

# IMAGE COMPRESSION BASED ON COMPRESSIVE SENSING USING WAVELET LIFTING SCHEME

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## ABSTRACT

*Many algorithms have been developed to find sparse representation over redundant dictionaries or transform. This paper presents a novel method on compressive sensing (CS)-based image compression using sparse basis on CDF9/7 wavelet transform. The measurement matrix is applied to the three levels of wavelet transform coefficients of the input image for compressive sampling. We have used three different measurement matrix as Gaussian matrix, Bernoulli measurement matrix and random orthogonal matrix. The orthogonal matching pursuit (OMP) and Basis Pursuit (BP) are applied to reconstruct each level of wavelet transform separately. Experimental results demonstrate that the proposed method given better quality of compressed image than existing methods in terms of proposed image quality evaluation indexes and other objective (PSNR/UIQI/SSIM) measurements.*

## KEYWORDS

*Compressive Sensing, CDF9/7, Basis Pursuit, Orthogonal Matching Pursuit, Lifting scheme.*

## 1. INTRODUCTION

Many researches are on orthogonal transforms; such as discrete cosine transform (DCT) and discrete wavelets transform (DWT) etc., which provides a unique representation of a given signal. These transforms have been widely employed in signal processing. Also, these transforms facilitate their practical applications, e.g., fast algorithm design for transforms and rate distortion optimization for image and video coding [1]. On the divergent, redundant transforms or dictionaries do not have a unique representation for a given signal. Expanding a signal raises an ill-posed problem under a redundant transform or dictionary, but provides an optimized solution, e.g., a sparse one, for a specific application. This property has benefited many applications such as time-frequency analysis [2] [3] [4], signal denoising [4] [5], image and video coding [6] [7] [8] , and compressive sensing [9].

In traditional imaging systems, images are uniformly sampled first at a high rate, and then most of the sampling data are thrown away for the purpose of compression. A common question is why we acquire this precious data, just to use a little part of it and discard the most. Over the past few years, a new sampling theory called compressed sensing (CS) has emerged. CS is a new pattern of obtaining signals, different from identical rate digitization followed by compression and often used for transmission or storage [10] [11]. CS delivers us a basis for acquiring and compressing signals simultaneously. The CS theory [11] states that if signals are sparse in some basis, then they will be recovered from a small number of random linear measurements via attractive convex optimization techniques. Innovation in CS have the potential to reduce the sampling rate

significantly in many signal processing and its applications such as cameras, medical scanners, fast analog to digital converter, and high-speed radar are widely used.

Compressive sensing comprehends three key problems: sparse representation, measurement matrix and reconstruction algorithm. In compressive sensing, the signal is passed first through some sort of transformed (Fourier transform (FT), wavelet transform (WT), discrete cosine transom (DCT)) into sparse domain or compressible, so that a measurement matrix irrelevant to transform basis can be designed to measure the signal, and the measures values can realize the exact or approximate signal reconstruction by solving numerical optimization techniques.

The main contribution of this paper is the development of a novel method for image compression based on compressive sensing using wavelet lifting scheme which is faster, simpler, and also keeping strong edge preservation. The method uses proposed sparse representation based on CDF9/7 wavelet transform. To guarantee exact recovery of every  $x$ - sparse signal, the measurement matrix needs to be one-to-one on all  $x$ -sparse vectors. Under this circumstance, Candes and Tao showed a slightly stronger condition RIP (Restricted Isometric Property) [12]. In this paper, compare the best fit of sparse representation of image by CDF9/7 wavelet transform, we use Gaussian measurement matrix [13], Bernoulli measurement matrix [14] and Random orthogonal measurement matrix [15] and image reconstruction by convex optimization techniques for reconstruction of image such as  $l_1$  norm which is called Basis Pursuit (BP) [16], and Orthogonal Matching Pursuit (OMP) algorithm [17]. For comparison purposed of this novel work, we have also used sparse basis DWT and DCT of this paper. Experimental results on open sources database [18] show that our proposed approach with sparse basis on CDF9/7 wavelet transform in CS have higher qualities compressed image than traditional approach with sparse DWT and DCT in terms an image quality assessment scheme [19] [20].

This paper is organized as follows. Section 2 gives a review of the compressive sensing problem statement. Section 3 describes sparse image representation by wavelet lifting scheme. Section 4 describes our proposed algorithm for image compression based on compressive sensing. Experimental results are reported in Section 5. The paper ends with a brief conclusion.

## 2. COMPRESSIVE SENSING

Shannon's Nyquist sampling theorem specifies that a signal should be sampled at a rate higher than twice the maximal frequency of the signal for fidelity of signal reconstruction. For high bandwidth signals, such as image and video, the required sampling rate becomes very high. Some small coefficients of the discrete cosine transform (DCT) or wavelet transform (WT) coefficients can be discarded with little affection the quality of the reconstructed signal significantly. This fundamental idea was used in most existing signal compression techniques. The concept of compressive sensing (CS) is to acquire significant information directly without first sampling the signal in the traditional sense. It has been shown that if the signal is "sparse" or compressible, then the acquired information is sufficient to reconstruct the original signal with a high probability [8] [10] [11]. Sparsity is defined with respect to an appropriate basis, such as DCT or WT for that signal. The theory of CS is also acquired measurements of the signal through a process that is incoherent with the signal. Incoherence makes convinced that the information developed is randomly spread out. In CS, a sensing technique should provide a sufficient number of CS measurements in a non-adaptive manner, so that enables near perfect reconstruction.

According to compressive sensing theory three main steps for CS application:

- a) Sparse representation of the signal.
- b) Design  $M \times N$  measurement matrix unrelated to transform basis to measure the signal and develop an  $M$ - dimensional measurement vector.

- c) Reconstruction the signal by  $M$ -dimensional measurement vector.

The three criteria of CS can be described as below:

The following notation [8] as we have  $f = \{f_1, \dots, f_N\}$  be  $N$  real-valued samples of a signal, which can be represented by the transform coefficients,  $x$ . That is,

$$f = \Psi x = \sum_{i=1}^N x_i \psi_i \quad (1)$$

where  $\Psi = [\psi_1, \psi_2, \dots, \psi_N]$  is an  $N \times N$  transform basis matrix, which determines the domain where the signal is sparse and also  $x = [x_1, x_2, \dots, x_N]$  is an  $N$ -dimension vector of coefficients with  $x_i = \langle x, \psi_i \rangle$ . We assume that  $x$  is  $S$ -sparse, meaning that there are only significant elements in  $x$  with  $S \ll N$ .

Suppose a general linear measurement process computes inner products  $M < N$  between  $f$  and a collection of vectors,  $\phi_j$  giving  $y_i = \langle f, \phi_j \rangle; j = 1 \dots M$ . If  $\Phi$  denotes the  $M \times N$  matrix with  $\phi_j$  as row vectors, then the measurements  $y = [y_1, y_2, \dots, y_M]$  are given by:

$$y = \Phi f = \Phi \Psi x = \Theta x \quad (2)$$

where  $y$  is  $M$ -dimensional observation vector and  $\Phi$  is the  $M \times N$  random measurement matrix and  $\Theta = \Phi \Psi$  is called sensing matrix. For reconstruction ability  $x$  is  $S$ -sparse, if  $\Theta$  satisfies the Restricted Isometric Property (RIP) [12]. Define the restricted isometric property constant  $\delta_k \in (0,1)$  for the sparse signal  $x$  for any  $S$  as the minimum value for the constitution of the following formula:

$$(1 - \delta_k) \|x\|_2^2 \leq \|\Theta x\|_2^2 \leq (1 + \delta_k) \|x\|_2^2 \quad (3)$$

Currently, researchers developed some measurements matrix are follows: Gaussian random matrix [13], binary random matrix (Bernoulli matrix) [14], Fourier random matrix [21], Hadamard matrix [22], random orthogonal matrix [15].

The signal reconstruction problem involves using  $y$  to reconstruct the  $N$ -length signal,  $x$  that is  $S$ -sparse, given  $\Phi$  and  $\Psi$ . Since,  $M < N$ , this is an ill-conditioned problem and there are infinite many  $x'$  that is satisfy  $\Theta x' = y$ . The conventional approach to solving ill-conditioned problems of this kind is to minimize the  $l_p$  norm. Define the  $l_p$  norm vector  $x$  as  $\|x\|_p = \sum_{i=1}^N |x_i|^p$  for  $1 \leq p < \infty$  and  $\|x\|_p = \max|x_i|$ . In practice, signals are often encountered that are not exactly sparse, but whose coefficients decay rapidly. As mentioned, compressible signals are that satisfying power law decay:

$$\|x^*\| \leq R i^{\frac{1}{q}}$$

Considered  $l_2$  norm to solve this ill-condition and the optimization problem is given by:

$$\hat{x} = \operatorname{argmin} \|x'\|_2 \text{ such that } \Theta x' = y \quad (4)$$

It has been demonstrated that this  $l_2$  minimization can only produce a non-sparse  $\hat{x}$  [10]. The purpose is that the  $l_2$  norm measures the energy of the signal, and signal sparsity properties could not be combined in this measure. The  $l_0$  norm counts the number of non-zero entries and, therefore, allows us to specify the sparsity requirement. The optimization problem using this norm can be stated as:

$$\hat{x} = \operatorname{argmin} \|x'\|_0 \text{ such that } \Theta x' = y \quad (5)$$

There is a high likelihood of obtaining a solution using only  $M = S + 1$  independent and identical distribution (i.i.d.) Gaussian measurements [11]. However, the produced solution is numerically unstable [10]. It turns out that optimization based on the  $l_1$  norm is able to exactly recover  $S$ -sparse signals with a high probability using only  $M \geq cS\log(N/S)$  i.i.d. Gaussian measurements [10] [24]. The convex optimization problem is given by:

$$\hat{x} = \arg\min \|x'\|_1 \text{ such that } \Theta x' = y \quad (6)$$

This can be reduced to a linear program. Algorithms based on basis pursuit (BP) [4] can be used to solve this problem with a computational complexity of  $O(N^3)$ .

A number of different algorithms have been developed to solve these three CS reconstruction problems. They include linear programming (LP) techniques [10] [24], greedy algorithms [2] [25] [26] [27], gradient-based algorithms [9] [28] and iterative shrinkage algorithms [29] [30]. Although  $l_1$  norm has strong guarantees of exact recovery, it has disadvantages in computational cost and implementation complexity. As a result, another line of research that seems valuable to explore is Orthogonal Matching Pursuit (OMP) algorithm. This recovery scheme is especially simpler to implement and potentially faster than Basis Pursuit (BP). In this paper we have used gradient-based algorithms as OMP algorithm [17] and basis pursuit (BP) algorithm [4] for proposed image compression applications.

### 3. SPARSE IMAGE REPRESENTATION USING WAVELET LIFTING SCHEME

DCT and DWT are the two most commonly used sparse basis in CS algorithms. Due to its periodicity, DCT can well describe the texture characteristics of an image, but it easily may lead to the block effect. DWT can be an excellent description of the edge characteristics of an image. Therefore, it can compensate for this shortcoming of DCT. The reconstruction effects of DWT sparse processing are mainly affected by wavelet function type and decomposition levels. According to the contrast experiments of different wavelet basis, references [10] [14] found that choosing symmlet wavelet function for image wavelet decomposition can produce the best reconstructed image. Currently, multi-resolution pyramid decomposition and synthesis algorithm, namely the Mallat algorithm, is the most commonly used in wavelet research area.

Cohen-Daubechies-Feauveau 9/7 (CDF 9/7) wavelet transform (WT) [31] is a lifting scheme based wavelet transform which can reduce the computational complexity. A lifting is an elementary modification of perfect reconstruction filters, which is used to improve the wavelet properties. The lifting scheme is a flexible technique that has been used in several different settings for easy construction and implementation of traditional wavelets and for construction of wavelets on arbitrary domains such as bounded regions, second generation wavelets or surface, spherical wavelets. To optimize the approximation and compression of signals and images, the lifting scheme has also been widely used to construct adaptive wavelet basis with signal-dependent lifting. The principle of lifting scheme is described as follows: consider an input image  $x$  fed into a  $\tilde{h}$  (low pass filter) and  $\tilde{g}$  (high pass filter) separately. The outputs of the two filters are then down sampled by  $2(\downarrow 2)$ . The resulting low-pass subband  $y_L$  and high-pass subband  $y_H$  are shown in Figure 1. The original signal can be reconstructed by synthesis filters  $h$  (low pass) and  $g$  (high pass), which take the up-sampled by  $2(\uparrow 2)$  for  $y_L$  and  $y_H$  as inputs [32] [33]. An analysis and synthesis system has the perfect reconstruction property if and only if  $x' = x$ .

The mathematical representations of  $y_L$  and  $y_H$  can be defined as

$$\begin{cases} y_L(n) = \sum_{i=0}^{N_L-1} \tilde{h}(i)x(2n-i) \\ y_H(n) = \sum_{i=0}^{N_H-1} \tilde{g}(i)x(2n-i) \end{cases} \quad (7)$$

Where  $N_L$  and  $N_H$  are the lengths of  $\tilde{h}$  and  $\tilde{g}$  respectively.

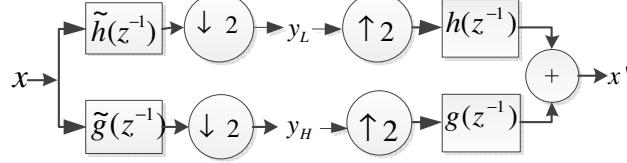


Figure 1: Discrete wavelet transform (or subband transform) analysis and synthesis system.

In this paper, we developed multi-layer lifting scheme WT with sparse basis, which decomposes an image  $x$  into 4 parts for each layer: low-low frequency LL1, high-low frequency HL1, low-high frequency LH1, and high-high frequency HH1. LL1 is a sub-band corresponding to low frequency in both vertical and horizontal direction; HL1 is a sub-band corresponding to high frequency in vertical and low frequency in horizontal direction; LH1 is a sub-band corresponding to low frequency in vertical and high frequency in horizontal direction; HH1 is a sub-band corresponding to high frequency in both vertical and horizontal direction. For this first layer, an image  $x$  is decomposed into 4 parts LL1, HL1, LH1 and HH1. This concept is also applied to the second and third level decompositions based on the principle of multiresolution analysis. For example the LL1 subband is decomposed into four smaller subbands: LL2, HL2, LH2, and HH2. The three layer subbands are sparse so we can adopt Orthogonal Matching Pursuit (OMP) algorithm [17] or Basis Pursuit (BP) to rebuild these parts directly.

#### 4. PROPOSED METHOD

For image processing application, we proposed an image compression based on compressive sensing using wavelet lifting scheme framework as shown in Figure 2. The objective of novel image compression based on compressive sensing process is to estimate the original image  $x$  with dimension  $N \times N$  pixels from the function  $f$  as show in Equation (1). The parameters are denoted as  $x$  is an image,  $\Psi$  is an  $N \times N$  transform basis matrix,  $\Phi$  is the  $M \times N$  measurement matrix, OMP is Orthogonal Matching Pursuit algorithm, BP is Basis Pursuit algorithm and  $x'$  reconstruction image. Our proposed algorithm is designed as shown in Figure 2.



Figure 2: Proposed image compression framework based on compressive sensing.

Proposed Algorithm:

Stage 1: *Compute  $\Psi$* :

- Takes sparsity of an image  $f = \Psi x = \sum_{i=1}^N x_i \psi_i$ , where  $x$  is the sparse vector with only  $S \ll N$  and  $x_i$  is none zero elements.
- Perform sparse domain CDF9/7 wavelet transform to signal  $f$  into several sparse directions to CDF9/7 wavelet subband  $f_{j,l}$ , where  $j$  is the decomposition level and  $l$  the number of direction level at each scale. In general the image  $f$  is transformed into a  $x$  sparse signal with only  $x_i$  non-zero elements. We are developed into three levels of the CDF9/7 wavelet transform coefficients are respectively expressed as

$$\hat{f}^1 = W^1 f = [\hat{f}_{LL}^1 \hat{f}_{LH}^1 \hat{f}_{HL}^1 \hat{f}_{HH}^1]^T$$

$$\hat{f}^2 = W^2 \hat{f}_{LL}^1 = [\hat{f}_{LL}^2 \hat{f}_{LH}^2 \hat{f}_{HL}^2 \hat{f}_{HH}^2]^T$$

and

$$\hat{f}^3 = W^3 \hat{f}_{LL}^2 = [\hat{f}_{LL}^3 \hat{f}_{LH}^3 \hat{f}_{HL}^3 \hat{f}_{HH}^3]^T$$

Where  $W^j, j \in \{1,2,3\}$  represents the 2D CDF9/7 wavelet transform (WT) matrix of level  $j$ .

Stage 2: *Compute  $\Phi$ :*

- a. Design  $M \times N$  dimension observation matrix  $\Phi$ , where  $M < N$ , then use  $\Phi$  to measure  $f_{N \times N}$  and develop the observation vector  $y_{M \times 1}$ .

Stage 3:

- a. Apply CS scheme to each direction and decomposition level as follows:

$$y = \Phi_{j,l} \Psi x_{j,l} \quad (8)$$

Stage 4: *Reconstruction:*

- a. Apply  $l_1$  norm approach is called Basis Pursuit (BP) or Orthogonal Matching Pursuit (OMP) to reconstruct the signal  $x$  from  $y$  of Equation (8).
- b. Reconstruction of subbands is transformed back to the spatial domain by inverse CDF9/7 wavelet transform to recovered image  $x'$ .

Stage 5: *Evaluation:*

To evaluate the compressed image using proposed quality indexes [19] [20] and others evaluation of image quality (EIQ) indexes.

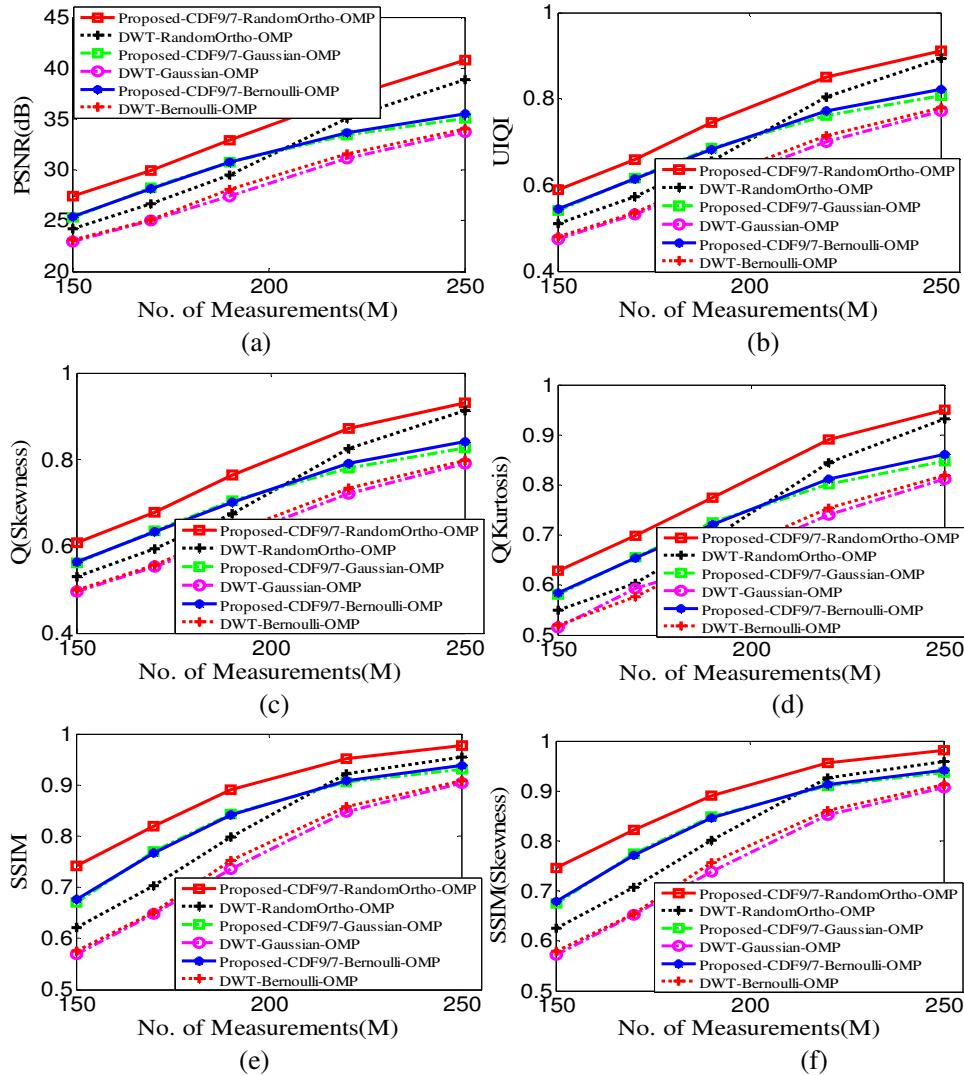
Stage 6: *Comparison:*

We use sparse basis DWT or DCT to decompose the image into feature by replacing of stage 1(b).

## 5. EXPERIMENTAL RESULTS

In this section, we illustrate the reconstruction performance of compressed sensing image; we deploy both the OMP and Basis Pursuit on proposed sparse basis CDF9/7 wavelet transform (WT), DWT and DCT using Gaussian matrix, Bernoulli matrix and random orthogonal matrix measurements. To evaluate the performance of proposed algorithm we used open source databases [18]. The images are for 8-bit  $256 \times 256$  pixels and 8-bit  $64 \times 64$  pixels. We used different image quality indexes: PNSR, UIQI, Q(Skewness), Q(Kurtosis), SSIM, SSIM(Skewness), SSIM(Abs-Skewness), SSIM(Kurtosis) [19] [20] to evaluate the qualities of the compressed images. In order to make clearer comparisons, besides our proposed sparse basis CDF9/7 WT with three different measurements matrix's and Basis Pursuit (BP) algorithms or greedy optimization techniques as OMP and are also used. We made experiments on 15 images in our proposed image compression based on CS methods. Comparing the evaluation of image quality (EIQ) indexes for the compressed image at several measurements  $M < N$  ( $N = 256$ ), we note that the higher measurements  $M$ , the better quality of image compression will be. Figures 3 & 4 show the quality indexes of image compression obtained with proposed sparse CDF 9/7 wavelet transform (WT) and sparse DWT with OMP algorithm performed on the human body

muscle cells and cellulase cells image from novel image compression framework based on CS as shown in Figure 2 . As example, in Figure 3 (a), the PSNR values of proposed sparse basis CDF 9/7 with three measurements matrix and OMP in proposed framework is showing higher values than sparse basis DWT with three measurements matrix and OMP in the range [150-250]. Similarly, we also observes that other quality index values including new image quality indexes of proposed sparse basis CDF 9/7 WT are higher values than others. However, the new image quality indexes also show higher value than original UIQI and SSIM. Comparing the evaluation of image quality (EIQ) indexes for the compressed image at several measurements  $M < N$  ( $N = 256$ ), we note that the higher the measurements  $M$  given the better the quality of image compression will be. Figures 5 & 6 also show the quality indexes of image compression obtained with proposed sparse basis CDF9/7 WT and other techniques of sparse basis DWT or DCT with BP algorithm were also used on the Brain (MR)-1 and Brain (MR)-2 images.



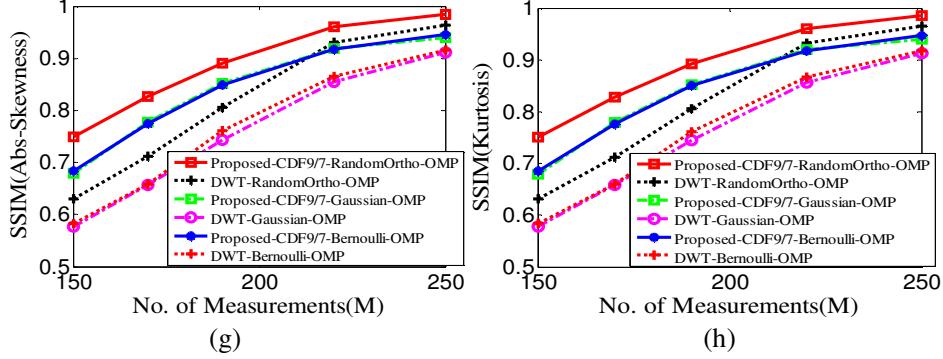
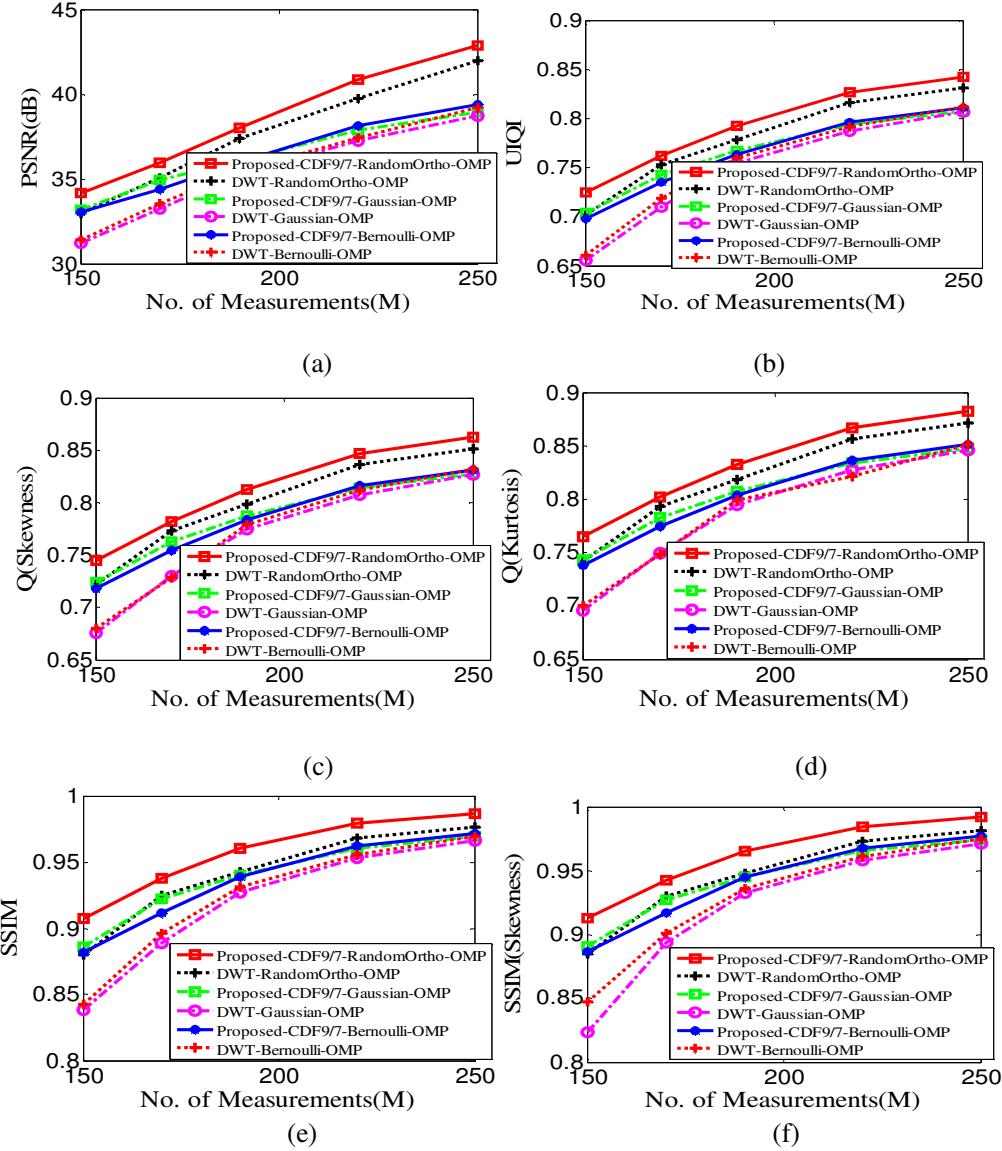


Figure 3: Plots of eight image quality indexes for human body muscle cells image versus number of measurements ( $M$ ) ( $M < N, N = 256$ ) in the range [150-250] for proposed sparse basis CDF9/7 wavelet transform with three measurements matrix and OMP including the sparse basis DWT.



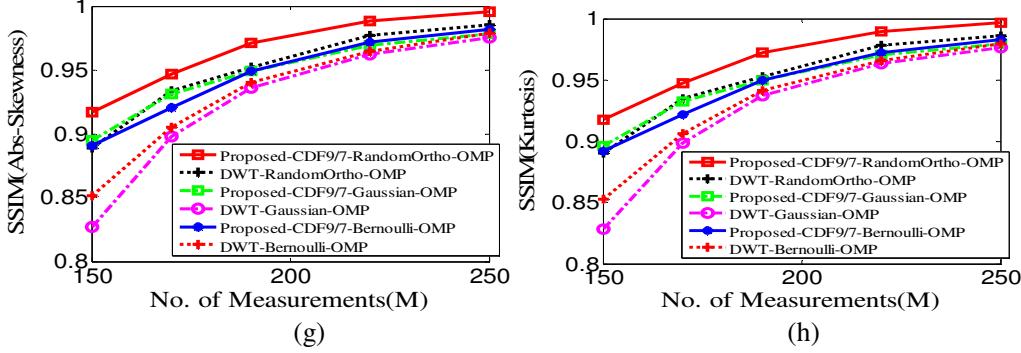
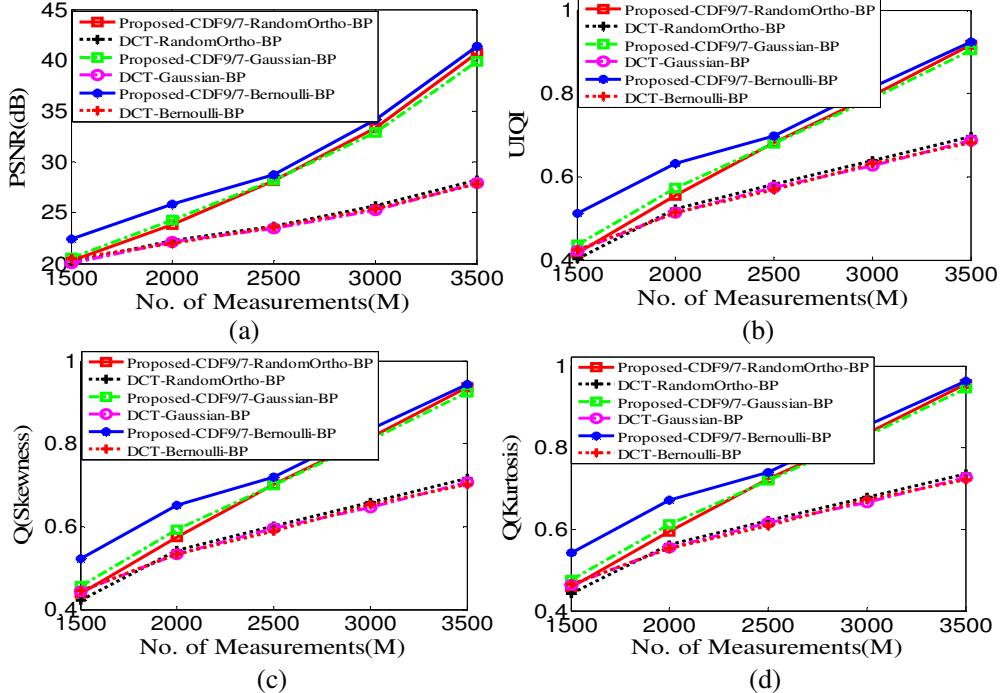


Figure 4: Plots of eight image quality indexes for human body celulas cell image versus number of measurements ( $M$ ) ( $M < N, N = 256$ ) in the range [150-250] for proposed sparse basis CDF9/7 wavelet transform with three measurements matrix and OMP including the sparse basis DWT.

The experimental results clearly show that the proposed method based on sparse basis CDF 9/7 WT by OMP with three different measurements matrix as Gaussian, Bernoulli and random orthogonal is out-performed than all other methods with various number of measures ( $M$ ) ( $M < N, N = 256$ ) in the range [150-250]. However the proposed sparse basis CDF9/7 WT with BP algorithm is also better over all other methods with different number of measurements ( $M$ ) ( $M < N, N = 4096$ ) in the range [1500-3500].

Figure 7 shows the visual comparison of compressed images. The comparison is clearer with data plotted in Figure 3, which shows the relationship between the image quality index UIQI, Q(Skewness), Q(Kurtosis), SSIM, SSIM(Skewness), SSIM(Absolute Skewness), SSIM(Kurtosis), for the different number of measurements ( $M$ ) values performed on an human body muscle image. Similarly for Brain (MR)-1 image, the visual comparison of the compressed image in Figure 8 with data plotted in Figure 5.



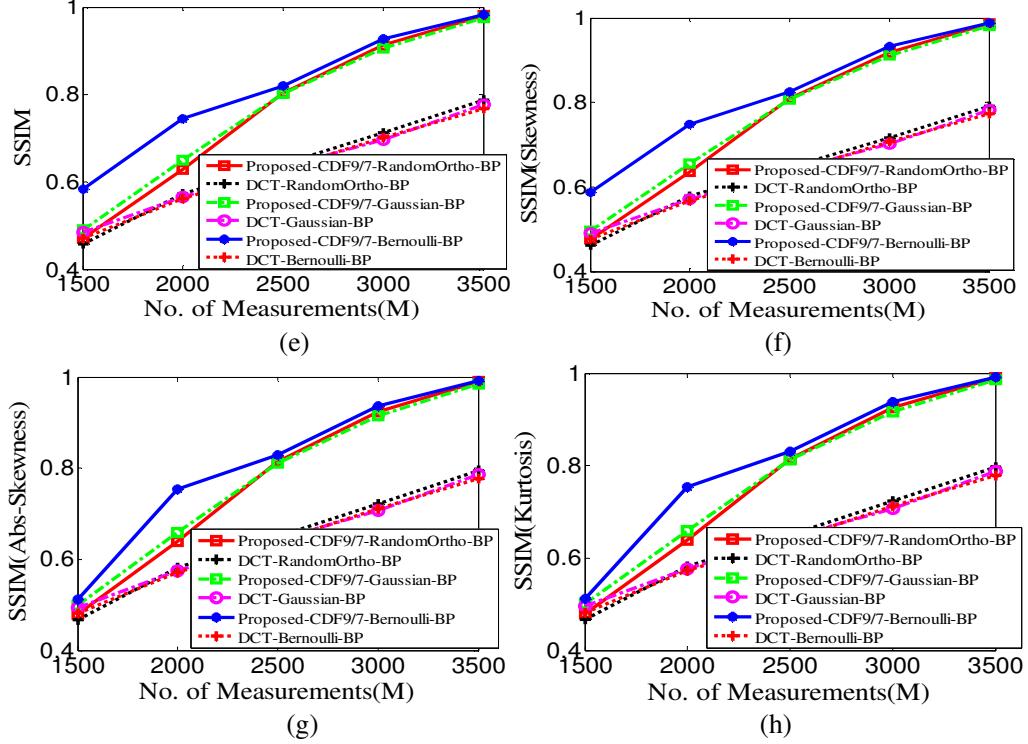
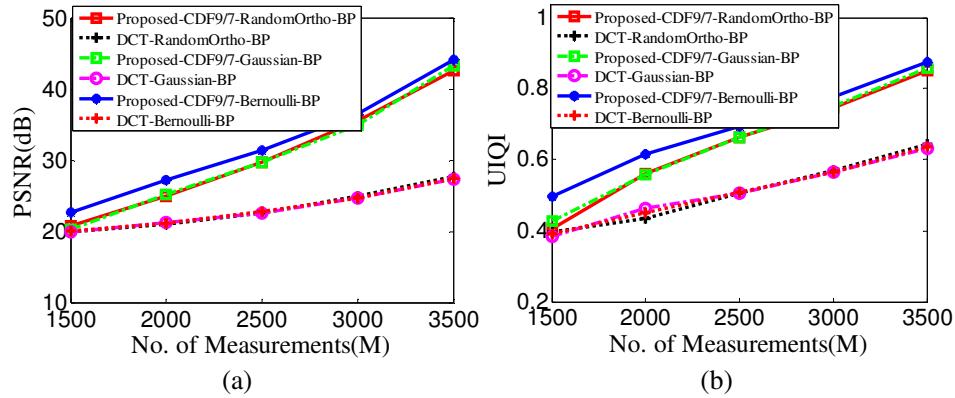


Figure 5: Plots of eight image quality indexes for brain (MR)-1 image versus number of measurements ( $M$ ) ( $M < N, N = 4096$ ) in the range [1500-3500] for proposed sparse basis CDF9/7 wavelet transform with three measurements matrix and BP including the sparse basis DCT.

In Figure 7, represent the visual comparison why does need proposed algorithm based on compressive sensing. The visual effect is showing the compressed image with the proposed algorithm (CDF9/7 WT with OMP) algorithm in Figure 7(b) random orthogonal measurement matrix, Bernoulli measurement matrix in Figure 7(c), and Gaussian measurement matrix (where  $N = 256$  and  $M = 190$ ) in Figure 7(d) higher quality as compared with compressed image of DWT with OMP algorithm for Figure 7(e) - (g).

Similarly, we can also compare of compressed image with proposed algorithm (CDF9/7 WT with BP algorithm) for Figure 8 (b) – (d) (where  $N = 4096$  and  $M = 3000$ ), therefore showing higher quality as compared with compressed image of DCT with BP algorithm for Figure 8(e) -(g).



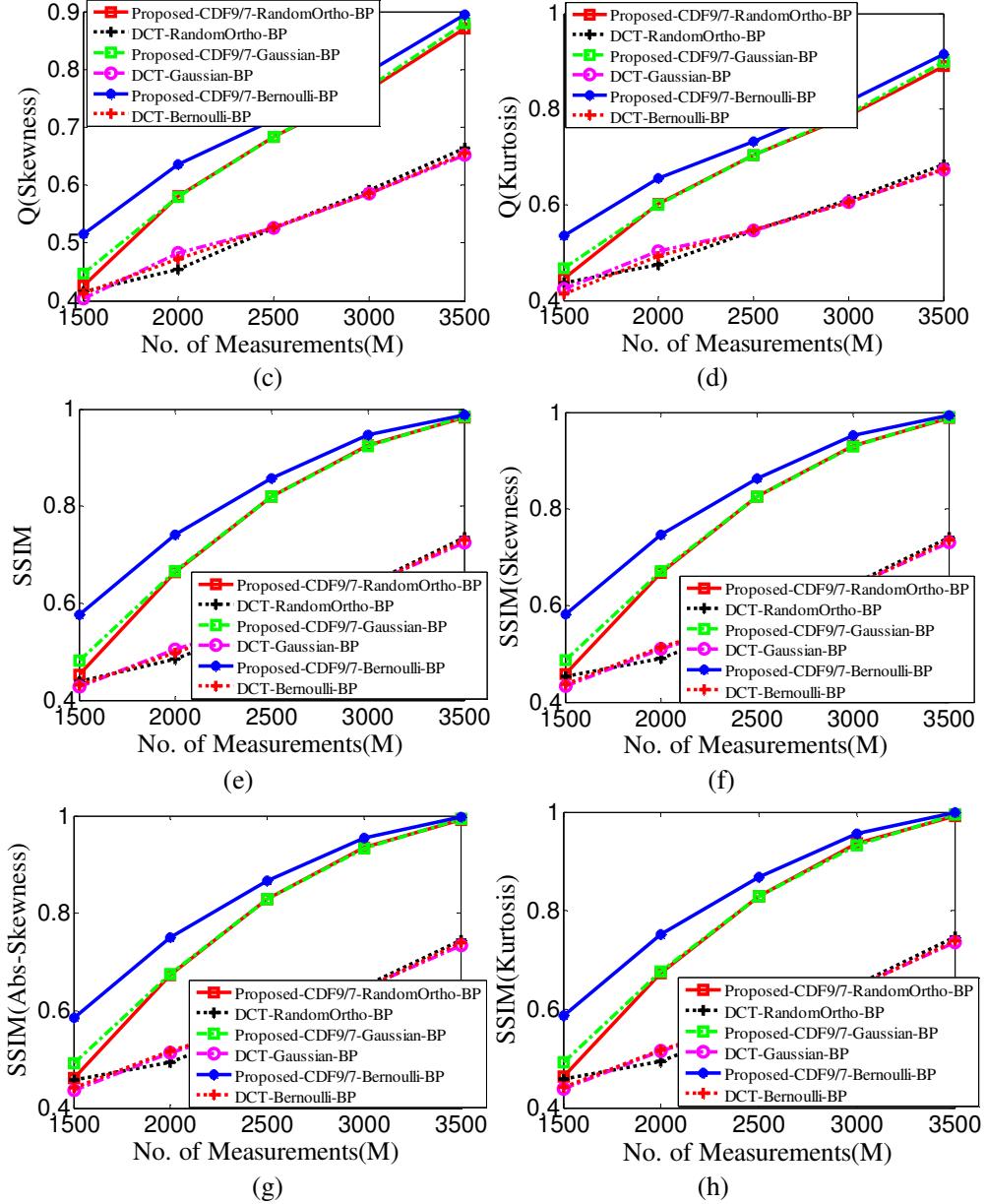


Figure 6: Plots of eight image quality indexes for brain-2 (MR) image versus number of measurements ( $M$ ) ( $M < N, N = 4096$ ) in the range [1500-3500] for proposed sparse basis on CDF9/7 wavelet transform with three measurements matrix and BP including the sparse basis on DCT.

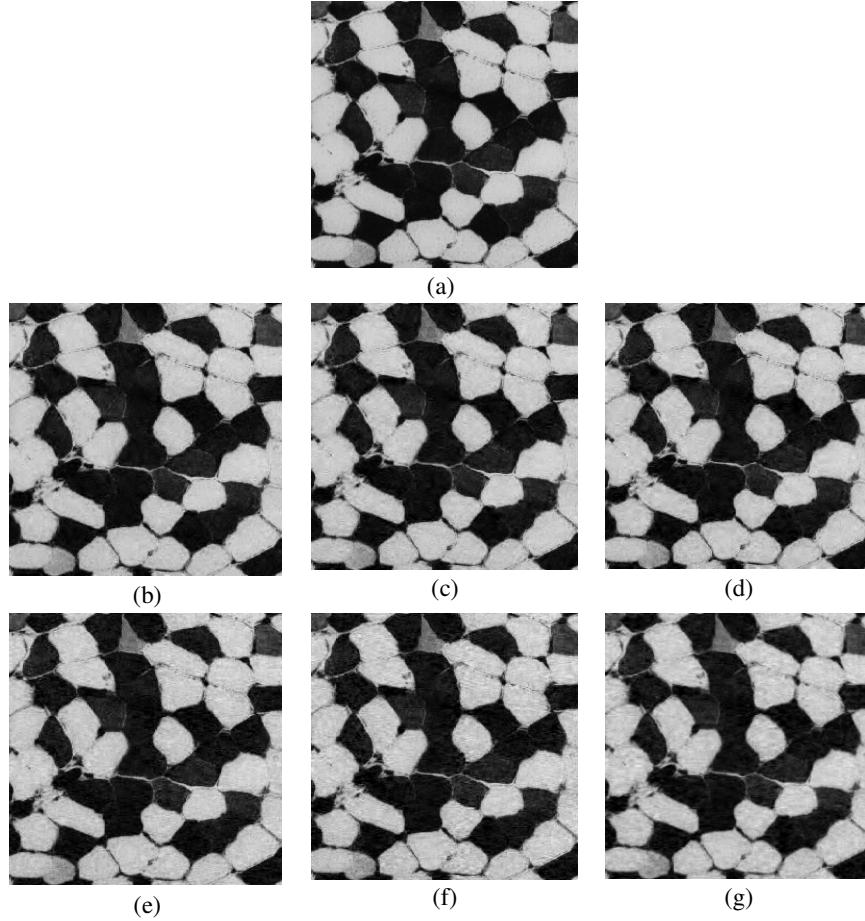


Figure 7: Compressed of human body muscle cells image (grey) (a) Original image ; compressed image with proposed algorithm (CDF9/7 WT with OMP) for (b) random orthogonal measurement matrix; (c) Bernoulli measurement matrix; (d) Gaussian measurement matrix; and compressed image of DWT with OMP for (e) random orthogonal measurement matrix; (f) Bernoulli measurement matrix; (g) Gaussian measurement matrix (where  $N = 256$  and  $M = 190$ ).

As can be seen from all figures, proposed algorithm has increased quality images significantly. The values of image quality index are better than others such as sparse DWT or DCT based algorithm. These shows the proposed algorithm is feasible and has distinct advantages over others. As we all know, image quality indexes value reflects the difference between the original image and reconstructed image and the higher is quality indexes, the better. The proposed algorithm is not only deals with higher frequency part, but also deals with low-frequency part to ensure reconstruction quality.

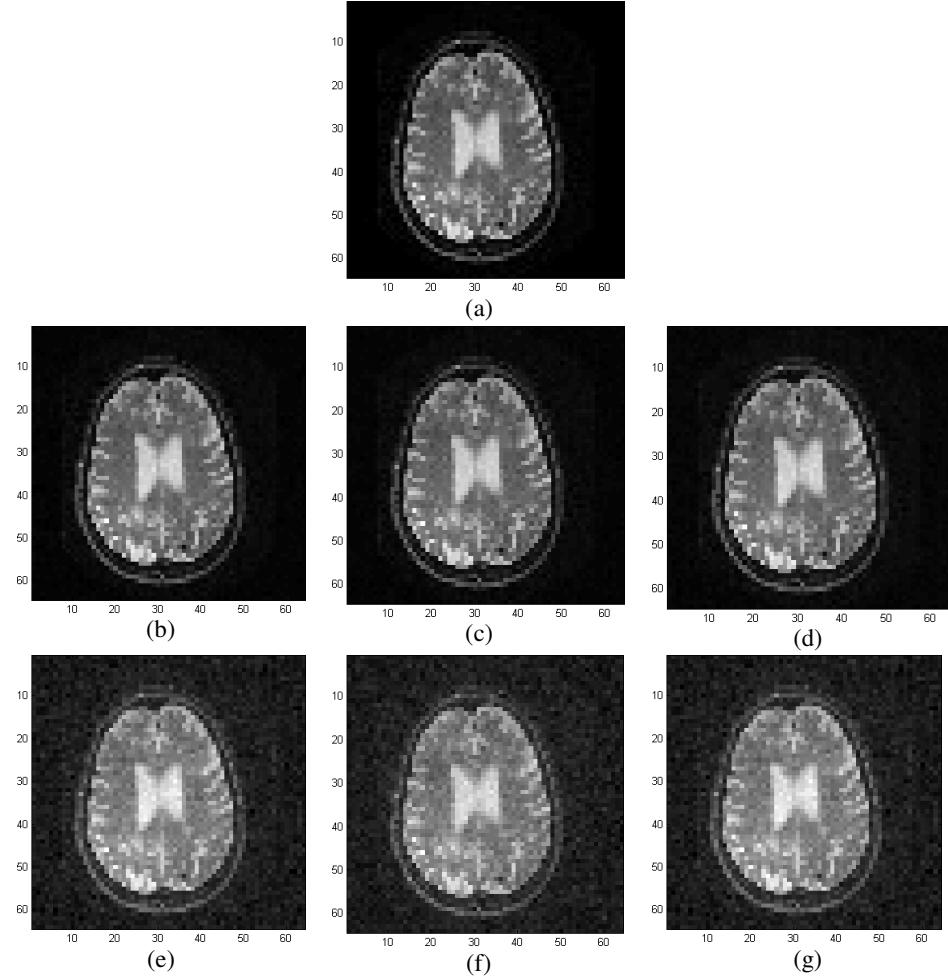


Figure 8: Compressed of human brain (MR) image (grey) (a) Original image ; compressed image with proposed algorithm (CDF9/7 WT with BP) for (b) random orthogonal measurement matrix; (c) Bernoulli measurement matrix; (d) Gaussian measurement matrix; and compressed image of DCT with BP algorithm for (e) random orthogonal measurement matrix (f) Bernoulli measurement matrix (g) Gaussian measurement matrix (where  $N = 4096$  and  $M = 3500$ ).

The main motivation was to develop a good image compression using proposed sparse CDF9/7 wavelet transform based on compressive sensing as much as possible and its quality evaluates by proposed image quality indexes and others evaluation of image quality (EIQ) indexes. This proposed algorithm can be used for compressive sensing image processing application.

## 6. CONCLUSION

In this paper, we proposed image compression based on compressive sensing using wavelet lifting scheme framework that addresses the best-compressed image components and preservation of high-frequency details in medical images. The proposed method is also compared with three different matrix, i.e. Gaussian, Bernoulli and random orthogonal measurement matrix and image reconstruction by convex optimization technique for reconstruction of the image via  $l_1 - \text{norm}$  which is called Basis Pursuit (BP) and greedy pursuit such as Orthogonal Matching Pursuit (OMP) algorithm. Experimental results demonstrate that the proposed sparse basis CDF9/7

wavelet transform can better compressed images than sparse DWT or DCT with OMP or Basis Pursuit (BP) algorithm in proposed framework with CS.

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