# MINDFUL MATH: THE METHOD, HOW TO USE IT AND HOW IT WORKS

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***ABSTRACT***

*Traditional mathematics education, based on standard textbooks, does not align with the humanistic values of the 21st century and fails to take into account the individual cognitive characteristics of learners. Strictly speaking, the traditional method of teaching math is not a technology and, consequently, cannot be fundamentally improved. This article presents the flow-based Mindful Math technology, which offers principles and tools for creating individual learning pathways for each student within the classroom setting through asynchronous learning. The key outcomes of this technology, confirmed by a 15-year longitudinal action research study, include a more stable and effective learning environment, as well as the ability to determine the statistical characteristics of the learning process. Mindful Math technology has been designed with scalability in mind and can be useful in situations of social crisis (war, epidemics, emigration, lack of accessible education); it can also be effectively transferred to a digital environment as an open-source platform and, potentially, adapted for teaching other subjects.*

***KEYWORDS***

*Mathematics Education, Education Technology, Zone of Proximal Development (ZPD), AsynchronousLearning, Curriculum Compression, Mastery Learning, Adaptive Learning Systems, Cognitive Load Optimization*

## 1. INTRODUCTION

When we talk about technology, we are talking about "...the application of conceptual knowledge to achieve practical goals, especially in a reproducible way"[1]. The concept of pedagogical technology[2] encompasses the reproducibility of techniques, the controllability of the process, and the guaranteed outcome. Furthermore, the structure of the technology is holistic; it functions as a blueprint, as it regulates the actions of both the pupil and the teacher. This distinguishes technology from a mere set of methods.

From this perspective, most textbooks is already obsolete.Just think – the methods used to structure curricula and compile teaching materials have remained fundamentally unchanged for centuries. They retain clear traces of the medieval craft guild systems (rigidly fixed volumes and paces, the process tied to the master's charisma), which leads to mediocrity and the impossibility of systematically replicating outstanding results[3]. Conversely, the technology employs a unique operating method that is ideally aligned with W. Edwards Deming' principles and can be improved through the Shewhart-Deming cycle (PDCA)[4].

Human cognitive abilities have developed over millions of years of evolution and have been directly linked to the movements of the hands and fingers[5]. In a world that has become increasingly digital, today's children spend most of their time focused on a tiny screen, performing micro-movements with their fingers. However, the areas of the brain responsible for deep understanding are activated when a child writes or draws by hand[6]. Not only children, but also contemporary adults, whose approach to information are now changing. We are forced to process a significant flow of information, much of which is likely to turn out to be noise. In the wake of this evolutionary upheaval, we need to preserve the methods of teaching mathematics. More precisely, we need to develop a technology.

## 2. THEORETICAL BACKGROUND

To ensure effectiveness, the starting point for learning must be individualised – not from the realm of current knowledge, but from the zone of proximal development – where the task is still achievable but not yet mastered[7]. As previously argued, the learning process must be documented. Bloom's Mastery Learning also requires us to adapt the pace of learning to students' abilities, and to incorporate feedback and individual error correction into the process[8]. It is difficult to imagine that all of this is possible simultaneously.

At this point, we appear to face a problem: how can we effectively teach children with divergent levels of prior knowledge within a single curriculum? The first answer: asynchronous learning. An experiment conducted in California in 2010 in collaboration with Khan Academy empirically demonstrated that the academic year provides sufficient time to cover the entire curriculum, though the pace of learning remains highly individual[9]. Some will spend more time on equations, whilst others will focus on simple fractions. And the ability to organise asynchronous learning within one or more classes depends on the availability of materials and the teacher's resources.
Progress within a new topic should be gradual, moving from simple to complex, whilst developing both mental and motor skills. It has been scientifically proven that if the framework of guidelines for the student is set correctly, the number of errors is minimised[10]. When the level of difficulty of the tasks matches the student's skills, a state of flow[11] arises, along with a sense of intellectual mastery resulting from the experience of solving new problems[12]. At this point, the presence of the teacher as a qualified tutor is crucial. They can provide feedback, help correct mistakes and encourage the student to improve their work to achieve academic exellence. At the moment of active learning, this dramatically improves the quality of the current learning process[13] and enhances attention and thinking skills, encouraging the formation of long-term cognitive connections[14]. In practical terms this involves applying the PDCA cycle: pointing out errors directly or indirectly and asking for corrections until more than 95% of the work has been completed.

The most likely counter-argument will be that this process is routine and boring. Yes, to prevent things from becoming monotonous and to ensure the children do not get bored (and bored children in a lesson are something of a critical mass), the level of difficulty must be unpredictable from time to time. Therefore, further work on a new topic can be approached through elaboration[15], Desirable Difficulties[16] and interleaving: alternating between sets of tasks from different topics within a single lesson[17]. This fosters the formation of strong neural connections and prevents the cognitive illusion of knowledge from arising. For high-quality, long-term learning, none of the key topics should be neglected – consequently, both new and old topics must be included in the rotation of task sets[18].

We must not forget about creativity as a means of applying mathematical knowledge in a practical way, nor the importance of independent discovery. Paul Lockhart[19] and Hans Freudenthal[20] have discussed this, and tasks of this kind are equally important for both potentially strong and seemingly weaker students. All these methods, when applied in combination, accurately replicate the non-linear proceeding of the brain's neural networks[21]. The mechanism underlying long-term potentiation (LTP) of synapses is biochemically cyclical and requires time not only for the formation of new connections but also for consolidation intervals[22].

An affective filter can hinder the acquisition of new knowledge[23]. An atmosphere of safety and trust within the group must designed to counteract its effects.

# 3. FLOW-BASED TECHNOLOGY

## 3.1. Principles

To summarise, we shall attempt to bring together the principle and the reference numbers, and then note their presence (or absence) in the traditional model (Table 1):

| Num | Principle | References | Traditional |
|---|---|---|---|
| 1 | Start with each student's prior knowledge in the classroom | 7 | No |
| 2 | The technology should encourage independent work on paper | 6 | Yes |
| 3 | Asynchrony within the group | 9, 24 | No |
| 4 | A linear increase in complexity | 10, 11, 24 | Not strictly |
| 5 | The possibility of developing a skill | 10, 11, 12 | Sometimes |
| 6 | Without fear and with opportunities for creativity | 19, 23 | Generally not |
| 7 | Non-linearity and the cross-fit | 15-18, 20, 25 | More often in tests |
| 8 | A comfortable learning environment for participants | 12, 14, 23 | No |
| 9 | The teacher as a tutor | 8, 13, 14 | No |

Table 1. Principles

There is a clear systemic imbalance in line 7, which renders the points in lines 6 and 8 structurally unattainable. Failure to strictly adhere to the principle in line 4 effectively negates the possibilities inherent in line 3 – which is why this idea may seem unattainable.
How can all these ideas be combined into a functioning system? Through the use of targeted teaching materials and systematically organised interactions.

## 3.2. Materials

All teaching materials are designed with the objectives of the learning stage in mind. The smallest structural unit within our framework is not a single isolated example or problem, but a calibrated set of tasks — a master-list (exceeding 20 tasks) or a problem unit (consisting of 5 to 10 tasks, equations, inequalities, or problems). A new topic begins with a minimalist explanation on the board (or on the computer screen for online frameworks) – a couple of lines, one or two definitions if necessary, and a diagram. Following two or three very simple examples and the ball is immediately in the students' court .

The first stage of working with a new topic is assimilation. Starting with the simplest and most self-evident tasks, with a strictly linear increase in difficulty. Everything strictly on paper. This is so that a mechanical skill begins to develop. There is no strict assessment of the number of iterations required; sometimes 21 repetitions are mentioned, but based on personal experience, I would say 20–70 tasks. The tasks utilized at this stage share the closest architectural proximity to

the Kumon system, their effectiveness in mathematics has been proven by a longitudinal study in Pakistan[24].
The second stage of work involves expansion, deepening and contextual cross-fit. After familiarizing ourselves with the topic through a long list, we move on to shorter units. As a rule, these consist of 5 or 10 tasks.

In most countries, academic grading scalesare based on the decimal system. Consequently, master-lists and problem units usually consist of a number of questions that is a multiple of 5. This makes marking more convenient. In my experience, during one academic hour, most students are able to correctly solve between 5 (the minimum problem unit size) and 50 tasks (for example, the long master-list at the start of the Logarithms topic). The units themselves should be manageable to solve. Within a single topic, the units gradually become more complex. Within each unit, the problems become more difficult (two-tiered scaling of difficulty). The situation may be slightly different when tackling the most complex topics in the school curriculum, such as problems involving cross-sections, trigonometric and logarithmic equations and inequalities. For those capable of solving these, a somewhat chaotic structure of the units is preferable.
And after the first 20–30 tasks within a topic, mixing must begin, providing each student with problem units from a few different topics. Furthermore, we change these topics constantly. If our aim is high-quality learning through spacing effect, none of the topics should be consigned to the archives[25].

Stage three: checking. Any arrangement of tasks and any type of task may be deployed. There are two restrictions: not only the types of tasks, but also the way in which they are arranged must be familiar (if the exam is in the form of a test, we teach students how to tackle tests in advance); the level of difficulty of the assessment task should remain slightly lower than what students routinely master during standard training sessions. This is not the end of the topic – alongside the previously covered substrate, it is systematically reintegrated in the form of units or incorporated into more complex advanced tasks in subsequent topics.

## 3.3. Interactions

**The student – the material.** The learner receives assignments – sometimes a single master-list, sometimes a few short problem units across distinct topics – and executes them. Processing the core algorithmic steps mentally, they write the answers straight onto the assignment sheet. The margins can be used for auxiliary calculations and drawings. If questions arise, they ask. The completed unit is taken to the teacher formative assessment, resulting in a preliminary grade. If the preliminary grade is 80% or higher, the student has a choice: to record this grade in the register or to improve it by correcting any errors. They take it away, correct it, and bring it back for marking. If the initial grade is below 80%, they must improve it and submit it for marking again. In both cases, the number of attempts is unlimited. Unfinished assignments may be completed at home or in the subsequent lesson, at the student's discretion. When several unfinished units have accumulated, students may choose not to take on new problem units, but to work on what has not yet been submitted.

Everyone has one exercise book (preferably a thick A4 format). It is used for writing down explanations, solutions and diagrams, or as a draft pad. Separate pages are set aside for formulas and the wording of theorems (with diagrams). My students referred to their exercise books as their personal reference guide.

Knowing your students' abilities, you can set them 2–3 units from different topics to work on during the lesson. In my experience, there have been learners for whom the cognitive shifting ability from one topic to another within a single lesson and to complete two sets of tasks was, in itself, a step forward and a real achievement.

**The teacher – the class.** Based on the overall curriculum, we need to continuously plan the rotation of four types of lessons: a) introduction to a new topic; b) standard asynchronous lessons; c) assessment lessons; d) intensive lectures (necessary in certain circumstances where the curriculum is accelerated or needs catching up; in my case, this was plane geometry), including overview lectures (e.g. Second-order Curves or The Study of Functions).

Any classroom interaction must be rely on each student's ZPD. For example, if not everyone knows their multiplication tables, it is not possible to explain square roots. By the time a new topic is explained, each student must, to the best of their ability, have mastered the logically preceding prerequisite framework. The explanation of a new topic is given at the blackboard, for everyone, with compulsory note-taking from dictation. Learners must simultaneously see with their eyes, hear and write, engaging their primary perceptual channels and, where possible, their memory. Ideally lasting between 2 to 5 minutes. If there is a lot of information, we are segmented into granular components or shifted into a lecture format .

When working on tasks related to the new topic, we answer any questions that arise in such a way that everyone can hear and see the feedback: 'Everyone, Anna has a question about task 17; here's a hint (written on the board)'. Because this question is systematically non-isolated and will come up again. I very rarely give a direct answer. More often, I provide a leading response. It might be paradoxical. It might take the form of a Socratic dialogue. Sometimes the guidance is general, sometimes individual. But for me, the best way to handle questions is to say: 'If anyone has any questions, please come to the blackboard. Write them down, ask them.' Perhaps the student will understand for themselves as they explain. Perhaps someone from the class will help. Perhaps I will need to give a precise answer or point them in the right direction. Importantly, no marks are given for work at the blackboard. The blackboard becomes a tool for learning and a place for discussion. One of the best moments of this approach was when, after a particularly tricky homework assignment in the final year, in response to my usual question: 'Is there anything you don't understand about the homework?', 11 out of 28 students rushed to the blackboard. They rushed because they were afraid there wouldn't be enough space for them.

About the blackboard. For questions and brief explanations. For diagrams. For formulas when covering a new topic. Under no circumstances for public self-flagellation. Not for answering questions at the blackboard (the ultimate form of torture, in my opinion).

**The teacher– the student.** Above all, we offer our support. When a student has finished the current section, they raise their hand, call out or come over themselves. We check their work and award an initial grade. In exam classes, it is useful to award a grade without specifying the incorrect answer; in lower years, we do indicate the mistake. If the preliminary grade is 80% orhigher, the student has a choice: to have the grade recorded in the register or to continue working until they reach 100%. I categorically refused to give grades below 80%. As far as I can recall, over the course of several years, there were very few instances where a student insisted on being given 60%. More often than not, they choose to continue working.

And I always say: your mistakes matter to me; I value your mistakes. They reveal where your personal blind spots actually lie.

**The teacher – the material.** Ideally, we work with existing teaching materials. In fact, this is the most critical aspect of school education. Textbooks lack good sequences of 20-30 similar examples in a row. Nor do they offer the variety needed for every student to work through them. Generally speaking, the difficulty level of the examples is consistent overall, but not in the finer

details; for a beginner, task 1 might be more difficult than task 5. This hinders the development of skills. Instead of repetition and building confidence, there are numerous intellectual dead ends. Internalisation becomes piecemeal rather than continuous. Some knowledge is retained, while some is not. Instead of a coherent and connected picture, only puzzle pieces remain in the mind. The inevitable consequence is a consistently poor outcome in mathematics education as a whole.
If there are no ready-made materials available, wemust create them ourselves. More on this below.

**The student – the student.** We can create situations that encourage productive interaction. Students can explain things to one another in a legitimate way, rather than simply giving away the answers. You can set up a worksheet so that it needs to be solved in pairs – one student tackles the even tasks, the other the odd ones. You can use peer-to-peer techniques – we prepare the setting, create the atmosphere, and we set the process in motion so that the students, rather than the teacher, do the bulk of the work in the classroom.

## 4. PRACTICAL CASE STUDIES

### 4.1. Work With Materials

Based on my own experience of teaching a class asynchronously, I can say that as long as there is sufficient teaching material, there is no cause for concern. Assess each student's ZPD. At the first stage, this is 1 or 3 topics, depending on their level of prior knowledge. For the next lesson, give each student an individual set of tasks: one topic for those with less prior knowledge, and three topics for those who are more knowledgeable and quicker. Do not try to impose rules on them regarding the order in which to tackle the topics. It is important to teach them how to solve problems within a topic. It also makes no difference which part of the program you start with. What is more important is that it is a familiar area for the student, where they feel most confident at the start.

Printed sheets with units are useful. If the problem tasks are printed on paper, we encourage the use of paper for calculations. For me, it's good practice to write by hand and use the margins as a draft pad. And it makes grades much easier. The most important thing to start with is that the first exercises should be as simple as possible. This immediately creates a sense of accessibility, provides a small taste of success and sparks an interest in continuing. A very gradual increase in difficulty (as in the well-known Kumon method, for example) makes the learning process self-directed. – And then each student progresses at their own pace within the overall workflow. If the increase in difficulty is linear and gradual, most of the work can be done independently. And this frees up a significant amount of the teacher's time during the lesson.

We can plan where to offer guidance – this is a way of continuing to explain new concepts without causing confusion. Alternatively, the teacher can state: Problem 18 is more difficult than the others; it is graded separately, so anyone who doesn't want to do it can skip it.
I'd like to add a few words about going round in circles and taking a leap aside. Now that we've decided on a teaching approach and have a sense of how students will be introduced to a new topic, we can move on to consolidation and assimilation. This will require different types of tasks. A linear master-list of 20–30–50 tasks is fine for an initial introduction. Next, we broaden our scope of familiarity and delve deeper into the topic. Here, problem tasks (usually 5 or 10) can be useful. The first three out of five will follow the master-list's principle: roughly the same level of difficulty or a small, predictable change. This 60% form the basis for confidence. To reach 80% (4 out of 5), a considerable effort is required. This may involve drawing on previously acquired knowledge. It may also involve a slight shift upwards or outwards.

For 5 out of 5, a shift in logic, a personal micro-discovery, or even the application of a completely different principle is required. I refer to the need to act differently as ‘sideways leaps’. In work for middle and senior school, ‘hippopotamus traps’ are useful. These are either tricky problems, or tasks which, when worked on in parallel within a group, deliberately slow everyone down. Or (for high school classes) tasks that have no solution.

So the difficulty will gradually increase. We’ll introduce unusual problems, more complex ones requiring more steps, and problems based on entirely different solution principles. Some formulas and theorems can be derived by the children themselves (for example, the difference of squares, the fundamental trigonometric identity via the Pythagorean theorem, the triangle inequality, and formulas for the areas of geometric shapes). We cannot expect everyone in the class to reinvent mathematics, but when solving problems of this kind, an understanding of the validity of theorems and formulas naturally emerges, which increases the level of trust in the system.

## 4.2. Work With Students

How classroom work might generally be organised. Personal experience. At the start of the lesson, we hand out the worksheets we’ve prepared for each student. This is convenient, and I’ll explain why below. Some students might refuse straight away: they’ve still got unfinished work on their desks. Everyone is busy with their work; it’s clear to everyone what they need to do. The teacher is free to move about.

Once a unit has been completed to the desired level of 80% or higher, the grade is entered into the register. Regardless of the number of attempts. If three units are completed during a lesson, we need to work out how to fit all three grades into the register. In my case, this was a problem: the number of grades for each student in the class exceeded the number of spaces in the register. If everyone completes between 5 and 50 tasks per lesson, how can we not grade them? And this is a wonderful micro-incentive. I have seen joy and pride in the students’ eyes many times: I solved this myself, I can do it! Bearing in mind that everyone learns at a different pace, I can also say that some students will master the entire curriculum within a year, while others will have to keep writing new problem tasks because their progress will be slow. But this will also lead to a difference in the final grade – five-minute tests, written exercises on formulas and large mixed-topic assignments drastically add lower scores to the otherwise excellent picture of general understanding. These are simply tools for monitoring the situation. But our common goal is to teach everyone to the best of their ability, taking their development into account.

## 4.3. Practice Of Interactions

The teacher’s role in these circumstances is to create a comfortable environment, provide gentle guidance, offer brief explanations and clarifications, grade work, devise individual learning pathways and gather feedback. All of this becomes incredibly interesting, because in these circumstances children begin to display completely unexpected qualities.

Here we come to the less obvious effects of using the Mindful Math technology. The simple, technically minimalist handouts create a sense of cognitive calm. It is easy to concentrate on them. The requirement to solve problems in a specific order structures the individual learning path. The ability to use the margins for calculations eliminates distractions. The absence of deadlines, and the resulting lack of a race against time, relaxes the learner and removes the underlying stress from the learning process. Whether or not a notebook or textbook is available is irrelevant: here’s a sheet of paper, get on with it. And the first task is a straightforward one. And there’s the dopamine reward, along with an easily earned grade.

What are the benefits for the teacher? Firstly, free time within the lesson. Secondly, a more relaxed atmosphere. Thirdly, a change of role. From being a low-ranking supervisor, they become a facilitator of a controlled learning environment. The process is manageable because, in an asynchronous learning setting, the most pressing constraint – time – disappears. All that is needed is to ensure the number of iterations required for success. For some, that is 21; for others, 70.
The reaction of a class using this technology to a new topic is remarkable. The students are like athletes before a competition. They are excited. After all, new knowledge (which is a dopamine-inducing experience in itself) is acquired through a light-hearted and collaborative game involving a simple master-list. And everyone has a chance to succeed in this topic. At the end of the lesson, the students with a more reflective cognitive tempo and more introverted students hand in their work with quiet joy and pride: they feel part of the general flow and are just as successful.

## 5. RESEARCH BACKGROUND

The narrative – and anecdotal – starting point of this study is, first and foremost, that it did not originate as a traditional academic inquiry based on classical pedagogical basis. Finding myself in a situation where time was severely limited, the learners urgently required preparation for the state mathematics examination, and the standard textbooks proved entirely dysfunctional, I began designing instructional materials. At first, these were master-lists and tests. Then came interesting and useful exercises to help with learning. Subsequently, I introduced targeted exercises to optimize the learning process, eventually structuring them into coherent problem units of increasing difficulty – purely to streamline my own classroom management.

All these years, I was absolutely certain that I was writing a workbook. It was only last winter, during a debate with an AI agent regarding the technical difficulties of transferring these paper-based structures online, that I realised the materials themselves are useless without the technology. Below, I will briefly describe the stages of the work, the participant`sprofiles and the longitudinal outcomes of this action research.

### 5.1. Start

**2010–2012.** Duration: 17 academic months. Location: municipal school. Number of participants: 90–100. Age: 13–17 years. Operational Challenge: urgent preparation for the state exam in classes where math was taught sporadically (teachers changed) or where there had been no math lessons for three academic months (there was not enough time left to strictly follow the recommended curriculum, and its scope could not be reduced due to the final exam).

I began writing master-lists and compiling exam preparation problemsin a sequence of increasing difficulty. I minimized my explanations at the blackboard. I stopped wasting precious time on students struggling at the blackboard. I began marking only written work. I redefined the purpose and content of workbooks and ceased grading them as a compliance metric. Through trial and error, I came to realise that not only the content but also the structure of the tests should be familiar to the students in advance. Overall, my work with the classroom was based on a synchronized combination of my own custom materials and the textbook mandated by the curriculum.

**Results.** I found that if the class was fully prepared for a new topic, they would work through the master-list almost without any mistakes, and the lessons that followed demonstrated a solid grasp of the new material. Students with a more reflective cognitive tempobegan to achieve results comparable to those of faster learners (including exam marks). The atmosphere in all groups remained controlled and friendly. The average attendance at my lessons was the highest among all teachers teaching the same groups. Parents of pupils in the the most academically challenged class requested that 'the teaching of plane geometry be organised in a way that is as easy, interesting and accessible as the teaching of algebra'... After that, I moved to the northern capital.

## 5.2. Second Stage

**2012–2017.** Duration: 42 academic months. Location: private school. Number of participants: 70–80. Age: 15–18 years. Problem: groups of vastly differing ability levels (very quick and bright pupils, learners with a more reflective cognitive tempo, and those who were completely neglected – all those whom the state school system could not cope with). New students joined the class right up until December (exams in early June). The timetable originally included fewer math lessons than the standard curriculum (4–5 instead of 6–8).

In one of the senior classes (16–19 years old), an unfortunate situation arose: our interaction began with their resistance (I was already the third teacher in six months), and as they had not studied math in the preceding months, none of them had a solid grasp of the subject, yet there was a great deal of resistance to learning. In this situation, I was unable to immediately pinpoint each student's Zone of Proximal Development (ZPD) and began teaching them blindly — following a plan similar to the standard curriculum, but using my own materials. As a result, time was wasted and exam preparation was geared towards a result above the pass grade but below average. An unforeseen factor worked against me: the previous year, the exam answers had been available to the children a day before the exam, and so this group had neither the desire nor the motivation. The result: all of them just scraped past the minimum pass grade on their own, and 2 out of 12 achieved the city's average .

From then on, work in the group began strictly with the ZPD, and the situation did not recur. For the first two years, I continued to set problem tasks for every lesson. Apart from lessons introducing new topics, the children's work was entirely self-directed. I began to actively mix up the topics for each of them.

At the same time, I found myself in a situation where one of the final-year classes, lacking sufficient knowledge of plane geometry, had to master the material of three years' worth of the curriculum in the six months remaining before the exam, to a standard sufficient for the exam (two or more correct answers out of five questions, one of which was a theory question, the rest on ready-made drawings). The solution was a format of lecture + absolutely homogeneous task sets on ready-made drawings. Each unit consists of an A4 sheet with 8–15 problems. The sheets are issued in order of increasing difficulty. A total of 24 sheets were produced; those who solved 17 or more pass the exam with confidence, with a minimum threshold of 3 out of 5 (the same result is consistent with individual tutoring). Testing of theoretical knowledge: students write out the formulations of theorems as a list (from 5 to 20, with varying degrees of complexity); to achieve 60%, they need not write them out in words, but simply draw the required number of diagrams accompanying the theorem.

**Results:** whether a student would pass or fail the exam became, for me, a statistical prediction based on the amount of work completed. Those who attended every week passed the exam with confidence. Very few students finished their work at home. The bulk of the work was done in class. In my second year of teaching, I almost stopped using textbooks, and from the third year onwards I taught without them. At the same time, teaching small groups freed up such a significant amount of time that, from my third year, I asked for the timetable to be organised into two groups: Grades 8 and 10, and Grades 9 and 11. Each class had just one separate lesson per week; for the rest, I worked with both age groups simultaneously. This wasn't any more difficult for me, because the students' work in class flowed naturally. This allowed them to solve more problems and was accompanied by a 'learning spillover' effect: the students absorbed knowledge beyond what was strictly necessary. The only clear shortcoming was the Ebbinghaus forgetting effect, if problem tasks on key topics were rarely revisited. This made my predictions for the exam results somewhat optimistic. No one failed. Geometry – consistently above the statistical average..

## 5.3. Second Stage. Same Time

**2015–2017.** Duration: 12 academic months. Location: a voluntary educational project for adults, with one session of two academic hours per week. Number of participants: 20–30. Participants' ages: 20–55. Context: non-formal math education for non-specialist adults with at least a school-leaving qualification. Those who attended were people who had retained an interest and wished to refresh their mental agility.

Each term (and there were three) I began with my own planimetry homogeneous task sets on ready-made drawings. It was easy, a little amusing, clear and effective. Previously acquired knowledge came back very quickly. Next, I selected unusual topics where a small amount of theory could be used to compile a vivid set of accessible problems. In particular: probability theory (up to Bayes), the basics of mathematical statistics (including an introduction to distributions), the basics of graph theory (based on Harari's classic book[26]), algebra of logic (up to problems on switching circuits), algebra and the basics of set theory.

All sessions took place in a very calm and friendly atmosphere. I was thanked mainly for stimulating their cognitive abilities and the enjoyment they derived from solving problems, and they were surprised to find that real mathematics is actually quite straightforward.

The end of the second stage for me was going on maternity leave. A significant milestone at that point was handing over my classes at the private school to a new teacher – a young postgraduate student. He listened to my explanations of the teaching process for about an hour, and then timidly asked: 'But, excuse me, does that mean that with your worksheets, any child can prepare for the plane geometry section of the exam in two months?' I thought for a moment and answered honestly: 'No. In three.'

## 5.4. Third stage

**2017–2026.** Duration: 9 years. Location: offline and online tutoring. Number of participants: 100–130. Age: 12–18 years, one 30+. Issues: a) individual preparation for state exams, b) resolving current problems and misunderstandings, c) preparation for non-standard tests and exams, d) stress relief, overcoming fear of math, the learning process and exams, e) support for learning under a different curriculum in a non-native language (children of emigrants), f) a unique case — preparing an adult with a degree in economics for admission to and study in a PHD programme in a foreign language, specialising in Big Data.

After my child was born, my teaching didn't stop: without any advertising on my part, neighbours' children, classmates of my neighbours' children, and relatives and acquaintances of those who had previously studied with me started coming to me. Every year, I teach 20–30 children. Around 10–15% of them leave after a few weeks or months, once their current situation has stabilised. I see this as a good sign. Some of the requests were unusual: preparation for an exam at a foreign college, preparation for postgraduate study (history, higher education obtained in Russian, doctoral studies in French in Belgium; she was admitted, defended her thesis, and now works in Antwerp. In response to my follow-up question about the quality of the knowledge she had gained from me, she said that for the first-year programme, the only thing she lacked was knowledge of Markov chains), recovery from the cognitive effects of a nervous breakdown (a very lazy girl from an educated family was driven to a breakdown by a school teacher who threatened that she would fail her exam; she scored 95% without much effort), moving to another country and adapting to a different language of instruction and a different curriculum[27].

During this period, my curiosity was piqued. Would the results of learning outside the group via master-lists be the same as those achieved within the group? After all, it’s clear that the flow state[11] is an effect of group work. Yes. A powerful flow state does not occur, and 2–3 times more questions are asked than when working in a group, but individual performance remains the same (on master-lists). The second practical point is the need for constant mixing of problem tasks from different topics – this counteracts the Ebbinghaus forgetting curve, sharply increases the sense of achievement (and reduces the fear of going to the blackboard in school lessons) and improves the quality of exam preparation.

The Covid pandemic has led to an increase in the number of online lessons. Currently, I conduct about 20–30% of my lessons online (previously via Skype, now via Zoom). I cover the same units as in face-to-face lessons – most of them are formatted in LaTeX/TikZ. I do not switch on my camera when teaching online – I provide only voice commentary. I explain new concepts by writing on the screen with a stylus. And I insist that everything be worked out in writing, not in the head; we say the answers out loud.

**Results:** The learning trajectory in individual tuition matches that of group tuition; cognitive self-esteem and confidence in one’s results increase significantly. Fear of the subject and of exams decreases, and authoritarian teachers are perceived as a failing element of the (educational) system. There are no instances of failing exams, even in very difficult cases. Children whose mathematical abilities are initially assessed as extremely low require a greater number of simple tasks on almost every topic, but even they inevitably encounter situations where a particular topic is grasped easily and quickly (at the level of their peers). Children with ADHD require greater attention during support and constant encouragement; at the same time, it is necessary to ensure that notes or diagrams are made while solving problems.

The second key benefit, apart from a comfortable learning environment, is time saved. A three-year course in plane geometry is easily completed using 17–24 sheets (totalling around 450 problems on ready-made diagrams) over 3–4 months. Average exam result: 60–85%. To improve the grade to 90–100%, another month is needed – to learn how to solve problems in several steps and write proofs. We are talking about the level of an intermediate school course, without additional theorems (Menelaus, Cheva, Euler), but with the application of the basics of trigonometry. For tutoring and preparation for the Grade 9 exam, one hour per week is usually sufficient for most children. In more challenging cases or to achieve a high grade, 2–3 hours a week. When preparing for the Grade 11 final exam: on average 2 hours a week (up to a 75% grade); for a higher grade or where knowledge is weak – 4 hours a week over the course of a year or two. The knowledge gained is sufficient for study at non-specialist higher education institutions (with a 1-2year course in advanced mathematics).

Third result: by using problem tasks as building blocks – akin to a child’s Lego set – it is easy to transfer learners to a different programme using other teaching materials and to bridge any potential gaps in knowledge (casee). In practice, when working with LaTeX PDF files on screen, children can use not only computers, laptops and tablets, but also smartphones without compromising the quality of their education (provided the condition of solving problems on paper and with the support of a qualified tutor is met).

## 6. CONCLUSIONS

The traditional system of teaching mathematics in schools is not a comprehensive, self-contained methodology, nor is it based on modern humanistic principles. Therefore, before making any changes, we must first establish a theoretical foundation and provide pedagogical and cognitive justification for the feasibility of developing such a methodology. We not only identify the possible principles underpinning its existence, but also propose for consideration an initial version, validated by 15 years of development, application and refinement (through the PDCA cycle) both within the sequence of processes and in the overall framework.

The initial results of its application include features unusual for math teaching contexts: asynchronous interaction within the group, participant comfort, a state of flow within the group, system flexibility and portability, a reduction in the teacher's workload, and significant time savings without compromising quality. This creates a more stable and effective learning environment. The principles and interactions described enable high-quality teaching of children and adults with different cognitive speeds, both in person and online, individually and in groups. There are theoretical and practical grounds for believing that methods and markers for the development of cognitive abilities are built into the proposed technology. The use of specially created materials is a prerequisite.

The empirical numerical estimates obtained can subsequently be used to apply a statistical approach to data on the learning process. Once the technology is fully developed and can be subjected to statistical analysis, it becomes possible to study it, accumulate data and subsequently improve it through the PDCA cycle. This is a direct path to creating modern, effective EdTech systems.

The classical approach to database programming rightly asserts that a clear data structure is essential prior to the coding stage[28]. Otherwise, programming a chaotic set leads to unstructureable, programmed chaos. Since the structure of traditional textbooks is linear where it should be at least two-dimensional (the sequence of topics does not account for inter-topic connections) and chaotic where it should be linear (the increasing complexity of tasks) – the overall data structure is unclear to both students and teachers. Transferring this data to platforms that encroach on e-learning does not lead to the emergence of educational technologies. One result of this digital chaos is the inability to distinguish between a bug in the digital platform and a bug in the dataset. Furthermore, in my opinion, most such projects inherently increase user anxiety (as an unintended consequence).

In contrast, the proposed approach to compiling sets of examples and exercises according to the described rules allows for the digital construction of individual and group learning pathways from ready-made units, similar to the Lego approach. This enables the initial set of examples and exercises to be quickly adapted to various educational objectives and programmes. It is also evident that the approach of constructing a curriculum from ready-made building blocks can be utilised in situations of social crisis (war, epidemics, emigration, lack of accessible education). We identify the following limitations to its application: a) the need to train tutors to work with the technology and materials; b) the precise systematic organisation of materials; c) the complexity of record-keeping when working with large groups – systems based on two-dimensional tables are inconvenient, as a high-quality analysis for each student requires taking into account: working topics; unit numbers; raw grades; and the start date of work on the unit.

The set of materials compiled by the author is a structured database comprising over 4,000 examples and exercises covering more than 70 topics. From simple fractions to the end of the school math curriculum. The sources were more than 40 different textbooks and collections, published from 1913 to the present day. This collection provides material for 90% of lessons in middle and high school, taking into account the need to fill accumulated gaps. I assume that for fully varied work, it should be 3–4 times larger.

This preliminary estimate is based on simple arithmetic calculations and could be refined using the concept of 'long-term cognitive capacity' – that is, the approximate number of examples and tasks a student can tackle at a comfortable pace over the course of an academic year without disrupting their daily life or development. I have to introduce this definition because, despite the existence of EdTech systems, the task of determining such a quantity does not have its own term. Based on my experience, I assume that the average values will range from 2,000 to 8,000. This is a fairly wide range, but it is suitable for preliminary estimates.

What might the development look like? First and foremost, the existing database needs to be moved online, while preserving the non-linear nature of the overall structure and the concise presentation of the materials. This is quite complex in itself and, it seems, can only be implemented on Drupal. Such a collaborative resource could be a living, evolving open-source project with a subscription-based professional guild for tutors...

The working title 'Mindful Math' may not be the most apt. At this point, I encountered a difficulty in translation. The original title in the author's native language is 'Mathematics by Mind', which uses the popular idiom 'to be well thought out' and carries a secondary meaning of 'in accordance with cognitive abilities'. In the English title, I wanted to emphasise completeness and fluidity. Perhaps I didn't quite succeed.

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