INTEGRATION OF MATHEMATICS INTO GAME DESIGN MECHANISMS FOR ENGAGING EDUCATION OF K-12

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ABSTRACT

This research demonstrates how integrating mathematics directly into game mechanics creates engaging educational experiences. We present Math Knight and Math Magics as complementary examples to demonstrate how core arithmetic operations can be embedded directly into gameplay mechanics, analyzing how mechanics such as fractional health systems, ability based weapons, and turn based combat contribute to the meaningful learning experience. Experimental results with elementary students show significant improvements in calculation accuracy, speed, and mathematical attitudes. The study illustrates how well designed games that embed learning objectives organically into gameplay can effectively address math anxiety, taking into account diverse learning styles and abilities, making mathematics more approachable for students who may struggle with traditional teaching methods. This inclusivity promotes educational equity, ensuring that every child, irrespective of their background or learning differences, has the opportunity to develop critical mathematical skills in a supportive and interactive environment.

KEYWORDS

Game-Based Learning, Math Integration, Roguelike Mechanics, Adaptive Difficulty, Arithmetic Proficiency, K-12 Education.

1. INTRODUCTION: TRANSFORMING MATH EDUCATION

Mathematics education faces a paradox. While recognized as fundamental to scientific and technological progress [1], traditional pedagogical approaches often engender anxiety and disengagement among K-12 students. The prevalent *"drill-and-practice"* methodology [2] has proven particularly ineffective at sustaining student interest, despite its systematic approach to skill development. This contradiction becomes increasingly problematic as society demands greater STEM literacy [3]. The emergence of digital learning tools presents an opportunity to redefine mathematics instruction. Research demonstrates that virtual environments can achieve educational outcomes comparable to traditional laboratories [4], while game-based approaches show particular promise for enhancing engagement [5]. However, most existing educational games commit a critical design flaw: they treat mathematical concepts as intrusive elements rather than organic gameplay components [6]. This fundamental misalignment between learning objectives and game mechanics undermines their pedagogical potential.

My cross-cultural educational experiences revealed a complementary insight. The Chinese system's rigorous practice regimen builds computational fluency but often suppresses creativity, while American pedagogy emphasizes engagement at the expense of systematic skill development [7]. This dichotomy suggested an opportunity. Could a game simultaneously deliver the motivational benefits of Western approaches with the methodological rigor of Eastern practice?

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Math Magics emerged from this synthesis. Unlike conventional educational games that interrupt play with math problems [8], our roguelike design embeds arithmetic operations directly into core mechanics, such as fractional health systems that require simplification to defeat enemies, weapon abilities that perform mathematical operations, and progressive level design introducing number sets from N to Q [9]. This approach aligns with Vygotsky's scaffolding theory [5] and the "Ongoing Learning Principle" [2], where concepts develop through graduated challenges. The game's reward system reinforces this progression, providing immediate feedback that maintains engagement while reinforcing mathematical understanding[10]. Preliminary studies with elementary students shown both improvement in arithmetic problem solving speed and reduction in computational errors, along with qualitative shifts from "math anxiety" to better perceptions of mathematics. These outcomes suggest that *Math Magics*' deep mechanical integration of mathematical concepts offers a viable alternative to traditional pedagogies. By transforming arithmetic operations, which are the most basic skills of mathematics, into gameplay elements, we create an environment where mathematical thinking emerges naturally from the desire to progress and succeed [11]. This paradigm shift shows positive results for addressing the challenges of chronic engagement in early mathematics education [12].

The following sections detail our challenges, technical implementation, methodology, and results, demonstrating how game mechanics can serve as powerful vehicles for studying mathematics when properly aligned with learning objectives and cognitive processes [13].

2. TECHNICAL AND PEDAGOGICAL HURDLES

Developing *Math Magics* presented significant challenges at the intersection of computational accuracy, pedagogical effectiveness, and game design. These hurdles required innovative solutions.

2.1. Computation Inaccuracy

A fundamental challenge was representing mathematical concepts accurately within digital systems. As noted by Merrikh-Bayat and Shouraki [14], computers inherently struggle with exact representation of rational numbers due to floating-point limitations. For instance, when a player divides 1 by 3, the result becomes 0.333... which when multiplied by 3 yields 0.999... rather than the mathematically precise value of 1. This discrepancy creates cognitive dissonance for learners [9]. Our solution implements an expression tree system that preserves fractions symbolically (e.g., representing 1/3 as $\frac{1}{3}$ rather than 0.333). This approach has many advantages. It maintains mathematical precision through symbolic computation; introduces fraction simplification mechanics organically; and Reinforces conceptual understanding of equivalence (e.g., $\frac{6}{4} = \frac{3}{2}$). As Wang et al. demonstrated [9], such tree-based representations align with human cognitive processing of mathematical operations, making them ideal for educational contexts.

2.2. Overcoming Mathematical Aversion

The pervasive perception of mathematics as "boring" or "difficult" presented a significant design challenge [2]. Research by Tobias et al. [5] confirms that negative attitudes toward math substantially impede learning outcomes. To address this, we implemented multiple engagement strategies:

- Narrative Integration: Mathematical operations became magic abilities. "subtraction wands" reduce enemy health while "division wands" fractionate opponents' health. This transforms abstract operations into tangible actions, creating what Potkonjak et al. [13] term "conceptual embodiment."

- Roguelike Motivation: The progression system leverages what Szabados et al. [10] identify as core roguelike engagement. Permanent death encourages experimentation, while incremental progression provides constant reinforcement. Each failed run becomes a learning opportunity rather than a punitive experience.
- Visual Scaffolding: Boss enemies model operations visually. For example, the boss with multiplication ability could spawn enemy duplicates, making abstract concepts concrete through what Alvarez-Rodr´ ´ıguez et al. [3] call "visual mathematics scaffolding."

2.3. Procedural Content Generation

Creating diverse gameplay experiences required extensive reward systems. As Oda et al. noted [15], procedural generation presents significant implementation challenges. To address that, our solution involved:

- A tiered reward system (Common/Rare/Legendary) with weighted probabilities
- A base RewardCard class enabling polymorphic behavior
- Contextual generation algorithms considering player state (health, level, resources)

This architecture reduced development time, while creating many unique reward effects in game, validating Deshpande and Huang's findings [16] about simulation game efficiency. The system dynamically adjusts to player progression, ensuring balanced challenge as noted in Mayer's educational game principles [11].

3. SOLUTION

Our team program the game *Math Knight* and *Math Magics* [17] using the unity game engine, which is coded entirely in the language of C#. *Math Knight* is a prototype, while *Math Magics* has more appealing graphics, both game uses the central idea discussed in the scope of this paper.

In the game, the player's and enemies' health are represented as strings, where death only occurs when health reaches exactly zero. This design allows for creative manipulations, including negative and fractional health values, as shown on right side of *Figure 1*. The player attacks the enemy by using one of the four weapons shown on the left side of *Figure 1*, which is subtraction sword, addition axe, multiplication mace, and division dagger. The weapons have integer levels, and they deal the amount of "damage" by its level. For example. If the player attacks with a level 5 subtraction sword, it will deal a damage of "-5". If the player attack with a level 3 multiplication mace, it will deal a damage of "×3". Once the player reduces an enemy's health to zero, a reward screen appears, enabling progression through an interactive strengthening system. The player then uses the reward to upgrade his weapons, which is increasing the weapon level. When the weapon reaches a certain amount of level, it makes the player easier to defeat the boss and progress to the next level, unlocking a new weapon.

Figure 1: How to play

This structure fosters adaptive learning, tailoring challenges to individual progression. Scaffolding arithmetic complexity would personalizes skill acquisition, enhancing engagement and conceptual mastery[18]. This is done in the levels of *Math Knight*. There are four levels in total, and each level the player would unlock a new weapon and encounter a new boss.

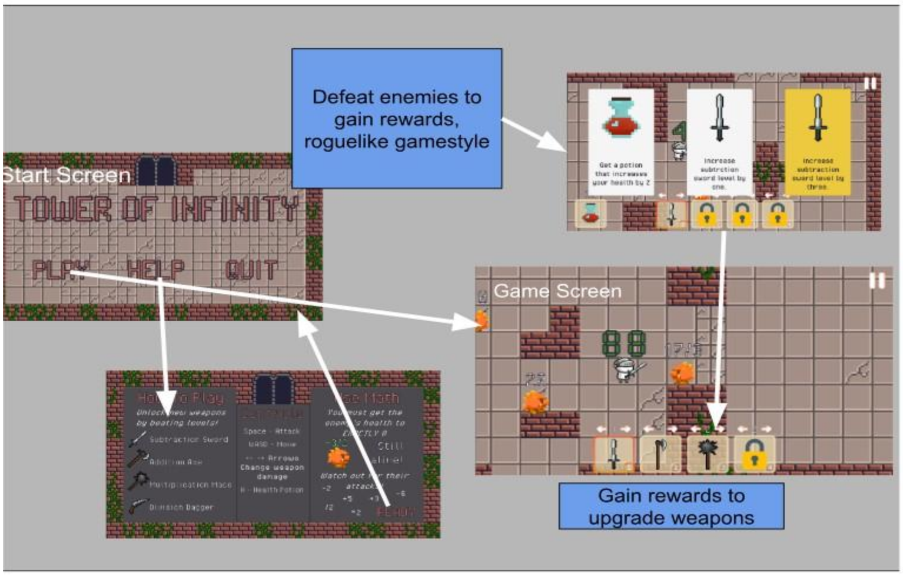


Figure 2: Overview of Math Knight

3.1. Level 1: Introduction to Natural Numbers (N)

The player begins with a *subtraction sword*, facing enemies with health values restricted to natural numbers, N = {1,3,7,66...}. This level familiarizes players with basic counting and subtraction. The boss introduces addition by healing itself, reinforcing arithmetic operations.

This design is supported because introductory game mechanics should limit variables to build confidence [19].

3.2. Level 2: Expansion to Integers (Z)

Upon acquiring the *addition axe*, enemies now have integer-based health, Z = {...,−7,−2,4,23,...}, introducing negative numbers. Players master addition and subtraction with positive and negative values. The boss multiplies smaller enemies' health, introducing multiplication as a new mechanic. These positive and negative weapons provide "mirrored mechanics" (e.g., tools for both adding and subtracting) to contextualize negatives [20].

3.3. Level 3: Rational Numbers and Strategic Operations (Q)

With the *multiplication mace*, enemies now possess fractional health $\mathbb{Q} = \{\ldots, -7, -\frac{1}{5}, \frac{3}{2}, 4, \frac{229}{5}, \ldots\}$. Players must strategically simplify fractions. For example, reducing $\frac{73}{7}$ to 3 via $(-10, \times 7)$ is more efficient than attacking 73 directly via (×7,−10,−10,−10,···). This teaches the players to prioritize order-of-operations [21]. The boss introduces division, further diversifying gameplay.

3.4. Level 4: Advanced Rational Numbers and Prime Factorization

The *division dagger* is introduced, though enemies retain health in Q with larger values (e.g., 999). Players optimize operations (e.g., +1, ÷5, ÷5, ÷5, ÷2, −4 would get 999 to zero) to defeat foes efficiently. Complex fractions such as $\frac{1}{105}$ require prime factorization (×5, ×3, ×7), reinforcing multiplicative concepts [22].

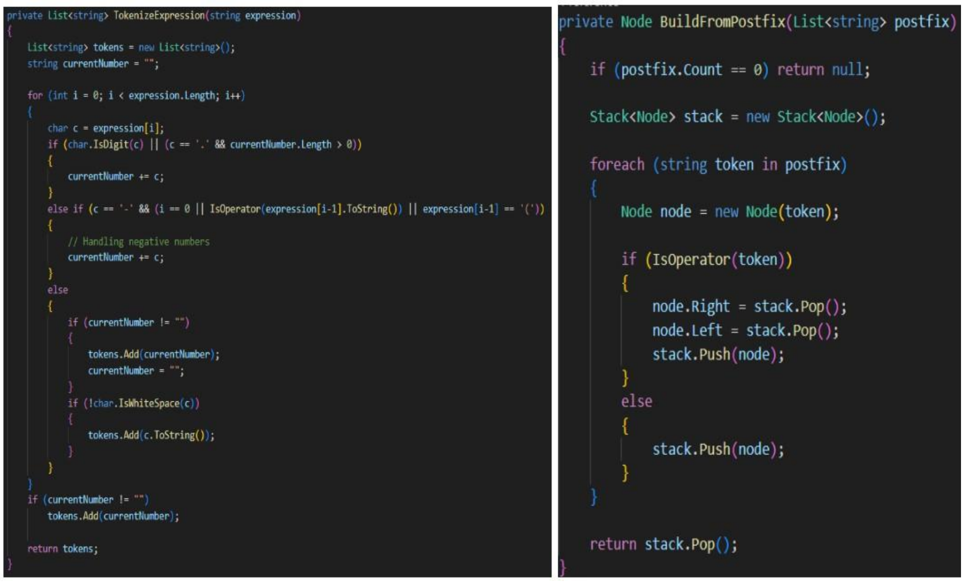
3.5. Health System Implementation Through Expression Trees

The health system in Math Knight works differently than most games. Instead of just tracking numbers, both players and enemies have their health represented as actual mathematical expressions. This means that health can be negative, fractional, or even complex equations that need to be solved. When you attack an enemy with 20 health using a "divide by 6", their health doesn't round down to 1 - it becomes the fraction , which the game simplifies to 3/2.

To implement this system robustly, I developed an expression tree data structure adapted from computer science principles of binary tree representation. In this implementation:

The game handles health calculations through a two-stage parsing system. When combat occurs – such as when an enemy divides the player's 6 health by 4 – the left code segment initiates the process by tokenizing the raw expression. This lexical analysis breaks "6 / 4" into its fundamental components: the operand 6, the operator '/', and the operand 4. These tokens preserve the exact mathematical relationship without any loss of precision.

The right code segment then takes these tokens and constructs the actual Expression Tree data structure. This tree maintains the expression in its unsimplified form during all internal calculations. Only when displaying health values to the player does the system reduce fractions to their simplest whole-number forms. This approach serves the dual purpose of maintaining mathematical accuracy (avoiding decimal approximation errors) while presenting clean, understandable values to players.



```
private List<string> TokenizeExpression(string expression)
{
    List<string> tokens = new List<string>();
    string currentNumber = "";

    for (int i = 0; i < expression.Length; i++)
    {
        char c = expression[i];
        if (char.IsDigit(c) || (c == '.' && currentNumber.Length > 0))
        {
            currentNumber += c;
        }
        else if (c == '-' && (i == 0 || IsOperator(expression[i-1].ToString()) || expression[i-1] == '('))
        {
            // Handling negative numbers
            currentNumber += c;
        }
        else
        {
            if (currentNumber != "")
            {
                tokens.Add(currentNumber);
                currentNumber = "";
            }
            if (!char.IsWhiteSpace(c))
            {
                tokens.Add(c.ToString());
            }
        }
    }
    if (currentNumber != "")
        tokens.Add(currentNumber);

    return tokens;
}
```

```
private Node BuildFromPostfix(List<string> postfix)
{
    if (postfix.Count == 0) return null;

    Stack<Node> stack = new Stack<Node>();

    foreach (string token in postfix)
    {
        Node node = new Node(token);

        if (IsOperator(token))
        {
            node.Right = stack.Pop();
            node.Left = stack.Pop();
            stack.Push(node);
        }
        else
        {
            stack.Push(node);
        }
    }

    return stack.Pop();
}
```

Figure 3: Expression Tree code example

Research in mathematical expression representation[9] and arithmetic learning systems[14] supports this approach as pedagogically sound to reinforce fundamental concepts while avoiding computational inaccuracies that might confuse learners.

3.6. Rouglike Reward System

The reward system is a central mechanic designed to enhance both engagement and replayability. After each enemy defeat, players are presented with a choice of three randomly generated rewards. These rewards are drawn from three distinct tiers, each containing a pool of unique items that provide different effects and varying levels of strength. Importantly, the generation of rewards is not purely random. The system also considers contextual factors such as the player's current health, potion value, and progression level. This adaptive layer ensures that rewards feel situationally relevant, offering players meaningful choices that directly interact with their circumstances in the game. By integrating randomness with adaptive logic, the reward system prevents repetition and creates a dynamic gameplay loop where every encounter feels fresh and strategically significant.

The technical implementation further supports flexibility and scalability. Each time the reward screen appears, the game generates three potential reward cards. For each card, the algorithm first selects one of the three tiers, then randomly draws an available reward from that tier's pool. To streamline development and avoid redundant code, I implemented a base reward class that defines shared attributes and functionality, with all specific rewards inheriting from this parent class. This object-oriented structure made it much easier to expand the system with new rewards, since each addition requires only minimal, customized logic. Beyond reducing code duplication, this design ensures long-term maintainability and provides a framework for quickly introducing new content. The result is a reward system that is both technically efficient and gameplay-rich, ensuring players remain motivated and engaged across multiple playthroughs.

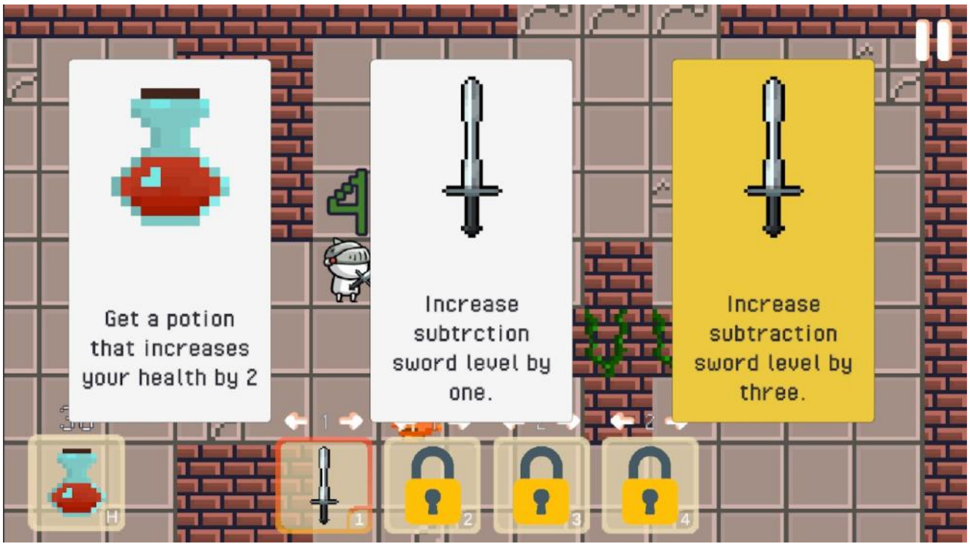


Figure 4: Rewards example

3.7. Turn Based Gameplay

The decision to implement a turn-based combat system in *Math Knight* is a critical design feature. What turn base means is that the enemies and player take turns to take action. If the player chooses not to take immediate action, the enemy will not attack or move, but it will remain in idle state. This mechanics allows players to take time and think about their actions before they play. It directly counteracts the rapid, anxious calculation by creating a low pressure environment for deep thought. This forced pause is crucial for developing metacognition[12], which is the ability to plan, monitor, and evaluate one's own strategy. It is crucial for players to have the time to think and calculate in order to master mathematics. Faced with an enemy health value like $\frac{69}{5}$, a player under time pressure would likely panic and act hastily, perhaps multiplying by 5 immediately and creating a harder problem (69). With time to think, they can instead strategize: subtracting 13 first to get $\frac{4}{5}$ is a more efficient solution. This process of evaluating different operational sequences builds real sense of numbers and strategic flexibility, turning each move into a deliberate practice session rather than a frantic guess.

Ultimately, the turn-based system ensures that the game is a tool for mathematics practice. It makes the player's primary weapon their intellect, not the speed of their button presses, fostering their learning of true mathematics proficiency.

4. EXPERIMENT

To evaluate the efficacy of Math Knight, two experiments were designed to measure its impact on both students' attitudes toward mathematics and their arithmetic abilities. The first experiment employed a qualitative approach to assess shifts in perception and engagement, while the second utilized a quantitative pre-test/post-test design to measure improvement in calculation accuracy and speed. Both studies were conducted during a summer camp program with 66 elementary school students from Orange County. Participants included 13 third graders, 20 fourth graders, 19 fifth graders, and 14 sixth graders. All students and parents consent to participate in this experiment, and *Figure 5* shows kids engaging with the game.

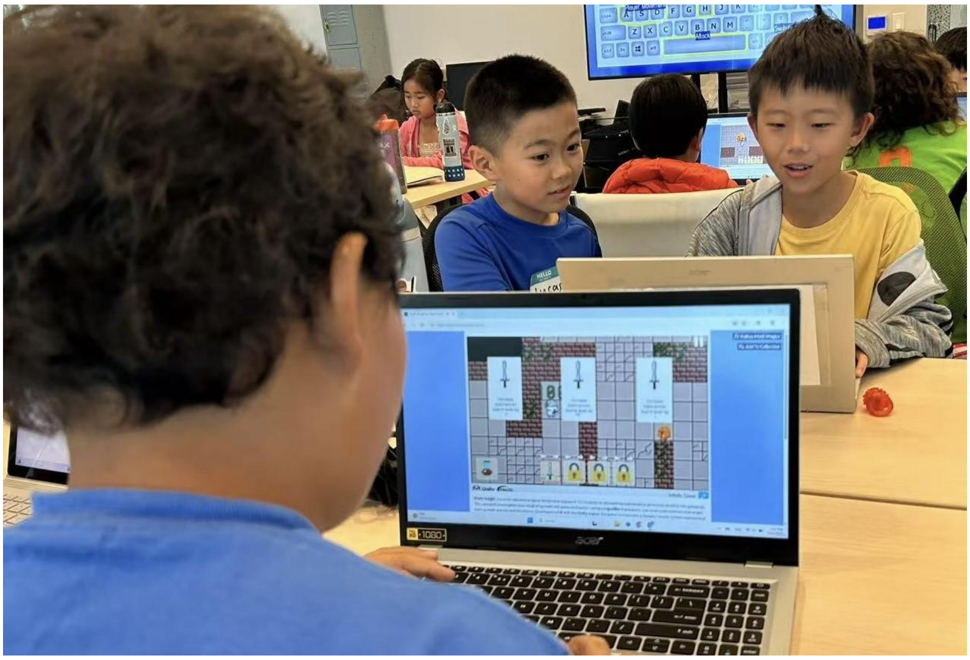


Figure 5: Participants of summer camp playing Math Knight

4.1. Experiment 1: Shifting Perceptions of Mathematics

At the start of the camp, students were asked to choose one word that best described their feeling about math. Their responses were collected and visualized in *Figure 6*.

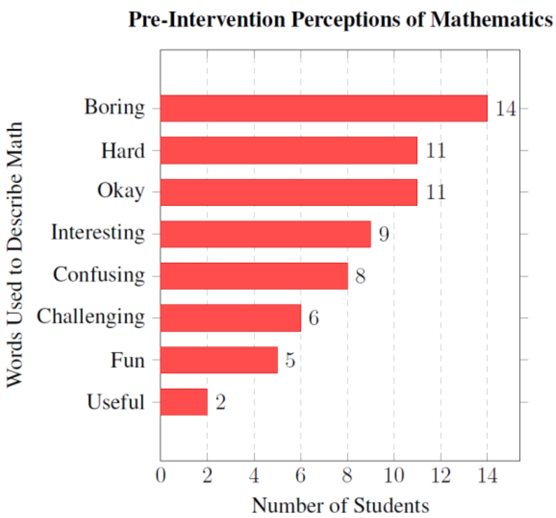


Figure 6: Word frequency before playing Math Knight

It is evident that these responses were overwhelmingly negative, with frequently occurring words as “boring” and “hard”. A result that appeared surprising at first was that “Challenging” only got 5 picks. We figured that this was because most students who felt math was challenging have chosen the word “hard” at first glance, because it is a more simple word that describes the same type of feeling as challenging. After engaging with *Math Knight* daily for one week during the summer camp, students were again asked to choose one word that describes their current feeling about math

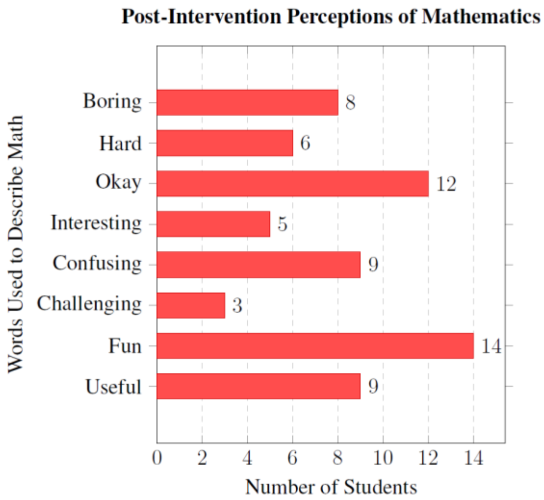


Figure 7: Word frequency after playing Math Knight for a week

The resulting word cloud *Figure 7* showed a positive shift. The most common descriptors became “Fun” and “Okay”, though some negative perceptions remained. It is also notable that the word “Useful” had a significant increase in occurrence, while the word “Boring” had a decrease in occurrence.

This clear change in word choice suggests that *Math Knight* successfully impacted students’ perception on mathematics. By embedding arithmetic practice into the core mechanics of the game, rather than interrupting gameplay with explicit math problems, the game fostered a more positive and intrinsically motivated learning experience.

4.2. Experiment 2: Assessing Arithmetic Improvement

The second experiment measured the game’s impact on students’ mathematical skills. The same 66 students began by taking a timed diagnostic test of simple arithmetic calculations. This provided a baseline for their computational skills prior to exposure to the game. After engaging with *Math Knight* for one week during the camp, students retook an equivalent form of the arithmetic test with comparable difficulty and content coverage but different specific numbers. Their post-intervention accuracy and completion times were recorded and compared to their baseline performance.

The data revealed substantial improvements across multiple dimensions of arithmetic proficiency. The overall accuracy rate increased from 64.2% on the pre-test to 82.6% on the posttest, representing a significant improvement of 18.4 percentage points ($p < 0.01$). Third graders showed the most dramatic gains, improving from 58% to 79% accuracy. Students also completed the arithmetic problems 23.1% faster on average after paying *Math Knight*, with mean completion time decreasing from 122.5 seconds to 94.3 seconds, showing that Students maintained higher correctness rates despite working more quickly. The standard deviation of scores decreased from ±13.8% to ±11.2%, indicating that lower-performing students made particular gains, helping to narrow the performance gap.

The results provide compelling evidence that thoughtfully designed game-based learning environments like *Math Knight* can make substantial contributions to mathematics education by engaging students in meaningful practice.

5. Related Works

The integration of game-based learning (GBL) in mathematics education has been explored through various methodologies, with varying degrees of success in engagement and educational efficacy. This section reviews three prominent methodological approaches, analyzing their strengths and limitations, while presenting how *Math Knight* addressed their limitations.

5.1. Methodology 1: Video Simulated Recall

The first approach uses video-stimulated recall to analyze student engagement and learning processes. A sample game from this 2012 study aimed to engage "digital natives" but was limited by its design, because learning was constrained as players could progress through trial and error without necessarily achieving conceptual mastery. The gameplay lacked a mechanism to ensure that learning happens alongside new challenges. Such methods often fail to adequately accommodate the "Ongoing Learning Principle." [2] This principle is vital for mathematics education, as it facilitates expansive learning, where students continuously revise prior schemas (e.g., understanding addition as a strictly increasing process) to incorporate new, complex concepts (e.g., integrating negative numbers). Effective scaffolding should expand learning to include anomalies, building more sophisticated understandings over time.

Math Knight directly addresses this by its very structure. The roguelike genre is inherently built on the Ongoing Learning Principle. As players level up and progress through tiers of difficulty (from natural numbers to integers to rational numbers), they are forced to continually revise and expand their arithmetic schemas. The game's mechanics provide experiential scaffolding, because a strategy that works on an enemy with health 7 (a natural number) fails against an enemy with health -2 (an integer) or $\frac{7}{3}$ (a rational number). This ensures that learning is not static, but a trajectory aligned with the principle described by Jorgensen and Lowrie[2].

5.2. Methodology 2: Reward and Curricular Relevance

The second approach concludes that while digital games have positive educational effects, their implementation often falls short in sustaining engagement and aligning with standard curricula. Ayaz & Smith[8] review several games exemplifying these limitations:

- *Dinner for Dogs*: A role playing game where players feed dogs with limited resources. While it incorporates mathematical reasoning, its feedback loop is not engaging because its reward for correct answers is just the dog being happy. The lack of a compelling, variable reward system leads to quick player disinterest, a critical flaw for a learning tool intended for repeated use.
- *The Last Chip*: A puzzle game based on mathematical reasoning similar to the classic game of Nim, which has a complete mathematical theory[23]. Although valuable for teaching strategic thinking, its core concept (binary numbers) is a niche topic that falls outside of the standard K-12 curriculum. Consequently, its direct benefit to everyday mathematics study is limited. Furthermore, it shares the same weakness in its reward structure, failing to provide long-term motivation.

Math Knight is designed to overcome these specific pitfalls. First, its reward system is a core mechanic. Using a tiered random reward system that provides tangible power-ups, it generates excitement, expectation, and a compelling reason for replay, directly countering the boredom associated with static feedback. Second, it focuses exclusively on the four fundamental arithmetic operations within the domain of rational numbers (Q), which form the absolute basis for all subsequent mathematics learning. This ensures that the skills practiced are directly relevant and beneficial to students' core academic work, unlike games based on more abstract mathematical concepts.

5.3. Methodology 3: Theoretical Guidance

A comprehensive review by Mayer[11] provides valuable theoretical frameworks for designing educational games, but it is short of delivering a concrete product. The review suggests:

- Value-Added Research: Recommends features like personalization, coaching, and self explanation to enhance learning.
- Cognitive Consequences Research: Proposes using specific game genres to train skills like perceptual attention.
- Media Comparison Research: Identifies mathematics as a promising area where games could be more effective than conventional media.

However, the article's primary limitation is its own concluding statement:"Future research is needed to pinpoint the cognitive, motivational, affective, and social processes that underlie learning with educational computer games"[11]. It serves as a call to action rather than a presentation of a validated solution.

Math Knight responds to this call by acting as a practical implementation that embodies many of Mayer's suggested features. It employs personalization through its adaptive reward system and adaptive difficulty as player progress through levels. It encourages self-explanation by forcing players to deduce the optimal sequence of operations. Most importantly, it is a tangible product designed explicitly to facilitate the learning processes Mayer highlights. While other attempts have been either too boring with the game or too divorced from the math curriculum, *Math Knight* integrates essential math directly into compelling game mechanics. The game is designed to be inherently enjoyable; even if the mathematical component were removed, the core loop of strategic combat and progression would remain engaging, ensuring that the learning is wrapped in a genuinely fun experience.

6. Future Vision

While *Math Knight* demonstrates impact in educational gaming, it is not without its limitations. The current visual design and animations are functional, but lack the polished appeal that would make the game more engaging for a younger audience. Improving the artwork would be a key priority for future development, which is why we are working on the game *Math Magics*. In the game *Math Magics*, we kept the core game mechanics established in *Math Knight*, while adding more appealing graphics and a story line with the game. We unified aesthetics of the game to Pixel Art, with some examples shown in *Figure 8*. Then, at the beginning of the game, we added animations that tells a story of a reincarnation of a magician: *The magician is the player, who has the four most powerful magic wands that could bring mathematical operations to real life. However, the magician tried to divide by 0, which exploded the world. Luckily, he reincarnated to his childhood where he just begun to learn magics. This time, he would learn mathematics along with obtaining his magical powers to ensure that nothing goes out of hand!* This new graphic and story line of *Math Magics* would serve as an appeal to students that are not inherently interested in mathematics.

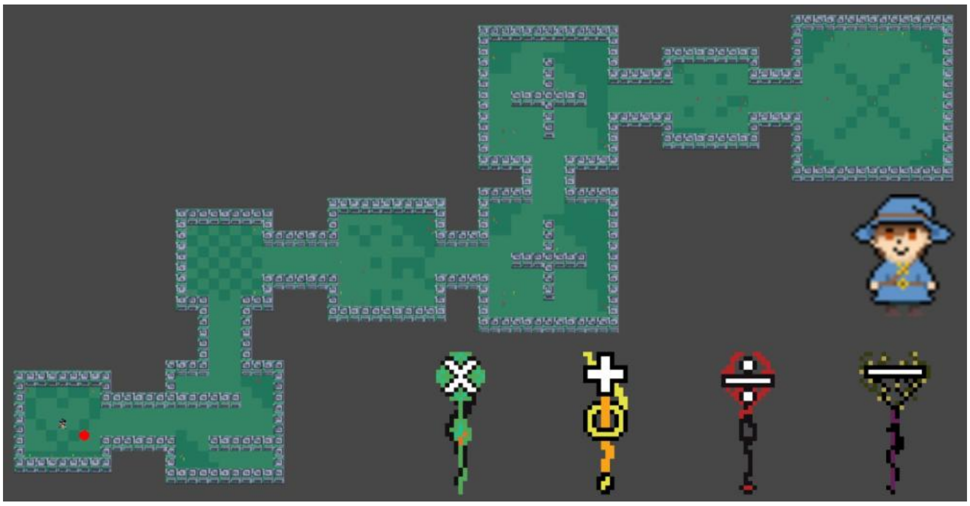


Figure 8: Sprites in Math Magics

Looking ahead, several more enhancements could deepen the educational impact of the game. To further incentivize strategic efficiency, we could implement an attack counter. This system would track and display the number of operations a player uses to complete a level, encouraging them to find the most elegant solution. Building on this, a competitive ranking system could be introduced, where players are scored and placed on a leaderboard based on their number of attacks. This layer of friendly competition could motivate practice and mastery.

A more advanced future development would be an adaptive AI system. This AI would analyze a player's performance to identify specific weaknesses, such as struggling with fractions or negative numbers. It could then generate enemies that target these weaknesses, providing customized practice.

Lastly, future improvements may include extending *Math Knight*'s reach into higher grade level mathematics. We can achieve this by introducing two new levels and two new weapons, which is exponent epee and radical rifle. The new levels would introduce irrational numbers, with the enemies health covering the range of real numbers R. The weapons could deal square damage and square root damage, and by similar logic, they could be upgraded to deal cube and cube root damages and so on.

7. Conclusion

Overall, the integration of mathematics into game design represents a new education method, moving beyond traditional drill-based approaches and interruptive math problems in games.

Math Knight serves as a compelling case study illustrating how core mathematical concepts can be organically embedded within game mechanics to create immersive learning experiences that simultaneously build proficiency and foster positive attitudes toward mathematics. The effectiveness of this approach lies in its alignment with pedagogical principles: scaffolding complexity through progressive level design, providing immediate feedback through symbolic computation systems, and creating low-stakes environments that encourage experimentation. By representing mathematical operations as tangible gameplay actions rather than abstract concepts, such games enable students to engage in meaningful problem solving, or playing.

Furthermore, this research highlights the importance of balancing engagement with educational rigor. Successful math-integrated games must not only captivate players but also provide good practices in mathematics. The implementation of expression trees for exact fraction representation, adaptive reward systems, and turn-based mechanics that promote metacognition exemplify how technical solutions can address both educational and motivational challenges. As digital platforms continue to transform educational landscapes, game based learning offers a way to make mathematics more accessible and equitable. By reducing anxiety, providing personalized challenges, and demonstrating the practical utility of mathematical thinking, well designed games can reach students who might otherwise disengage from traditional instruction. Future work in this domain should continue to explore the synergies between game design principles and mathematics education.

Ultimately, *Math Knight* and *Math Magics* represents more than just a novel teaching tool. It is a blueprint for reconciling joy with rigor in STEM education.

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