

A MODEL FOR PRICING USED PRODUCTS AND REPLACEMENT PARTS IN REVERSE LOGISTICS

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ABSTRACT:

A unique specification in remanufacturing is the uncertainty of returned flows. This makes the coordination between supply and demand difficult for the firm. As a result, remanufacturers typically use pricing tools to control the return flow of used products.

In this study, a model is presented for optimal quantity and price of used products and the price of used products with replacement parts after collection and consolidation based on their quality levels. This model was developed from the perspective of the remanufacturer and the consolidation center. When the consolidation center receives the remanufacturer's demand, the consolidation center and the remanufacturer use the proposed model for evaluating the optimal quantity and the acquisition price of used products as well as the price provided by the remanufacturer to the consolidation center so that they both reach maximum profit. The supply of used products is random. The presented model is an integer nonlinear programming (INLP) model. Consequently, due to the complexity of the problem, The SA and GA metaheuristic methods are used to solve the model.

KEYWORDS:

Reverse logistics, remanufacturing, pricing, used products, recycle, replacement parts.

1. INTRODUCTION

In recent years, reverse logistics (RL) has been considered due to environmental directives and legislation, consumer concerns and social responsibilities for the environment, awareness of the limits of natural resources and economic potential. Pokharel and Mutha (2009) categorized reverse logistics into various aspects of product recovery such as redesign, collection, reuse, recycle and remanufacture.

One of the most profitable product recovery options is remanufacturing which is the process of remanufacturing used parts and products in order to achieve similar conditions with new products (Zhou et al., 2011). Remanufacturing is used successfully in many industries such as cell phones, computers, cameras, photocopiers, telecommunications equipment and automotive parts. Remanufacturing provides environmental benefits while increasing profitability and reducing

production costs (Ferrer and Whybark, 2000). For example, the remanufacturing cost is typically 40% to 60% of the cost of manufacturing any new product by consuming only 20% of energy (Mitra, 2007; Guide and Jayaraman, 2000).

The first step in remanufacturing is the acquisition of used products. It affects all other activities and complicates the problem due to the high uncertainty in the quantity, time and quality of returns (Guide, 2000; Guide and Wassenhove, 2001).

Due to the high uncertainty in the timing and quantity of returned products and the demand for remanufactured products, supply and demand are rarely coordinated (Xiong et al., 2014). One of the best ways to manage the return flow of used products is to use pricing tools.

In this study, four elements are considered as the major players in remanufacturing:

1. Users or customers: who hold renewable products.
2. Collection centers: receive used products from users.
3. A consolidation center: receives used products from collection centers in a pre-determined time interval.
4. A remanufacturer: A company that receives a combination of used products and replacement parts.

This paper focuses on assessing the optimal quantity and acquisition price by the consolidation center and the price offered by the remanufacturer to the consolidation center.

2. LITERATURE REVIEW

Study on reverse logistics has been developing since 1960's. Research on the RL models and strategies can be seen in the journals of 1980's onwards. Most studies have focused only on limited issues of RL, such as network design, production planning and environmental programs (Pokharel and Mutha, 2009). Fleischmann et al. (1997) studied reverse logistics from the perspective of distribution planning, inventory control and production planning. Carter and Ellram (1998) focused on the transportation and packaging, procurement and environmental aspects. Rubio et al. (2008) reviewed articles published between 1995 and 2005, focusing on recovery management, production planning and inventory management, and supply chain management.

Pokharel and Mutha (2009) presented a study on reverse logistics. They used content analysis method for displaying a comprehensive view of the reverse logistics. This study shows that mathematical modeling in RL focuses on deterministic methods, and limited articles considered random demand for remanufactured products and the random supply of used products. In addition, the formulation of pricing policies for attracting used products is still being developed. In another study in 2009, they presented a mathematical model for designing a RL network in order to manage the returned products. The purpose of this paper is to reduce RL costs by determining the number, location and capacity of various facilities and assigning material flows between them.

Liang et al. (2009) presented a model for evaluating the acquisition price of used products in the open market. The proposed model relates the acquisition price of used products to the sales price

of remanufactured ones and assumes other costs deterministic such as logistical and remanufacturing costs. The model also assumes that the sales price fluctuations of remanufactured products follow the Geometric Brownian Motion (GBM).

Darabi et al. (2011) proposed a mixed integer nonlinear programming model for the integrated network design of a multi-level and multi-period forward-reverse supply chain for the recovery of raw materials of returned products.

Qiaolun et al. (2011) studied the price of collecting used products in reverse supply chains. They also considered a recovery channel whereby the remanufacturer either collects/processes used products itself or outsources them to a retailer or third party. Shi et al. (2011) examined a closed-loop system in which the manufacturer has two channels to meet the demand: manufacturing new products and remanufacturing returns as new products. In this paper, a mathematical model was developed which aimed to maximize the overall profit of the system by simultaneous determination of the sale price, the production quantity of new and remanufactured products and the acquisition price of used products.

Pokharel and Liang (2012) presented a quantitative model to evaluate the optimal quantity and the acquisition price of used products based on their quality levels. This model was developed from the view of consolidation center as the decision maker that receives the price and demand information from the remanufacturer and then evaluates the optimal quantity and acquisition price of used products based on the available data on the random quantity and quality of returns and the predefined cost of replacement parts and offers them to collection centers.

Jing and Huang (2013) examined optimal pricing policies with replacement purchase and varying return rates (deterministic or random). Sun et al. (2013) examined a dynamic acquisition pricing problem and the remanufacturing quantity in a multi-period problem with random returns sensitive to price.

Chen and Chang (2013) presented an unconstrained static model and two constrained dynamic models using the Lagrangian procedures and the dynamic programming and the pricing strategy in multi-period settings.

Mahmoudzadeh et al. (2013) applied a robust optimization procedure for a dynamic manufacturing and pricing problem in a multi-period hybrid manufacturing and remanufacturing system, faced with returns and demand uncertainty.

Xiong et al. (2014) provided a study on dynamic pricing of used products for a car engine remanufacturer faced with price-dependent random returns and random demand.

3. MODEL DESCRIPTION

Using its own tools, the remanufacturer predicts the demand for used products and orders a fixed quantity to the consolidation center. The assessment of the remanufacturer demand is outside the scope of this research. In addition, this study assumes that there is one consolidation center and many collection centers. When the consolidation center receives the demand of the remanufacturer, the consolidation center and the remanufacturer use the proposed model for evaluating the optimal quantity and acquisition price of used products and also the price offered

by the remanufacturer to the consolidation center so that they both achieve maximum profit. Moreover, a set of replacement parts needed for remanufacturing used products at a certain quality level along with used products are sent to the remanufacturer. Therefore, the acquisition price of the consolidation center also depends on the cost of replacement parts.

This study assumes that the consolidation center can predict the supply parameters (mean and standard deviation). Since supply is random, the quantity of used products obtained from the collection centers may not correctly correspond with demand. To provide business continuity, it is assumed that the consolidation center receives all used products collected by collection centers and recycles the surplus after meeting the remanufacturer's demand. But when faced with a shortage in supply, the consolidation center buys used products directly from the market to meet the remanufacturer's demand. Therefore, the proposed model minimizes surplus and shortage. Figure 1 shows the conceptual model used in this study based on transactions on the consolidation center and the remanufacturer.

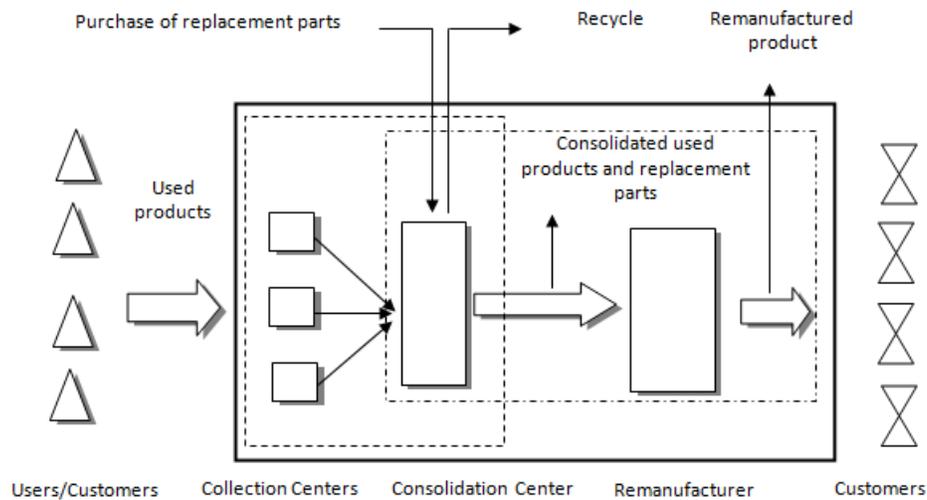


Figure 1: Conceptual model of the study

3.1. Model Assumptions:

1. The supply of used products is random, but the remanufacturer's demand for used products and the quantity and sales price of remanufactured products is constant.
2. The supply of used products follows the normal probability density function with known mean and standard deviation.
3. There is no initial inventory, and transactions are limited to a specified period.
4. Used products at any quality level need predetermined parts for remanufacturing, so a coefficient is predefined for calculating the cost of replacement parts for each used product at each quality level.

4. MODEL DEVELOPMENT

The symbols used in the model are as follows:

Parameters

n : The quality level of used products, which is defined by the remanufacturer $n = 1, 2, \dots, k$.

b_n : Coefficient between 0 and 1 for calculating the total cost of replacement parts for quality level n in ascending order.

p_0 : Coefficient between 0 and 1 for calculating penalty price.

r_0 : Coefficient between 0 and 1 for calculating recycle price, which is smaller than P_0 .

d : Deterministic demand of the remanufacturer for used products.

S_n : Expected supply of used product at quality level n .

$f(S_n)$: Probability density function of supply for used product at quality level n .

$N(\mu_n, \sigma_n)$: Mean and standard deviation of the normal distribution

σ_n : Standard deviation of the supply of used product with quality level n from collection centers to the consolidation center.

q_{s_n} : Sales quantity of the remanufacturer.

x_n : Selling price of the remanufacturer.

c_n : Remanufacturing costs of the remanufacturer.

$G_u(k)$: Loss function.

Decision Variables

P_n : Price of a used product with quality level n , offered by the remanufacturer to the consolidation center.

L_n : Acquisition price of used product with quality level n , offered by the consolidation center to the collection centers. Costs of inspections and logistics are included in this price.

q_n : Acquisition quantity of used product with quality level n by the consolidation center.

Dependent Variables

$b_n L_n$: Total cost of pre-defined replacement new parts needed, for each used product unit with quality level n , offered by the consolidation center to the remanufacturer.

$p_0 p_n$: Penalty price for shortage of used products with quality level n which is the same as the price of new product.

r_0p_n : Recycle price for the surplus of used products with quality level n .

4.1. Mathematical Modeling

Using the above symbols, the integer nonlinear programming model for pricing is as follows:

$$Max \Pi = \sum_{n=1}^k [(q_n p_n) + (q_{s_n} x_n)] - \sum_{n=1}^k \left\{ [S_n L_n + q_n (b_n L_n) + p_0 p_n \int_{q_n}^{q_n} (q_n - S_n) f(S_n) ds_n - r_0 p_n \int_{q_n}^{\infty} (S_n - q_n) f(S_n) ds_n] + [(q_n p_n) + c_n] \right\} \quad (1)$$

$$\sum_{n=1}^k q_n = d \quad (2)$$

$$\sum_{n=1}^k q_{s_n} = d \quad (3)$$

$$r_0 p_n < L_n < p_0 p_n - b_n L_n \quad (4)$$

$$L_n < p_n < x_n \quad (5)$$

$$L_n, q_n, p_n > 0 \quad (6)$$

Expression (1) represents the objective function of the model, which maximizes the total profit composed of total incomes minus total costs. Incomes include the sum of income of the consolidation center and the remanufacturer. Costs include all costs of the consolidation center (e.g. the acquisition cost of the consolidation center) and the remanufacturer costs (including acquisition and remanufacturing costs). Constraint (2) is a constraint on the total acquisition quantity of used products with different quality levels. Constraint (3) is a constraint on the total sales for the remanufacturer with different quality levels. Constraint (4) defines the acquisition price of used products by the consolidation center. Constraint (5) defines the scope of the acquisition price of the remanufacturer. Constraint (6) is related to the type of decision variables of the problem.

The simplification of the two integrals is listed in the appendix. Finally, the proposed objective function after mathematical simplification of the two integrals is shown as follows:

$$Max \Pi = [(q_n p_n) + (q_{s_n} x_n)] - \left\{ \left[S_n l_n + q_n (b_n l_n) + p_0 p_n \left(\frac{q_n}{2} - \frac{\sigma_n}{\sqrt{2\pi}} e^{-\frac{\mu_n^2}{2\sigma_n^2}} - \frac{\mu_n}{2} + \sigma_n G_u(k) \right) - r_0 p_n (\sigma_n G_u(k)) \right] + [(q_n p_n) + c_n] \right\}$$

5. SOLUTION METHOD

In this paper, metaheuristic methods were used to solve the model, given that the model is integer nonlinear programming and complicated. Metaheuristic algorithms used in this study include a single-solution based algorithm (SA) and a population-based algorithm (GA).

5.1. Simulated Annealing Algorithm

The simulated annealing (SA) algorithm originates from the works of Kirkpatrick and Cerny et al. in 1983 and 1985. Kirkpatrick et al. were specialists in the field of statistical physics. They proposed a method based on the annealing technique to solve difficult optimization problems. The annealing technique is used to achieve a state in which a solid substance is well sorted and its energy is minimized. This technique involves placing a substance at a high temperature and then gradually lowering the temperature. The SA algorithm is a simple and effective metaheuristic search algorithm for solving combinatorial optimization problems. Next, the main components of the simulated annealing algorithm are described.

5.1.1. The main components of the simulated annealing algorithm

5.1.1.1. Initial solution

The initial solution is produced randomly. Here is a sample of solutions generated from variables.

1) [105 94 93 77 73 66]

The first matrix represents the price of a used product, which is offered by the remanufacturer to the consolidation center. The second matrix represents the acquisition price of the used product, which is offered by the consolidation center to the collection center, and the third matrix represents the acquisition quantity of used product by the consolidation center.

5.1.1.2. Acceptance Function

The algorithm can exit by the probability of accepting solutions worse than local optimum. The acceptance probability depends on the temperature T (initial temperature) and the variation of the objective function (energy ΔE). The acceptance function in the SA algorithm is defined as follows:

$$p = e^{\frac{-\Delta E}{T}} > R$$

where E is the objective function value and ΔE is the difference between the objective function value of the current solution and the neighbor solution. T is current temperature and R is a random number between zero and one. In the above function, if the ΔE value is less than zero, then the neighbor solution is accepted. Otherwise, if the random number generated at each iteration is less than the probability value, then the neighbor solution is accepted even if it is worse.

5.1.1.3. Initial Temperature

If the initial temperature is too high, searching becomes less and more random. Otherwise, when the temperature is too low, searching becomes somewhat a local search. Thus we must create balance between the two states.

5.1.1.4. Neighborhood structure and motion mode

Neighborhood structures are insertion, switching and inversion. In this study, these three neighborhood structures are used, and the way they are selected in the algorithm is defined randomly.

5.1.1.5. Annealing Function

In the algorithm SA, temperature gradually decreases in a way that temperature becomes greater than zero in each iteration and $\lim_{i \rightarrow \infty} T_i = 0$. Solution quality and annealing speed have reverse relationship. In this study, a geometric function is used to reduce temperature. In the geometric annealing function, temperature is updated by the following equation:

$$T = \alpha T$$

where $\alpha \in (0,1)$. This approach is the most common annealing function. Experience has shown that α should be in the range [0.5, 0.99].

5.1.1.6. Stopping Criterion

Different Stopping criteria can be applied to a simulated annealing algorithm. In this study, reaching a predetermined number of iteration in which a percentage of neighbors is seen at each temperature is used as a criterion to stop the algorithm.

In this study, the SA algorithm starts with a set of initial responses (nPop) and consequently examines more neighborhoods (a set of neighborhoods (nMove)). In this case, the SA algorithm is converted to the Multi Point SA.

5.2. Genetic Algorithm

In 1960, imitation of organisms for use in powerful algorithms for optimization problems was considered, which were called evolutionary computation techniques. The genetic algorithm was first introduced by John Holland et al. in Michigan University in 1962-1965 while presenting a course called adaptive systems. In their research, they focused on the adaptation process in natural systems and tried to model it in artificial systems, which must have the ability of natural systems. The genetic algorithm was the result of this effort.

The main advantages of the genetic algorithm are multilateral search and working on a population of variables. The genetic algorithm starts in a population of solutions with a set of solutions rather than a single solution. So instead of finding an appropriate solution, appropriate scopes in the space of variables are identified.

5.2.1. The main components of genetic algorithm

5.2.1.1. Initial solution

The initial solution is generated randomly. Below is a sample of the solutions generated from variables.

1) [276 251 217 203 182]

5.2.1.2. Crossover Operator

The crossover operator is applied at the same time on two chromosomes and creates two offspring by combining the structure of two chromosomes. In this study, a simple recombination operator is used to perform the crossover operation.

5.2.1.3. Mutation Operator

This operator makes unplanned random changes on different chromosomes and enters genes into the population that did not exist in the initial population. The study uses the non-uniform mutation operator as the mutation operator.

5.2.1.4. Fitness Function

Obviously, there should be an index to evaluate chromosomes and detect most suitable chromosome. For optimization functions problems, this index is usually the objective function value.

5.2.1.5. Selection Mechanism

To choose the best solutions to reproduce a new population, a method should be used that selects the best solution. The selection mechanism used in this study is the roulette wheel algorithm.

5.2.1.6. Stopping Criterion

Different stopping Criteria can be applied to a genetic algorithm. The stopping condition for this algorithm is to reach the pre-specified number of iteration.

It should be noted that this study uses the Taguchi method to set the parameters of both algorithms. The results of mean S/N ratio for each level of the factors in the SA algorithm are shown in Figure 2.

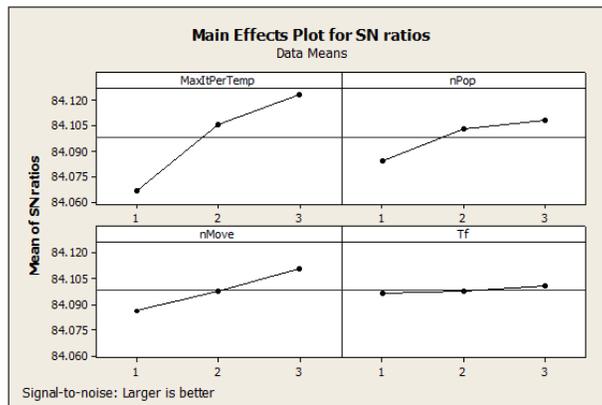


Figure 2: Mean S/N ratio for each level of the factors in the SA algorithm

According to Figure 2, the optimal level of factors MaxItPerTemp, nPop, nMove and T_f for the SA algorithm is equal to: MaxItPerTemp (3) = 30 , NPop (3) = 20, nMove (3) = 20 and T_f (3) = 0.001.

The results of the mean S/N ratio for each level of the factors in the GA algorithm are shown in Figure 3.

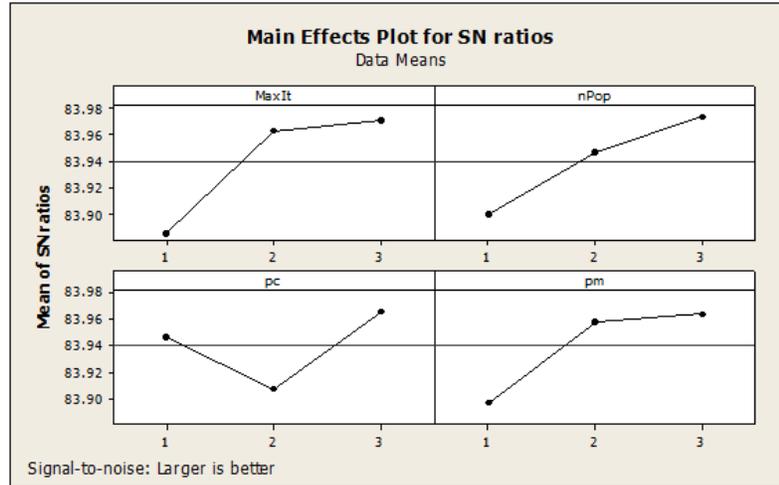


Figure 3: Mean S/N ratio for each level of the factors in the GA algorithm

According to Figure 3, the optimal level of factors MaxIt, nPop, pc and pm for the GA algorithm are: MaxIt (3) = 200, nPop (3) = 120, pc (3) = 0.9 and .Pm (3) = 0.3.

6. NUMERICAL RESULTS

In this section, 30 numerical examples are presented to evaluate and verify metaheuristic algorithms. The examples are divided into three sections (small, medium and large) based on the remanufacturer's demand. The information related to these examples is shown in Table 1. The results of solving the SA algorithm and the GA algorithm are presented in Tables 2 and 3, respectively. In addition, these results are the best solutions in several times of running the algorithm.

Table 1: Data generated for the problem under study

Number	Range of demand	d	n	r0	p0	qs	b	c
1	d<1000	200	6	0/1	0/9	[60 40 20 30 20 30]	[0/1 0/2 0/3 0/4 0/5 0/6]	[30 35 40 50 55 60]
2		275	4	0/2	0/7	[80 100 40 55]	[0/2 0/4 0/48 0/6]	[40 50 68 72]
3		300	8	0/1	0/8	[20 40 70 30 50 20 40 30]	[0/1 0/2 0/35 0/45 0/6 0/62 0/7 0/72]	[15 25 30 34 40 48 60 70]
4		350	3	0/3	0/8	[150 100 100]	[0/3 0/4 0/5]	[70 80 110]
5		500	7	0/2	0/6	[100 150 90 70 40 20 30]	[0/2 0/4 0/5 0/6 0/65 0/7 0/75]	[30 45 55 60 80 87 95]
6		600	5	0/35	0/8	[200 180 70 80 70]	[0/2 0/35 0/45 0/5 0/7]	[70 80 100 130 150]
7		700	5	0/3	0/7	[200 100 150 150 100]	[0/2 0/35 0/45 0/57 0/68]	[70 90 110 150 180]
8		800	10	0/2	0/75	[100 200 70 80 58 90 62 40 50 50]	[0/1 0/15 0/2 0/24 0/3 0/34 0/4 0/5 0/58 0/7]	[25 38 50 100 150 180 200 220 238 250]
9		880	4	0/2	0/7	[300 200 200 180]	[0/3 0/4 0/5 0/58]	[70 80 110 150]
10		900	2	0/3	0/8	[400 500]	[0/3 0/5]	[100 180]
11	1000<d<2000	1000	3	0/25	0/85	[450 350 200]	[0/2 0/4 0/6]	[80 150 180]
12		1250	4	0/2	0/7	[250 300 350 350]	[0/2 0/35 0/5 0/6]	[70 80 95 110]
13		1400	5	0/28	0/84	[300 250 280 440 130]	[0/2 0/3 0/4 0/48 0/58]	[50 70 80 110 150]
14		1550	2	0/35	0/88	[1000 550]	[0/3 0/5]	[100 180]
15		1680	6	0/2	0/65	[400 200 380 350 250 100]	[0/1 0/2 0/35 0/4 0/58 0/7]	[70 80 95 110 130 150]
16		1700	4	0/3	0/85	[400 510 380 410]	[0/2 0/35 0/48 0/6]	[85 100 120 150]
17		1800	3	0/2	0/6	[650 580 570]	[0/3 0/5 0/68]	[100 180 220]
18		1840	7	0/3	0/85	[280 250 300 200 290 280 240]	[0/1 0/2 0/35 0/48 0/55 0/68 0/75]	[80 100 120 150 180 210 240]
19		1900	2	0/3	0/8	[980 920]	[0/3 0/5]	[100 200]
20		1980	3	0/2	0/85	[700 800 480]	[0/2 0/4 0/6]	[100 170 200]
21	d>2000	2100	3	0/2	0/7	[680 750 670]	[0/2 0/4 0/68]	[100 150 185]
22		2500	5	0/3	0/85	[480 520 370 600 530]	[0/2 0/3 0/5 0/58 0/7]	[120 150 180 205 220]
23		2800	4	0/28	0/7	[640 700 740 720]	[0/2 0/38 0/45 0/6]	[100 120 160 190]
24		3000	2	0/3	0/84	[1800 1200]	[0/3 0/5]	[115 150]
25		3500	4	0/2	0/65	[900 940 870 790]	[0/2 0/35 0/48 0/67]	[140 165 180 200]
26		3800	3	0/3	0/75	[1450 1700 650]	[0/3 0/5 0/68]	[110 170 195]
27		4000	6	0/3	0/75	[780 720 680 710 690 420]	[0/2 0/35 0/42 0/57 0/68 0/8]	[110 170 195 210 250 280]
28		4550	2	0/2	0/68	[2500 2050]	[0/3 0/58]	[150 198]
29		4800	4	0/3	0/8	[1500 1200 1350 750]	[0/2 0/38 0/45 0/6]	[100 145 170 195]
30		5000	5	0/35	0/84	[1200 1150 1100 900 650]	[0/2 0/35 0/5 0/68 0/8]	[120 150 180 200 215]

Continued of table 1

Number	Range of demand	d	n	r0	p0	μ	σ	x
1	d<1000	200	6	0/1	0/9	[40 30 60 20 30 20]	[4 3 5 2 4 2]	[140 135 130 110 95 80]
2		275	4	0/2	0/7	[50 30 60 80]	[5 3 2 4]	[150 130 110 90]
3		300	8	0/1	0/8	[40 60 30 70 40 30 50 40]	[5 3 2 2 4 5 4 2]	[250 220 200 180 150 135 120 100]
4		350	3	0/3	0/8	[100 90 80]	[2 4 3]	[200 170 150]
5		500	7	0/2	0/6	[100 80 70 90 60 50 50]	[4 7 5 3 2 4 5]	[300 265 234 210 185 165 140]
6		600	5	0/35	0/8	[100 120 95 80 90]	[4 3 5 2 5]	[320 280 245 220 205]
7		700	5	0/3	0/7	[180 100 170 150 100]	[2 5 4 3 4]	[320 298 270 243 210]
8		800	10	0/2	0/75	[100 150 110 80 70 100 90 65 80 40]	[6 4 3 2 2 5 6 2 6 5]	[520 485 460 425 389 354 330 318 297 275]
9		880	4	0/2	0/7	[350 180 250 220]	[5 4 3 5]	[300 275 225 184]
10		900	2	0/3	0/8	[480 420]	[4 3]	[285 222]
11	1000<d<2000	1000	3	0/25	0/85	[300 550 150]	[4 3 5]	[280 250 210]
12		1250	4	0/2	0/7	[300 400 350 200]	[5 4 3 2]	[300 280 250 200]
13		1400	5	0/28	0/84	[280 220 300 350 150]	[5 4 3 2 5]	[250 220 200 185 174]
14		1550	2	0/35	0/88	[900 850]	[5 4]	[350 280]
15		1680	6	0/2	0/65	[350 300 280 380 200 170]	[5 4 3 2 5 3]	[300 250 230 210 200 188]
16		1700	4	0/3	0/85	[500 380 420 500]	[4 5 3 2]	[300 280 220 200]
17		1800	3	0/2	0/6	[550 480 600]	[4 3 2]	[350 300 280]
18		1840	7	0/3	0/85	[250 300 200 280 270 300 240]	[4 5 3 2 6 4 5]	[380 350 320 310 300 280 260]
19		1900	2	0/3	0/8	[800 700]	[4 5]	[350 300]
20		1980	3	0/2	0/85	[810 750 500]	[5 4 3]	[350 318 280]
21	d>2000	2100	3	0/2	0/7	[780 640 680]	[4 3 5]	[300 280 245]
22		2500	5	0/3	0/85	[600 580 540 480 500]	[5 4 3 6 2]	[380 350 310 290 270]
23		2800	4	0/28	0/7	[800 720 680 600]	[5 4 3 4]	[320 300 285 260]
24		3000	2	0/3	0/84	[1550 1270]	[5 3]	[300 225]
25		3500	4	0/2	0/65	[930 910 880 840]	[5 4 3 5]	[370 340 300 270]
26		3800	3	0/3	0/75	[1650 1300 850]	[4 5 3]	[350 310 260]
27		4000	6	0/3	0/75	[800 690 720 810 700 510]	[5 4 3 6 2 4]	[398 370 360 340 310 300]
28		4550	2	0/2	0/68	[1980 2300]	[5 3]	[380 300]
29		4800	4	0/3	0/8	[1100 1350 1200 900]	[4 5 3 4]	[385 310 280 220]
30		5000	5	0/35	0/84	[1400 1000 900 850 850]	[4 5 3 2 4]	[395 360 310 290 260]

Table 2: Results of solving the SA algorithm

Number	Range of demand	p (SA)	L (SA)	q (SA)	Cost (SA)	Cpu (SA)
1	d<1000	[98 94 91 77 66 56]	[10 9 8 7 6 5]	[1 4 12 7 54 122]	16066/2908	248/182338
2		[105 91 77 62]	[21 19 16 13]	[3 6 16 250]	22346/682	262/32994
3		[175 154 140 125 105 94 84 70]	[18 16 14 13 11 10 9 7]	[3 2 5 1 84 38 30 137]	34435/5556	253/364402
4		[140 118 105]	[42 36 32]	[1 2 347]	31592/0432	252/255495
5		[210 185 163 147 129 115 98]	[42 37 33 30 26 23 20]	[19 23 19 41 11 23 364]	79728/4576	258/450414
6		[224 196 171 154 143]	[79 69 60 54 51]	[3 7 58 471 61]	78587/4124	255/948211
7		[224 208 189 170 147]	[68 63 57 51 45]	[4 7 25 67 597]	92985/2852	260/952554
8		[364 339 322 297 272 247 230 222 207 192]	[73 68 65 60 55 50 46 45 42 39]	[0 28 4 37 29 9 159 122 217 195]	195856/185	264/932832
9		[210 192 157 128]	[42 39 32 26]	[3 7 58 812]	134258/1456	258/716979
10		[199 155]	[60 47]	[0 900]	99721/5856	255/159075
11	1000<d<2000	[196 175 147]	[49 44 37]	[10 3 987]	126414/738	253/454663
12		[210 196 175 140]	[42 40 35 28]	[1 27 41 1181]	187126/8956	245/477268
13		[175 154 140 129 121]	[50 44 40 37 34]	[12 18 17 123 1230]	137276/6046	248/685566
14		[244 196]	[86 69]	[0 1550]	181475/5488	252/265054
15		[210 175 161 147 140 131]	[42 35 33 30 28 27]	[2 12 64 1066 429 107]	242169/789	257/470372
16		[210 196 154 140]	[63 59 47 42]	[15 5 6 1674]	189906/1552	256/193603
17		[244 210 196]	[49 42 40]	[3 1796 1]	339135/0472	253/079488
18		[266 244 224 217 210 196 182]	[80 74 68 66 63 59 55]	[82 624 499 24 7 9 595]	238790/0104	258/415101
19		[244 210]	[74 63]	[0 1900]	296767/0656	250/599045
20		[244 222 196]	[49 45 40]	[2 2 1976]	328502/5464	263/483556
21	d>2000	[210 196 171]	[42 40 35]	[2093 4 3]	324648/7452	263/702699
22		[266 244 217 203 189]	[80 74 66 61 57]	[39 35 8 576 1842]	310516/7608	274/451122
23		[224 210 199 182]	[63 59 56 51]	[2739 17 29 15]	397226/4356	270/093682
24		[210 157]	[63 48]	[0 3000]	382765/2096	265/369392
25		[259 237 210 189]	[52 48 42 38]	[14 27 441 3018]	662093/5542	271/473571
26		[244 220 182]	[74 66 55]	[11 5 3784]	548753/9552	267/590201
27		[278 259 251 237 217 210]	[84 78 76 72 66 63]	[2872 182 315 74 530 27]	601623/677	277/744208
28		[266 210]	[54 42]	[0 4550]	927607/008	264/264068
29		[269 217 196 154]	[81 66 59 47]	[54 28 102 4616]	767936/732	269/977373
30		[276 251 217 203 182]	[97 88 76 72 64]	[101 26 571 297 4005]	610985/9088	278/934735

Table 3: Results of solving the GA algorithm

Number	Range of demand	p (GA)	L (GA)	q (GA)	Cost (GA)	Cpu (GA)
1	d<1000	[98 94 91 77 66 56]	[10 9 8 7 6 5]	[5 5 5 9 77 99]	15966/8408	14/348476
2		[105 91 77 62]	[21 19 16 13]	[5 7 24 239]	22272/792	12/034807
3		[175 154 140 125 105 94 84 70]	[18 16 14 13 11 10 9 7]	[5 8 7 8 48 42 82 100]	34112/4856	15/902107
4		[140 118 105]	[42 36 32]	[4 4 342]	31553/0432	12/005139
5		[210 185 163 147 129 115 98]	[42 37 33 30 26 23 20]	[8 12 16 21 56 152 235]	79423.5576	14.419838
6		[224 196 171 154 143]	[79 69 60 54 51]	[10 11 32 469 78]	78517/7124	13/405609
7		[224 208 189 170 147]	[68 63 57 51 45]	[22 16 19 54 589]	92834/2452	13/068956
8		[364 339 322 297 272 247 230 222 207 192]	[73 68 65 60 55 50 46 45 42 39]	[8 11 17 10 26 111 129 164 162 162]	195455/31	17/153259
9		[210 192 157 128]	[42 39 32 26]	[8 11 20 841]	134456/0256	12/540969
10		[199 155]	[60 47]	[4 896]	99673/1856	10/799835
11	1000<d<2000	[196 175 147]	[49 44 37]	[4 7 989]	126436/088	11/256011
12		[210 196 175 140]	[42 40 35 28]	[31 22 31 1166]	186857/3956	12/076952
13		[175 154 140 129 121]	[50 44 40 37 34]	[11 23 41 356 969]	136806/1466	13/323324
14		[244 196]	[86 69]	[4 1546]	181425/8688	10/474227
15		[210 175 161 147 140 131]	[42 35 33 30 28 27]	[17 42 52 1122 179 268]	242123/414	13/858836
16		[210 196 154 140]	[63 59 47 42]	[21 18 232 1429]	188767/077	12/977453
17		[244 210 196]	[49 42 40]	[17 1751 32]	339018/4472	12/094361
18		[266 244 224 217 210 196 182]	[80 74 68 66 63 59 55]	[64 557 533 28 34 40 584]	238500/0704	15/118298
19		[244 210]	[74 63]	[4 1896]	296749/8656	10/87934
20		[244 222 196]	[49 45 40]	[6 5 1969]	328462/5964	12/207746
21	d>2000	[210 196 171]	[42 40 35]	[1594 187 319]	323601/6452	12/322986
22		[266 244 217 203 189]	[80 74 66 61 57]	[36 56 71 578 1759]	310106/2008	14/470943
23		[224 210 199 182]	[63 59 56 51]	[2214 93 194 299]	395280/0656	13/640212
24		[210 157]	[63 48]	[6 2994]	382662/2496	11/489342
25		[259 237 210 189]	[52 48 42 38]	[43 59 356 3042]	661778/0892	13/298843
26		[244 220 182]	[74 66 55]	[17 15 3768]	548623/5262	12/434618
27		[278 259 251 237 217 210]	[84 78 76 72 66 63]	[1200 1888 423 120 293 76]	595755/362	15/349338
28		[266 210]	[54 42]	[4 4546]	927563/488	11/551518
29		[269 217 196 154]	[81 66 59 47]	[87 132 229 4352]	762594/362	13/288935
30		[276 251 217 203 182]	[97 88 76 72 64]	[82 95 1291 102 3430]	610742/9088	13/847878

6.1. Comparison of solutions and metaheuristic algorithms used to solve the model

In order to compare solutions and algorithms, we used three methods: comparison of considering cpu time, comparison considering the obtained solutions and their difference, and comparison by using hypothesis test of equal means of two populations.

6.1.1. Comparison of solving methods considering cpu time

According to Chart 1, the GA metaheuristic algorithm has a lower solving time than the SA metaheuristic algorithm. So if the criterion is cpu time, the GA algorithm is better than another.

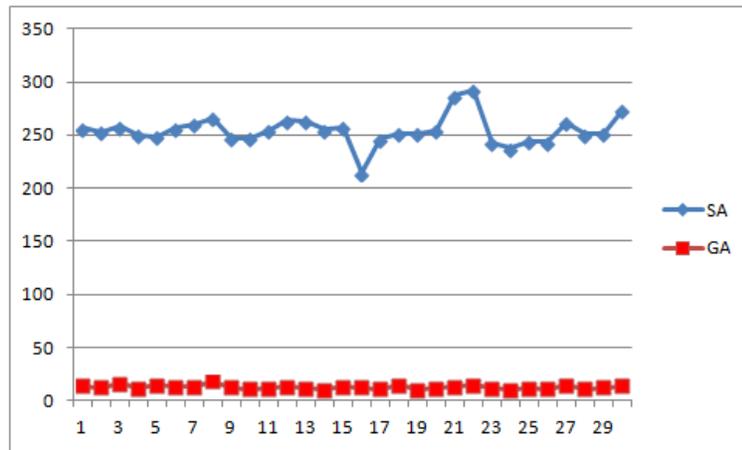


Chart 1: Comparison of metaheuristic methods based on cpu time

6.1.2. Comparison of solution methods considering the obtained solutions and their difference

As is evident in Chart 2, the results differ very little from each other, so one cannot say that which method is better. In order to better display the differences between the two algorithms, the result difference chart is shown in Chart 3.

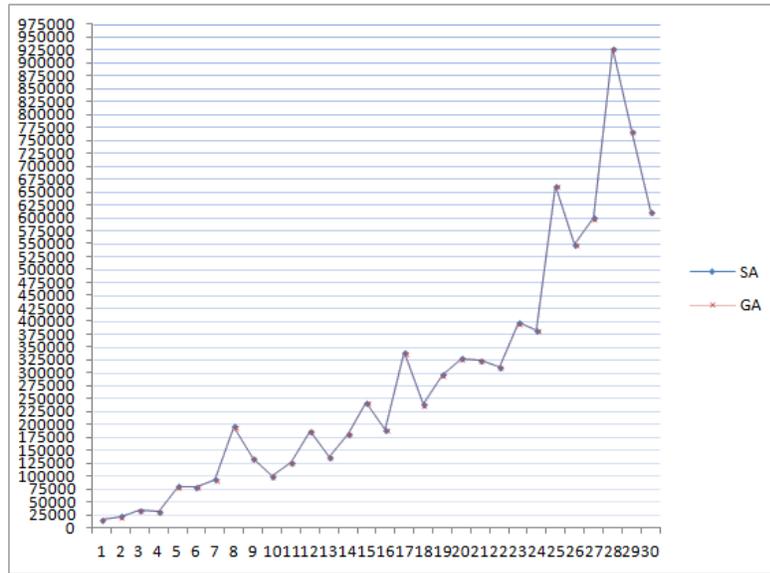


Chart 2: Comparison of metaheuristic methods based on the obtained results

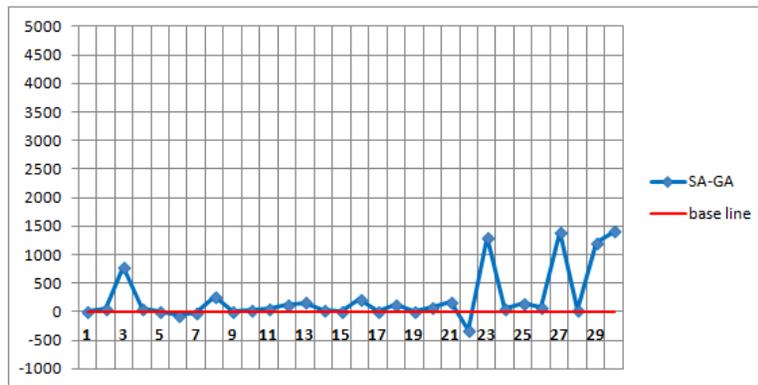


Chart 3: Comparison of metaheuristic methods based on the difference between the results

6.1.3. Comparison by using hypothesis test of equal means of two populations.

In this study, we sought to test the hypothesis $H_0: \mu_1 = \mu_2$ or the average equality of the two populations against $H_1: \mu_1 \neq \mu_2$. In all statistical software such as Minitab, assessment of hypothesis test is based on a criterion called P-value. If the P-value is greater than or equal to the probability of Type 1 error (α), H_0 is accepted, otherwise, it is rejected. Minitab was used in this study, and the results of this test can be found below.

Two-Sample T-Test and CI: SA, GA

Two-sample T for SA vs GA

N Mean StDev SE Mean

SA 30 286477 237130 43294
 GA 30 286022 236645 43205

Difference = mu (SA) - mu (GA)
 Estimate for difference: 455
 95% CI for difference: (-122024, 122933)
 T-Test of difference = 0 (vs not =): T-Value = 0.01 P-Value = 0.994 DF = 57

The P-value for *t* test is 0.994 which is greater than α (0.05). As a result, the null hypothesis is failed to reject. Thus one cannot say that their averages are different.

CONCLUSIONS

This paper presents a quantitative model to evaluate the optimal acquisition price and quantity of used products and the price of used products along with replacement parts after their collection and consolidation, based on their quality levels. This model was developed from the perspective of the remanufacturer and the consolidation center. The supply of used products is random. In addition, this study considers the profit function of the remanufacturer along with the consolidation center, and the goal is to maximize their joint profits. The presented model is an integer nonlinear programming (INLP) model. Consequently, The SA and GA metaheuristic methods were used to solve the model due to the complexity of the problem. The comparison of numerical results shows that if the criterion is Cpu time, the GA algorithm has the better solution in comparison with the SA algorithm. But if the criterion is the obtained solutions, they differ very little from each other, so one cannot say which method is better.

The suggestions for future research are as follows: development of the model for the multi-period state, assessment of price sensitivity of market for remanufactured products, randomness of demand, and combination of pricing used products with remanufactured products.

Appendix

In this section, simplification of $\int_{q_n}^{\infty} (S_n - q_n) f(S_n) dS_n$ and $\int_0^{q_n} (q_n - S_n) f(S_n) dS_n$ are done.

Since the supply of used products follows the normal probability distribution with known mean and standard deviation, its probability density function is defined as follows:

$$S_n \sim N(\mu_n, \sigma_n^2) \quad f(S_n) = \frac{1}{\sqrt{2\pi} \sigma_n} e^{-\frac{1}{2} \left(\frac{S_n - \mu_n}{\sigma_n}\right)^2}$$

Part I:

$$\int_{q_n}^{\infty} (S_n - q_n) f(S_n) dS_n = \int_{q_n}^{\infty} (S_n - q_n) \frac{1}{\sqrt{2\pi} \sigma_n} e^{-\frac{(S_n - \mu_n)^2}{2\sigma_n^2}} dS_n$$

To solve the integral, we consider the following variable change, and the integral range will change as follows:

$$\left\{ \begin{array}{l} \frac{q_n - \mu_n}{\sigma_n} = k \rightarrow q_n = k\sigma_n + \mu_n \rightarrow \left\{ \begin{array}{l} S_n = \left| \begin{array}{l} \infty \\ q_n \end{array} \right. \rightarrow u = \left| \begin{array}{l} \infty \\ \frac{q_n - \mu_n}{\sigma_n} \end{array} \right. = k \\ \frac{S_n - \mu_n}{\sigma_n} = u \rightarrow S_n = u\sigma_n + \mu_n \rightarrow dS_n = \sigma_n du \end{array} \right. \\ \rightarrow \int_{q_n}^{\infty} (S_n - q_n) \frac{1}{\sqrt{2\pi}\sigma_n} e^{-\frac{(S_n - \mu_n)^2}{2\sigma_n^2}} dS_n \end{array} \right.$$

As a result, we have:

$$= \int_k^{\infty} (u\sigma_n + \mu_n - k\sigma_n - \mu_n) \frac{1}{\sqrt{2\pi}\sigma_n} e^{-\frac{1}{2}u^2} \sigma_n du$$

By simplifying the above equation, we have:

$$= \int_k^{\infty} \sigma_n (u - k) \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} du = \sigma_n \int_k^{\infty} (u - k) \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} du$$

Note: $G_u(k) = \int_k^{\infty} (u - k) \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} du$ is the loss function, and its numbers are calculated in the respective table of $G_u(k)$. Thus, the surplus quantity can be shown as follows:

$$\int_{q_n}^{\infty} (S_n - q_n) f(S_n) ds_n = \sigma_n G_u(k)$$

Part II:

In this part, in order to simplify the integral, we separate it into two ranges:

$$\int_0^{q_n} (q_n - S_n) f(S_n) ds_n = \int_0^{q_n} (q_n - S_n) f(S_n) dS_n - \int_{q_n}^{\infty} - (S_n - q_n) f(S_n) ds_n \\ = \int_0^{q_n} (q_n - S_n) f(S_n) dS_n + \int_{q_n}^{\infty} (S_n - q_n) f(S_n) ds_n$$

In the previous part, $\int_{q_n}^{\infty} (S_n - q_n) f(S_n) dS_n$ was simplified, now we must simplify

$\int_0^{q_n} (q_n - S_n) f(S_n) dS_n$. Then we expand the integral, thus:

$$\rightarrow \int_0^{q_n} (q_n - S_n) f(S_n) dS_n = \int_0^{q_n} q_n f(S_n) ds_n - \int_0^{q_n} S_n f(S_n) ds_n$$

$$= \frac{q_n}{2} - \int_0^{\infty} S_n \frac{1}{\sqrt{2\pi}\sigma_n} e^{-\frac{(S_n-\mu_n)^2}{2\sigma_n^2}} dS_n$$

To solve the integral, we consider the following variable change, and the integral range will change as follows:

$$S_n - \mu_n = w \rightarrow dS_n = dw, \quad S_n = w + \mu_n, \quad \begin{cases} S_n - \mu_n = w \\ S_n = \begin{cases} \infty \\ 0 \end{cases} \rightarrow w = \begin{cases} \infty \\ -\mu_n \end{cases} \end{cases}$$

As a result, we have:

$$= \frac{q_n}{2} - \int_{-\mu_n}^{\infty} (w + \mu_n) \frac{1}{\sqrt{2\pi}\sigma_n} e^{-\frac{w^2}{2\sigma_n^2}} dw$$

We factor the integral constant, and expand the integral, we have:

$$= \frac{q_n}{2} - \frac{1}{\sqrt{2\pi}\sigma_n} \left[\int_{-\mu_n}^{\infty} w e^{-\frac{w^2}{2\sigma_n^2}} dw + \int_{-\mu_n}^{\infty} \mu_n e^{-\frac{w^2}{2\sigma_n^2}} dw \right]$$

$$= \frac{q_n}{2} - \frac{1}{\sqrt{2\pi}\sigma_n} \left\{ \left[-\sigma_n^2 e^{-\frac{w^2}{2\sigma_n^2}} \right]_{-\mu_n}^{\infty} + \int_0^{\infty} \mu_n e^{-\frac{(S_n-\mu_n)^2}{2\sigma_n^2}} ds_n \right\}$$

$$= \frac{q_n}{2} - \frac{1}{\sqrt{2\pi}\sigma_n} \left[\sigma_n^2 e^{-\frac{\mu_n^2}{2\sigma_n^2}} + \mu_n \int_0^{\infty} e^{-\frac{(S_n-\mu_n)^2}{2\sigma_n^2}} ds_n \right]$$

$$= \frac{q_n}{2} - \frac{1}{\sqrt{2\pi}\sigma_n} \times \sigma_n^2 e^{-\frac{\mu_n^2}{2\sigma_n^2}} - \mu_n \int_0^{\infty} \frac{1}{\sqrt{2\pi}\sigma_n} e^{-\frac{(S_n-\mu_n)^2}{2\sigma_n^2}} ds_n$$

$$= \frac{q_n}{2} - \frac{\sigma_n}{\sqrt{2\pi}} e^{-\frac{\mu_n^2}{2\sigma_n^2}} - \frac{\mu_n}{2}$$

Thus the shortage value can be shown as follows:

$$\int_0^{q_n} (q_n - S_n) f(S_n) dS_n = \frac{q_n}{2} - \frac{\sigma_n}{\sqrt{2\pi}} e^{-\frac{\mu_n^2}{2\sigma_n^2}} - \frac{\mu_n}{2} + \sigma_n G_u(k)$$

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