ANALYSIS OF FRACTIONALLY SPACED WIDELY LINEAR EQUALIZATION OVER FREQUENCY SELECTIVE CHANNELS WITH MULTIPLE INTERFERENCES

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ABSTRACT

This paper deals with the performance analysis of both linear (LE) and widely linear (WLE) equalization processes studied in the symbol spaced mode (SSE) as well as in the fractionally spaced one (FSE). This analysis is evaluated - using Matlab software- in terms of bit error rate (BER) and mean square error (MSE) in a system using a rectilinear modulation of type Pulse Amplitude Modulation (PAM) over a frequency selective channel and corrupted by multiple interferences. Moreover, the impact of the number of external interferers on the behavior of the different equalizers is studied. Thus, simulation results show the outstanding performance of the fractionally spaced mode when compared to the symbol spaced one besides the out-performance of the widely linear processing in both modes. Furthermore, results show the performance degradation for the different studied equalizers when the number of interferences increases.

KEYWORDS

Fractionally Spaced Equalizer, widely linear, mean square error, bit error rate, rectilinear modulation, interferences, frequency selective channel.

1.INTRODUCTION

It is well known that in wireless communication system, the transmitted signal may not arrive properly at the receiver side. Indeed, du to the wireless propagation channel [1], it may suffer from some deformation and distortion, fading, etc,.. Thus, in order to avoid signal loss at the receiver side, an extra processing should be implemented. This processing is called equalization [2], [3] and it is used in order to suppress interferences and recover the original transmitted signal.

Moreover, the equalization process should take into consideration the characteristics of the signal whether it is second order circular (SOC) or non second order circular (NSOC) [4], [5]. Indeed, it is well known, that for SOC signals, the information is only carried by the auto-correlation function since the pseudo-auto-correlation function is null; hence, the classical linear processing (LE) is sufficient.

However, when the signal is characterized as NSOC, the pseudo-auto-correlation function is non-null and the information is carried by both the auto-correlation function as well as the pseudo-auto-correlation one [6], [7]. In this latter case, the linear equalization is not an optimal solution and the widely linear (WLE) [8, 9, 3, 10] processing is required and it outperforms the classical one as demonstrated in [1], [4], [11], [12], [13] and [14].

On the other hand, it is also known that using fractionally spaced equalizers (FSE) [15, 16, 17] can enhance the system performance when compared to the symbol spaced equalizers (SSE).

Thus, in this paper, we combine the study of the widely linear equalizer when using the criterion of minimizing the mean square error (MSE) applied in the fractionally spaced mode in a system using a rectilinear modulation corrupted by many external interferences and ISI. Moreover, we study the impact of the number of interferences on the behavior of the FSE. For our knowledge, no such study in literature has been done before.

This paper is organized as follows: A brief overview of related works is presented in Section 2. Then, the system model is introduced in Section 3. Section 4 develops the expressions of MMSE-LE and MMSE-WLE in both SSE and FSE modes. Simulation results are given in Section 5, followed by the conclusion in Section 6.

In this article, we use bold capital letters to represent matrix, bold lowercase letters to denote vectors and scalars are represented by lowercase letters. $\mathbb{E}[\cdot]$ is the expectation operator, $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^H$ denote respectively the conjugate, transpose and the hermitian operations. $\star$ represents the convolution operation.

2. RELATED WORKS

The concept of equalization has been widely considered as the main topic in many research articles. Indeed, the necessity of this concept mainly in recovering the transmitted symbols leads to study and analyze the equalization performance according to the signal characteristics. In this context, the out-performance of the widely linear processing and the FSE have encouraged researchers to deal with these two themes. Indeed, authors in [9] have evaluated the performance of both linear (LE) and widely linear (WLE) equalizers in terms of MSE and BER for a system using rectilinear modulation corrupted by inter-symbol-interference (ISI) and external interferences. Besides, authors in [1] have evaluated the MSE in a system where the modulation is improper over a frequency selective channel. In [14], authors, have given an approximation of symbol error rate (SER) with WL receivers in single-input-multiple-output (SIMO) systems in presence of co-channel interferers. In [12], the impact of the number of interferences on the behavior of the equalizers has been highlighted. The performance evaluation studied in [9], [1], [14] and [12], has been done with the use of linear and widely linear equalizers applied on the symbol spaced mode. Regarding the FSE mode, authors in [16], have compared the blind linear equalizer based on two criteria (MMSE as well as zero-forcing). Besides, the convergence rate of the FSE has been studied in [18]. The study of linear fractionally spaced equalizer has been also studied in [19] and [17].

3. SYSTEM MODEL

We consider the system model given by Figure 1 with K external interferences.
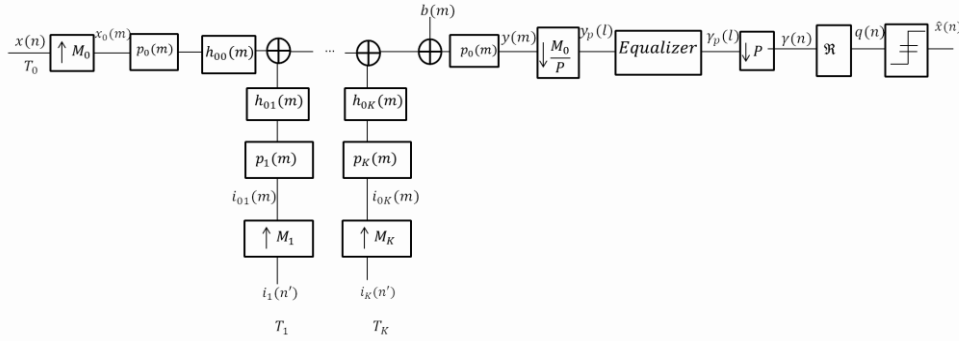
Figure 1: System model with K interferences.

In Figure 1, we transmit a useful rectilinear signal ($x(n)$) (also called signal of interest (SOI)) corrupted by K rectilinear interferences. Without loss of generality, the considered rectilinear modulation is chosen to be an $M-$ary Pulse Amplitude Modulation ($M-$PAM). The PAM signal amplitudes belong to the set $\{\pm A, \pm 3A, .., \pm(M-1)A\}$. Thus, the useful $M-$PAM symbols $x(n)$ are emitted with variance σ_x^2 and symbol-duration T_0. $x_0(m)$ is then a series of Dirac impulses with multiplicative coefficients equal to the $M-$PAM symbols $x(n)$, and spaced by T_0. This signal is then filtered by the shaping filter with impulse response $p_0(m)$ followed by $h_{00}(m)$ which represents the frequency selective channel of the SOI. Likewise, $i_k(n')$, for $k = 1, .., K$, represent the $M-$PAM symbols transmitted by the kth interferer, for $k = 1, .., K$, with variance $\sigma_{i_k}^2$ and symbol-duration equal to T_k. In the following, without loss of generality, we assume that all interferer signals are emitted with the same symbol duration (i.e., $T_i = T_j$, $i, j \in [1, K]$), and they have the same variance (i.e., $\sigma_{i_k}^2 = \sigma_{i_{k'}}^2 = \sigma_i^2$). Furthermore, We assume that $T_0 = M_0 T_e$ and $T_1 = M_1 T_e$, where T_e is the sampling time and $M_1 > M_0$. The kth interferer signal $i_{0k}(m)$ is then a series of Dirac impulses with multiplicative coefficients equal to the $k^{th} M-$PAM interferer symbols, $i_k(n')$, and spaced by T_1. This signal is also filtered by a shaping filter with impulse response $p_k(m)$, followed by a frequency selective channel h_{0k}. The channels output is then corrupted by a circular complex Additive White Gaussian Noise (AWGN) $b(m)$, with variance σ_b^2. The receiver front end is composed of a filter matched to the shaping filter of the useful signal, $p_0(m)$. The output of this matched filter, $y(m)$, will be sampled with P samples per symbol before processing by the equalizer. Finally, after equalization and decision process, the signal $\hat{x}(n)$ represents the received $M-$PAM symbols.

The received signal $y(m)$ can be written as follows:

$$y(m) = p_0 \star [h_{00} \star p_0 \star x_0(m) + \sum_{k=1}^{K} h_{0k} \star p_k \star i_{0k}(m) + b(m)]. \qquad (1)$$

Let us consider $h_0(m) = p_0 \star h_{00} \star p_0(m)$ (of length L_0) and $h_k(m) = h_{0k} \star p_k \star i_{0k}(m), k \in 1, .., K$ (of length L_k). Eq. (1) becomes equivalent to the following equation:

$$y(m) = \sum_{l=0}^{L_0-1} h_0(l) x_0(m-l) + \sum_{k=1}^{K} \sum_{l=0}^{L_k-1} h_k(l) i_{0k}(m-l) + b(m). (2)$$

4. EXPRESSIONS OF THE EQUALIZERS

In order to calculate the expressions of the different equalizers, we adopt the criterion of minimizing the mean square error (MSE) between the transmitted signal and the equalized one. We recall that

there are two types of the MSEs [9], one is used in order to compute the expressions of the equalizers and we note it MSE $_\gamma$, and the other one is to evaluate the system performance and we note it MSE $_q$. These two MSEs are given as follows:

$$MSE_\gamma = \mathbb{E}[|\gamma(n) - x(n - \Delta)|^2], \tag{3}$$

$$MSE_q = \mathbb{E}[|\Re(\gamma(n)) - x(n - \Delta)|^2]$$
$$= \mathbb{E}[|q(n) - x(n - \Delta)|^2], \tag{4}$$

where Δ is the delay for the decision procedure [20].

On the other hand, in Figure 1, the down-sampling procedure before equalization process defines whether the equalizers are either implemented in the symbol spaced mode (SSE) or in the fractionally spaced one (FSE). Indeed, by taking P samples per symbol in the equalization process, if $P = 1$, then the equalizers are implemented in the SSE mode and if $P > 1$ the linear (LE) and the widely linear (WLE) equalizers are called fractionally spaced ones (P FSE).

It is worth noting that since in the FSE mode we use P times more samples per symbol than in the SSE mode, we have P times more input information in the equalization process. Therefore, the number of the FSE taps is P times higher than the number of the symbol spaced equalizer.

In the following, we note L_e the length of the classical linear equalizer (LE). $L_e = L_s$ when the equalizer is a symbol spaced linear one and $L_e = L_f$ for the fractionally spaced linear equalizer. Thus, $L_f = PL_s$. Let us, also, consider the vector $\boldsymbol{y_p}(l)$ of length L_e, defined as $\boldsymbol{y_p}(l) = [y_p(lT_0), y_p(lT_0 - T_e), \ldots, y_p(lT_0 - (L_e - 1)T_e)]^T$.

4.1. Classical Linear equalizer (LE)

For the classical linear equalization [1], [9], [21], the equalizer input is the signal $y_p(l)$. We recall that the linear process exploits only the information carried by the auto-correlation function of the signal. Thus, its expression is given as follows [9]:

$$\boldsymbol{w_{LE}} = \boldsymbol{r_{xy_p}} \boldsymbol{R_{y_p}^{-1}}, \tag{5}$$

where $\boldsymbol{r_{xy_p}} = \mathbb{E}[x(n)\boldsymbol{y_p^H}(l)]$ is a vector $\in \mathbb{C}^{1 \times L_e}$ which represents the inter-correlation vector between the transmitted symbols and the equalizer input. $\boldsymbol{R_{y_p}} = \mathbb{E}[\boldsymbol{y_p}(l)\boldsymbol{y_p^H}(l)]$ represents the auto-correlation matrix $\in \mathbb{C}^{L_e \times L_e}$ of the equalizer input signal.

Moreover, Eq. (5) can be developed and finally expressed as follows:

$$\boldsymbol{w_{LE}} = \sigma_x^2 \mathbf{1}_\Delta \boldsymbol{H_0}^H \left[\sigma_x^2 \boldsymbol{H_0} \boldsymbol{H_0}^H + \sum_{k=1}^K \sigma_i^2 \boldsymbol{H_k} \boldsymbol{H_k}^H + \sigma_b^2 \boldsymbol{F_0} \boldsymbol{F_0}^H\right]^{-1}, \tag{6}$$

where the different matrices and vector, for SSE and FSE respectively, in Eq. 6 are defined as follows:

SSE:

Let us consider $c_k(n) = h_k(m)_{\downarrow M_0}$, $k = 0, \ldots, K$, which represents the equivalent channel (for SOI when $k = 0$ and the kth interferer when $k = 1, \ldots, K$) between the transmitter and the receiver in

the SSE mode. Let us also consider $q_k = length(c_k(n)) - 1$.

Therefore, the vector $\mathbf{1}_\Delta$ is a vector of length $q_0 + L_e$ defined by:

$$\mathbf{1}_\Delta(l) = \begin{cases} 1 & for l = \Delta \\ 0 & else. \end{cases}$$

H_k, for $k = 0,..,K$, is a matrix $\in \mathbb{C}^{L_e \times (L_e + q_k)}$ and is constructed as follows:

$$H_k = \begin{pmatrix} c_k(0) & \dots & c_k(q_k) & 0 & 0 & \dots & 0 \\ 0 & c_k(0) & \dots & c_k(q_k) & 0 & \dots & 0 \\ & & & & & & \\ 0 & \dots & 0 & 0 & c_k(0) & \dots & c_k(q_k) \end{pmatrix} \quad (7)$$

F_0 stands for the correlation matrix of the Noise. It is a function of the received matched filter $p_0(m)_{\downarrow M_0}$ and it is generated as done for the other matrices H_k.

P-FSE

We recall that for the case of the fractionally spaced equalizer, we take P samples per symbol of the received signal in the equalization process. Without loss of generality, we assume that $P = 4$. Therefore, as done in the case of SSE, let us consider the following parameters: $c_{k\prime}(l) = h_k(m)_{\downarrow \frac{M_0}{P}}$, $k = 0,..,K$. $c_{k\prime}(l)$ represents the equivalent channel of the studied system ($k = 0$ stands for the SOI, for $k = 1,..,K$ it is related to the kth interferer signal). Hence, H_k, $k = 0,..,K$ in the FSE mode is a matrix $\in \mathbb{C}^{(P \times L_e) \times (L_e + q_k)}$ and is generated as follows:

$$H_k =$$
$$\begin{pmatrix} c_{k\prime}(0) & c_{k\prime}(4) & \dots & c_{k\prime}(4q_k) & 0 & 0 & 0 & \dots & 0 \\ 0 & c_{k\prime}(3) & c_{k\prime}(7) & \dots & c_{k\prime}(4q_k+3) & 0 & 0 & \dots & 0 \\ 0 & c_{k\prime}(2) & c_{k\prime}(6) & \dots & c_{k\prime}(4q_k+2) & 0 & 0 & \dots & 0 \\ 0 & c_{k\prime}(1) & c_{k\prime}(5) & \dots & c_{k\prime}(4q_k+1) & 0 & 0 & \dots & 0 \\ 0 & c_{k\prime}(0) & c_{k\prime}(4) & \dots & c_{k\prime}(4q_k) & 0 & 0 & \dots & 0 \\ 0 & 0 & c_{k\prime}(3) & c_{k\prime}(7) & \dots & c_{k\prime}(4q_k+3) & 0 & \dots & 0 \\ 0 & 0 & c_{k\prime}(2) & c_{k\prime}(6) & \dots & c_{k\prime}(4q_k+2) & 0 & \dots & 0 \\ 0 & 0 & c_{k\prime}(1) & c_{k\prime}(5) & \dots & c_{k\prime}(4q_k+1) & 0 & \dots & 0 \\ & & & & & & & & \\ 0 & \dots & 0 & 0 & 0 & c_{k\prime}(1) & c_{k\prime}(5) & \dots & c_{k\prime}(4q_k+1) \end{pmatrix} \quad (8)$$

Likewise, the matrix of correlation of the Noise is generated as well as the matrix H_k.

4.2 . Widely linear equalizer (WLE)

Unlike the classical linear processing, the widely linear equalization (WLE) [1], [8], [3], [11] exploits the information carried by both the auto-correlation function and the pseudo-auto-correlation one, meaning that it uses the information contained in the second order properties of the signal.

Since the widely linear processing is applied on the received signal as well as its complex conjugate version, let us consider the following vector $\tilde{y}_p(l)$ of length $2L_e$ and defined as: $\tilde{y}_p(l) = [y_p^T(l), y_p^*(l)]^T$ which represents the WL equalizer input.

The expression of the widely linear equalizer is given by the following expression [11]:

$$\widetilde{w}_{WL} = r_{x\widetilde{y}_p} R_{\widetilde{y}_p}^{-1}, \tag{9}$$

where $r_{x\widetilde{y}_p}$ is a vector $\in \mathbb{C}^{1\times 2L_e}$ and represents the inter-correlation vector between the transmitted symbols and the equalizer input in its widely linear version (i,e: $\widetilde{y}_p(l)$). Besides, $R_{\widetilde{y}_p}$ is a matrix $\in \mathbb{C}^{2L_e\times 2L_e}$, called the auto-correlation matrix of the signal $\widetilde{y}_p(l)$.

As done in the classical linear case, Eq. (9) can be developed and expressed as follows:

$$w_{WLE} = \sigma_x^2 1_\Delta \widetilde{H}_0^H \left[\sigma_x^2 \widetilde{H}_0 \widetilde{H}_0^H + \sum_{k=1}^K \sigma_i^2 \widetilde{H}_k \widetilde{H}_k^H + \sigma_b^2 \widetilde{F}_0 \widetilde{F}_0^H \right]^{-1}, \tag{10}$$

where $\widetilde{H}_k = \begin{bmatrix} H_k \\ H_k^* \end{bmatrix}$, $k = 0,..,K$ and $\widetilde{F}_0 = \begin{bmatrix} F_0 & 0 \\ 0 & F_0^* \end{bmatrix}$, and H_k and F_0 are the matrix defined in the previous item.

It is worth to mention that Eq. (10) stands valid for SSE mode as well as the FSE one.

5. SIMULATION RESULTS

In this section, the performance evaluation of the linear (LE) (Eq. (6)) and widely linear (WLE) (Eq. (10)) equalizers for both SSE and FSE mode is studied. Besides, the impact of the number of the external interferences on the equalizers' behavior is analyzed. The different simulation realizations are done using Matlab software.

Thus, for these simulations, the different shaping filters (see Figure 1) are chosen to be square root raised cosine (SRRC) filters with a roll off factor equal to 1. Regarding the symbol durations, all the interferences have the same symbol duration. Without loss of generality, the interferer symbol duration is chosen to be twice the SOI one (i.e., $T_1 = 2T_0$). The different channel impulse responses of all signals (SOI + interferers) have 60 taps, having each one a Rayleigh modulus and a phase uniformly distributed in $[0,2\pi]$. Likewise, these taps are kept constant during the transmission of $10^4 M -$PAM symbols for $M = 2$ as well as for $M = 16$. The length of the classical linear equalizer in the SSE mode is equal to $L_e = 15$ taps; thus, the linear equalizer in the FSE mode has $4 \times L_e = 60$ taps. The presented simulation results are the average of 100 different channel realizations.

Figures 2 and 3 compare the system performance of both linear and widely linear equalizers implemented in the SSE mode as well as in the FSE one. This comparison is evaluated in terms of MSE and BER in a system where only one interferer is present ($K = 1$) and using respectively $2 -$PAM modulation for Figure 2 and $16 -$PAM modulation for Figure 3.
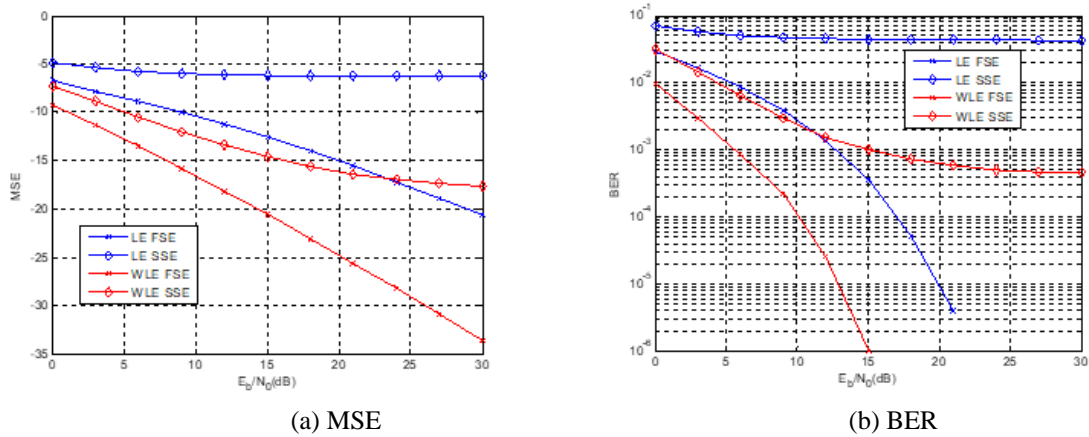
(a) MSE

(b) BER

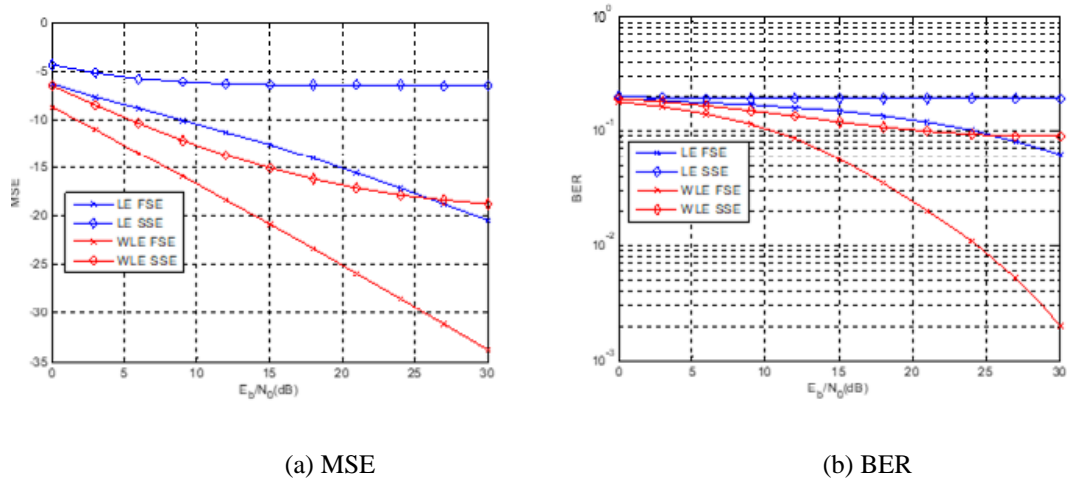Figure 2: Error performance for $K = 1$ using $2-$PAM modulation.

(a) MSE

(b) BER

Figure 3: Error performance for $K = 1$ using $16-$PAM modulation.

From results depicted in Figures 2 and 3, we can deduce that the widely linear equalizer (WLE) outperforms the classical linear one (LE) and that for both modes: SSE and FSE. Indeed, this out-performance can be explained via two reasons. The first one is the non-circularity characteristic of the transmitted signal. The second one is related to the presence of the inter-symbol-interferences (ISI) caused by the frequency selective channels.

Moreover, the equalizers implemented in the FSE mode give better performance when compared to the SSE mode. Indeed, the fractionally equalizers input signals use more information than the SSE leading to a better performance. Furthermore, the worst behavior is given by the classical linear equalizer used in the SSE mode, where it can not recover the transmitted symbols.

Focusing on the analysis of the fractionally spaced equalizers behavior as a function of the number of external interferers K, Figures 4 and 5 present the impact of K on the equalizers performance in terms of MSE and BER in presence of 1 interferer ($K = 1$) and more than one interferer ($K = 3$) and that using respectively, $2-$PAM (Figure 4) and $16-$PAM modulation (Figure 5).
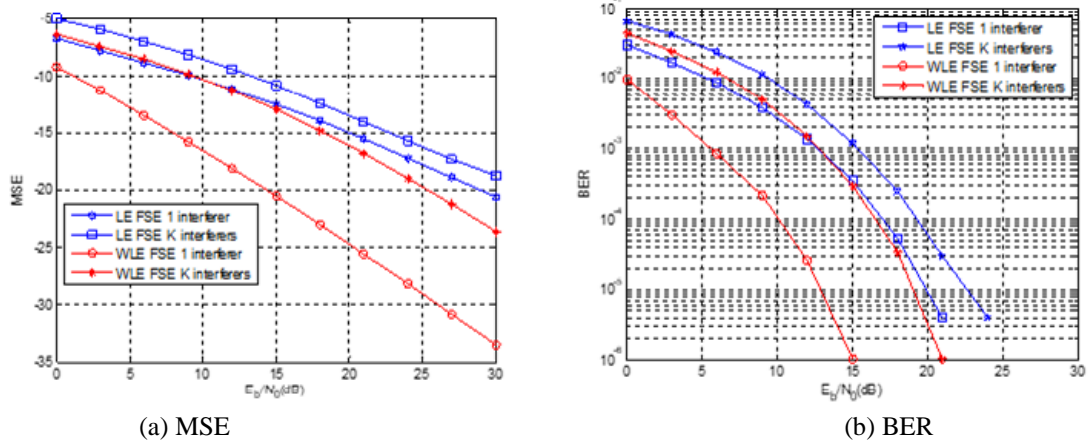
(a) MSE	(b) BER

Figure 4: Impact of K on fractionally spaced equalizers performance for $2-$PAM modulation.
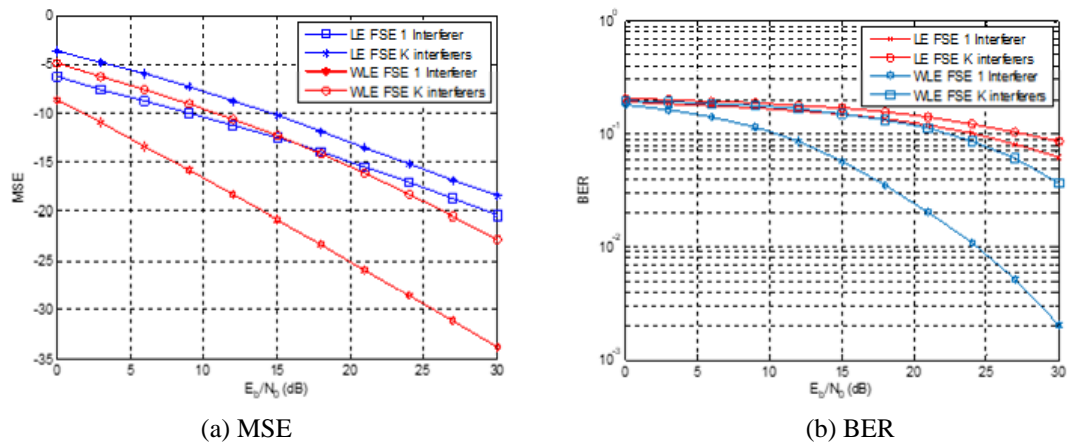

(a) MSE	(b) BER

Figure5: Impact of K on fractionally spaced equalizers performance for $16-$PAM modulation.

Results depicted from Figures 4 and 5, show the out-performance of the widely linear equalizer when compared to the classical one in presence of $K = 1$ and $K = 3$ interferences and for $2-$PAM and $16-$PAM modulations. Furthermore, $\forall M$, when the number of interferences (K) increases, the performances of the fractionally equalizers (LE and WLE) decrease. Besides, the worst performance is provided by the linear equalizer for any PAM order modulation ($\forall M$) and $\forall K$.

6.CONCLUSION

In this article, we have analyzed the performance of both linear and widely linear equalizers implemented in the symbol spaced mode and the fractionally spaced one. These equalizers are studied in a system using rectilinear modulation of type $M-$PAM modulation, over frequency selective channels and in presence of K NSOC interferers. We have shown that the fractionally spaced widely linear processing provides the best performance -$\forall K$ and $\forall M$- when compared to the classical linear equalization process. Moreover, we have shown that when the number of interferences (K) increases, the performances of the fractionally spaced equalizers decrease but they still provide better performance than the symbol spaced equalizers.

ACKNOWLEDGMENT

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