Media bias and the Hotelling game

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We present a game theoretic model of media bias based on the Pure Location Hotelling Game. Contrary to the usual restrictive assumption of inelastic demand, we allow that demand is elastic and introduce in this context the concepts of quite inelastic and quite elastic demand. The real world interpretation of the media bias model is explained in detail for its discrete variant for unlimited media providers and arbitrary distribution of individuals.

Keywords: Hotelling game, media bias, quite (in-)elastic demand, potential game

1 Introduction

Game theory very benefited by analysing concrete games like Hotelling games. This makes that such games have quite a high status in the game theoretic literature. By 'Hotelling games' one understands a variety of games that appeared in the literature after the seminal article of Hotelling [1] dealing with a continuous model concerning the location and price problem of two competing retailers selling a homogeneous product in a 1-dimensional geographic market of consumers. In modern treatments where prices are contingent on location choice, this classical model has become a two-stage game where each stage concerns a game in strategic form with real intervals as strategy sets; the first stage concerns the retailers location choice and the second their price choice. The game in the first stage usually is referred to as a pure location Hotelling game.

In the mathematical structure of various Hotelling games the pure location game can be identified in some sense, while its real world interpretation may be quite different from the classical one. Using Nash equilibrium as solution concept, the central theme in the classical pure location Hotelling game is the 'Principle of Minimum Differentiation' which states that the retailers, with maximising market share as goal, locate in the

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middle of the market. It is important to note that this result, as shown in [2, 3] depends on the fact that in this game demand is inelastic.

One of the powers of Hotelling games is that they can cope with various real world phenomena. For example, interpreting in the pure location Hotelling game the (common) strategy set, i.e. the set of locations, as a range of political opinions varying from extreme left to extreme right, one becomes a game theoretic model on electoral competition (see, e.g., [4, 5]). Again using Nash equilibrium as a solution concept, the central theme there is the 'Median Voter Theorem', which states an outcome where candidates are located at the median of the voter distribution. Another game based on the pure location Hotelling game deals with strategic forecasting ([6]).

For the present article, the articles [7, 8] where media bias is modelled with continuous Hotelling games with two players are relevant. [8] deals with a two-stage game (modelling opinion and pricing) and [7] with a three-stage game (modelling opinion, pricing and advertising). The solution concept of these games is the more sophisticated concept of subgame perfect Nash equilibrium. The media are important sources of information. In democracies, media like newspapers and television networks are supposed to be independent and to disseminate information as objectively as possible. However, in reality, media are very often politically biased, meaning that they take sides in favour of or against policies, political parties, candidates for public offices, ... For some literature dealing with these issues we refer to for instance [9, 10].

So, modelling media bias with Hotelling games is not new. The present article deals with this topic by using merely the pure location Hotelling game (that just has one stage). As far as we know such an variant is missing in the literature on media bias. As our article heavily relies on abstract results for the pure location Hotelling game, we use the opportunity to improve upon these results there. To one of our results we refer as the Media Bias Theorem (i.e. Theorem 4). This theorem provides an explanation for the real world observation that the Centric Media Principle (i.e. the pendant of the well-known Principle of Minimum Differentiation) is not so realistic. The Centric Media Principle uses the new concepts of quite inelastic and quite elastic demand which are motivated by the results in [2, 3]. It shows that the Centric Media Principle fails if demand is quite elastic.

The organisation of the article is as follows. As the set up and real world interpretation of our model is more transparent for a discrete one (having a finite number of individuals) than for a continuous one (having a continuum of individuals), we start in Section 2 with a 1-dimensional discrete model with an arbitrary finite number of players and explain in detail the (not so trivial) formula for the payoff functions and its real-world interpretation. Such a discrete model also is more realistic as, as a matter of fact the number of individuals in the real world is finite. Next, as the discrete model is very difficult to analyse, we transform this model into a continuous model with a general continuous decreasing demand function; we also assume (one or) two players. Choosing between a discrete or continuous model is not completely a matter of taste as the results may be quite different (see [11]).

Section 4 deals with two players (duopoly and cartel). Section 3 analysis the very simple case of one player (monopoly) whose results will be compared with the cartel situation. Our results heavily rely on those in [3, 12]. New results concern fully cooperative strategy profiles, Pareto efficiency and the price of anarchy.

2 Media Bias Game

2.1 Discrete model

In this subsection we set up the formal structure of our game theoretic model for media bias and explain in detail its real-world interpretation. The main real-world ingredients of the model are political opinions, media, (media) providers, individuals and subscriptions. As explained in the introduction, we start with a discrete model.

We assume that there are $m + 1$ political opinions $0, 1, \ldots, m$ where $m \geq 1$. One may interpret them from "extreme left" (i.e. political opinion 0) to "extreme right" (i.e. political opinion m). We refer to the $\mathcal{S} := \{0, 1, \ldots, m\}$ as political opinion space. For each political opinion $s \in \mathcal{S}$ we assume that there are $\lambda(s)$ individuals with this political opinion. We assume that there are $n \geq 1$ providers that provide media with a unique political opinion to which individuals can subscribe and that the amount of a subscription is represented by a number in $]0, 1]$. So a subscription is supposed to be a continuous good; an amount 1 represents a full subscription. Let $\mathcal{N} := \{1, 2, \ldots, n\}$ be the set of providers. The goal of each provider is the maximisation of subscriptions.

If $x_i \in \mathcal{S}$ is a political opinion choice of media provider $i \in \mathcal{N}$, then $\mathbf{x} = (x_1, x_2, \ldots, x_n) \in \mathcal{S}^n$ is referred to as a political opinion choice profile and we let $X := \{x_1, x_2, \ldots, x_n\}$, i.e. the political opinions offered by the providers. Next, given $\mathbf{x}$, we are going to consider to which media providers an individual subscribes and subsequently we explain the amount of these subscriptions. Well, we assume that an individual with political opinion $s \in S$ subscribes to providers who provide media with a political opinion closest to s; here distance of s to $x \in X$ is given by $|s - x|$. Let

$$N(s)$$

be the set of these providers. We assume, which is natural, that the amount of this subscription to each such provider is the same and that this amount only depends on the provider distance $d(s) := \min_{x \in X} |x - s|$ as follows: it is

$$f(d(s))$$

where $f : \{0, 1, \ldots, m\} \to \mathbb{R}$ is a given positive function with $f(0) = 1$ which is constant or strictly decreasing. Thus the amount of subscription of individuals with political opinion s is

$$\frac{\lambda(s) f(d(s))}{\# N(s)}$$

to each provider in $N(s)$. Allowing for a non-constant f makes the model much more realistic. We refer to f as demand function. We distinguish between two cases: the case of constant f is referred to as the inelastic case and the other one as elastic case.

The next thing to explain is total amount of subscription to provider $i \in N$. Here first the so-called hinterlands come into the picture. The hinterland of provider i is the set of individuals that subscribe to this provider; we denote it by H_i. Thus the amount of subscription to provider i equals

$$u_i = \sum_{s \in H_i} \frac{\lambda(s) f(d(s))}{\# N(s)}. \tag{1}$$

Assuming that the providers take their actions independently and simultaneously, and also that there is perfect information, the above model can be considered as a game in strategic form with $\mathcal{N}$ as player set, with $\mathcal{S}$ as (pure) strategy set for each player and with $u_i : \mathcal{S}^n \to \mathbb{R}$ given by (1) the payoff function of player i. We refer to this game as the discrete Media Bias Pure Location Hotelling Game.

Formula 1 for the payoff functions is very good for simulations with computer programmes like Maple and Netlogo. However, for a theoretical analysis as we intend here it is less this formula can be made much more explicit as we show now.

Well, the fundamental observation is that, because of the one dimensionality of the political opinion space $\mathcal{S}$, it holds that the set

$$X(s) := \{x_k \mid k \in N(s)\},$$

i.e. the set of political opinions s subscribes to has one or two elements. We refer to this number as the type of individual s. For $s \in \mathcal{S}$ and $k = 1, 2$, let

$$H_{i;k} := \{s \in H_i \mid s \text{ is of type } k\}.$$

It is not difficult to show that the above formula for u_i becomes

$$u_i(\mathbf{x}) = \sum_{s \in H_{i;1}} \frac{\lambda(s) f(d(s))}{\# N(x_i)} + \sum_{s \in H_{i;2}} \frac{\lambda(s) f(d(s))}{\# \cup_{x \in X(s)} N(x)}. \tag{2}$$

A discrete Media Bias Pure Location Hotelling Game is a symmetric game; we also refer to this property as player symmetry. That this is true is intuitively clear from its description; a formal proof is straightforward. Besides player symmetry, it is very important to realize that in case $\lambda(s) = \lambda(m - s)$ for all s there is another type of symmetry that we refer to as political opinion symmetry, meaning that for each medium provider i and political opinion choice profile $(x_1, \ldots, x_n)$

$$u_i(x_1, \ldots, x_n) = u_i(m - x_1, \ldots, m - x_n).$$

2.2 Continuous model

Handling the discrete Media Bias Game of the previous subsection is very difficult Besides the fact that we cannot use continuous optimization techniques, the second contribution in the formula in (2) complicates. This contribution concerns the payoffs of individuals which belong to two different hinterlands. We now shall consider the continuous variant of the discrete Media Bias Game.

In order to obtain a continuous model, we replace the political opinion space $\mathcal{S}$ and the set of individuals $\mathcal{S}$ by $[0,1]$. We refer to the political opinion $\frac{1}{2}$ as centrist. Next we replace the formula in (2) for the amount of subscription to provider i by

$$u_i(\mathbf{x}) = \int_{H_i} \lambda(z) \frac{f(|x_i - z|)}{\#N(x_i)} \, dz. \tag{3}$$

Here $H_i = \{z \in [0,1] \mid |z - x_i| \leq |z - x_j| \ (j \in N)\}$, $N(x_i) = \{j \in \mathcal{N} \mid x_j = x_i\}$ and the demand function $f : [0,1] \to \mathbb{R}$ is positive, continuous with $f(0) = 1$ and constant or strictly decreasing. Again we distinguish between two cases: the inelastic case (constant f) and the elastic case (strictly decreasing f). Note that in the formula in (4) there is no pendant of the second contribution in the formula in (2) which simplifies considerably. The reason is that this contribution is 0 due to the continuous nature of the model.

Also cases where there are more than 2 players and cases where individuals are not evenly distributed complicate. Therefore we further in this article shall deal 1 or 2 players and, by taking $\lambda(z) = M$, evenly distributed individuals. We simply refer further to this game as the Media Bias Game. The formula (3) now becomes

$$u_i(\mathbf{x}) = \frac{M}{\#N(x_i)} \int_{H_i} f(|x_i - z|) \, dz. \tag{4}$$

The interpretation of M is the mass of all individuals together. In the discrete variant this mass is $m + 1$, the number of individuals, and plays an important role. In the continuous variant it does not and one may take $M = 1$. But we shall not do this because of more natural interpretations of the results.

As in the discrete case, player and political opinion symmetry holds.

For handling the formula (4) for the payoff functions and the presentation of our results, it will be useful to define the function $\mathcal{L} : [0,1] \to \mathbb{R}$ by

$$\mathcal{L}(x) := \int_0^x f(z) \, dz. \tag{5}$$

In the inelastic case, $f = 1$ holds and therefore $\mathcal{L}(x) = x$.

As f is supposed to be positive and continuous, the function $\mathcal{L}$ is strictly increasing and differentiable, with derivative

$$\mathcal{L}' = f. \tag{6}$$

Thus in the case where f is elastic, $\mathcal{L}'$ is strictly decreasing and therefore $\mathcal{L}$ is strictly concave.

Lemma 1. *Suppose f is elastic.*

1. $\mathcal{L}(\frac{1}{2}) < 2\mathcal{L}(\frac{1}{4})$.
2. $\mathcal{L}(\frac{1}{4}) > \frac{1}{2}\mathcal{L}(x^\star) + \frac{1}{2}\mathcal{L}(\frac{1}{2} - x^\star)$. $\diamond$

Proof.— $\mathcal{L}$ is, by (6), strictly concave.

1. As $\frac{1}{4} = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot \frac{1}{2}$, we obtain as $\mathcal{L}(0) = 0$, as desired, $\mathcal{L}(\frac{1}{4}) > \frac{1}{2}\mathcal{L}(0) + \frac{1}{2}\mathcal{L}(\frac{1}{2}) = \frac{1}{2}\mathcal{L}(\frac{1}{2})$.
2. As $\frac{1}{4} = \frac{1}{2}x^\star + \frac{1}{2}(\frac{1}{2} - x^\star)$ and, by (13), $x^\star \neq \frac{1}{2} - x^\star$, we obtain, $\mathcal{L}(\frac{1}{4}) > \frac{1}{2}\mathcal{L}(x^\star) + \frac{1}{2}\mathcal{L}(\frac{1}{2} - x^\star)$. Q.E.D.

Our main results deal with a general demand function f. We shall illustrate these results for the specific demand function

$$f_w(z) := w^z \text{ with } 0 < w \leq 1. \tag{7}$$

So this function depends on a parameter w to which we refer as distance factor. For f_w, we have

$$\mathcal{L}(x) = \begin{cases} \frac{1}{\ln(w)}(w^x - 1) \text{ if } w \neq 1, \\ x \text{ if } w = 1. \end{cases} \tag{8}$$

As $f_w(z)$ is for $z \neq 0$ a strictly increasing function of w, it follows that $\mathcal{L}(x)$ is for $x \neq 0$ a strictly increasing function of w.

Below, when we illustrate with figures, we always shall assume $M = 1000$.

3 Monopoly

Consider the case of a monopoly, i.e. of one medium provider.

Lemma 2. $u_1(x_1) = M(\mathcal{L}(x_1) + \mathcal{L}(1 - x_1))$. $\diamond$

Proof.— As there is one provider, we have $\#N(x_1) = 1$ and $H_i = [0, 1]$. Thus the formula (4) becomes $u_1(x_1) = M \int_0^1 f(|x_1 - z|)\, dz = M(\int_0^{x_1} f(x_1 - z)\, dz + \int_{x_1}^1 f(z - x_1)\, dz) = M(\int_0^{x_1} f(z)\, dz + \int_0^{1-x_1} f(z)\, dz) = M(\mathcal{L}(x_1) + \mathcal{L}(1 - x_1))$. Q.E.D.

Theorem 1. *The Media Bias Game with one medium provider has a Nash equilibrium. Even:*

1. *in the inelastic case, each political opinion choice is a Nash equilibrium (and fully cooperative) with M subscriptions.*
2. *in the elastic case, only the centrist political opinion choice $\frac{1}{2}$ is a Nash equilibrium and has $2M\mathcal{L}(\frac{1}{2})$, which is less than M, subscriptions.* $\diamond$

Proof.— Of course, a Nash equilibrium is in this one player case nothing else than a maximiser of the payoff function u_1 given by Lemma 2. In the inelastic case, we have $u_1 = M$, and so every $x_1 \in [0, 1]$ is a maximiser and has payoff M. Now consider the elastic case. By Lemma 2, u_1 is, being a sum of strictly concave functions, strictly concave. This implies that u_1 has at most one maximiser. By (6) we have $u_1'(x_1) = M(f(x_1) - f(1 - x_1))$. Therefore $u_1'(x_1) = 0$ if and only if $f(x_1) = f(1 - x_1)$. As f is injective, it follows that $u_1'(x_1) = 0$ if and only if $x_1 = 1 - x_1$, i.e. if and only if $x_1 = \frac{1}{2}$. It follows that $x_1 = \frac{1}{2}$ is the unique maximiser of u_1. So the payoff at this x_1 is $2M\mathcal{L}(\frac{1}{2}) < 2M\frac{1}{2} = M$. Q.E.D.

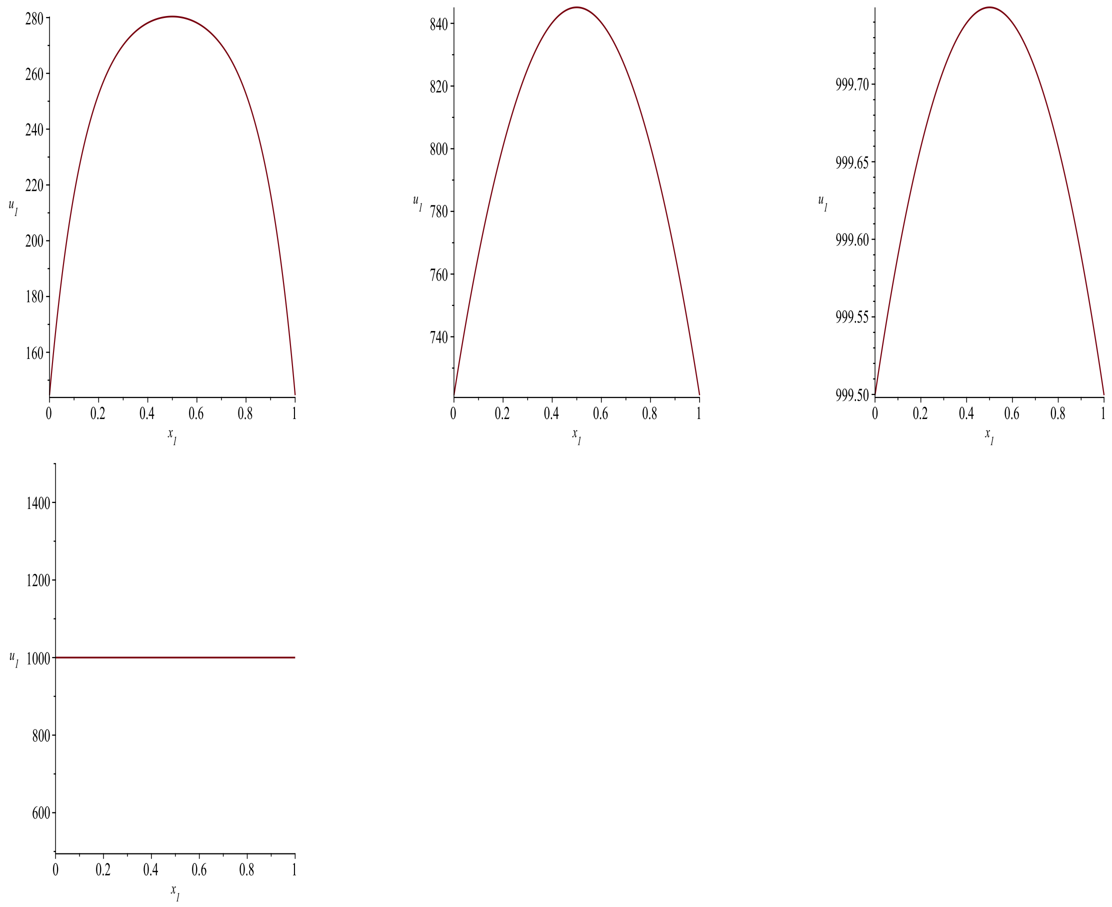

Figure 1: Monopoly. Graph of payoff function u_1 for (from left to right) $w = 0.001, w = 0.5, w = 0.999, w = 1$.

Thus in the monopolistic situation the centrist political opinion choice always is a Nash equilibrium.

For the special case of the demand function f_w in (7), the payoff function, according to formula in Lemma 2 and (8), is

$$u_1(x_1) = \begin{cases} M \frac{w^{x_1} + w^{1-x_1} - 2}{\ln(w)} \text{ if } w \neq 1, \\ M \text{ if } w = 1. \end{cases}$$

Figure 1 presents the graph of this payoff function for four values of w.

4 Cartel

Consider the case of a cartel, i.e. of two medium providers who collaborate in such way that they maximize the total amount of subscription. So their goal is to maximise the total payoff function, i.e. the function

$$U(x_1, x_2) = u_1(x_1, x_2) + u_2(x_1, x_2) = u_1(x_1, x_2) + u_1(x_2, x_1)$$

where u_1 is given by (10). So we obtain $U(x_1, x_2) =$

$$\begin{cases} \mathcal{L}(x_1) + \mathcal{L}(1 - x_2) + 2\mathcal{L}(\frac{|x_1 - x_2|}{2}) & \text{if } x_1 < x_2, \\ \mathcal{L}(x_2) + \mathcal{L}(1 - x_1) + 2\mathcal{L}(\frac{|x_1 - x_2|}{2}) & \text{if } x_1 > x_2, \\ \mathcal{L}(x_1) + \mathcal{L}(1 - x_1) & \text{if } x_1 = x_2. \end{cases} \quad (9)$$

This formula together with $\mathcal{L}(0) = 0$ implies that U is continuous. Therefore, as each strategy set is compact, the Weierstrass theorem implies that the game has a fully cooperative political opinion choice profile. But more can be said:

Theorem 2. *Consider the Media Bias Game with two collaborating medium providers.*

1. *In the inelastic case, each political opinion choice profile is fully cooperative with M total subscriptions.*
2. *In the elastic case, $(\frac{1}{4}, \frac{3}{4})$ and $(\frac{3}{4}, \frac{1}{4})$ are the fully cooperative political opinion choice profiles with $4M\mathcal{L}(\frac{1}{4})$, which is less than M, total subscriptions.* $\diamond$

Proof.— 1. Here $U = M$, which implies the desired result.

2. Let Δ_- be the set of strategy profiles (x_1, x_2) satisfying $0 \leq x_1 \leq x_2$ and denote by $\tilde{U}_-$ the restriction of U to Δ_-. Also let Δ_+ be the set of strategy profiles (x_1, x_2) satisfying $0 \leq x_2 \leq x_1$ and denote by $\tilde{U}_+$ the restriction of U to Δ_+. Using (9) and $\mathcal{L}(0) = 0$, we have

$$\tilde{U}_-(x_1, x_2) = \mathcal{L}(x_1) + 2\mathcal{L}(\frac{x_2 - x_1}{2} + \mathcal{L}(1 - x_2)).$$

First, we prove that $(1/4, 3/4)$ is a unique maximiser of $\tilde{U}_-$ with maximum $4M\mathcal{L}(\frac{1}{4})$. Well, $(1/4, 3/4)$ is an interior point of the domain of $\tilde{U}_-$ and in this point $\tilde{U}_-$ is partially differentiable with $D_1\tilde{U}_-(\frac{1}{4}, \frac{3}{4}) = Mf(\frac{1}{4}) - f(\frac{3/4-1/4}{2})) = 0$ and $D_2\tilde{U}_-(\frac{1}{4}, \frac{3}{4}) = M(-f(1- 3/4) + f(\frac{3/4-1/4}{2})) = 0$. As $\mathcal{L} : [0, 1] \to \mathbb{R}$ is by (6) strictly concave, it follows that $\tilde{U}_-$ is strictly concave. Therefore, it follows that $(\frac{1}{4}, \frac{3}{4})$ is a unique maximiser of $\tilde{U}_-$ with $\tilde{U}_-(\frac{1}{4}, \frac{3}{4}) = 4M\mathcal{L}(\frac{1}{4})$.

Having the above result and noting that $U(x_1, x_2) = U(x_2, x_1)$, it follows that $(3/4, 1/4)$ is a unique maximiser of $\tilde{U}_+$ with maximum $4M\mathcal{L}(\frac{1}{4})$. Now, it follows that the maximisers of U are $(\frac{1}{4}, \frac{3}{4})$ and $(\frac{3}{4}, \frac{1}{4})$ with $\max(U) = 4M\mathcal{L}(\frac{1}{4})$. Finally, noting that $\mathcal{L}(x) \leq x$ and that this inequality is for $x \neq 0$ strict if f is elastic, we obtain, as desired, $4M\mathcal{L}(\frac{1}{4}) < M$. Q.E.D.

Note that in the elastic case the two fully cooperative political choice profiles are independent of the specific demand function!

The above cartel game situation is formally equivalent to the previous monopolistic one where the medium provider now sells not one medium but two media. Comparing Theorem 2 with Theorem 1 shows that in doing so the monopolist can not improve the amount of subscription in the inelastic case, but can in the elastic case as, by Lemma 1(1), $2\mathcal{L}(\frac{1}{2}) < 4\mathcal{L}(\frac{1}{4})$.

5 Duopoly

Next consider the case of a duopoly, i.e. of two medium providers. As the game is symmetric, we have the formula

$$u_2(x_1, x_2) = u_1(x_2, x_1)$$

This symmetry makes that, below we often can restrict ourselves to deal with medium provider 1. Choice symmetry here gives the formula

$$u_i(x_1, x_2) = u_i(1 - x_1, 1 - x_2).$$

As $\#N(x_i) = 1$ if $x_1 \neq x_2$, $\#N(x_i) = 2$ if $x_1 = x_2$, the hinterland of provider 1 becomes

$$H_1 = \begin{cases} [0, x_1] \cup [x_1, \frac{x_1+x_2}{2}] & \text{if } x_1 < x_2, \\ [\frac{x_1+x_2}{2}, x_1] \cup [x_1, 1] & \text{if } x_1 > x_2, \\ [0, x_1] \cup [x_1, 1] & \text{if } x_1 = x_2. \end{cases}$$

Having this, for provider 1, the formula (4) becomes $u_1(x_1, x_2)$

$$\begin{cases} = M(\int_0^{x_1} f(x_1 - z)\, dz + \int_{x_1}^{\frac{x_1+x_2}{2}} f(z - x_1)\, dz) \\ M(\int_{\frac{x_1+x_2}{2}}^{x_1} f(x_1 - z)\, dz + \int_{x_1}^{1} f(z - x_1)\, dz) \\ \frac{M}{2}(\int_0^{x_1} f(x_1 - z)\, dz + \int_{x_1}^{1} f(z - x_1)\, dz) \end{cases}$$

$$= \begin{cases} M(\mathcal{L}(x_1) + \mathcal{L}(\frac{|x_1-x_2|}{2}) & \text{if } x_1 < x_2, \\ M(\mathcal{L}(1 - x_1) + \mathcal{L}(\frac{|x_1-x_2|}{2}) & \text{if } x_1 > x_2, \\ \frac{M}{2}(\mathcal{L}(x_1) + \mathcal{L}(1 - x_1)) & \text{if } x_1 = x_2. \end{cases} \tag{10}$$

So the payoff of medium provider 1 does not only depends on his own choice, but also of that of his opponent. This makes that this situation becomes, contrary to the monopoly situation in the previous section, a real game theoretic one.

Below we shall distinguish between the relatively simple inelastic case and the much more difficult elastic case. Among other things we shall analyse the conditional payoff functions. Remember that the conditional payoff function $u_i^{(z)}$ of medium provider i is the function which describes his payoff as a function of his own choice x_i, given a choice z of his opponent j. So $u_1^{(z)}(x_1) = u_1(x_1, z)$ and $u_2^{(z)}(x_2) = u_2(z, x_2)$. Because of player symmetry, we have the formula $u_1^{(z)} = u_2^{(z)}$ and because of choice symmetry we have the formula $u_i^{(1-z)}(x) = u_i^{(z)}(1 - x)$. And the best-response correspondence of player i is defined as the correspondence $B_i : [0, 1] \multimap [0, 1]$, that assigns to every choice of his opponent j the set of best choices of player i. Because of player symmetry, we have

$$B := B_1 = B_2.$$

And political opinion choice symmetry makes that

$$B_i(x_j) = 1 - B_i(1 - x_j).$$

5.1 Inelastic case

Consider the inelastic case, i.e. the case where f is constant. As now $\mathcal{L}(x) = x$, (10) becomes

$$u_1^{(x_2)}(x_1) = \begin{cases} M\frac{x_1+x_2}{2} & \text{if } x_1 < x_2, \\ M(1 - \frac{x_1+x_2}{2}) & \text{if } x_1 > x_2, \\ \frac{M}{2} & \text{if } x_1 = x_2. \end{cases} \tag{11}$$

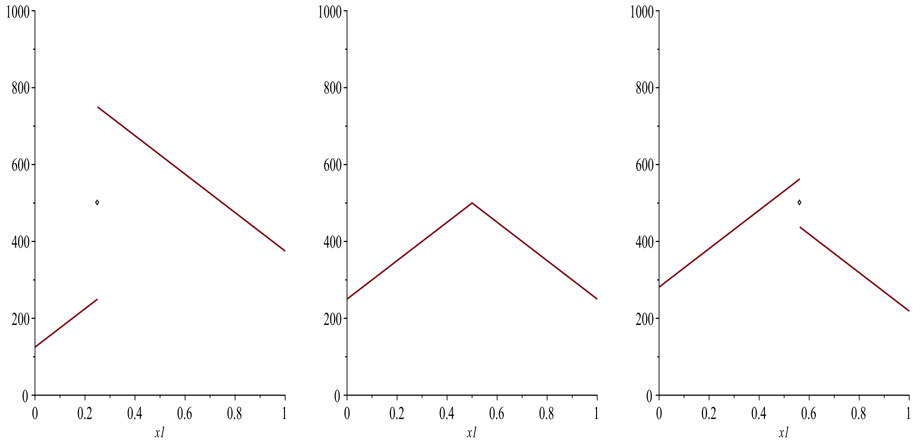

Figure 2: Duopoly; inelastic case. Graph of conditional payoff function $u_1^{(x_2)}$ for $w = 1$ with (from left to right) $x_2 = 1/4, x_2 = 1/2$, $x_2 = 9/16$.

From this formula we see that the conditional payoff function $u_1^{(x_2)}$ is continuous if and only if $x_2 = 1/2$, i.e. if and only if the opponent provides a medium with a centrist political opinion. In addition if $x_2 \neq 1/2$, then this function is only is discontinuous at $x_1 = x_2$. This discontinuity complicates the analysis of the game (although for the inelastic case here, things still are relatively easy). Also we see that $u_1^{(x_2)}$ is strictly increasing on $[0, x_2\,[$ and strictly decreasing on $]x_2, 1]$. Figure 2 illustrates these results with the graph of the conditional payoff functions $u_1^{(1/4)}, u_1^{(1/2)}$ and $u_1^{(3/4)}$.

Now let us have a look to the best-response correspondence B. There are three possibilities for this set: (1) it is a singleton, i.e. consists of one element, (2) it contains more than one element, (3) it is empty.

By analysing formula (11) one finds

$$B(x) = \begin{cases} \emptyset \text{ if } x \neq \frac{1}{2}, \\ \{\frac{1}{2}\} \text{ if } x = \frac{1}{2}. \end{cases} \tag{12}$$

Note that this finding is supported by Figure 2. Having this result for the best-response correspondence, we are ready to determine the Nash equilibria.

The result in the next theorem can be seen as the pendant of the Principle of Minimum Differentiation for the classical Hotelling pure location game or as the pendant of the

Medium Voter Theorem for the Hotelling voting game. In order to formulate this theorem we introduce the following terminology for a Media Bias Game with two medium providers: the Centric Media Principle holds if the game has $(1/2, 1/2)$ as unique Nash equilibrium.

Theorem 3. *For the Media Bias Game with two medium providers in the inelastic case the Centric Media Principle holds and in the unique equilibrium* $(1/2, 1/2)$ *each medium provider has* $\frac{M}{2}$ *subscriptions.* $\diamond$

Proof.— Remembering that (x_1, x_2) is a Nash equilibrium if and only if $x_2 \in B(x_1)$ and $x_1 \in B(x_2)$, the result follows from (12) and (11). Q.E.D.

Quite inelastic and quite elastic cases

As may be expected, game theoretic results for the Media Bias Game depend on the properties of the demand function f. The previous section dealt with the simple case of an inelastic, i.e. constant, f. In the present subsection we also consider the elastic case, i.e. the case where f is strictly decreasing. The analysis of this case is much more difficult. Therefore we here often will refer to the literature for a proof of our statements.

It turns out to be useful to introduce for a Media Bias Game the following further terminology. We speak of

- the quite inelastic case if $f(\frac{1}{2}) \geq \frac{1}{2} f(0)$;
- the quite elastic case if $f(\frac{1}{2}) < \frac{1}{2} f(0)$.

Thus the inelastic case is a special case of a quite inelastic case, but the elastic case is compatible with both the quite inelastic as quite elastic case. For the demand function $f_w(z) = w^z$ the quite inelastic case comes, with

$$w_c := 1/4$$

down to $w \geq w_c$ and the quite elastic case to $w < w_c$.

The in the previous subsection mentioned monotonicity properties for the inelastic case do not continue to hold for the elastic case: See Figure 3 for the quite inelastic case and Figure 4 for the quite elastic case.

Define the function $F : [0, \frac{1}{2}] \to \mathbb{R}$ by

$$F(x_1) := f(x_1) - \frac{1}{2} f(\frac{1}{2} - x_1).$$

Note that $F(0) > 0$, F is decreasing, and strictly decreasing if f is not constant. Thus F has at most one zero. If $f(\frac{1}{2}) \leq \frac{1}{2} f(0)$, then $F(0) \leq 0$ which implies that then F has a unique zero; we denote this zero by

$$x^\star.$$

As $F(\frac{1}{4}) = \frac{1}{2} f(\frac{1}{4}) > 0$, we obtain

$$x^\star \begin{cases} \in \,]\frac{1}{4}, \frac{1}{2}[\text{ if } f(\frac{1}{2}) < \frac{f(0)}{2}, \\ = \frac{1}{2} \text{ if } f(\frac{1}{2}) = \frac{f(0)}{2}. \end{cases} \tag{13}$$

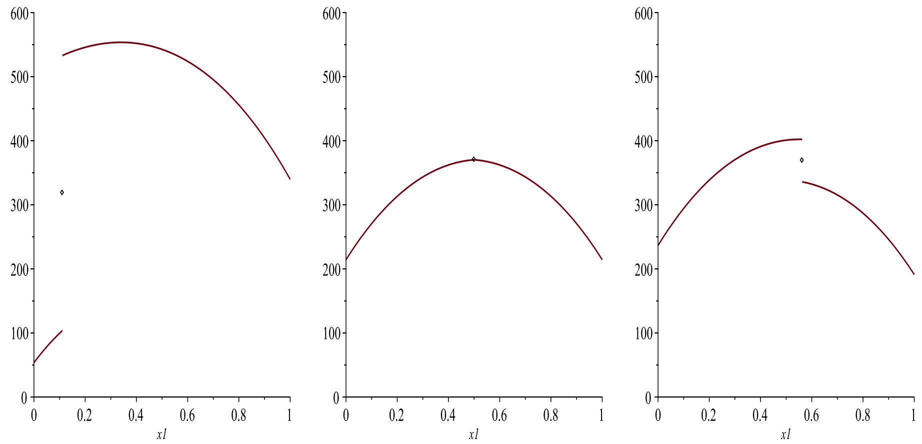

Figure 3: Duopoly. Graph of conditional payoff function $u_1^{(x_2)}$ for $w = 9w_c/8$ with (from left to right) $x_2 = 1/9, x_2 = 1/2$, $x_2 = 9L/16$.

In order to prove the next theorem in [12] the best-response correspondences are analysed.

Theorem 4. *Consider the Media Bias Game with two medium providers. This game has a Nash equilibrium. Even:*

1. *in the quite inelastic case, the Centric Media Principle holds and in the unique Nash equilibrium* $(\frac{1}{2}, \frac{1}{2})$ *each medium provider has* $M\mathcal{L}(\frac{1}{2})$ *subscriptions;*

2. *in the quite elastic case there are two Nash equilibria:* $(x^\star, 1-x^\star)$ *and* $(1-x^\star, x^\star)$*. In each of these equilibria each medium provider has* $M(\mathcal{L}(x^\star) + \mathcal{L}(\frac{1}{2} - x^\star))$ *subscriptions.* $\diamond$

Proof.— Theorem 5 in [12] gives the results for the Nash equilibria. Next, the subscriptions follow from (10). Q.E.D.

We refer to Theorem 4 as the Media Bias Theorem. Theorem 4 shows that for a continuous f, the Centric Media Principle holds if and only if the demand function is quite inelastic. In reality it is not so clear that this principle often holds; the Media Bias Theorem may provide a better description of reality.

Corollary 1. *Consider the Media Bias Game with two medium providers.*

1. *For each Nash equilibrium* (e_1, e_2) *it holds that* $e_1 + e_2 = 1$.

2. *For each Nash equilibrium* (e_1, e_2) *it holds that* $e_1, e_2 \in \,]\frac{1}{4}, \frac{3}{4}\,[$

3. *If* (e_1, e_2) *and* (e'_1, e'_2) *are Nash equilibria, then* $u_1(e_1, e_2) = u_2(e_1, e_2) = u_1(e'_1, e'_2) = u_2(e'_1, e'_2)$. $\diamond$

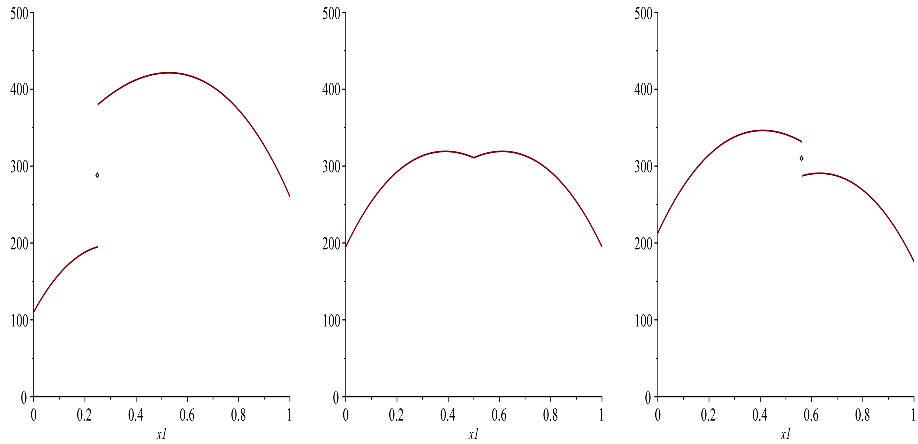

Figure 4: Duopoly; quite elastic case. Graph of conditional payoff function $u_1^{(x_2)}$ for $w = w_c/2$ with (from left to right) $x_2 = 1/4, x_2 = 1/2$, $x_2 = 9/16$.

Proof.— By Theorem 4 and (13). Q.E.D.

Again consider the special demand function f_w. Theorem 4(1) together with (8) imply that each medium provider has, for $w_c \leq w < 1$, $M\frac{w^{1/2}-1}{\ln(w)}$ subscriptions and $\frac{M}{2}$ subscriptions for $w = 1$. And for $0 < w \leq w_c$, we have that $x^\star$ in Theorem 4(2) equals $x_\star(w)$ where

$$x_\star(w) := \frac{1}{2}(\frac{1}{2} - \frac{\ln(2)}{\ln(w)}) = \frac{1}{2}(\frac{1}{2} - \log_w 2)$$

Note that $x_\star$ is a strictly decreasing function of w with $\lim_{w\downarrow 0} x_\star(w) = 1/4$. Thus for $w \downarrow 0$, the equilibrium set tends to $\{(\frac{1}{4}, \frac{3}{4}), (\frac{3}{4}, \frac{1}{4})\}$, i.e. to the two fully cooperative political opinion choice profiles (see Theorem 2(2)). Figure 5 shows the Nash equilibrium $(x_\star(w), 1-x_\star(w))$ as function of w. We clearly see that for $w < w_c$, the Nash equilibrium strategies of the players are different and that for $w \geq w_c$ both strategies are the same.

Remember that by Theorem 4 the amount of subscription to a medium provider is the same for each provider. By Theorem 4 and (8), for $w \geq w_c$ it equals

$$M\mathcal{L}(\frac{1}{2}) = \begin{cases} \frac{M}{\ln(w)}(w^{1/2} - 1) \text{ if } w \neq 1, \\ \frac{M}{2} \text{ if } w = 1 \end{cases}$$

and for $w < w_c$ it equals $M(\mathcal{L}(\frac{1}{4} - \frac{\log_w 2}{2}) + \mathcal{L}(\frac{1}{4} + \frac{\log_w 2}{2})) =$

$$\frac{M}{\ln(w)}\Big(\frac{w^{1/4}}{\sqrt{2}} + w^{1/4}\sqrt{2} - 2\Big). \tag{14}$$

Figure 6 shows this amount of subscription as a function of w.

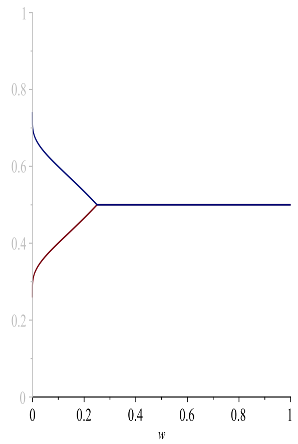

Figure 5: Duopoly. The Nash equilibrium $(x_\star(w), 1 - x_\star(w))$ as function of w.

Pareto efficiency

In economic games Nash equilibria are in general not Pareto efficient. In particular this holds for oligopolistic games. The next result picks up this question for our Media Bias Game.

Theorem 5. *Consider the Media Bias Game with two medium providers.*

1. *In the quite inelastic case, the unique Nash equilibrium* $(1/2, 1/2)$ *is strongly Pareto efficient.*

2. *In the quite elastic case, each Nash equilibrium is weakly Pareto inefficient. Even:* $(\frac{1}{4}, \frac{3}{4})$ *is an unanimous Pareto improvement of the equilibrium* $(x^\star, 1 - x^\star)$ *and* $(\frac{3}{4}, \frac{1}{4})$ *is an unanimous Pareto improvement of the equilibrium* $(1 - x^\star, x^\star)$. $\diamond$

Proof.— 1. As $u_1 + u_2 \leq M$ and $u_1(1/2, 1/2) = u_2(1/2, 1/2) = M/2$, it follows that $(1/2, 1/2)$ is fully cooperative and therefore strongly Pareto efficient.

2. By political opinion choice symmetry, it is sufficient to prove that $(\frac{1}{4}, \frac{3}{4})$ is an unanimous Pareto improvement of the equilibrium $(x^\star, 1 - x^\star)$. And for this in turn it is by player symmetry and political opinion choice symmetry sufficient to prove that $u_1(\frac{1}{4}, \frac{3}{4}) > u_1(x^\star, 1 - x^\star)$.

Well, by Theorem 4(2), $u_1(x^\star, 1 - x^\star) = M(\mathcal{L}(x^\star) + \mathcal{L}(\frac{1}{2} - x^\star))$ and by (10), $u_1(\frac{1}{4}, \frac{3}{4}) = 2M\mathcal{L}(\frac{1}{4})$. So the proof is complete by Lemma 1(2). Q.E.D.

Price of anarchy

The price of anarchy is a quite modern concept in economics and game theory ([13]). It measures how the efficiency of a system degrades due to selfish behaviour of its agents. It is a general notion that can be extended to diverse systems and notions of efficiency.

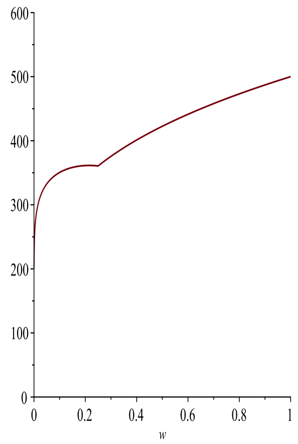

Figure 6: Duopoly. Nash equilibrium payoff of a medium provider at a Nash equilibrium as function of w.

In terms of our terminology, the price of anarchy comes down to the total amount of subscription in a fully cooperative political opinion choice profile divides by the minimal total amount of subscription in Nash equilibria.

With Theorem 2 and Theorem 4 we find that the price of anarchy, denoted by p_a, in the quite inelastic case equals

$$2\mathcal{L}(\frac{1}{4})/\mathcal{L}(\frac{1}{2}).$$

and in the quite elastic case

$$2\mathcal{L}(\frac{1}{4})/\Big(\mathcal{L}(x^\star) + \mathcal{L}(\frac{1}{2} - x^\star)\Big).$$

For the special demand function f_w, this gives, using (8) and (14), for p_a as a function of w

$$p_a(w) = \begin{cases} \frac{2(w^{1/4}-1)}{\frac{w^{1/4}}{\sqrt{2}}+w^{1/4}\sqrt{2}-2} \text{ if } w < w_c, \\ 2\frac{w^{1/4}-1}{w^{1/2}-1} = \frac{2}{w^{1/4}+1} \text{ if } w \geq w_c. \end{cases}$$

Figure 7 presents the graph of this function. We see that the price of anarchy is maximal at $w = w_c$.

5.2 Potentials

Recently, it has been investigated in [3] in which sense the continuous Hotelling pure location game is a potential game. We apply here some of the results in this article to our Media Bias Game.

Potential games are a special class of games in strategic form. There are many types: exact potential games, generalized ordinal potential games, best response potential games, quasi potential games, ... Potential games have various interesting properties, especially concerning improvement and best-response dynamics. Below we shall

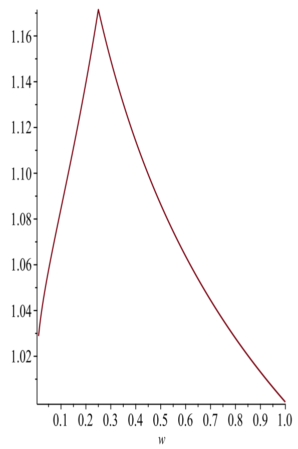

Figure 7: Duopoly. Graph of price of anarchy as a function of w.

only encounter best-response and quasi potential games. For the readers' convenience we now first remember various definitions and results for potential games in the case of two players.

For a game in strategic form with two players, strategy sets X_1 and X_2 and best-response correspondences $B_1 : X_2 \multimap X_1$ and $B_2 : X_1 \multimap X_2$, a function $P : X_1 \times X_2 \to \mathbb{R}$ is

- a best-response potential if for every $x_2 \in X_2$ it holds that $B_1(x_2) = \mathrm{argmax}_{x_1 \in X_1} P(x_1, x_2)$ and for every $x_1 \in X_1$ it holds that $B_2(x_1) = \mathrm{argmax}_{x_2 \in X_2} P(x_1, x_2)$.
- a quasi potential if the set of Nash equilibria coincides with the set of maximisers of P.

In these cases, one calls the game 'best-response potential game', quasi-potential game and respectively.

For more on potential games we refer to [14].

With $\mathcal{L}$ given by (5), define the function $P^\bullet \colon [0,1] \times [0,1] \to \mathbb{R}$ by $P^\bullet(x_1, x_2) :=$

$$\mathcal{L}(\min\{x_1, x_2\}) + \mathcal{L}(L - \max\{x_1, x_2\}) + \mathcal{L}(\frac{|x_2 - x_1|}{2}).$$

Note that $P^\bullet$ is continuous.

Theorem 6. *1. a) If $f(\frac{1}{2}) \leq \frac{1}{2} f(0)$, so in particular in the quite elastic case, $P^\bullet$ is a best-response potential.*

b) In the quite inelastic case, there does not exist a continuous best-response potential.

2. a) In the elastic case, $P^\bullet$ is a quasi-potential.

b) In the inelastic case, $P(x_1, x_2) = -(|\frac{1}{2} - x_1| + |\frac{1}{2} - x_2|)$ *is a continuous quasi-potential.* ⋄

Proof.— See Theorem 6 in [3]. Q.E.D.

6 Concluding Remarks

1. Hotelling Games have many applications. We reconsidered an under developed one: modelling media bias with pure location Hotelling games which allow for elastic demand. We set up such a game and referred to it as 'Media Bias Game'.

2. The Media Bias game has $[0, 1]$ as political opinion space, ranging from "extreme left" to "extreme right". For each political opinion there are individuals with this political opinion; they are evenly distributed on this space. We supposed one or two media providers who provides media with a unique political opinion (being the strategies) and whose goal is the maximisation of subscriptions. An individual subscribes to the providers who provide a political opinion closest to the own one.

3. For strategy profiles we investigated: Nash equilibria, being fully cooperative and Pareto efficiency. In addition we studied the price of anarchy and in which sense the game admits a potential.

4. For the demand function f, we used the notion of quite inelastic (i.e. $f(1/2) \geq f(0)/2$) and quite elastic (i.e. $f(1/2) < f(0)/2$) demand.

5. Each Media Bias Game has a Nash equilibrium (Theorems 1 and 4), but this may not be unique.

6. Theorems 1 and 2 show that a monopolistic provider can improve the amount of subscription in the elastic case by selling two instead of one medium, but cannot do so in the inelastic case.

7. The Centric Media Principle reads: there is a unique equilibrium and in this equilibrium each medium provider provides a medium with centrist (i.e. 1/2) political opinion. However, this principle only holds for the inelastic case.

8. We formulated the Media Bias Theorem (Theorem 4) which provides and explanation for the real world observation that the Centric Media principle (being the pendant of the classic Principle of Minimum Differentiation) may not hold. Sufficient and necessary for the Centric Media Principle to hold is that demand is quite elastic.

9. If (e_1, e_2) is an equilibrium, then $e_1 + e_2 = 1$ and $e_1, e_2 \in \,]1/4, 3/4\,[$ (Corollary 1). So extreme left and extreme right opinions cannot occur in an equilibrium.

10. In the quite inelastic case, the unique equilibrium $(1/2, 1/2)$ is strongly Pareto efficient and in the quite elastic case, each equilibrium is weakly Pareto inefficient (Theorem 5).

11. We studied the price of anarchy for the special demand function $f_w(z) = w^z$. This price is maximal for $w = 1/4$ which equals the value that distinguishes between quite inelastic and quite elastic demand.

12. The Media Bias game has a continuous quasi-potential. As strategy sets are compact this again implies that the game has a Nash equilibrium Theorem 6. Even: each maximiser of such a potential is a Nash equilibrium.

13. It would be interesting to redo the analysis for the more realistic case of a finite number of individuals; the model for this has been provided in Subsection 2.1. Recent results on abstract discrete pure location Hotelling games can be found in [15, 11] Dealing with more than two medium providers will be very challenging.

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