EFFECT OF TWO EXOSYSTEM STRUCTURES ON OUTPUT REGULATION OF THE RTAC SYSTEM

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ABSTRACT

This paper presents results on the output regulation of a single-input multi-output (SIMO) rotational-translational actuator (RTAC) system. The results focus primarily on stability and robustness, which are studied in light of the presence of externally generated exogenous input signals. Two exosystem types were investigated and tested. Obtained results answers the question of asymptotic stabilization and tracking of a desired trajectory in the presence of a dynamic exosystem. The results confirmed the working theory of robust stabilization using output feedback techniques, borne out of differential-geometric observer design principles. The utilized design showed good stability results which compares favourably with existing works on RTAC stabilization.

KEYWORDS

RTAC,Eexogenous Signals, Feedback Regulation, Asymptotic stability, Differential Geometry, Observer

1. INTRODUCTION

The RTAC system finds applicability in systems such as dual-spin stabilized spacecraft, drill-type systems which require stabilization in the presence of moving platforms. The RTAC system has also been used as a benchmark system for testing various control law designs. Stabilization of the RTAC benchmark system has been done using different approaches some of which include; output feedback techniques [1], [2], backstepping approaches such as feedback linearization [3], input-output feedback linearization [4], integrator backstepping [5], cascade control with coordinate changes [6], energy and entropy based hybrid framework [7] amongst others.

Output regulation-based control is defined by tracking and stabilization goals [8] [9]. This control paradigm targets the convergence of all or selected system states of the system towards a suitably defined equilibrium (zero or otherwise) as time goes to infinity [10] [11]. Some of the standard techniques which exist for output regulated systems include the use of observers as internal model, application of separation principle and certainty principle [8] [12]. Usually, the specification on the system to be regulated is made in presence of external disturbances and other uncertainties which the output regulation technique defines by considering and incorporating an external system (exosystem) to be the generator of these external signals (references and disturbances) to be tracked or rejected. The problem can further be cast as either a reference input tracking or disturbance input rejection problem [8] [13] [14].

In this paper, consideration has been made of two different exosystem structures, such as constant and oscillator type signal generators. The exosystem signals are of either reference or disturbance inputs which are modelled as part of the system and should be tracked or rejected in accordance with the problem specification being either a disturbance rejection problem or a tracking problem

DOI: 10.5121/ijctcm.2019.9201

[13][15]. Exosystems could also in practice be known or unknown, they could also have constant or oscillator type signal profiles [16]. This variability in types of potentially realistic disturbance and reference signal profiles, add to the difficulty in developing nonlinear output feedback controllers for dynamic systems [17][18].

This difficulty is encountered as part of the solution process for rejecting the exogenous signal disturbances or tracking the reference signals from the exosystem in practice but in theory this difficulty is in finding a closed form solution to the regulator (or Francis-Byrnes-Isidori) equations [12] [19] for different system structures such as linear or nonlinear, square or nonsquare, minimum or non-minimum phase, regular or non-regular [14] [20].

This work considers the RTAC system as a strictly SIMO model in all simulation experiments. Focus is placed on the stabilization via output feedback regulation of the nonlinear RTAC dynamic system that remains an actively researched benchmark system within the control community. The results for the two tested exosystem structures is also presented.

The organization of the rest of the paper is as follows: Section 2 introduces the RTAC system and the mathematical model, section 3 summarizes the output feedback problem for nonlinear system. Section 4 addresses the exosystem structure, section 5 treats the output regulator in detail, section 6 touches on the stable regulator or controller design, section 7 presents the important results and section 8 concludes.

2. THE RTAC SYSTEM

Practical research interest in the RTAC system stems from the fact that it is an underactuated coupled nonlinear system with four states and a single input. This paper treats the RTAC as a SIMO system under observer based output regulation. It focuses on the internal model component and observer estimator. The main contribution was in the regulator formulation without squaring the plant. Figure 1 shows the structure of the RTAC and some of its parameters.

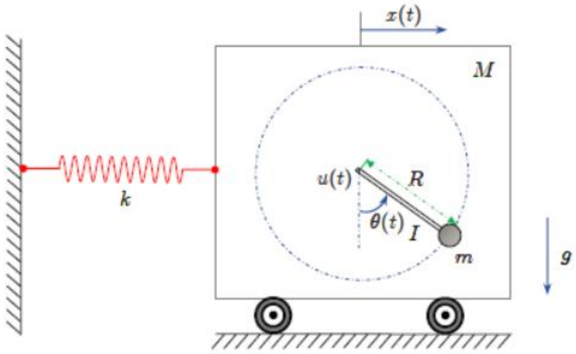

Fig. 1: RTAC System

The pendulum is the actuated member while the proof mass is driven forward and backward. This pendulum position is determined by the designed controller while the proof mass is placed at specific positions with the influence of disturbances on the proof mass taken into account.

2.1. RTAC EQUATIONS

The nonlinear RTAC model used by [21] was modified in [22] as follows

$$
\begin{aligned}
&(M+m)\ddot{q}+kq = ml\sin\theta\dot{\theta}^2 - ml\cos\theta\ddot{\theta}+F \\
&(I+ml^2)\ddot{\theta} = ml\cos\theta\ddot{q}+N
\end{aligned} \tag{1}
$$

where M, is the translation or proof mass, m, the mass of pendulum bob, l, the length of the pendulum arm, I, mass moment of inertia of the pendulum arm, g, acceleration due to gravity, θ, the displacement angle from the vertical and x, is the translational displacement of M. The nonlinear equation (2) was rearranged into the following form:

$$
\begin{aligned}
&\dot{x}_1 = x_2 \\
&\dot{x}_2 = -\frac{M_2 k x_1}{D_{nl}} + \frac{mlM_2 x_4^2 \sin x_3}{D_{nl}} - \frac{mlN\cos x_3}{D_{nl}} + \frac{M_2 F}{D_{nl}} \\
&\dot{x}_3 = x_4 \\
&\dot{x}_4 = \frac{mlkx_1\cos x_3}{D_{nl}} - \frac{m_2 l_2 x_4^2 \sin x_3 \cos x_3}{D_{nl}} - \frac{M_1 N}{D_{nl}} - \frac{mlF\cos x_3}{D_{nl}}
\end{aligned} \tag{2}
$$

Subsequent linearization of the above nonlinear equation was used for the initial analysis and design of the output feedback regulator.

$$
\begin{aligned}
&\ddot{q} = \frac{-M_2 kq}{Dl} - \frac{mlN}{Dl} + \frac{-M_2 F}{Dl} \\
&\ddot{\theta} = \frac{mlkq}{Dl} + \frac{-M_1 N}{Dl} - \frac{-mlF}{Dl}
\end{aligned} \tag{3}
$$

also expressed in state space form as

$$
\begin{aligned}
&\dot{x}_1 = x_2 \\
&\dot{x}_2 = -\frac{M_2 k x_1}{D_{nl}} - \frac{mlNx_3}{D_{nl}} + \frac{M_2 F}{D_{nl}} \\
&\dot{x}_3 = x_4 \\
&\dot{x}_4 = \frac{mlkx_1}{D_{nl}} - \frac{M_1 N}{D_{nl}} - \frac{mlF}{D_{nl}}
\end{aligned} \tag{4}
$$

The linearization was necessary to engage in any constructive development of the output feedback regulator. The working matrices were obtained from the RTAC structure in (4).

3. OUTPUT REGULATION SETUP

As presented in [23], the generalized nonlinear system for output regulation was given by

$$
\begin{aligned}
&\dot{x} = f(x,u,w) \\
&\dot{w} = s(w) \\
&y = h(x,u,w)
\end{aligned} \tag{5}
$$

with analytic, input affine form given as

$$
\begin{aligned}
&\dot{x} = f(x) + g(x)u + g(x)w \\
&\dot{w} = s(w) \\
&y = h(x) + q(w)
\end{aligned} \tag{6}
$$

A particular linear equivalent of the setup in equation (6) can be taken as presented by [24]

$$\begin{aligned} \dot{x} &= Ax + B_u u + P_w v \\ \dot{v} &= Sv \\ y &= Cx + Du + Qv \end{aligned} \tag{7}$$

Equation (7) represent plant dynamics, exosystem and output respectively whose working parameters discernible from equation (4). The variables x, u, v in equation (7) are in general vector valued with sizes $x \in R^n$, $u \in R^m$ and $v \in R^q$. While the matrices $A \in R^{nxn}$, $B \in R^{nxm}$, $P \in R^{nxq}$, $S \in R^{qxq}$, $C \in R^{pxn}$, $D \in R^{pxm}$, $Q \in R^{pxq}$. Here the exosystem plays the role of a signal generator, which encapsulates the possible class of reference and disturbance signals to be tracked or rejected by the system. In [24] the class of reference and disturbance signals generated with the different initial states of the exosystem were shown to depend on the operator S. This dependence of the system response on the exosystem structure will be explicitly shown in later part of this paper through analysis and results.

3.1 Output Feedback Regulator for the RTAC

The RTAC analysis utilized the following parameters taken from Sun *et al*., (2016) [9]

Table 1: Table of Parameters for the CIP.

Parameter	Value	Unit
Proof mass (M)	3.82	Kg
Pendulum mass (m)	0.5	Kg
Pendulum length(l)	0.12	m
Mass MOI (I)	0.0003186	Kg/m2
Spring constant (ks)	427	
Acc. due Gravity (g)	9.81	m/s2

3.2. State Space Realization of the RTAC

From the linear system given by equation (7), the working matrices for the values in Table 1 are derived as follows:

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 111.167 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 887.347 & 0 & 0 & 0 \end{pmatrix}, B = \begin{pmatrix} 0 \\ -2.0781 \\ 0 \\ 149.623 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 0 & 1 & 1 \end{pmatrix}$$

The uncertain disturbance term on the internal state dynamics and the output is captured by the matrices P and Q respectively

$$P = \begin{pmatrix} -0.649 & -0.8456 \\ 1.1812 & -0.5727 \\ -0.7585 & -0.5587 \\ -1.1096 & 0.1784 \end{pmatrix}, Q = \begin{pmatrix} -0.1969 & 0.8003 \\ 0.5864 & -1.5094 \\ -0.8519 & 0.8759 \end{pmatrix}$$

the disturbance P and Q are selected to be random noise terms with the requirement being that they must not be zero or sparse so as to avoid singular values or not-a-number (NaN) errors during simulation.

4. EXOSYSTEM SELECTION

The exosystem generates the reference signals and disturbances fed into the system. Certain conditions must however be satisfied for the exosystem to be acceptable as a signal generator for the output regulated setup. These conditions have been extensively discussed in the work of [24]. Generally, the selected exosystem must be antistable. One specific example of this antistable behaviour means the exosystem satisfies the Poisson stability criterion [10] [11]. However, other types of exosystem structures have been considered and used such as anti-Hurwitz stable (having non-negative real part for its eigenvalue) [25], non-negative real part with semi-simple map [26].

An often utilized form of the exosystem employs a linear time invariant (LTI) dynamic system [27]

$$\begin{aligned} \dot{v} &= Sv; v(0) = v_0 \in W \\ w &= Ev \\ y_{ref} &= -Fv \end{aligned} \tag{8}$$

Where $\dot{v}$ is the dynamic ODE representing the exosystem, w is the disturbance signal generated from the exosystem dynamic and y_{ref} is the reference signal generated accordingly. The system in (8) generates the reference signals and disturbances and have S, E and F appropriately sized. The exosystem map S, satisfies the condition given by $\mathrm{Re}(\sigma(S)) = 0$, being read as; the spectrum of the matrix S having all real parts equals zero. Implying, all eigenvalues of the chosen map S, must be Poisson stable or have their occurrence on the imaginary axis of the complex plane.

Although linear exosytems structures such as equation (8) are easier to work with and more tractable in solving the full system regulator equations [11], practical occurrences of the exosystem demand that nonlinear structures be also considered [28]. Also considered in certain implementations of the regulation problem is the availability or observability of the exosystem states under constrained conditions. The observed states usually used in feedforward arrangement [9] [25]. The current treatment has however investigated linear time invariant exosystem structures.

This work tested two different exosystem structures. The first is a constant-type disturbance generator, the second was a 2x2 dimension disturbance generator model. The first exosystem candidate selected was $\dot{w} = 0$

$$\begin{pmatrix} \dot{w}_1 \\ \dot{w}_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} \tag{9}$$

Here $S_{exo} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, which has a constant matrix value.

Another exosystem candidate has the oscillator of the form

$$\begin{pmatrix} \dot{w}_1 \\ \dot{w}_2 \end{pmatrix} = \begin{matrix} w_2 \\ -w_1 \end{matrix} \tag{10}$$

With $S_{exo} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. In both cases, the states w_1 and w_2 can be used for the disturbance and reference signal profiles respectively to be injected into (8).

5. Output Feedback Regulator Analysis

A regulator was constructed having the output error feedback form. The synthesis requires the construction of a compensator of the form

$$\begin{aligned} \dot{z} &= G_1 z + G_2 e \\ u &= Kz \end{aligned} \tag{11}$$

Where K = [K_x K_v]. The following set of parameters were designed for K_x, K_v, L1, L2, G1, G2, so that the following conditions hold:

i. The pair A, B is stabilizable by Kx s.t. is Hurwitz

ii. There exists a unique solution for X and U to the regulator equation pair

$$\begin{aligned} XS_{exo} &= AX + Bu + P \\ 0 &= CX + Q \end{aligned} \tag{12}$$

Such that the disturbance feedback gain Kv was computable as: $K_v=U-K_xX$

iii. finally the internal model-based observer was synthesized using the parameters obtained from the regulator equations in equations (29) and the selected minimal subset of the matrix

$$A_l = \begin{pmatrix} A - L_1 C & P - L_1 Q \\ -L_2 C & S - L_2 Q \end{pmatrix} \tag{13}$$

for a solution to L1, L2 and subsequently G1 and G2 for a complete solution to the controller design.

iv. All the previous computations led to the closed loop system given by

$$A_{clsys} = \begin{pmatrix} A & BK_{xv} & P \\ G_2 C & G_1 & -G_2 Q \\ 0 & 0 & S_{exo} \end{pmatrix} \tag{14}$$

, from which we derived the subset matrices

$$A_{cllp} = \begin{pmatrix} A & BK_{xv} \\ G_2C & G_1 \end{pmatrix} \tag{15}$$

and Sexo (for any selected exosystem structure) whose closed loop internal stability is deduced by checking the Hurwitz condition of (15) such that $spec(Acllp) \subset C^{-}$ and $spec(S_{exo}) \subset C^{0}$.

6. Stability Analysis and Controller Design

Considering the RTAC system as a SIMO model with target output selected as $y = (x_1,\ x_3,\ x_4)$, suitable generalized scalar Lyapunov function for the chosen outputs becomes

$$\mathrm{V(x)} = x_1^T P x_1 + x_3^T P x_3 + x_4^T P x_4 \tag{16}$$

satisfying the Lyapunov equation PA+A^{T}P+I=0, where P is a positive definite and symmetric (PDS) matrix, *I* is the identity matrix of suitable size and A, the stable transmission matrix from linearization of (2). However the RTAC system is considered a SIMO model with more specific Lyapunov function given by

$$V(x,t) = \frac{1}{2}(x_{1r} - x_1)^2 + \frac{1}{2}(x_{3r} - x_3)^2 + \frac{1}{2}(x_{4r} - x_4)^2 \tag{17}$$

Equation (17) met the following *conditions* for Lyapunov function construction in continuous time non-autonomous systems (CTNAS):

1. $V(0,t_0) = 0$
2. $V(x, t) > 0 \quad \forall x \neq 0 \in D,\ t \geq t_0$
3. there exists class K functions α(.), β(.) and γ(.) which satisfy the inequality

$$\alpha(\|x\|) \leq V(x,\ t) \leq \beta(\|x\|), \forall t \geq t0$$

4. and finally $\dot{V}(x,\ t)$ satisfies the inequality

$$\dot{V}(x,t) \leq -\gamma(\|x\|) < 0, \forall t \geq t0$$

Equation (17) satisfied conditions (1)-(4) and satisfactorily generated desired control signals.

7. Experiments and Results

The computed feedback gain that stabilized the closed loop system $A + BF$ was obtained as F= [237.3198, 17.3309, 91e-003, 48.2e-003]. The FBI regulator equations were solved for X and U and the following results were obtained as solution to the FBI equations with the oscillator exosystem

$$X_{osc} = \begin{pmatrix} 196.8614e-003 & -800.321e-003 \\ 0.0 & 0.0 \\ -586.4426e-003 & 1.509 \\ 851.887e-003 & -875.874e-003 \end{pmatrix}$$

$$U_{osc} = \begin{pmatrix} -1.1519e+000 & 4.7416e+000 \end{pmatrix}$$

with the constant exosystem, U was obtained as

$$U_{const} = \begin{pmatrix} -1.1577 & 4.7359 \end{pmatrix}$$

these solutions were unique and proved the existence of the regulator. The computed solutions also made possible the computation of a unique value for the internal model gain K_v, which was obtained as K_v = [−111.2459 452.2505] when oscillator-type exosystem was used and K_v = [−111.2517 452.2448] when constant-type exosystem was applied. The input vector was formed as K_{xv} = [K_x K_v] in both cases. Other parameters computed include the dynamic output gain L1, L2, G1, G2, which for space constraint has been moved to the appendix. Simulated experiments were made to show the uncompensated, compensated and combined responses of the RTAC under initial perturbation.

Starting with an initial condition, IC = (0.5 0 0.75 0), the unforced closed loop response was unstable. This necessitated development of a feedback regulator.

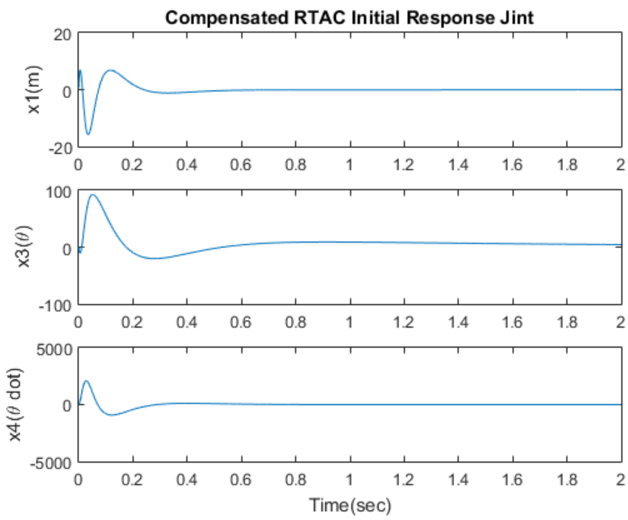

Fig. 2: Compensated Initial Response for IC = (0.5 0 0.75 0)

Fig. 2, shows the compensated response with the feedback and output gains acting to make the internal model and plant stable.

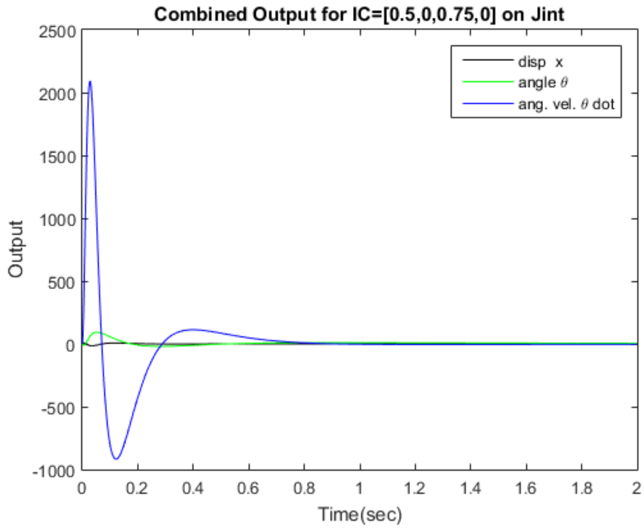

Fig. 3: Combined Initial Response for IC = (0.5 0 0.75 0)

Fig. 3, shows the combined compensated response with the internal model acting and all states stabilizing under 1.175sec.

For IC = (1 0 − 0.5 0), a second experiment was made to test for internal stability of the designed controller.

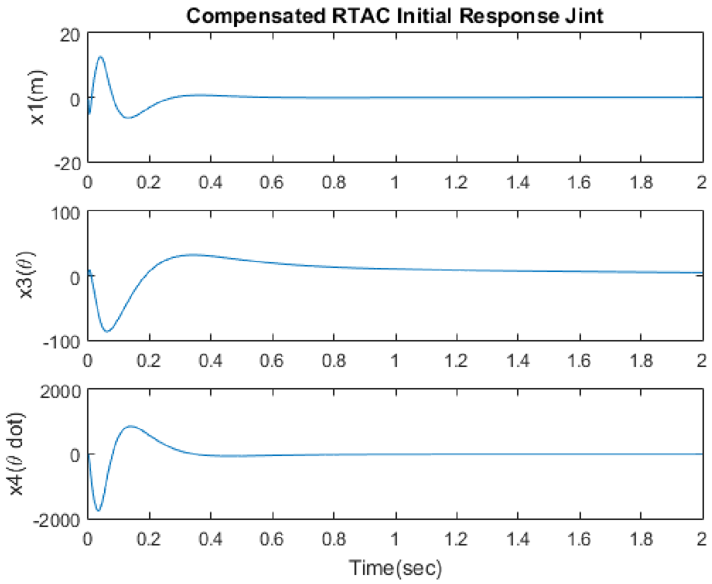

Fig. 4: Compensated Initial Response for IC = (1 0 − 0.5 0)

Fig. 4 gives the initial response when the states were perturbed by (1.0m 0 π/4rad 0rad/s).

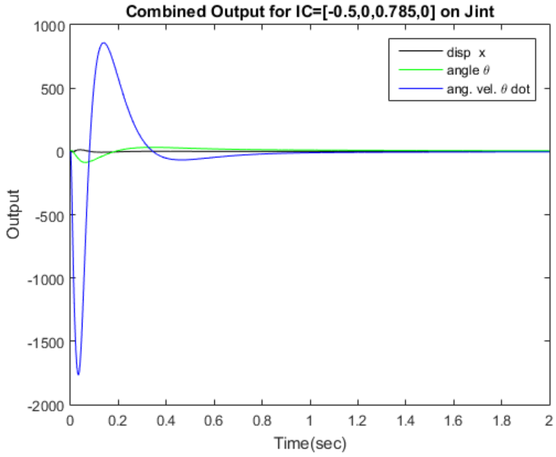

Fig. 5: Combined Initial Response for IC = (1.0m 0 π/4rad 0rad/s).

Fig. 5 shows the combined plot of the initial perturbation response of the RTAC system showing relative settling times of the various states.

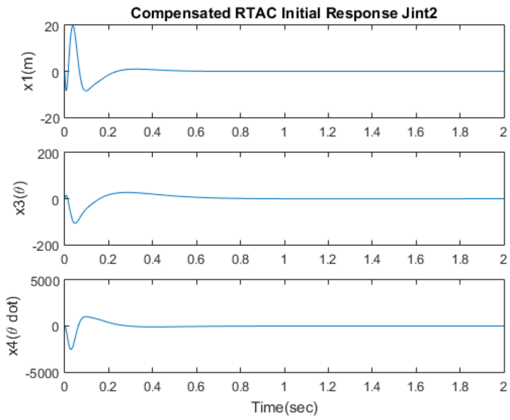

Fig. 6: Compensated Initial Response for IC = (1.0m 0 π/4rad 0rad/s)

Fig. 6 gives the initial response when the states were perturbed by (1.0m 0 π/4rad 0rad/s)

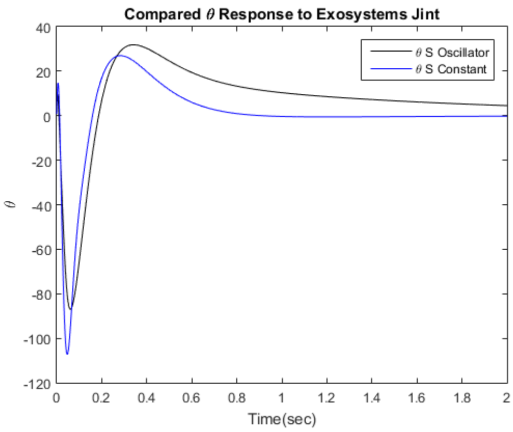

Fig. 8: Angular Position Response for Two Exosystem Types

The effect of selecting either the oscillator or constant type exosystem is viewed through the angular response when an initial perturbation was applied. The oscillator type exosystem showed a slowly decaying behavior compared to the constant type exosystem response which converged faster from within a local region of the origin. Fig. 8, shows the angular response of the RTAC system from application of the two types of exosystem structures. The constant exosystem showed faster convergence to the equilibrium in 2.5sec. While the oscillator-type exosystem had a slow convergence profile which only settled to the equilibrium after 45sec in the experiments conducted.

8. CONCLUSION

The RTAC benchmark control system has been experimented with in simulation. An output feedback regulator stabilizer was synthesized and put in place to stabilize the system internal dynamics. It was seen that from any starting condition, the system settles down to its stable equilibrium within 1.175sec from start of the simulation. This stability has been achieved with the aid of carefully designed gains which utilized both pole placement and polynomial matching for its synthesis. The system was therefore made stable by output regulation. The effect of a constant and oscillator-type exosystem were also considered. The constant type exosystem in equation (9) settled down faster to its equilibrium in 2.5sec while the oscillator-type exosystem described in equation (10) settled down after 45sec. However both attained asymptotic stability in their response profiles.

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APPENDIX A: INTERNAL MODEL GAINS

The Luenberger observer parameters for the oscillator-type exosystem experiment is as given by the variables G1 and G2

$$G1 = \begin{pmatrix} -168 & 1 & 0 & 0 & 32.424 & -135.3 \\ -8172.2 & -105.96 & -2.139 & -0.777 & 1633.7 & -6637.2 \\ 0 & 0 & -49 & 0 & -28.642 & 72.53 \\ 83892.7 & 7628.8 & 154.01 & -77.041 & -16.36e3 & 66840.6 \\ -15.35 & 0 & -11.46 & -34.913 & 20 & -22.65e-15 \\ -35.45 & 0 & 41.67 & 35.71 & -3.11e-15 & -60 \end{pmatrix}$$

$$G2 = \begin{pmatrix} 168 & 0 & 0 \\ 7.12e3 & 0 & 0 \\ 0 & 49 & 1 \\ 887.35 & 0 & 133 \\ -15.35 & 11.46 & 34.912 \\ 35.45 & -41.674 & -35.71 \end{pmatrix}$$

While the corresponding Luenberger observer parameters for the constant-type exosystem experiment is as given by the variables G1c and G2c

$$G1c=\begin{pmatrix} -168 & 1 & 0 & 0 & 32.424 & -135.3 \\ -8172 & -105.96 & -2.14 & -777.21e-3 & 1.634e3 & -6.64e3 \\ 0 & 0 & -49 & 0 & -28.64 & 72.53 \\ 83.893e3 & 7.63e3 & 154.01 & -77.04 & -16.36 & 66.84 \\ -400 & 0 & 0 & 0 & 78.745 & -320.13 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -144 & 0 & -84.45 & 217.35 \\ 0 & 0 & 0 & -61.42 & 52.323 & -53.796 \end{pmatrix}$$

$$G2c=\begin{pmatrix} 168 & 0 & 0 \\ 7118.2 & 0 & 0 \\ 0 & 49 & 1 \\ 887.35 & 0 & 133 \\ 400 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 144 & 0 \\ 0 & 0 & 61.42 \end{pmatrix}$$