values α_{2i} by standard relation [6, t]

$$\alpha_{2i} = \frac{1}{\Theta}\int_0^\Theta \left[P_{2i}(z,t) - P_{1i}(z,t)\right] d\,t\,. \qquad (8)$$

Substitution of the first- and the second-order approximations of probabilities in the relation (8) gives equations to determine average values α_{2i} in the following form

$$\begin{cases} \Theta\,\alpha_{20} = -(\alpha_{20}-\alpha_{10})\int_0^\Theta(\Theta-t)a(t)\,d\,t - \\ \qquad -(\alpha_{21}-\alpha_{11})\int_0^\Theta(\Theta-t)b_1(t)\,d\,t - (\alpha_{22}-\alpha_{12})\int_0^\Theta(\Theta-t)b_2(t)\,d\,t + 1 \\ \Theta\,\alpha_{21} = -(\alpha_{20}-\alpha_{10})\int_0^\Theta(\Theta-t)a(t)\,d\,t - \\ \qquad -(\alpha_{21}-\alpha_{11})\int_0^\Theta(\Theta-t)b_1(t)\,d\,t - (\alpha_{22}-\alpha_{12})\int_0^\Theta(\Theta-t)b_2(t)\,d\,t \\ \Theta\,\alpha_{22} = -(\alpha_{20}-\alpha_{10})\int_0^\Theta(\Theta-t)a(t)\,d\,t - \\ \qquad -(\alpha_{21}-\alpha_{11})\int_0^\Theta(\Theta-t)b_1(t)\,d\,t - (\alpha_{22}-\alpha_{12})\int_0^\Theta(\Theta-t)b_2(t)\,d\,t \end{cases} \qquad 9)$$

Solution of equations (9) could be written as [8]

$$\alpha_{20}=\Delta_4/\Delta,\ \alpha_{21}=\Delta_5/\Delta,\ \alpha_{22}=\Delta_6/\Delta, \qquad (10)$$

where

$$\Delta_4 = \Bigg\{\left[\Theta + \int_0^\Theta(\Theta-t)b_1(t)\,d\,t\right]\left[\Theta + \int_0^\Theta(\Theta-t)b_2(t)\,d\,t\right] - \int_0^\Theta(\Theta-t)b_1(t)\,d\,t\int_0^\Theta(\Theta-t)\times$$

$$\times\, b_2(t)\,d\,t\Bigg\}\,\alpha - \Theta(\beta-\gamma)\int_0^\Theta(\Theta-t)b_1(t)\,d\,t + \Theta(\beta-\gamma)\left[\int_0^\Theta(\Theta-t)b_2(t)\,d\,t\right]^2,$$

$$\Delta_5 = (\gamma-\beta)\times$$

$$\times\int_0^\Theta(\Theta-t)b_2(t)\,d\,t\int_0^\Theta(\Theta-t)a(t)\,d\,t - \alpha\,\Theta\int_0^\Theta(\Theta-t)a(t)\,d\,t + \beta\,\Theta\left[\Theta + \int_0^\Theta(\Theta-t)a(t)\,d\,t\right],$$

$$\Delta_6 = -(\gamma-\beta)\int_0^\Theta(\Theta-t)b_1(t)\,d\,t\int_0^\Theta(\Theta-t)a(t)\,d\,t + \Bigg\{\gamma\int_0^\Theta(\Theta-t)b_1(t)\,d\,t - \beta\int_0^\Theta(\Theta-t)b_1(t)\,d\,t +$$

$$+\,\Theta(\gamma-\beta)\Bigg\}\left[\Theta + \int_0^\Theta(\Theta-t)a(t)\,d\,t\right] - \alpha\,\Theta\int_0^\Theta(\Theta-t)\,a(t)\,d\,t\,,$$

$$\alpha = \alpha_{10}\int_0^\Theta(\Theta-t)a(t)\,d\,t + 1 +$$

$$+\alpha_{11}\int_0^{\Theta}(\Theta-t)b_1(t)\,dt+\alpha_{12}\int_0^{\Theta}(\Theta-t)b_2(t)\,dt+1,$$

$$\alpha=\alpha_{10}\int_0^{\Theta}(\Theta-t)a(t)\,dt+\alpha_{11}\int_0^{\Theta}(\Theta-t)\times$$

$$\times b_1(t)\,dt+\alpha_{12}\int_0^{\Theta}(\Theta-t)b_2(t)\,dt+1,$$

$$\beta=\alpha_{10}\int_0^{\Theta}(\Theta-t)a(t)\,dt+\alpha_{11}\int_0^{\Theta}(\Theta-t)b_1(t)\,dt+\alpha_{12}\times$$

$$\times\int_0^{\Theta}(\Theta-t)\,b_2(t)\,dt+\alpha_{11}\int_0^{\Theta}(\Theta-t)b_1(t)\,dt+\alpha_{12}\int_0^{\Theta}(\Theta-t)b_2(t)\,dt,$$

$$\gamma=\alpha_{10}\int_0^{\Theta}(\Theta-t)a(t)\,dt+$$

$$+\alpha_{12}\int_0^{\Theta}(\Theta-t)b_2(t)\,dt-(\alpha_{21}-\alpha_{11})\int_0^{\Theta}(\Theta-t)b_1(t)\,dt.$$

Analysis of changing of the considered probabilities in time has been done analytically by using the second-order approximation in the framework of the method of averaging of function corrections. The approximation is usually sufficient to obtain a qualitative analysis and to obtain some quantitative results. The results of analytical calculations were verified by comparing them with the results of numerical simulation.

3. Results of Analysis

In this section we analyzed changing of considered probabilities in time. Figs. 1 and 2 shows typical dependences of probability P_0 on time. With changing of conditions of medical service of patients of a hospital positions for patients could be released faster or slower. With increasing of quantity of requests on treatment, quality of service of patients could be improved or reduced to a minimum.

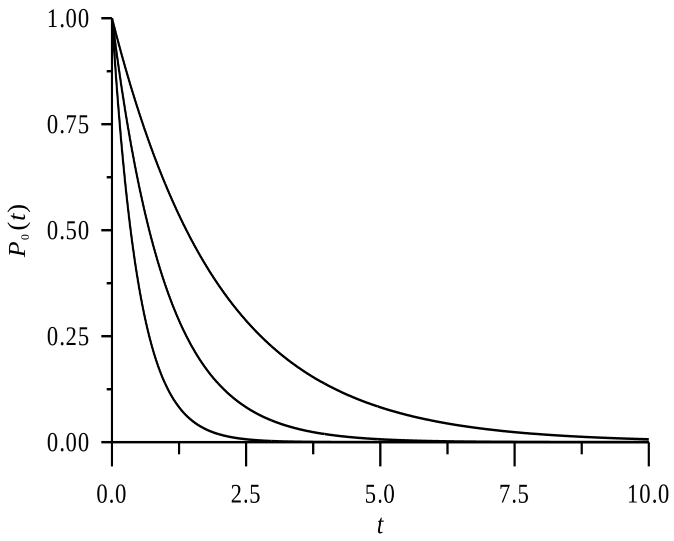

Fig. 1. Typical dependences of probability of freeing up of position of patient at increasing of quantity of requests of service

CONCLUSION

We introduce a model for prognosis of intensity of service of patients in a medical organization. Based on the model it is possible to make prognosis of intensity of service of patients with account of changing of situation with the considered service. We introduced an analytical approach of solution of considered equations in the framework of the above model.

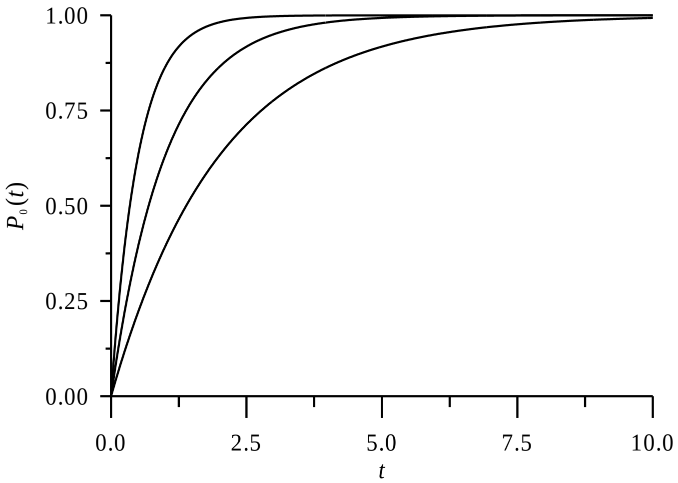

Fig. 2. Typical dependences of probability of freeing up of position of patient at decreasing of quantity of requests of service

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