ENERGY EFFICIENT OPTIMUM SAMPLING RATE FOR ANALOGUE SIGNALS WITH EXTREMELY WIDE BANDWIDTH USING COMPRESSIVE SENSING

Sylvester Ajah[1], Ifenyiwa, E. Achumba[2],
Nkwachukwu Chukwuchekwa[3], Nosiri Onyebuchi[4]

[1,3,4] Department of Electrical /Electronic Engineering,
Federal University of Technology Owerri (FUTO), Imo State, Nigeria
[2]Departement of Information and Communication Technology (ICT), Federal University of Technology Owerri (FUTO), Imo State, Nigeria

ABSTRACT

The natural signals are mostly analogue in nature, but because of the benefits of digital processing of these signals: flexibility, accuracy, storage and low cost; processing these signals digitally is often preferred. But the existing analogues to digital converters are efficient in processing signals with small to medium bandwidths, but inefficient for signals with large bandwidths. The real-time processing of these signals with large bandwidths are done analogically or optically at the cost of the aforementioned advantages of digital processing of these signals. This paper is aimed at solving the real-time challenge of processing these extremely wide bandwidth signals digitally using a compressive sensing (CS) algorithm, with specific detail on the ways the application of CS will enhance the energy efficiency of wireless communication devices. Consequently, determine the throughput at which the use of CS is energy efficient for wireless devices using energy-efficient compressive sensing throughput (EECST) model. The simulation results show that the throughput requirements for introducing CS in any machine to machine (M2M) / internet of things (IoT) communication application to be energy efficient are minimum of 54bits per second and 317 bits per second when the required number of clock cycles for performing various device applications is 20,000 and 50000 respectively.

KEYWORDS

Analog Signals, Analogue to Digital Converters, Compressive Sensing, Digital Signals, Digital Signal Processing, Energy Efficiency, Extremely Wide Bandwidth Signals, Optimum Sampling Rate.

1. INTRODUCTION

Most signals in nature are analogue signals, to process these signals digitally, they have to be converted to digital form using analogue – to – digital (A/D) converter. This conversion is done via sampling, quantization, and coding the quantized sample into digital numbers. The conversion of analogue signals to digital form result in distortion that inhibits the reconstruction of the original analogue signals fully. The control of this distortion can be achieved by proper choice of the sampling rate, and the precision in the quantization process. Sampling is the conversion of continuous – time signal into a discrete - time signal gotten via taking samples of the continuous time signals at discrete – time instants [1]. Figure 1 shows A/D conversion process.

However, many communication systems involve high bandwidth, but sparse radio frequency (RF) signals. These features necessitate the application of compressive sensing technique in enhancing the energy efficiency [2]. Compressive sensing (CS) is a signal processing strategy for

DOI:10.5121/ijwmn.2022.14305

efficient signal acquisitions and reconstructions, which sample the signals at approximately information rates. It can also be described as signal acquisition method that collects few samples of signal of interest and use optimization technique to reconstruct the original signal from incomplete measurement. This technique has provided the window of opportunities for enhancing the energy efficiency of machine to machine (M2M) / IoT communication systems.

CS combines the signal sampling and compression into a single process, with low sensitivity to packet loss and graceful degradation of signal in the event of unusual sensor readings [3]. It also reduces the time spent on data acquisition by the nodes via intelligently picking the coefficients of the non-zero part of signals to be sensed. Hence reducing the energy cost of sensing and communications. Also, CS represents a paradigm shift in which the number of measurements is reduced during acquisition so that no additional compression will be needed [4]. Furthermore, CS exploits the information rate with any signal, hence removes redundancy in the signal during sampling process. Leading to lower effective sampling rate which reduces the energy cost of sampling in the nodes. Also, most computations take place in the base stations (sink) in CS, hence elongating the life span of M2M devices [3]. Note that many of these M2M / IoT communication devices that are deployed or will be deployed will be only depend on their internal battery capacities for processing their data, while their servers (receivers) in the data centres will have access to power sources, hence the durability of the M2M / IoT communication networks will mainly depend on the durability of the deployed M2M / IoT communication devices that are batteries powered. This energy efficient communication paradigm has been applied in signals processing, statistics, optimization and many other applications including wireless communications [5]. Consequent to the aforementioned points, the imperativeness energy efficient algorithms to elongate the battery life span of these devices.

This technique has provided the window of opportunities for enhancing the energy efficiency of IoT communication systems, and optimum sampling of radio frequency signals with extremely high bandwidth. It also reduces the time spent on data acquisition by the wireless devices via intelligently picking the coefficients of the non-zero part of signals (frequency bandwidth) to be sensed, hence reducing the energy cost of sensing and communications simultaneously. Also, most computations take place in the base stations (sink) in CS algorithm, hence elongating the life span of M2M communication devices [3]. These enumerated points make CS an effective technique for enhancing the energy efficiency of wireless devices, and optimum sampling of the extremely bandwidth analogue signals.

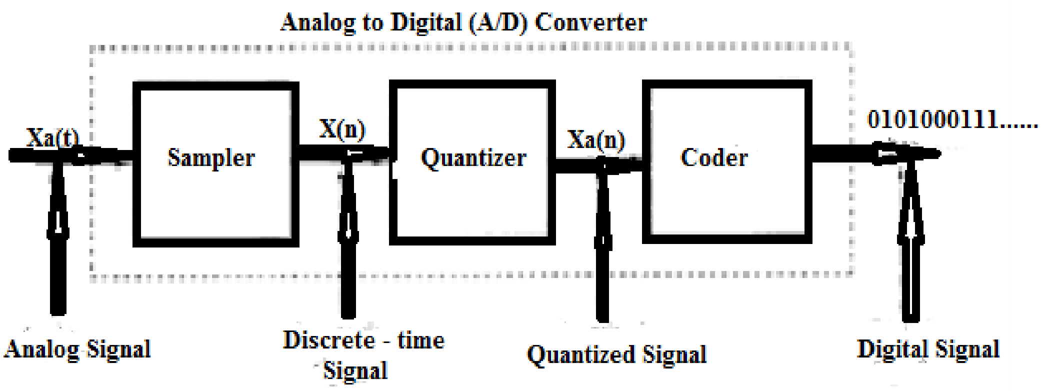

Figure 1. Analogue - to – digital conversion Process

In figure 1, the analogue signal is represented by $x_a(t)$, when the signal is passed via sampler, it resulted to discrete time signal represented by $x_a(nT) \equiv x(n)$, where T is the sampling interval. The faster the sampling rates, the lower the sampling periods, the lower the distortion of the analogue signal during reconstruction. But this result to more expensive A/D converter based on the existing technology [1]. Hence this paper hope to address this challenge using a more energy efficient paradigm in analogue to digital conversion process that is based on the existing technology called compressive sensing. The discussion on this paper is limited to periodic sampling as described in equation (1).

$$x(n) = x_a(nT) \qquad -\infty < n < \infty \tag{1}$$

The reciprocal of the sampling period T is called the sampling rate and it is represented in equation (2).

$$\frac{1}{T} = F_s \tag{2}$$

The unit of sampling rate is samples per second or the sampling frequency in Hertz. The relationship between the time variable (t) of the periodic analogue signal to (n) of the discrete time signal is shown in equation (3).

$$t = nT = \frac{n}{F_s} \tag{3}$$

From equation (3), there is a relationship between the frequency in the analogue signal F, and the frequency variable in the discrete – time signals f. To establish this relationship, consider analogue sinusoidal signal of the form shown in equation (4).

$$x_a(t) = Acos(2\pi Ft + \theta) \tag{4}$$

Where A is the amplitude of the sinusoidal signal, and θ is the phase angle. Which when they are sampled periodically at a rate F_s, yields equation (5).

$$x_a(nT) \equiv x(n) = Acos(2\pi FnT + \theta) = Acos\left(\frac{2\pi nF}{F_s} + \theta\right) \tag{5}$$

A discrete time sinusoidal signal can be expressed in equation (6).

$$x(n) = Acos\,(\omega n + \theta)\,, \qquad -\infty < n < \infty \tag{6}$$

Where $\omega \equiv 2\pi f$

Therefore,

Therefore,

$$x(n) = Acos(2\pi fn + \theta),\ for\ -\infty < n < \infty \tag{7}$$

Comparing equations (5) and (7) yields equation (8). It can be observed that the frequency variable in the analogue signal F, and that of the digital signal f are linearly related as given in (8).

$$f = \frac{F}{F_s} \quad \equiv \omega = \Omega T \tag{8}$$

Where f is the frequency of the digital signal, F is the frequency of the analogue signal and F_s is the sampling frequency. Equation (8) is called relative or normalized frequency. Equation (8) implied that the frequency of the digital frequency can be established from the frequency of the analogue frequency if the F_s is known.

If the analogue signal is converted to digital signal via sampling the analogue signal at discrete time instants, obtaining a discrete – time signals as explained above, the process of converting this discrete continuous time signal into discrete values is called quantization, which is approximating the discrete time signals to a finite value. In equation (7), the variation of continuous – time sinusoidal signals varies from $-\infty < n < \infty$ but discrete – time sinusoidal signals, the variation is as given in equation (9).

$$-\frac{1}{2} < f < \frac{1}{2} \equiv -\pi < \omega < \pi \tag{9}$$

The fundamental difference between continuous – time signals and discrete – time signals is the range values of the frequency variables F and f. Periodic sampling of continuous – time signals implies the mapping of the infinite frequency range for the variable F into a finite frequency range variable f. This is because the highest frequency in a discrete – time signal $\omega = \pi \quad or \quad f = \frac{1}{2}$. This follows that, with F_s the corresponding highest values of F and Ω are given in equation (10) [1].

$$F_{max} = \frac{F_s}{2} = \frac{1}{2T} \quad and \quad \Omega_{max} = \pi F_s = \frac{\pi}{T} \tag{10}$$

The paper is organized as fellows; section 2 discussed the basis of sampling theorem, section 3 discussed sampling theorem, section 4 discussed the theory of comprehensive sensing, section 5 discussed energy consumption in wireless devices, section 6 discussed Energy Efficient Compressed Sensing Throughput (EECST) Model, and section 7 concludes the work.

2. BASIS OF SAMPLING THEOREM

Given any analogue signal, how to select the sampling rate is core to be able to effectively reconstruct with minimal distortion. Assume that any analogue signal can be represented as the sum of sinusoids of different amplitudes, frequencies and phases as given in equation (11).

$$x_a(t) = \sum_{i}^{N} A_i \cos(2\pi F_i t + \theta_i) \tag{11}$$

Where N is the number of the frequency components. All the signals lend themselves to representation in equation (11) over short time segment. The amplitude, frequencies and phases usually change slowly with time from one-time segment to another. Moreover, assuming that the frequencies do not exceed some known frequencies like F_{max}. Example, the F_{max} for some speech signal is 3000Hz, while the F_{max} for television signal is 5000Hz for television signals. Since the maximum frequency may vary slightly from different realization among signals of any given class, they may be wish to ensure that F_{max} does not exceed some predetermined value by passing the analogue signal through a filter that severely attenuates the frequency components above F_{max} [1].

From the knowledge of F_{max}, the appropriate sampling rate can be selected. From Nyquist theorem, the highest frequency in an analogue signal can be reconstructed when the signal is sampled at a rate $F_S = \frac{1}{T}$ is $\frac{F_S}{2}$. Any frequency above $\frac{F_S}{2}$ or below $-\frac{F_S}{2}$ results to a sample that are identical with corresponding frequency in the range $-\frac{F_S}{2} \leq F \leq \frac{F_S}{2}$. To avoid ambiguity resulting from aliasing, the sampling rate that is sufficiently high must be selected. That is, $\frac{F_S}{2}$ must be greater than F_{max}. Thus to avoid the problem of aliasing, F_s is selected so that equation (12) is observed;

$$F_s > 2F_{max} \tag{12}$$

As mentioned earlier, $F_{max}$is the highest frequency component in the analogue signal. With the sampling rate selected in this manner, any frequency component say $|F_i| < F_{max}$in the analogue signal is mapped into a discrete – time sinusoid with a frequency as given in equation (13).

$$-\frac{1}{2} \leq f_i = \frac{f_i}{F_s} \leq \frac{1}{2} \equiv -\pi \leq \omega_i = 2\pi f_i \leq \pi \tag{13}$$

Since $|f| = \frac{1}{2}$ or $|\omega| = \pi$ is the highest (unique) frequency in a discrete – time signal, the choice of sampling rate according to equation (12) avoids the problem of aliasing [1]. Alternatively, the condition $F_s > 2F_{max}$ensure that all the sinusoidal components in the analogue signal are mapped into discrete – time frequency components with frequencies in the fundamental interval. Thus analogue signals can be reconstructed without distortion from the sample values using appropriate interpolation formula.

3. SAMPLING THEOREM

If the highest frequency contained in an analogue signal $x_a(t)$ is $F_{max} = B$, and the signal has a rate $F_s > 2F_{max} \equiv 2B$, then $x_a(t)$ can exactly be recovered from its sample values using interpolation function as given in equation (14).

$$g(t) = \frac{sin2\pi Bt}{2\pi Bt} \tag{14}$$

Hence, $x_a(t)$ may be expressed as given in equation (15).

$$x_a(t) = \sum_{n=-\infty}^{\infty} x_a\left(\frac{n}{F_s}\right) g\left(t - \frac{n}{F_s}\right) \tag{15}$$

Where $x_a\left(\frac{n}{F_s}\right) = x_a(nT) \equiv x(n)$is the sample of $x_a(t)$

When the sampling of $x_a(t)$ is performed at the minimum sample rate $F_s = 2B$, the reconstruction formula in equation (15) becomes as given in equation (16).

$$x_a(t) = \sum_{n=-\infty}^{\infty} x_a\left(\frac{n}{2B}\right)\frac{\sin 2\pi B\left(t-\frac{n}{2B}\right)}{2\pi B\left(t-\frac{n}{2B}\right)} \quad (16)$$

The sample rate $F_N = 2B = 2F_{max}$ is called Nyquist rate and the above theorem is called Nyquist sampling theorem, which is basically the theorem used in most digital signals processing.

4. THEORY OF COMPRESSIVE SENSING (CS)

The basic principle of CS is to transform code the analogue signal of interest x into the basis or frame that will provide the compressible and sparse representation of the signal [6]. Transform coding is done using Fourier transform (FT), fast Fourier transforms (FFT), discrete cosine transforms (DCT), wavelet transforms (WT), etc. The sparse representation of the signal of length n, entails that it can be represented with k nonzero coefficients, where $k \ll n$. While the compressible representation of the signal entails that the signal can be well-approximated to those k none zeros part of the signal n. The CS algorithm can represent the signal of interest with high fidelity by preserving the k none zeros value of n and their locations. This method is called sparse approximation, which is the foundation of transform coding schemes, that used the signal sparsity and compressibility like JPEG, MPEG, MP3 and JPEG2000 standards [6]. The number of non-zeros coefficient of x is less than or equal to k. With this technique, the extremely wide bandwidth signals can be reduced to size in which the existing A/D can effectively and efficiently sample the signals.

$$||\boldsymbol{x}||_0 \leq \boldsymbol{k} \quad (17)$$

The signal **x** is encoded into a smaller vector say **b** with the aid of a sensing matrix $\boldsymbol{A} \in \boldsymbol{R}^{\boldsymbol{m}\times\boldsymbol{n}}$**,** where $m < n$ and it is chosen independently of **x** [5]. The CS coded signal can be represented as given in (17). The CS approach involves directly acquiring the compressed samples without going through the intermediate stages, the compressive measurement through linear projections as given in (18) [7].

$$\boldsymbol{Ax} = \boldsymbol{b} \quad (18)$$

In the applications of CS in M2M / IoT communication, the encoding of **x** is not calculated by a computer or microcontroller, but obtained by certain electrical or electromagnetic, physical, or optical measuring means, depending on the application. Also, because of the condition in equation (19), **b** is the compression of **x**.

$$\begin{cases} \boldsymbol{k} \leq \boldsymbol{m} \\ \boldsymbol{m} < n \end{cases} \quad (19)$$

Furthermore, on the application of this technique on M2M communication paradigm, **b** is recorded by the wireless M2M device and becomes digitally available to the decoder. Though equation (18) above is an underdetermined equation system and has infinite number of solutions, **x** is recovered from **b** by finding the sparsest solution of equation (18) by solving equation (20).

$$\min_{x} ||\boldsymbol{x}||_0 \ \boldsymbol{subject\ to\ A = b} \quad (20)$$

Equation (20) above is called l_0 norm, and though the combinational equation of (20) is Non-deterministic Polynomial-time hard (NP Hard), and the method of trying all the possible supports of cardinality **k** is computationally intractable. To make it tractable, $\boldsymbol{l_0}$ norm is replaced by $\boldsymbol{l_1}$ norm as given in equation (21).

$$\min_x ||x||_1 \; \textbf{subject to } Axb \qquad (21)$$

Equation (21) above is a convex program and has several fast solvers than equation (20). In ideal case, it is better to recover **x** from (21) when **m** equals **2k**, **x** is uniquely determined by **k** indices, and **k** values of its non-zeros entries. In summary, the whole processes of compressive sensing consist of the following stages; signal sparse representation using the transform coding techniques discussed above, linear encoding measurement collection using sensing matrix discussed in the subsection that follows, and compressed signal recovery also discussed in subsection 4.3. For linear encoding measurement collection, the types sensing matrix **A** as it is in equation (18) is invaluable in determining the error free compressing and recoverability of the signals **b.** The types of sensing matrices is as discussed in the subsection 4.2.

4.1. Sensing Matrices

The A in equation (18) denotes sensing matrix, which is $M \times N$ matrix, and represents a dimensionality reduction because it maps R^N to R^M, where $M \ll N$. In designing the sensing matrix, they are two fundamental theoretical questions that need to be addressed; how to design the sensing matrix A to ensure that it preserves the information in the signal **x**, and how to recover the original signal **x** from the compressed measurement **b**. To effectively address the above fundamental questions, they are features A is expected to have to effectively address the aforementioned challenges. These features are discussed below.

4.1.1. Null Space

The NSP of A which is given in equation (22) is very important in the recovery of the compressed signal x' from b using l_1-minimization. The null space of all A vectors **x**, which are mapped to 0 is given in equation (22).

$$\aleph(A) = \{x: Ax = 0\} \qquad (22)$$

This entails that two different k-sparse signals $x \neq x'$do not result to same measurement vector **b**. This means that their difference is not part of the null space of the measuring matrix as given in equation (23).

$$A(x - x' \neq 0 \leftrightarrow x - x' \notin \aleph(A) \qquad (23)$$

For instance, the difference between two k-sparse vectors is at most 2k-sparse. Hence a k-sparse **x** is uniquely defined if $\aleph(A)$ contains no 2k-sparse vectors. This entails that any 2k columns of A are linearly independent, which results to lower bound of the number of measurements, that is the necessary condition that reconstruction is possible, and is as given in equation (24) [8].

$$M \geq 2k \qquad (24)$$

This is because to be able to recover all the sparse signals **x** from the measurement **b**, then it is imperative that $Ax \neq Ax'$ so that it will be possible to distinguish x from x' based on **b** [9]. The sensing matrix $A \in R^{M \times N}$ is said to satisfy the null space property (NSP) of order k with constant $\gamma \in (0,1)$ if the condition in equation (25) is met by A.

$$||\eta T||_1 \leq \gamma||\eta Tc||_1 \quad (25)$$

Where $T \subset \{1, \ldots\ldots\ldots\ldots, N\}$ $and\ T \leq k, for\ all\ \eta \in \ker A \equiv$ set of the k-largest entries of **x** in the absolute value [10]. Though, there are many ways of characterizing this property of A , the most used property in characterizing the NS property is the spark ($\sigma = spark(A)$) of the sensing matrix A. The spark of A is the minimum numbers of columns of A that are linearly dependent [11]. For instance, when there are non-zero columns in A, then $\sigma \geq 2$, with equality only when two columns in A are linearly dependent [12].

4.1.2. Restricted Isometry Property (RIP)

This is the property of the sensing matrix that aids it to recover the compressed signal that is contaminated with noise. A sensing matrix A is said to satisfy the RIP of order K if there exist a restricted isometry constant $\partial_K \in (0,1)$ of a matrix $A \in R^{M\times N}$ that is the smallest number, such that the condition in equation (26) holds for all $x \in \sum_K = \{x: ||x||_0 \leq K$. RIP is also called uniform uncertainty principle[13].

$$(1 - \partial_K)\Big||x||_2^2 \leq \Big||Ax||_2^2 \leq (1 + \partial_K)\Big|\Big| x||_2^2 \quad (26)$$

If the matrix A satisfies the RIP of the order 2K, then equation (26) can be explained that A approximately preserve the distance between any pair of K-sparse vectors. In the definition of the RIP above, bounds that are symmetric about 1 was assumed. In practice, arbitrary bounds as given in equation (27) could be used.

$$\alpha\Big||x||_2^2 \leq \Big||Ax||_2^2 \leq \beta\Big|\Big| x||_2^2 \quad (27)$$

where $0 < \alpha \leq \beta < \infty$. Assuming any value within the bounds above, the sensing matrix A can be scaled, so that it satisfies the symmetric bound about 1 in equation (27). For instance, multiplying A by $\sqrt{2/(\beta+\alpha)}$ will result in A' that satisfies equation (26), with constant $\partial_K = \frac{\beta-\alpha}{\beta+\alpha}$[9].

4.1.3. Incoherence

This is the property of the sensing matrix A that aids to determine the recovery ability of A [14][12]. It is specifically used to determine the sufficient condition for $L_0\ and\ L_1$ unique solutions [7]. The coherence of the sensing matrix A ($\mu(A)$ is the largest absolute inner products two columns $A_i\ and\ A_j$ of the sensing matrix A as given in equation (28)[9].

$$\mu(A) = \max_{1\leq i\leq j\leq N} \frac{|< A_i, A_j >|}{||A_i||_2||A_j||_2} \quad (28)$$

It has been established that A with coherence coefficient μ satisfies the RIP of s with $\delta_s \leq \mu(s-1)$ whenever $s < \frac{1}{\mu} + 1$, where s is the sparcity level of the sparse signal [15] [16].

The aforementioned inequality states that a small μ results in a high value of recovered s; entailing that if the coherence coefficient is reduced, one can reconstruct a sparse signal with a large value of s with a constant M. Hence, matrices with low coherence are desirable. The minimum coherence of an arbitrary matrix $A_{M\times N}$ is stated by the Welch bound that is given in equation (29)[17]. This bound asymptotically tends to $\frac{1}{\sqrt{M}}$ whenever $M \ll N$ [16].

$$\mu \geq \sqrt{\frac{N-M}{M(N-1)}} \quad (29)$$

The incoherence property of A is easier to evaluate than RIP of A, reason being that the computational complexity of evaluating $\mu(A)$ scales exponentially with the number of columns in A[18]. The relationship between coherence and spark of A is given in equation (30). If (30) holds, then for each measuring vector $b \in R^M$ there exist at most one signal $x \sum_k$ ⬚ such that equation (18) holds.

$$k < \frac{1}{2}\left(1 + \frac{1}{\mu(A)}\right) \quad (30)$$

4.2. Types of Sensing Matrices

Compressive sensing involves three main steps: sparse representation, measurement and sparse recovery. The sensing or measurement matrix are used to sample only the components that best represent the sparse signal. Hence, the choice of the sensing matrix affects the success of the sparse recovery process. Therefore, the design of an accurate sensing matrix is a vital process in compressive sensing [19]. The authors in [20] stated that design of measuring matrix and development of an efficient sparse recovery algorithms are the two main problems that must be addressed by compressive sensing theory.

Sensing matrices are broadly classified into two groups – random sensing matrices and deterministic sensing matrices. Random sensing matrices are generated at random by identical or independent distributions (iid) like normal, Bernoulli random and Fourier ensembles, they are easy to construct and they satisfy the RIP with high probability. Random sensing matrices are further classified into two types – structured and unstructured sensing matrices as shown in figure 2. While the deterministic sensing matrices are constructed deterministically to satisfy the RIP, or to have small mutual coherence. Most of the deterministic sensing matrices are made based on employing polynomials of finite fields [16].

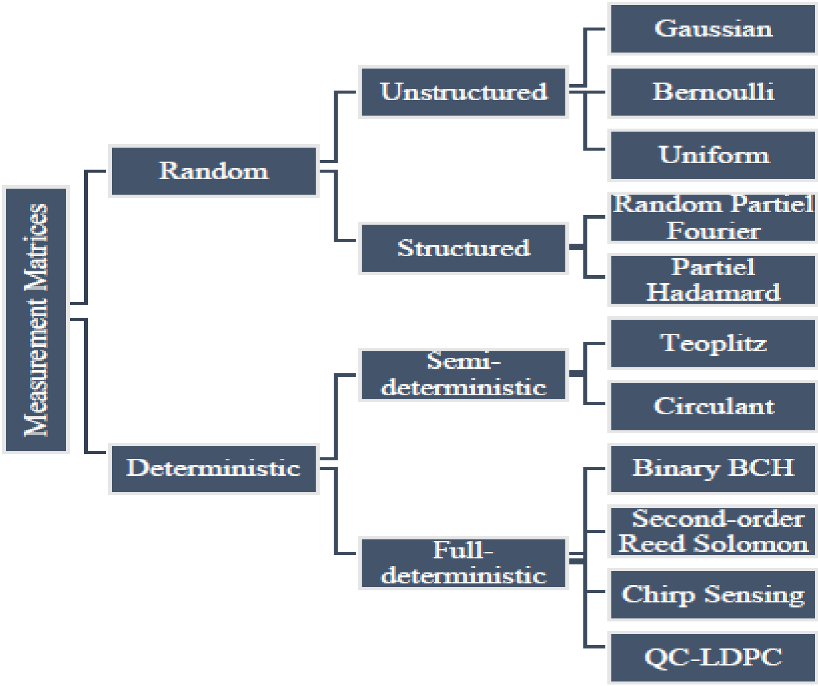

Figure 2. Classification of Sensing Matrices [19]

4.3. Compressed Signals Recovery

The recovery of the compressed signal sample is the critical part in determining its adoption in M2M communications paradigm. This sub-section provides the summary of the algorithms that are used for sparse compressed signals recovery. The compressed signal recovery is NP – hard (Non deterministic polynomial-time hard), hence can be recovery using alternative algorithms. These recovery algorithms can broadly be classified into three; the convex and relaxation algorithms, the Bayesian algorithms and Greedy algorithms. The convex and relaxation algorithms solve the sparse signals recovery via convex relaxation algorithms; the examples are as shown in figure 3. The Greedy algorithms solve the sparse signals recovery via iterative processes, examples of this type of algorithm are also shown in figure 3. Bayesian algorithms solve the sparse signals recovery via taking into account a prior knowledge of the sparse distributions, examples are also shown in figure 3.

5. Energy Consumption in Wireless M2M Communications Devices

Equation (31) summarizes various energy costs associated with wireless communication, and figure 4 gives the percentage of various operational energy costs within the wireless devices. These energy costs are based on MAC protocol, because it has a major impact on the power consumption of the wireless devices, and it is based on a single sampling period [4].

$$\boldsymbol{E}_T = \boldsymbol{E}_{start-up} + \boldsymbol{E}_{ramp} + \boldsymbol{E}_{sensing} + \boldsymbol{E}_{logging} + \boldsymbol{E}_{sampling} + E_{computing} + \boldsymbol{E}_{communication} \quad (31)$$

Where $\boldsymbol{E_T}$ is the total operational energy consumption costs in the wireless devices; $\boldsymbol{E_{start-up}}$ is the device initial energy consumption during start-up; $\boldsymbol{E_{ramp}}$ is the energy cost of switching to different energy states for various devices' operations; $E_{sensing}$ is the energy used by the wireless devices for sensing signals; $\boldsymbol{E_{logging}}$ is the energy cost of storing the sensed data; $\boldsymbol{E_{sampling}}$ is the data sampling energy cost, $\boldsymbol{E_{Computing}}$ is the energy used by the wireless devices in internal processing of the sensed signals; and $\boldsymbol{E_{communication}}$ is the communication energy cost used for transmitting digital signals to the receiver.

Being that start-up energy cost in the wireless device is negligible (< 1%) as can be seen in figure 4, its effect will not be considered further. The ramp energy cost in the micro-controller unit (MCU) of the wireless devices is negligible too, but the ramp energy cost in the radio is significant and it constitutes 10% of the total energy cost in the wireless device. Equation (32) is used for calculating the radio ramp energy cost.

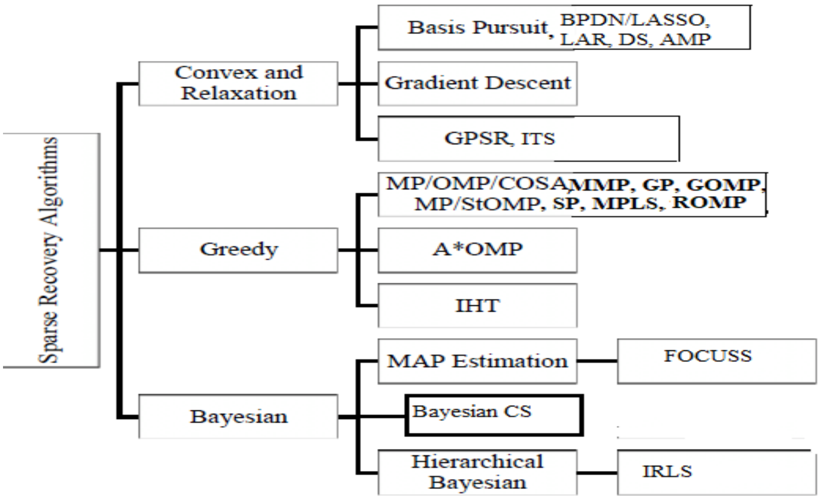

Figure 3. Classifications and examples of sparse recovery algorithms

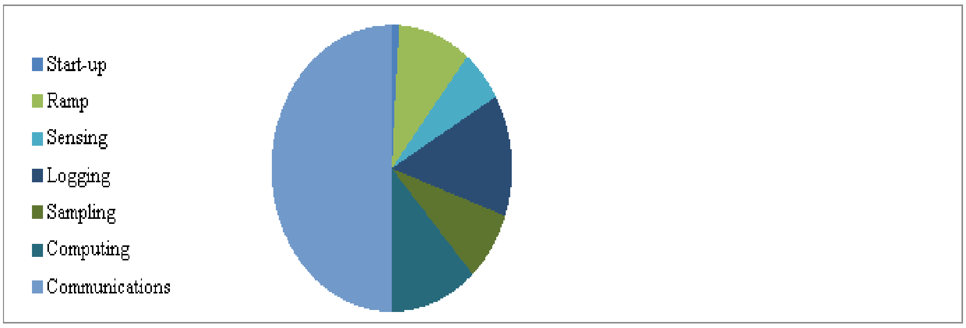

Figure 4. Various operational energy costs in the wireless device

$$\boldsymbol{E_{ramp}} = \frac{|(\boldsymbol{I_{st2}} - \boldsymbol{I_{st1}})| \times \boldsymbol{T_{st12}} \times \boldsymbol{V_{dc}}}{\boldsymbol{2}} \qquad (32)$$

Where $\boldsymbol{I_{st2}}$ is the current consumed in the state switched to, $\boldsymbol{I_{st1}}$ is the consumed current in the current state, $\boldsymbol{T_{st12}}$ is time used in switching from state 1 to state 2, and $\boldsymbol{V_{dc}}$ is the voltage consumed.

The sensing energy cost is about 6% of the total energy cost in the wireless device as can be seen in figure 2, and the energy cost of sensing b bits of data can be calculated using equation (33).

$$E_{sensing} = b \times V_{dc} \times I_{Sensing} \times T_{sensing} \quad (33)$$

Where $I_{Sensing}$ and $T_{sensing}$ are the current consumed in sensing and the sensing time respectively.

The logging energy cost is the energy used by the wireless device for reading b bit packet data and writing it into the memory [21]. Equation (34) shows how to evaluate the energy cost of logging **b**bits' data size per cycle.

$$E_{logging}(b) = E_{read} + E_{write} = \frac{b \times V_{dc}}{8}(I_{read} \times T_{read} + I_{write} \times T_{write}) \quad (34)$$

The sampling energy cost $E_{sampling}$ is the energy the wireless device spent on sampling **b** bits of data. The $E_{sampling}$ is greatly dependent on the type of data and the rate of sampling. Let the period for data sampling be T, time spent on sampling data be t_{data}, current used for data sampling be I_{data} and voltage used to be V_{dc}. The energy spent on data sampling $E_{sampling}$ is given in (35).

$$t_b = t_{data} \times \left(\frac{1}{T}\right) E_{sampling} = t_b \times I_{data} \times V_{dc} \quad (35)$$

The energy cost of computing $E_{Computing}$ in a wireless communication device is a key constituent of total energy cost in a wireless device. The $E_{Computing}$ consists of MCU's active and other modes. The computation energy $E_{computation}$ is given in (36) in two states (active and sleep).

$$E_{computation} = V_{dc} \times I_{active} \times T_{active} + V_{dc} \times I_{sleep} \times T_{sleep} \quad (36)$$

The communications energy costs consist of the energy cost of data transmission and the cost of data reception is given in equation (37).

$$E_{communication} = E_{tx} + E_{rx} \quad (37)$$

Where E_{tx} is the energy cost of transmission of packets of data, and E_{rx} is the energy cost of receiving data packets. Equations (38) and (39) represents E_{tx} and E_{rx} respectively.

$$E_{tx} = V_{dc} \times I_{tx} \times B_{ltx} \times T_{btx} \quad (38)$$

$$E_{rx} = V_{dc} \times I_{rx} \times B_{lrx} \times T_{brx} \quad (39)$$

Where I_{rx} *and* I_{tx} are the current consumed in the reception and transmission mode respectively; B_{ltx} *and* B_{lrx} are the bit length of the transmitted, and received packets along with their preambles respectively; T_{btx} *and* T_{brx} are the time for transmitting and receiving single bit of data. However in wireless communication devices, the energy costs of various operations are evaluated in terms of the number of clock cycles required to perform such operations [22]. To this end, the energy cost of performing various operations in wireless communication devices as given in equation (31) are evaluated in terms of the number of clock cycles required to perform such operations, which varies from one operation to another.

6. ENERGY EFFICIENT COMPRESSED SENSING THROUGHPUT (EECST) MODEL

Equation (40) gives the fundamental communication model through which any communication system follows, hence to have an energy efficient system, the effect of the energy cost of variables in equation (40) have to be considered.

$$\boldsymbol{y} = \boldsymbol{Hb} + \boldsymbol{n} \tag{31}$$

Where **y** is the signal at the receiver, **H** is the channel properties, **b** is the compressed transmitted signal as given in equation (18), and **n** is the channel noise. In (40), H (the channel characteristics) has the most significant effect on the received signal **y**. The sensitivity of the receiver is the major determining factor in knowing the amount of power required to get a signal **b** from transmitting station to the receiver when the H properties have been ascertained using the appropriate models. The receiver's sensitivity is the minimum input signal (S_{min}) required to produce a specific output signal with a specified signal-to-noise ratio (S/N) and is given in equation (41) [23].

$$\boldsymbol{S_m} = (\boldsymbol{S}/\boldsymbol{N})_{\boldsymbol{min}} \times \boldsymbol{K} \times \boldsymbol{T_o} \times \boldsymbol{B} \times (\boldsymbol{NF}) \tag{41}$$

Where $(\boldsymbol{S}/\boldsymbol{N})_{\boldsymbol{min}}$ is the minimum signal-to-noise ratio needed to detect a signal, NF is the noise factor, K is Boltzmann's constant =$1.38 \times 10^{-23} Joule/°K$, T_o is the absolute temperature of the receiver input ($°Kelvin) = 290°K$ and B is the receiver bandwidth (Hz) [24]. A typical receiver's sensitivity is around -110dBm, though it is dependent on device type [25].

As stated in [26], that sub 1GHz spectrum provides the energy efficient medium through which M2M devices can be connected, let the channel frequency be Sub 1GHz Industrial, Scientific and Medical (ISM) radio band (902 – 928) MHz with the centre frequency of 915MHz and bandwidth of 26MHz. Now, to determine the power required to transmit the signal from the transmitter to the receiver say 4 kilometres in a large city scenario, the path loss normally follows Rayleigh distribution. For this discussion, let assume that the value of n as contained in (40) is zero. Also, let the base station (BS) height be 40 meters, and mobile station height be 2 meters. Using Hata path loss model [27] in path loss evaluation, using the above variables will result to a path loss of 144.55dB.

However, the channel losses between a transmitter, and a receiver in wireless channel is given in equation (42).

$$Losses = Path\ loss + penetration\ loss + other\ losses \tag{42}$$

The penetration loss can be evaluated using equation (43).

$$\boldsymbol{L_{pl}}(\emptyset) = \sqrt{\boldsymbol{d} - \boldsymbol{e}(\emptyset - \boldsymbol{f})^{\boldsymbol{2}}} \tag{43}$$

Where d, e, f are empirical parameters; and $\emptyset$ is the angle of the signal inclination [28]. Now using the experimental results for the empirical parameters as contained in [28] for computation. The penetration loss at 60° signal reception angle is 23.69dB. Also, other losses as contained in (33) are gotten by calculating 10% of the summation of the above losses. Therefore, other loss is 16.82dB. Hence, a Total loss as given (42) is approximately 185dB.
The power of the received signal at the receiver P_r is given in (44).

$$\boldsymbol{P_r = P_t + G_t - Losses} \quad (44)$$

Where P_t is the transmit power at the transmitter and G_t is the total gains in both the transmitter and the receiver. Now using P_r as -110dBm, assuming that the total gains between the transmitter and receiver at 2dBm each is 4dB, then the required P_t is 71dBm. This value of P_t is the minimum required to get any signal across 4 kilometres, assuming all the variables are constant.

Given that there are numerous anticipated M2M / IoT applications across all sectors, the throughput requirement for each anticipated application also vary. Based on the trade-off between the energy gains from mainly communication energy cost and computing energy cost when CS algorithm is applied to the M2M communication devices, it is imperative to ascertain the throughput at which the applications of CS algorithm are energy efficient.
The $E_{tx}>E_{rx}$ for a typical wireless ad hoc network and based on the fact that the wireless device to be used for M2M communications / IoT are expected to be dumped in order to elongate the life span of the devices [29]. It can be deduced that most of the M2M / IoT communication devices will rarely receive packets except for updates in terms of the available spectra for communication. Hence, in this EECST model, the assumptions made are summarized as follows;

a) M2M communication cellular structure as proposed by Weightless Special Interest Group [30], hence, the major communication cost will be on transmitting the data packets to the base station, which do most processing and scheduling in the M2M communication networks.
b) The battery power of the devices is very limited, hence the need to know the throughput at which the applications of CS algorithm is energy efficient.
c) The channels of communication are sub 1GHz spectra (902 – 928) MHz ISM Band, hence have limited bandwidths as proposed in [31].
d) The data processing is done on the base station
e) Multipath fading does not exist
f) Binary phase shift keying (BPSK) signal is used in the model

Let the amount of energy required to transmit a single bit of data be $\boldsymbol{e_{tb}}$, which is equivalent to the number of clock cycles required to transmit a single bit of data $\boldsymbol{n}$ and let the energy cost per clock cycle be $\boldsymbol{e_{cc}}$. To transmit **x** bit(s) of data without compression, the $\boldsymbol{E_{tx}}$ is evaluated as given in (45). As mentioned above, the application of CS algorithm increases the energy cost of computing / processing. Let the number of clock cycle required to execute CS algorithm on the data sample be C_n, and the $\boldsymbol{E_{tx}}$ when CS algorithm is applied is given in equation (45).

$$\boldsymbol{E_{tx} = x \times e_{tb} \equiv x \times n \times e_{cc}} \quad (45)$$

For the benefit of the context of this discussion, it is imperative to split the total number of bits to be transmitted into the preamble which is assumed to be constant for both compressed and uncompressed signal. Let the number of bits in the preamble be $\boldsymbol{x_1}$ and the remaining number of bits in the packet be $\boldsymbol{x_2}$, putting the above assumptions in (45) will result to equation (46).

$$\boldsymbol{E_{tx1} = (x_1 + x_2) \times e_{tb} \equiv (x_1 + x_2) \times n \times e_{cc}} \quad (46)$$

To be able to evaluate $\boldsymbol{E_{tx}}$ when CS algorithm is implemented on the M2M communications device, let the number of clock cycle required to perform data compression using CS algorithm be $\boldsymbol{n_c}$ and the percentage of compression be $\boldsymbol{\eta}$, then the $\boldsymbol{E_{tx}}$ using CS algorithm is given in equation (47).

$$\boldsymbol{E_{tx2}} = \boldsymbol{x_1} \times \boldsymbol{e_{tb}} + \eta \times \boldsymbol{x_2} \times \boldsymbol{e_{tb}} + \boldsymbol{n_c} \times \boldsymbol{e_{cc}} \equiv (\boldsymbol{x_1} + \eta \times \boldsymbol{x_2}) + \boldsymbol{n_c} \times \boldsymbol{e_{cc}} \quad (47)$$

Equation (46) and (47) above are suitable for single – input to single – output (SISO) form of communication. For multiple input multiple output (MIMO) form of communication, the above equations will not be suitable. This is as a result of the increase in circuit complexity which will increase the computing / processing energy cost, coupled with ℵ number of bits that is being transmitted simultaneously. However, Zimran R et al [32] stated based on their analysis that, using MIMO for data transmission is more energy efficient for transmission across long distance. While across short distance, $\boldsymbol{E_{computation}}$ is more than $\boldsymbol{E_{tx}}$ as a result of the circuit complexity. In order to evaluate $\boldsymbol{E_{tx}}$ when MIMO is used, let the energy cost of transmitting ℵ bits of data using MIMO per transmission be $\boldsymbol{e_{\aleph}}$, and the number of clock cycles required by a MIMO circuit to transmit x bits of data be $\boldsymbol{n_{\aleph}}$,then the energy cost of transmitting x bits of data is as given in equation (48).

$$\boldsymbol{E_{tx3}} = \left(\frac{\boldsymbol{x_1} + \boldsymbol{x_2}}{\aleph}\right) \times \boldsymbol{e_{\aleph}} \equiv \left(\frac{\boldsymbol{x_1} + \boldsymbol{x_2}}{\aleph}\right) \times \boldsymbol{n_{\aleph}} \times \boldsymbol{e_{cc}} \quad (48)$$

Then the $\boldsymbol{E_{tx}}$ on the application of CS algorithm is as given in equation (49).

$$\boldsymbol{E_{tx4}} = \left(\frac{(\boldsymbol{x_1} + \eta \times \boldsymbol{x_2})}{\aleph}\right) \times \boldsymbol{e_{\aleph}} + \boldsymbol{n_{\aleph}} \times \boldsymbol{e_{cc}} \quad (49)$$

Considering SISO scenario as given in (46) and (47), let's take the energy cost of MSP430 serial micro-processor in active state as the yardstick for evaluation, in which $e_{tb} = 230nj$ and $e_{cc} = 0.729nj$ for a WSN with a range of 100 meters [22]. Also as mentioned earlier, the energy cost of processing CS algorithm in the node(s) is determined by the number of clock cycles required by the devices to perform the compression operations. Table 1 shows the number of clock cycles required by MSP430 micro-processor to perform certain floating-point operations.

Table 1. The number of clock cycles for specific floating-point operations [22]

FLOATING POINT OPERATION(S)	NUMBER OF CLOCK – CYCLES
Addition	184
Subtraction	177
Multiplication	395
Division	405
Comparison	37

The type of the CS algorithm and the size of the sensing matrix will determine the number of clock cycle required by the micro-processor to perform the compression operation on the device. For instance, a micro-processor with an impeded Sub-threshold (Sub-V_T) CS processor will require 8460 clock cycles to apply 50% compression on 512 samples of ECG data [33]. The A in (18) is constructed by 12 random indices per column, and the sampling rate of the signal is 125Hz. Also, assuming that the length of the MAC addresses for both the compress and un-compressed signal samples are same, then only x_2 in (46) and (47) is considered in the computation. Figure 5 shows the data rates at which compression becomes energy efficient.

Based on the input variables as stated, the throughput at which compression becomes energy efficient is at 53.63 ≈ 54 bits. Though the number of clock cycles used for compression is based on Sub-threshold (Sub-V_T) CS processor, the number of clock cycles required for performing CS compression algorithm varies from one micro-controller to another. This is because of the variations in the inherent properties of various micro-controllers like speed, energy requirement per clock cycle, etc. and the variations on the various computation requirements of various CS algorithms.

Consequent to the variations in the capabilities of different microprocessor, and the number of clock cycles required to perform CS algorithms, Figure 5 shows the minimum data rates that are required for the application of CS algorithm on M2M / IoT communication devices to be energy efficient. Figures 5 and 6 are derived from equation (46) and (47) which can be used for evaluating the data rates in a BPSK scenario, while equation (48) and (49) can be used in evaluating the data rates at which the applications of CS algorithm on M2M / IoT communication devices become energy efficient in MIMO scenarios. As can be seen in figure 6, when the number of clock cycles is increased to 20000, the data rate at which compression becomes energy efficient is 127 bits and at 50000, the data rate becomes approximately 317 respectively.

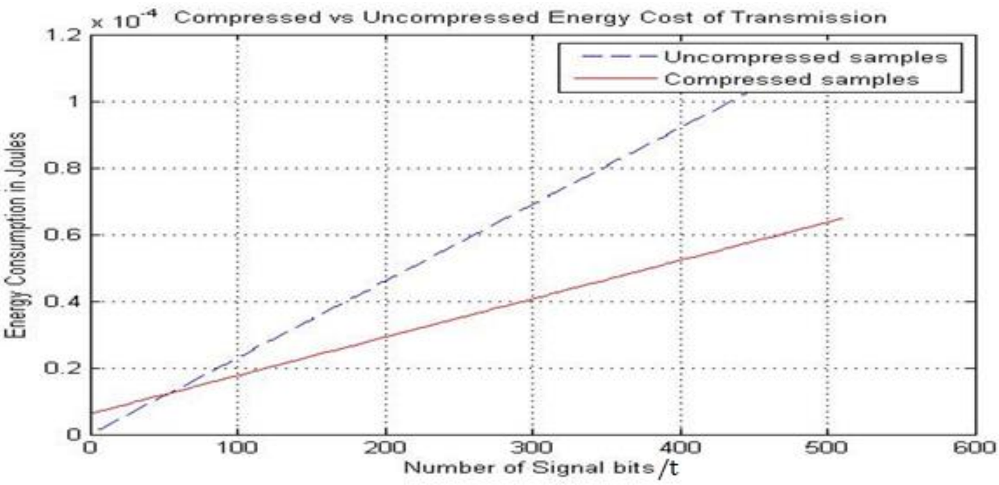

Figure 5. The CS compressed Vs uncompressed transmitted energy cost.

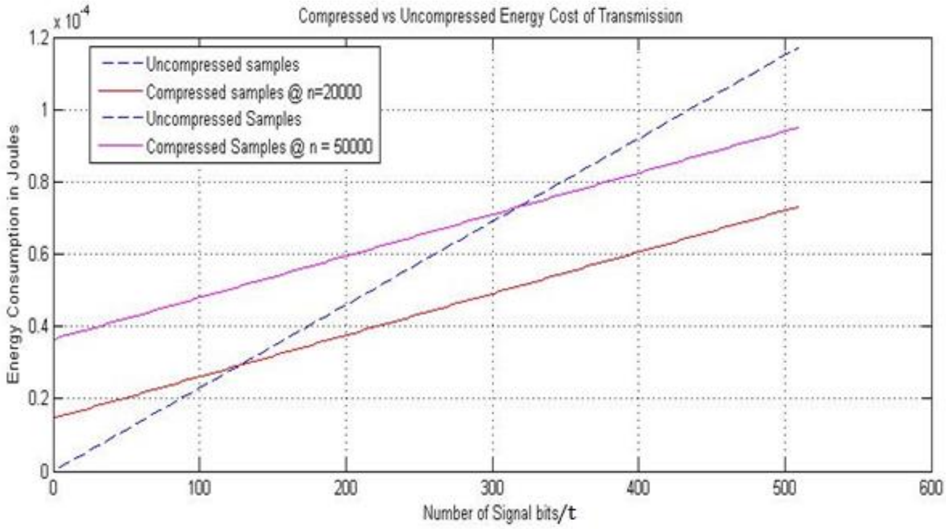

Figure 6. The CS compressed Vs uncompressed transmitted energy cost @ n = 20,000 and 50,000

7. Conclusion

The EECST model has become handy for M2M / IoT communication experts to use in evaluation based on the throughput requirement of various M2M / IoT communication applications, the rates at which the application of the CS algorithm on the wireless devices becomes energy efficient. The higher the computational energy cost for executing CS algorithm on the wireless device, the higher the throughput at which the application of CS algorithm becomes energy efficient. The values used for $e_{tb}$is based on WSN with a range of 100 meters, the value is more in the range of 4 kilometres. Though, it is imperative to note that the application of CS on any of the applications is subject to the recoverability error performance of such signal on the receiver. Also, the type of the sensing matrix used for compression, and the type of compressed signal recovery algorithm is very vital in determining the difference between the original signals, and recovered signals.

References

[1] G. P. John and G. M. Dimitris, Digital Signal Processing: Principle, Algorithms, and Applications - Third Edition, New Jersey: Prentice Hall International Inc., 1996.

[2] M. H., "Reducing ADC Sampling Rate with Compressive Sensing," 2015.

[3] A. R. Mohammad and D. Simon, "Energy - Efficient Sensing in Wireless Sensor Networks Using Compressive Sensing," Sensors, vol. 14, no. 2, pp. 2822 - 2859, 2014.

[4] Y. Zhang, "Theory of Compressive Sensing via L 1-Minimization: a Non-RIP Analysis and Extensions," Journal of the Operational Research Society of China, vol. 1, no. 1, pp. 79 - 105, 2013.

[5] H. Zhu, L. Husheng and Y. Wotao, Compressive Sensing for Wireless Networks, Cambridge: Cambridge University Press, 2013.

[6] A. D. Mark, F. D. Marco, Y. C. E. and K. Gitta, Introduction to Compressive Sensing, Stanford: Stanford.edu, 2011.

[7] L. Heung-No, Introduction to Compressed Sensing: With Coding Theoretic Perspective, GIST Korea, 2011.

[8] M. Patrick, "VLSI Architectures for Compressive Sensing and Sparse Signal Recovery," ETH, Zurich, 2012.

[9] B. Richard, A. D. Mark, F. D. Marco and H. Chinmay, Texas: Rice University, Houston, Texas., Houston Texas: Rice University (Rechard, B.; Mark, A. D.; Marco, F. D.; Chinmay, H. ed.), 2011.

[10] F. &. H. R. Massimo, Compressive Sensing, Ricam Austria: Linz, 2010.

[11] A. D. Mark, F. D. Marco, C. E. Yonina and K. Gitta, Introduction to Compressed Sensing, Stanford: Standord.edu, 2011.

[12] L. D. David and E. Michael, “Optimally Sparse Representation in General (Nonorthogonal) Dictionaries Via L1 Minimization,” National Academy of Science of the United States of America, vol. 100, no. 5, p. 2197–2202, 2002.
[13] J. C. &. T. T. Emmanuel, “Decoding by Linear Programming,” IEEE Transactions on Information Theory, vol. 51, no. 2, pp. 4202 - 4215, 2005.
[14] A. T. Joel, “Greed is Good: Algorithm Results for Sparse Approximation,” IEEE Transactions on Information Theory, vol. 50, no. 10, pp. 2231 - 2242, 2003.
[15] J. Bourgain, S. J. Dilworth, K. Ford, S. Konyagin and D. Kutzarova, “Explicit Constructions of RIP Matrices and Related Problems,” Duke Mathematics Journal, vol. 159, no. 1, pp. 145 - 185, 2011.
[16] M. M. Mohades, A. Mohades and A. Tadaion, IEEE , 2014.
[17] L. Welch, “Lower Bounds on the Maximum Cross Correlation of Signals,” IEEE Transaction on Information Theory, vol. 20, no. 3, pp. 397 - 399, 1974.
[18] S. Dennis, “ Compressed Sensing Algorithms and Applications,” Royal Institute of Technology (KTH), Stockholm, 2012.
[19] Y. Arjoune, N. Kaabouch, H. El-Ghazi and A. Tamtaoui, “A Performance Comparison of Measurement Matrix in Compressive Sensing,” International Journal of Communication Systems, pp. 1 - 22, 2017.
[20] E. C. Marques, N. Maciel, L. C. H. Naviner and J. Yang, “A Review of Sparse Recovery Algorithms,” IEEE Access: Special Section on Theory, Algorithms, and Applications of Sparse Recovery, vol. 7, pp. 1300 - 1322, 2018.
[21] S. Stan, K. Saleem and M. Q. W. Halpem, “Wireless Sensor Networks,” University of Melbourne Lecture Notes, Australia, 2005.
[22] Z. Davide, M. Borja, V. Ignasi and R. Michele, “To Compress or Not To Compress: Processing vs Transmission Tradeoffs for Energy Constrained Sensor Networking,” arXiv Ad Hoc Networks, pp. 1 - 14, 2012.
[23] Department of Physics & Astronomy - University of Hawaii , “Receiver Sensitivity / Noise,” Department of Physics & Astronomy - University of Hawaii at Manoa, Manoa.
[24] Department of Physics & Astronomy - University of Hawaii at Manoa, Manoa, “Receiver Sensitivity / Noise,” Department of Physics & Astronomy - University of Hawaii , Hawaii.
[25] Cooper Crouse-Hinds, “Wireless Connectivity,” Cooper Crouse-Hinds.
[26] S. Ajah, A. Al-Sherbaz, S. Turner and P. Picton, “Sub 1 GHz M2M Communication Standardization: The Advancement in White Space Utilization in Enhancing the Energy Efficiency,” in PGNET, Liverpool, 2014.
[27] N. A. A. E. Nkwachukwu C., “Optimization of Hata Model for 2.1 GHz Communication Networks in Owerri Metropolis Nigeria,” International Journal of Advances in Scientific Research and Engineering, vol. 5, no. 4, pp. 12 - 17, 2019.
[28] H. Jaroslav and P. Pavel, “Penetration Loss Measurement and Modelling for HAP Mobile Systems in Urban Environment,” EURASIP Journal on Wireless Communications and Networking, vol. 2008, no. 543290, pp. 1 - 7, 2008.
[29] Weightless SIG, “Weightless Core Specification V1.0,” Weightless SIG, Cambridge, 2013.
[30] Weightless SIG, “Weightless Core Specification V1.0,” Weightless SIG, Cambridge, 2013.
[31] Cooper Crouse-Hinds, “Wireless Connectivity,” Cooper Crouse-Hinds.
[32] R. Zimran, s. Boon-Chong and A. Al-Anbuky, “Perforamce Analysis of Cooperative Virtual MIMO Systems for Wireless Sensor Networks,” Sensors, vol. 13, pp. 7033 - 7052, 2013.
[33] C. Jeremy, D. Ahmed, A. Oskar and M. Pascal, “TamaRISC-CS: An Ultra-Low_Power Application-Specific Processor for Compressed Sensing,” Santa Cruz, CA, 2012.